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A Spatial Nash Equilibrium Model

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Summary and Conclusions

Markets of most primary commodities and manufactured goods lie somewhere between two extreme cases: perfectly competitive and monopolistic. Various models have been developed which incorporate the behavioral assumptions of those two markets over space. However, we are unaware of any attempt to develop a model built on oligopolistic assumptions to simulate the behavior of these kinds of markets over space.

This paper presents a spatial equilibrium model based on assumed oligopolistic market behavior, characterized as a Nash non-cooperative game. More specifically, producers in any region will maximize profits by assuming that their decisions do not affect those of producers in other producing countries. The model, developed in the Takayama-Judge model framework, consists of M regions that consume a given commodity and N regions that produce it. The model solves for the price, demand and supply quantities in each region and for the trade flows among regions. The model can be directly applied to commodity markets, most of which are characterized as oligopolistic

In this paper results from the Nash equilibrium model are compared with results from the perfectly competitive model and the monopoly model, that are also built on the same spatial equilibrium framework. It is shown that the interregional price relationships differ among these three models. In the perfectly competitive model, when trade takes place between regions, the price differentials between exporting and importing regions are made up only of transportation costs. In the Nash equilibrium model the price differentials include additional margins that are in proportion to import quantities; while in the monopoly model the margins are in proportion to total demand quantities

of importing regions. Thus, the price differentials in the Nash equilibrium model are greater than in the perfectly competitive model but smaller than in the monopoly model. The different interregional price relationships in turn generate different trade flows, demand and supply quantities, and prices among the three models.

The applicability of the spatial Nash equilibrium model is limited because it is based on a rather specific assumption about oligopolistic behavior. However, the author believes that the model not only sheds light on a special type of oligopolistic market, both conceptually and numerically, but that it also constitutes a useful starting point for the extension of the spatial pricing and allocation approach to oligopolistic markets.

1. Introduction

The importance of space in economic activities has been long recognized by economists, and various efforts have been made to incorporate space into economic models. The development of activity analysis models by Koopmans (1949) and Dantzig (1951) opened up a new approach to the spatial pricing and allocation problem. Samuelson (1951) pointed out that there exists an objective function whose maximization guarantees fulfillment of the conditions of perfectly competitive equilibria among spatially separated markets. This prompted uses of mathematical programming to simulate market behavior. Operational effectiveness of the mathematical programming approach for the solution of market equilibria over space was significantly enhanced by the Takayama and Judge quadratic programming (QP) formulation of spatial price and allocation models (1964).

Later, Takayama and Judge (1971) presented two versions of the spatial pricing and allocation models: a perfectly competitive market model and a monopoly model. However, markets of most primary commodities and manufactured goods lie somewhere between these two extremes, taking on some form of oligopoly. Therefore, neither version of the Takayama-Judge model is able to provide appropriate solutions for the equilibrium conditions in the actual markets of most commodities. This paper presents a spatial equilibrium model that describes behavior of an oligopolistic market, characterized as a Nash non-cooperative game (Nash [1951]). The structure of the paper is as follows. In the first three sections, we present a spatial Nash equilibrium model as a QP problem and discuss some prominent features of the model. In the following sections, we compare the spatial Nash equilibrium model with the two versions of the Takayama-Judge model mentioned above.

2. Preparation for the Spatial Nash Equilibrium Model

a. Basic Setting

It is assumed there are M regions that consume a given commodity and N regions that produce it. It is further assumed that each producing region has a linear marginal cost function and that each consuming region has a linear demand function. All the functions are known to all the participants. The commodity is assumed to be traded freely from any producing region to any consuming region. Producers in each producing region determine their export and production quantities in accordance with the Nash non-cooperative rule: to maximize their profits by assuming that their decisions do not affect those of producers in other producing regions. In determining import and consumption quantities, however, consumers in each consuming region are assumed to behave as if they were in a perfectly competitive market. The model is static; thus, decisions in any region involve neither inventories nor investment in productive capacity.

b. Notation

The notation listed below will also be used in the perfectly competitive and monopoly models discussed later.

