WPS3480

Capital Accumulation and Resource Depletion:
A Hartwick Rule Counterfactual

Kirk Hamilton*, Giovanni Ruta* and Liaila Tajibaeva+

Abstract. How rich would resource-abundant countries be if they had actually followed the Hartwick Rule (invest resource rents in other assets) over the last 30 years? We employ time series data on investment and rents on exhaustible resource extraction for 70 countries to answer this question. The results are striking: Venezuela, Trinidad and Tobago, and Gabon would all be as wealthy as South Korea, while Nigeria would be five times as well off as it is currently. We also derive a specific rule for sustainability – maintain positive constant genuine investment – and use this to obtain further empirical results.

World Bank Policy Research Working Paper 3480, January 2005

---

* Environment Department, World Bank. The comments of Jean-Christophe Carret and Roberto Martin-Hurtado are gratefully acknowledged – the usual caveats apply. Corresponding author: Kirk Hamilton (khamilton@worldbank.org), fax 1-202-522-1735, World Bank, 1818 H St. NW, Washington DC 20433.
+ Department of Applied Economics, University of Minnesota. Funding by the Swedish International Development Agency is acknowledged with thanks.

Introduction

There is by now a substantial empirical literature documenting the 'resource curse' or 'paradox of plenty.'[1] Resource-rich countries should enjoy an advantage in the development process, and yet these countries experienced lower GDP growth rates post-1970 than less well endowed countries. A number of plausible explanations for this phenomenon have been suggested: inflated currencies may impede the development of the non-oil export sector ('Dutch disease'); easy money in the form of resource rents may reduce incentives to implement needed economic reforms; and volatile resource prices may complicate macroeconomic management, exacerbating political conflicts over the sharing and management of resource revenues.

In the most extreme examples, levels of welfare in resource-rich countries are lower today than they were in 1970 – development has not been sustained by Pezzey's (1989) definition. The Hartwick Rule (Hartwick 1977) offers what Solow (1986) termed a 'rule of thumb' for sustainability in exhaustible resource economies – a maximal constant level of consumption can be sustained if the value of investment equals the value of rents on extracted resources at each point in time. For countries dependent on such wasting assets this rule offers a prescription[2] for sustainable development, a prescription that Botswana in particular has followed with its diamond wealth (Lange and Wright 2004).

Drawing on a 30-year time series of resource rent data underlying the *World Development Indicators* (World Bank 2004), we construct a 'Hartwick Rule counterfactual': how rich would countries be in the year 2000 if they had followed the Hartwick Rule since 1970? The results are, in many cases, striking.

Our empirical work draws upon new results[3] showing that the Hartwick Rule is a special case of a more general rule for sustainability. We extend the results in Hamilton and Withagen (2004) to determine the properties of a constant net saving rule – constant positive net savings entails a path for consumption that rises without bound. We then apply this rule and the standard Hartwick Rule to our historical data on investment and resource rents covering 1970-2000.

The Generalized Rule for Sustainability

For a quite general model of a dynamic economy, Hamilton and Withagen (2004) establish that if the economy is competitive (households maximize utility while firms maximize profits) and if externalities are internalized through Pigovian taxes, then utility U, consumption C, net (genuine) saving G and interest rate r are related as follows:

$$\dot{U} = U_C\left(rG - \dot{G}\right). \qquad (1)$$

[1] See Auty 2001, Ch. 1 for a good overview. One of the earliest studies was Sachs and Warner (1995).
[2] Note that Asheim *et al.* (2003) question whether the Hartwick rule is truly prescriptive. This is partly because a commitment to invest resource rents *now* cannot commit future generations to do the same.
[3] Hamilton and Hartwick (forthcoming) and Hamilton and Withagen (2004).

This relates the current change in utility to the sign and rate of growth of genuine saving. But, since optimality is not assumed, it also provides the basis for a general rule for sustainability (non-declining utility). If the policy rule is to hold $G = 0$ for all time, then this is just the standard Hartwick rule, yielding constant utility. If $G > 0$ and $\dot{G}/G < r$ for all time, then utility is everywhere increasing[4].

For our purposes we assume a simple economy with an exhaustible resource that is essential for production, as in Dasgupta and Heal (1979). For capital *K* and resource extraction *R*, the production function is Cobb-Douglas, $F = K^{\alpha} R^{\beta}$, $\alpha + \beta = 1$. Output is divided between consumption and investment, so that $F(K,R) = C + \dot{K}$. Utility is given by $U = U(C)$. Resource extraction is assumed to be efficient, implying that $S_0 = \int_0^{\infty} R\, ds$ – the initial resource stock S_0 is exhausted over the infinite time horizon. The initial endowment of produced capital is K_0.

Competitiveness in the Dasgupta-Heal economy implies that the resource price is equal to F_R, that the interest rate is F_K, and that the Hotelling Rule is satisfied,

$$\dot{F}_R / F_R = F_K . \tag{2}$$

Genuine saving is given by,

$$G \equiv \dot{K} - F_R R . \tag{3}$$

Hamilton and Hartwick (forthcoming) establish the following basic proposition for the Dasgupta-Heal economy, analogous to expression (1):

Proposition 1: In the competitive Dasgupta-Heal economy, $\dot{C} = F_K G - \dot{G}$.

Hamilton and Hartwick also prove the following wealth accounting result. It will be useful in proving the main proposition below.

Proposition 2: Under constant returns to scale total wealth is given by

$$W = K + F_R S = \int_t^{\infty} C e^{-\int_t^s F_K d\tau} ds .$$

The following proposition characterizes a particular instance of a generalized sustainability rule in the Dasgupta-Heal economy, an instance we will exploit empirically in the next section.

Proposition 3: If $\alpha > \beta$ then $G = \overline{G} > 0 \ \forall t$ for constant $\overline{G} < \alpha F(0)$ is a feasible program for rising consumption. Initial consumption will be lower than on the Hartwick Rule ($\overline{G} = 0$) path,

---

[4] Dixit *et al.* (1980) derive expression (1) in the proof of their main proposition, where they show that utility will be constant if either $G = 0 \ \forall t$ or $\dot{G}/G = r \ \forall t, G_0 > 0$.

but consumption increases without bound. Wealth on this path is greater than under the standard Hartwick Rule, and maximum wealth is independent of the initial resource stock S_0.

Proof: See Annex I.

Having established the properties of this specific sustainability rule for the Dasgupta-Heal economy, we now turn to the empirical application of the rule to historical data.

Hypothetical Estimates of Capital Stocks

While the foregoing theory can be shown to apply to rules for *saving* in an open economy, we will limit ourselves to *investment* rules in the empirical application. All of the countries which are highlighted in the empirical results had significant net foreign debts in 2000. Rather than looking at the more complex question of whether resource rents could have been used to either pay down foreign debt or invest in domestic assets, we limit ourselves to comparing an estimate of the current stock of produced capital with a hypothetical estimate of how large this stock could be if resource rents had been invested in produced capital. We assume that all resource rents are invested in produced capital for simplicity, although the theory suggests more generally that resource rents could be invested in a range of assets, including human capital – if any of the countries highlighted below had in fact been investing their resource rents in human capital (quite unlikely given the observed levels of per capita income) then our methodology would produce a biased picture of their investment performance.

In order to examine a variety of counterfactuals, we derive four estimates of produced capital stock using empirical data covering 1970-2000: (i) a baseline capital stock derived from investment series and a Perpetual Inventory Model (PIM); (ii) a capital stock derived from strict application of the standard Hartwick Rule; (iii) a capital stock derived from the constant net or genuine investment rule; and (iv) a capital stock derived from the maximum of observed net investment and the investment required under the constant genuine investment rule. All investment and resource rent series are measured in constant 1995 US dollars at nominal exchange rates.