Y_j = demand quantity in region j (j=1, ..., M),

X_i = supply quantity in region i (i=1, ..., N),

X_{ij} = the quantity shipped from region i to region j

PD_j = demand price in region j

MC_i = marginal production cost in region i,

$\overline{FC}_i$	=	fixed cost portion of total production cost in region i
MR_i	=	marginal revenue in region i,
Π_i	=	net profit in region i,
NSP	=	net social payoffs,
ASP	=	net social payoffs adjusted to the Nash equilibrium model,
MNP	=	monopolist's net profit,
α_j	=	intercept value of the inverse linear demand function in region j,
β_j	=	slope coefficient of the inverse linear demand function in region j,
ν_i	=	intercept value of the linear marginal production cost function in region i,
η_i	=	slope coefficient of the linear marginal production function in region i,
t_{ij}	=	unit transportation cost from region i to region j.

c. Demand and Supply Functions

In the basic setting of the model it is assumed that each consuming region has the following inverse linear demand function:

$$PD_j = \alpha_j - \beta_j Y_j \text{ for all } j. \tag{1}$$

Similarly, it is assumed that each producing region has the following inverse linear supply function:

$$MC_i = \nu_i + \eta_i X_i \text{ for all } i. \tag{2}$$

3. Formulation of the Spatial Nash Equilibrium Model

The spatial Nash equilibrium model will be formulated in two stages. In the first stage, we define each producing region's rule for determining its exports and production. In the second stage, we present the entire model, including all producing and consuming regions, and show that the model satisfies the Nash non-cooperatve rule for each producing region as well as the ordinary spatial equilibrium conditions.

a. Producing Region's Rule for Exports and Production

Based on the supply function described above, region i's net profit can be written as follows:

$$\Pi_i = \sum_j PD_j X_{ij} - \left(\int MC_i \, dX_i + \overline{FC}_i\right) - t_{ij} X_{ij}$$

$$= \sum_j \left\{\alpha_j - \beta_j \sum_k X_{kj}\right\} X_{ij} - \int (\nu_i + \eta_i X_i) \, dX_i - \overline{FC}_i - t_{ij} X_{ij} \qquad (3)$$

Then, the Nash equilibria for region i's exports and production can be obtained by solving the following maximization problem:

$$\text{maximize } \Pi_i \qquad (4)$$

$$\text{subject to } X_i \geq \sum_j X_{ij} \qquad (5)$$

$$X_i \geq 0 \text{ and } X_{ij} \geq 0 \text{ for all i and j.} \qquad (6)$$

Defining the Lagrangean for this maximization problem as L_i, we can derive the Kuhn-Tucker conditions (Kuhn and Tucker [1951]) as follows:

$$\frac{\partial L_i}{\partial X_{ij}} = \{\alpha_j - \beta_j \sum_k X_{kj}\} - \beta_j X_{ij} - t_{ij} - \phi_i \leq 0$$

$$\text{and } \frac{\partial L_i}{\partial X_{ij}} \cdot X_{ij} = 0 \text{ for all } j \tag{7}$$

$$\frac{\partial L_i}{\partial X_i} = -\nu_i - \eta_i X_i + \phi_i \leq 0 \quad \text{and} \quad \frac{\partial L_i}{\partial X_i} \cdot X_i = 0 \tag{8}$$

$$\frac{\partial L_i}{\partial \phi_i} = X_i - \sum_j X_{ij} \geq 0 \quad \text{and} \quad \frac{\partial L_i}{\partial \phi_i} \cdot \phi_i = 0 \tag{9}$$

where ϕ_i (≥ 0) stands for the Lagrangean multiplier for Condition (5), which can be interpreted as the equilibrium marginal production cost.

These conditions describe the Nash non-cooperative rule for producing region i. Condition (7) states that the sum of the marginal production cost in producing region i and the transportation cost plus the margin expressed by $\beta_j X_{ij}$ must be equal to the price in consuming region j -- as long as trade takes place from region i to region j. Condition (8) states that the marginal production cost in region i must correspond to that described by Equation (2), as long as a positive quantity is produced in that region.

b. The Spatial Nash Equilibrium Model

A spatial Nash equilibrium model can be formulated as the following maximization problem:

$$\text{maximize} \quad ASP = \sum_j \int \{\alpha_j - \beta_j Y_j\} dY_j - \sum_i [\int \{\nu_i + \eta_i X_i\} dX_i + \overline{FC}_i]$$

$$- \sum_i \sum_j \beta_j X_{ij} - \sum_i \sum_j t_{ij} X_{ij} \qquad (10)$$

$$\text{subject to} \quad \sum_i X_{ij} \geq Y_j \text{ for all } j, \qquad (11)$$

$$X_i \geq \sum_j X_{ij} \text{ for all } i, \qquad (12)$$

All endogenous and counterpart variables are non-negative. (13)

Defining the Lagrangean of this maximization problem as L_N, we can derive the following Kuhn-Tucker conditions:

$$\frac{\partial L_N}{\partial Y_j} = \alpha_j - \beta_j Y_j - \rho_j \leq 0 \text{ and } \frac{\partial L_N}{\partial Y_j} \cdot Y_j = 0 \text{ for all } j, \qquad (14)$$

$$\frac{\partial L_N}{\partial X_i} = - \nu_i - \eta_i X_i + \phi_i \leq 0 \text{ and } \frac{\partial L_N}{\partial X_i} \cdot X_i = 0 \text{ for all } i, \qquad (15)$$

$$\frac{\partial L_N}{\partial X_{ij}} = \rho_j - \beta_j X_{ij} - t_{ij} - \phi_i \leqslant 0$$

$$\text{and } \frac{\partial L_N}{\partial X_{ij}} \cdot X_{ij} = 0 \text{ for all i and j,} \tag{16}$$

$$\frac{\partial L_N}{\partial \rho_j} = \sum_i X_{ij} - Y_j \geqslant 0 \text{ and } \frac{\partial L_N}{\partial \rho_j} \cdot \rho_j = 0 \text{ for all j,} \tag{17}$$

$$\frac{\partial L_N}{\partial \phi_i} = X_i - \sum_j X_{ij} \geqslant 0 \text{ and } \frac{\partial L_N}{\partial \phi_i} \cdot \phi_i = 0 \text{ for all i,} \tag{18}$$

where ρ_j ($\geqslant 0$) and ϕ_i($\geqslant 0$) stand for the Lagrangean multipliers for Conditions (11) and (12) respectively.

The task here is to examine whether these conditions satisfy the Nash non-cooperative rule as well as the ordinary spatial equilibrium conditions.

First, among the conditions listed above, Conditions (14) and (17) stipulate the equilibrium conditions for consuming regions. Condition (14) assures that both the demand quantity and price in each consuming region satisfy the given demand function, as long as a positive quantity is consumed. Condition (17) represents the material balance in each consuming region. These conditions are familiar to any spatial equilibrium model. Next, Conditions (15) and (18) stipulate the equilibrium conditions for producing regions. Condition (15) assures that the supply quantity and the marginal production cost satisfy the given marginal cost function, as long as a positive quantity is produced. This condition corresponds to Condition (8). Condition (18) specifies the material balance in each producing region, which corresponds to Condition (9). These conditions are also familiar to any spatial equilibrium model.

Finally, Condition (16), which specifies the interregional price relationships, is unique to the Nash equilibrium model; thus, a more careful examination is required for conformity of this condition with Condition (7).

First, let us examine the regular case where $Y_j>0$ and $\rho_j>0$ for all j´s. Because $Y_j>0$ and $\rho_j>0$, Condition (14) and (17) hold with equality; thus, Condition (16) is equivalent to Condition (7).