Details of the PIM are given in Hamilton (2002). For each country the estimate of baseline capital stock is given by,

$$K_t = \sum_{s=0}^{T-1} I_{t-s} (1-\gamma)^s .$$

Here I is gross investment, the average asset service life T is assumed to be 20 years and the depreciation rate γ is 5% – these are held constant across countries and over time. We use the year 2000 as our basis for comparison of capital stocks.

For genuine investment I^G, net investment N, depreciation of produced capital D and resource depletion R we have the following basic identities at any point in time:

$$I^G \equiv I - D - R$$

$N \equiv I - D = I^G + R$

Note that, given a base year capital stock estimate, it is possible to estimate capital stocks beyond the base year by simply accumulating net investment in each period. Therefore, for constant $\bar{I}^G$, we estimate the counterfactual series of produced capital for each country as:

$$K^*_{2000} = K_{1970} + \sum_{i=1971}^{2000} \left(\bar{I}^G + R_i\right)$$

$$K^{**}_{2000} = K_{1970} + \sum_{i=1971}^{2000} \max\left(N_i, \bar{I}^G + R_i\right)$$

We calculate two versions of K^* in what follows – one with $\bar{I}^G = 0$ (the standard Hartwick rule), and a second with $\bar{I}^G$ equal to a constant 5% of 1987 GDP. The choice of a particular level of genuine investment for the analysis is obviously arbitrary. We use 5% of 1987 GDP for the following reasons: (i) there is some logic to choosing the mid-point of our time series of data from 1970-2000, but 1987 is a slightly better choice, falling after the early 1980's recession, after the collapse of oil prices in 1986, and before the early 1990's recession; and (ii) a 5% genuine investment rate is roughly the average achieved by low income countries over time. Since the elasticity of output with respect to produced capital α is implicitly greater than 0.5 in the theoretical model of the preceding section, the choice of 5% of GDP ensures that the feasibility condition $\overline{G} < \alpha F(0)$ of Proposition 3 is satisfied.

Resource depletion is estimated as the sum of total rents on the extraction of the following commodities: crude oil, natural gas, coal, bauxite, copper, gold, iron, lead, nickel, phosphate, silver and zinc. While the underlying theory suggests that scarcity rents are what should be invested under the Hartwick rule (i.e. price minus marginal extraction cost), the World Bank data do not include information on marginal extraction costs. This gives an upward bias to the hypothetical capital stock estimates under the genuine investment rules.

There is another clear divergence between our empirical methods and the theory of the preceding section. In the autarkic Dasgupta-Heal economy presented above, the choice of policy rule also determines the level and path of resource rents (F_R). By using historical rents in our calculations we are clearly diverging from the theory. However, in most instances we would expect resource exporters to be price takers, which favors using historical rents. If resource prices change exogenously, a further adjustment to saving to reflect future capital gains is required (see Vincent *et al.* 1997), but Hamilton and Bolt (2004) show that the adjustment is typically small if historical price trends are extrapolated.

When comparing estimates of the stock of produced capital for different countries, it is worth noting that the Perpetual Inventory Model underestimates the capital stock for countries with very old infrastructure, as in most European countries. The value of roads, bridges and buildings constructed many decades and even centuries ago is not captured by the PIM. Pritchett (2000) makes a different point, that low returns on investments imply that the PIM overestimates the value of capital in developing countries. Our methodology assumes that both the PIM and cumulated net investments are in fact adding up productive investments. To the extent that this is not the case, our estimated capital stock levels should be lower in developing countries – but we are primarily interested in relative stock levels, which makes the point less salient.

Empirical Results

How rich would countries be in the year 2000 had they followed the Hartwick Rule since 1970? Based on the preceding methodology, Table A1 presents the year 2000 produced capital stock and the changes in this stock which would result from the alternative investment rules. The countries shown in this table are those having both exhaustible resources and a sufficiently long time series of data on gross investment and resource rents. For reference, the table also shows the average share of resource rents in GDP over 1970-2000. Note that negative entries in this table imply that countries actually invested more than the policy rule suggests.