Next, let us examine the so-called irregular case, where $Y_j=0$. This case can be examined by separating two sub-cases. First, let us consider the sub-case $X_{ij}=0$. Because $Y_j=0$, the market equilibrium price ρ_j may deviate from the linear demand function (1), and it can be strictly greater than α_j. Let us write the market equilibrium price as $\rho_j = \alpha_j'(\geqslant \alpha_j)$. Then, the first term in the maximand of (3) in the Nash producer rule, $\{\alpha_j - \beta_j \sum_k X_{kj}\} X_{ij}$ can be expressed as $\alpha_j' X_{ij}$. Thus, Condition (7) can be rewritten as:

$$\alpha_j' - \beta_j X_{ij} - t_{ij} - \phi_i \leqslant 0 \text{ and}$$

$$(\alpha_j' - \beta_j X_{ij} - t_{ij} - \phi_i) \cdot X_{ij} = 0 \qquad (7')$$

It is obvious that Condition (16) corresponds to (7´), because $\rho_j = \alpha_j'$.

Second, let us examine the sub-case : $Y_j = 0$ and $X_{ij} > 0$. Because there exists an excess supply in region j, Condition (17) has a positive slack; thus $\rho_j = 0$. Therefore, the first term in the maximand of (3) can be replaced by 0 (= ρ_j). Consequently, Condition (7) can be rewritten as:

$$- \beta_j X_{ij} - t_{ij} - \phi_i \leqslant 0 \text{ and}$$

$$(-\beta_j X_{ij} - t_{ij} - \phi_i) \cdot X_{ij} = 0 \qquad (7")$$

It is obvious that Condition (16) corresponds to (7") because $\rho_j = 0$.

In conclusion, Conditions (14) through (18) conform with the Nash rule for producing region's exports and production as well as the ordinary spatial equilibrium conditions in both regular and irregular cases.

4. Comparison with the Perfectly Competitive and Monopoly Models

The perfectly competitive model can be developed as a maximization problem of the so-called net social payoffs under a given set of quantity balances. The problem can be described as follows:

maximize
$$NSP = \sum_j \int (\alpha_j - \beta_j Y_j)\, dY_j - \sum_i [\int (\nu_i + \eta_i X_i)\, dX_i + \overline{FC}_i]$$
$$-\sum_i \sum_j t_{ij} X_{ij} \qquad (19)$$

subject to
$$\sum_i X_{ij} \geqslant Y_j \text{ for all } j, \qquad (20)$$
$$X_i \geqslant \sum_j X_{ij} \text{ for all } i, \qquad (21)$$

All endogenous and counterpart variables are non-negative. (22)

Defining the Lagrangean of this maximization problem as L_C, we can derive the perfectly competitive market equilibrium conditions as follows:

$$\frac{\partial L_C}{\partial Y_j} = \alpha_j - \beta_j Y_j - \rho_j \leqslant 0 \text{ and } \frac{\partial L_C}{\partial Y_j} \cdot Y_j = 0 \text{ for all } j, \qquad (23)$$

$$\frac{\partial L_C}{\partial X_i} = -\nu_i - \eta_i X_i + \phi_i \leqslant 0 \text{ and } \frac{\partial L_C}{\partial X_i} \cdot X_i = 0 \text{ for all } i, \qquad (24)$$

$$\frac{\partial L_C}{\partial X_{ij}} = \rho_j - t_{ij} - \phi_i \leqslant 0 \text{ and } \frac{\partial L_C}{\partial X_{ij}} \cdot X_{ij} = 0 \text{ for all } i \text{ and } j, \qquad (25)$$

$$\frac{\partial L_C}{\partial \rho_j} = \sum_i X_{ij} - Y_j \geqslant 0 \text{ and } \frac{\partial L_C}{\partial \rho_j} \cdot \rho_j = 0 \text{ for all } j, \tag{26}$$

$$\frac{\partial L_C}{\partial \phi_i} = X_i - \sum_j X_{ij} \geqslant 0 \text{ and } \frac{\partial L_C}{\partial \phi_i} \cdot \phi_i = 0 \text{ for all } i. \tag{27}$$

where the Lagrangean multipliers ρ_j ($\geqslant 0$) and ϕ_i ($\geqslant 0$) can be interpreted as the equilibrium demand price and the equilibrium marginal production cost, respectively.

The conditions listed above are the perfectly competitive market equilibrium conditions. The difference from the Nash equilibrium model lies only in the interregional price relationship. In the perfectly competitive market model, the difference between the demand price in consuming region j and the marginal production cost in producing region i is equal to the transportation cost, as long as trade takes place between these two regions. In the Nash equilibrium model, the difference is equal to the sum of the marginal production cost and the transportation cost plus the margin described by $\beta_j X_{ij}$.

The spatial monopoly model is based on an assumption that outputs over all the producing regions are controlled by one producer. 1/ The model can be developed as a maximization problem of the monopolist's net profit in the following:

$$\text{maximize} \quad MNP = \sum_j PD_j\, Y_j - \sum_i [\int(\nu_i + \eta_i X_i)dX_i + \overline{FC}_i] - \sum_i \sum_j t_{ij} X_{ij}, \tag{28}$$

1/ The model is equivalent to a multi-plant monopoly model.

subject to $\sum_i X_{ij} \geq Y_j$ for all j, (29)

$X_i \geq \sum_j X_{ij}$ for all i, (30)

All endogenous and counterpart variables are non-negative. (31)

Defining the Lagrangean of this maximization problem as L_M, we can derive the spatial monopoly equilibrium conditions as follows:

$$\frac{\partial L_M}{\partial Y_j} = \alpha_j - 2\beta_j Y_j - \lambda_j \leq 0 \text{ and } \frac{\partial L_M}{\partial Y_j} \cdot Y_j = 0 \text{ for all j,} \quad (32)$$

$$\frac{\partial L_M}{\partial X_i} = \nu_i - \eta_i X_i + \phi_i \leq 0 \text{ and } \frac{\partial L_M}{\partial X_i} \cdot X_i = 0 \text{ for all i,} \quad (33)$$

$$\frac{\partial L_M}{\partial X_{ij}} = \lambda_j - t_{ij} - \phi_i \leq 0 \text{ and } \frac{\partial L_M}{\partial X_{ij}} \cdot X_{ij} = 0 \text{ for all i and j,} \quad (34)$$

$$\frac{\partial L_M}{\partial \lambda_j} = \sum_i X_{ij} - Y_j \geq 0 \text{ and } \frac{\partial L_M}{\partial \lambda_j} \cdot \lambda_j = 0 \text{ for all j,} \quad (35)$$

$$\frac{\partial L_M}{\partial \phi_i} = X_i - \sum_j X_{ij} \geq 0 \text{ and } \frac{\partial L_M}{\partial \phi_i} \cdot \phi_i = 0 \text{ for all i,} \quad (36)$$

where the Lagrangean multiplier of Constraint (29), $\lambda_j (\geq 0)$, can be intepreted as the equilibrium marginal revenue in consuming region j, instead of the equilibrium demand price.

To facilitate comparison with the other two models, let us rewrite Condition (34), the interregional price relationship. Because $\lambda_j = \alpha_j - 2\beta_j Y_j$, Condition (34) can be written as follows:

$$\frac{\partial L_M}{\partial X_{ij}} = \rho_j - \beta_j Y_j - t_{ij} - \phi_i \leq 0$$

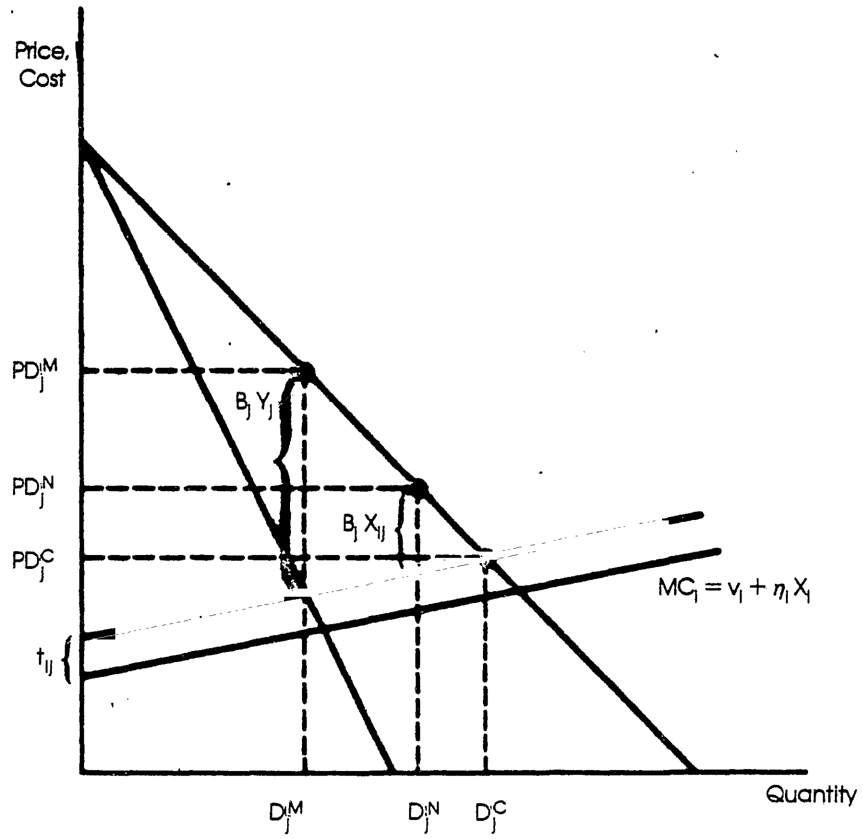
$$\text{and } \frac{\partial L_M}{\partial X_{ij}} \cdot X_{ij} = 0 \text{ for all i and j.} \quad (34')$$

The interregional price relationship in the monopoly model is such that the difference between the demand price in consuming region j and the marginal production cost in producing region i is equal to the sum of the transportation cost plus the margin described by $\beta_j Y_j$. Because of Condition (29), the margin in the monopoly model is greater than or equal to the margin in the Nash equilibrium model, $\beta_j X_{ij}$.

The differences in equilibria among these three spatial models -- for a case with one consuming and one producing region — are illustrated in Figure 1. Because of the differences in the interregional price relationships stated above, the perfectly competitive model generates the lowest demand price, the highest marginal production cost and the largest quantity demanded among the three models. It also generates the largest value in the objective function (the area of consumer and producer surplus minus transportation costs). The monopoly model is exactly opposite to the perfectly competititive model, with the highest price, the lowest marginal production cost, the lowest quantity demanded and the smallest value in the objective function. The Nash equilibrium model lies between the two extremes.

To show how the three models work in the setting of more than one region for each of consumption and production, a sample model with three consuming and three producing regions is developed for each model. The assumed coefficients are given in Tables 1 and 2. Tables 3 through 5 show the coefficient matrices for the three models, through which the differences in

Figure 1: **Comparison of Model Equilibria**



Note:
Superscripts of N, C & M for D & PD represent demand quantitites & price in the Nash equilibrium, perfectly competitive & monopoly models, respectively.

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Table 1: ASSUMED COEFFICIENTS OF INVERSE LINEAR DEMAND AND MARGINAL COST FUNCTIONS

	Region (j, i)		
	1	2	3
α_j	16	12	24
β_j	1	.5	.25
ν_i	2	1	1.5
η_i	2	.25	.5

Table 2: ASSUMED TRANSPORTATION COSTS /a

to	Region (j)		
from	1	2	3
Region (i)			
1	0	1	3
2	1	0	2
3	3	2	0

/a Consuming region i is assumed to be located so closely to producing region i that the transportation cost between these two regions is assumed to be zero.

Table 3: Tableau of the Spatial Nash Equilibrium Model

	Y_1	Y_2	Y_3	X_1	X_2	X_3	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	ρ_1	ρ_2	ρ_3	ψ_1	ψ_2	ψ_3	
WY_1	β_1															1						α_1
WY_2		β_2															1					α_2
WY_3			β_3															1				α_3
WX_1				η_1															−1			ν_1
WX_2					η_2															−1		ν_2
WX_3						η_3															−1	ν_3
WX_{11}							β_1									−1			1			t_{11}
WX_{12}								β_2									−1		1			t_{12}
WX_{13}									β_3									−1	1			t_{13}
WX_{21}										β_1						−1				1		t_{21}
WX_{22}											β_2						−1			1		t_{22}
WX_{23}												β_3						−1		1		t_{23}
WX_{31}													β_1			−1					1	t_{31}
WX_{32}														β_2			−1				1	t_{32}
WX_{33}															β_3			−1			1	t_{33}
$W\rho_1$	−1						1			1			1									
$W\rho_2$		−1						1			1			1								
$W\rho_3$			−1						1			1			1							
$W\psi_1$				1			−1	−1	−1													
$W\psi_2$					1					−1	−1	−1										
$W\psi_3$						1							−1	−1	−1							

Variables in the fartherst left column, which begin with the letter W, represent counterpart variables.

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Table 4: Tableau of the Spatial Perfectly Competitive Model

	Y_1	Y_2	Y_3	X_1	X_2	X_3	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	ρ_1	ρ_2	ρ_3	ψ_1	ψ_2	ψ_3	
WY_1	β_1															1						α_1
WY_2		β_2															1					α_2
WY_3			β_3															1				α_3
WX_1				η_1															−1			ν_1
WX_2					η_2															−1		ν_2
WX_3						η_3															−1	ν_3
WX_{11}																−1			1			t_{11}
WX_{12}																	−1		1			t_{12}
WX_{13}																		−1	1			t_{13}
WX_{21}																−1				1		t_{21}
WX_{22}																	−1			1		t_{22}
WX_{23}																		−1		1		t_{23}
WX_{31}																−1					1	t_{31}
WX_{32}																	−1				1	t_{32}
WX_{33}																		−1			1	t_{33}
$W\rho_1$	−1						1			1			1									
$W\rho_2$		−1						1			1			1								
$W\rho_3$			−1						1			1			1							
$W\psi_1$				1			−1	−1	−1													
$W\psi_2$					1					−1	−1	−1										
$W\psi_3$						1							−1	−1	−1							

Variables in the fartherst left column, which begin with the letter W, represent counterpart variables.

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Table 5: Tableau of the Spatial Monopoly Model

	Y_1	Y_2	Y_3	X_1	X_2	X_3	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	λ_1	λ_2	λ_3	ψ_1	ψ_2	ψ_3	
WY_1	$2\beta_1$															1						α_1
WY_2		$2\beta_2$															1					α_2
WY_3			$2\beta_3$															1				α_3
WX_1				η_1															−1			ν_1
WX_2					η_2															−1		ν_2
WX_3						η_3															−1	ν_3
WX_{11}																−1			1			t_{11}
WX_{12}																	−1		1			t_{12}
WX_{13}																		−1	1			t_{13}
WX_{21}																−1				1		t_{21}
WX_{22}																	−1			1		t_{22}
WX_{23}																		−1		1		t_{23}
WX_{31}																−1					1	t_{31}
WX_{32}																	−1				1	t_{32}
WX_{33}																		−1			1	t_{33}
$W\lambda_1$	−1						1			1			1									
$W\lambda_2$		−1						1			1			1								
$W\lambda_3$			−1						1			1			1							
$W\psi_1$				1			−1	−1	−1													
$W\psi_2$					1					−1	−1	−1										
$W\psi_3$						1							−1	−1	−1							

Variables in the fartherst left column, which begin with the letter W, represent counterpart variables.

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structure among them can be understood visually. It should be noted that the tableau of the Nash equilibrium model has positive figures in the diagonal elements that correspond to X_{ij}'s. The solutions of the major variables are presented in Tables 6 through 8. The model solutions indicate more complexities in models with many regions than in the illustrative case with one consuming and one producing region. For example, the trade pattern is different among the models. (Trade takes place from producing region 1 to consuming region 3 in the Nash equilibrium model only). Furthermore, the demand price in consuming region 2 is the lowest and the demand quantity in the same region is the highest in the monopoly model, while the other region's results conform to the illustrative case.

5. Concluding Remarks

Markets of most primary commodities and manufactured goods lie somewhere between two extreme cases: perfectly competitive and monopolistic. Various models have been developed to simulate behavior of these two markets over space. However, we are unaware of any attempt to model the behavior of oligopolistic markets over space.

In this paper, an attempt has been made to articulate a spatial oligopolistic market model within the spatial pricing and allocation modeling framework. The model assumes the Nash non-cooperative rule; that is, producers in any region will maximize profits by assuming that their decisions do not affect those of producers in other producing regions. The spatial equilibria based on this assumption result in interregional price relationships which differ from those of the perfectly competitive spatial model and those of the

Table 6: SOLUTIONS - NASH EQUILIBRIUM MODEL

DEMAND			SUPPLY			TRADE	
Region (j)	Demand price	Demand quantity	Region (i)	Marginal production cost	Supply quantity	Region (i → j)	Trade quantity
1	11.70	4.30	1	10.51	4.26	1 → 1	1.19
						1 → 3	3.07
2	9.79	4.42	2	7.58	26.32	2 → 1	3.12
						2 → 2	4.42
3	14.28	38.89	3	10.02	17.04	2 → 3	18.79
						3 → 3	17.04

Value of the objective function 472.5

Table 7: SOLUTIONS - PERFECTLY COMPETITIVE MODEL

DEMAND			SUPPLY			TRADE	
Region (j)	Demand price	Demand quantity	Region (i)	Marginal production cost	Supply quantity	Region (i → j)	Trade quantity
1	10.67	5.33	1	10.67	4.33	1 → 1	4.33
						2 → 1	1.00
2	9.67	4.67	2	9.67	34.67	2 → 2	4.67
						2 → 3	29.00
3	11.67	49.33	3	11.67	20.33	3 → 3	20.33

Value of the objective function 596.3

Table 8: SOLUTIONS - MONOPOLY MODEL

DEMAND			SUPPLY			TRADE	
Region (j)	Demand price a/	Demand quantity	Region (i)	Marginal production cost	Supply quantity	Region (i → j)	Trade quantity
1	11.85	4.15	1	7.70	2.85	1 → 1	2.85
						2 → 1	1.30
2	9.35	5.30	2	6.70	22.80	2 → 2	5.30
						2 → 3	16.20
3	16.35	30.60	3	8.70	14.40	3 → 3	14.40

Value of the objective function 390.3

a/ Calculated from the solved marginal revenue.

monopoly spatial model. When trade takes splace between regions in the perfectly competitive model the price differentials between exporting and importing regions are made up only of transportation costs. In the Nash equilibrium model the price differentials include additional margins that are in proportion to import quantities. In the monopoly model the margins are in proportion to total demand quantities of importing regions. Thus, the interregional price differentials in the Nash equilibrium model are greater than in the perfectly competitive model but smaller than in the monopoly model. The different interregional price relationships in turn generate different trade flows, demand and supply quantities, and prices among the three models.

Needless to say, the applicability of the spatial Nash equilibrium model is limited because it is based on a rather specific assumption about oligopolistic behavior. However, the author believes that the model not only sheds light on a special type of oligopolistic market, both conceptually and numerically, but also constitutes a starting point for the extension of the spatial pricing and allocation approach to oligopolistic markets.

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