For the standard Hartwick rule Figure 1 scatters resource dependence, expressed as the average share of exhaustible resource rents in GDP, against the percentage difference between actual capital accumulation and counterfactual capital accumulation. Using 5% of GDP as the threshold for high resource dependence, Figure 1 divides countries into the four groups shown.

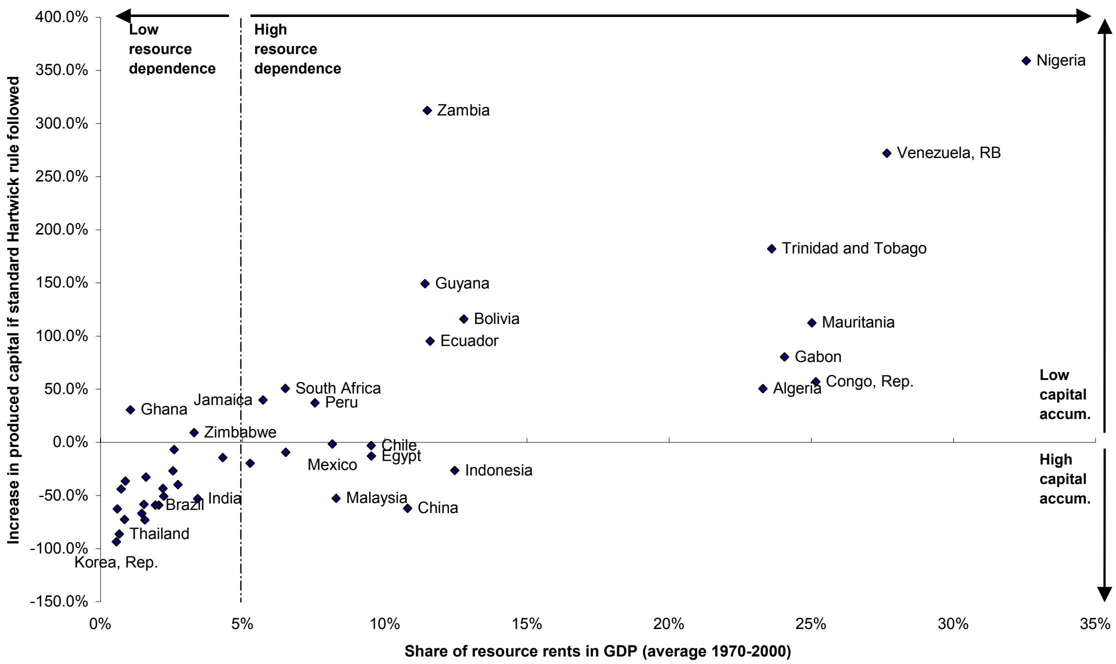


Figure 1 – Resource abundance and capital accumulation (standard Hartwick rule)

The top-right quadrant of the graph displays countries with high resource dependence and a counterfactual capital stock that is higher than the actual (baseline) capital stock. The bottom-left quadrant of the displays countries with low natural resource dependence and baseline capital stock that is higher than would be obtained under the Hartwick rule. These two quadrants include most of the countries in our sample, indicating a high negative correlation between resource abundance and the difference between baseline and counterfactual capital accumulation – a simple regression shows that a 1% increase in resource dependence is associated with a 9% increased difference between counterfactual and actual capital. Clearly the countries in the top-right quadrant have not been following the Hartwick rule. Economies with very low levels of capital accumulation despite high rents include Nigeria (oil), Venezuela (oil), Trinidad and Tobago (oil and gas), and Zambia (copper) – with the exception of Trinidad and Tobago, all of these countries experienced declines in real per capita income over 1970-2000. In the opposite quadrant, economies with low exhaustible resource rent shares but high levels of capital accumulation include Korea, Thailand, Brazil and India. A number of high income countries are also in this group.

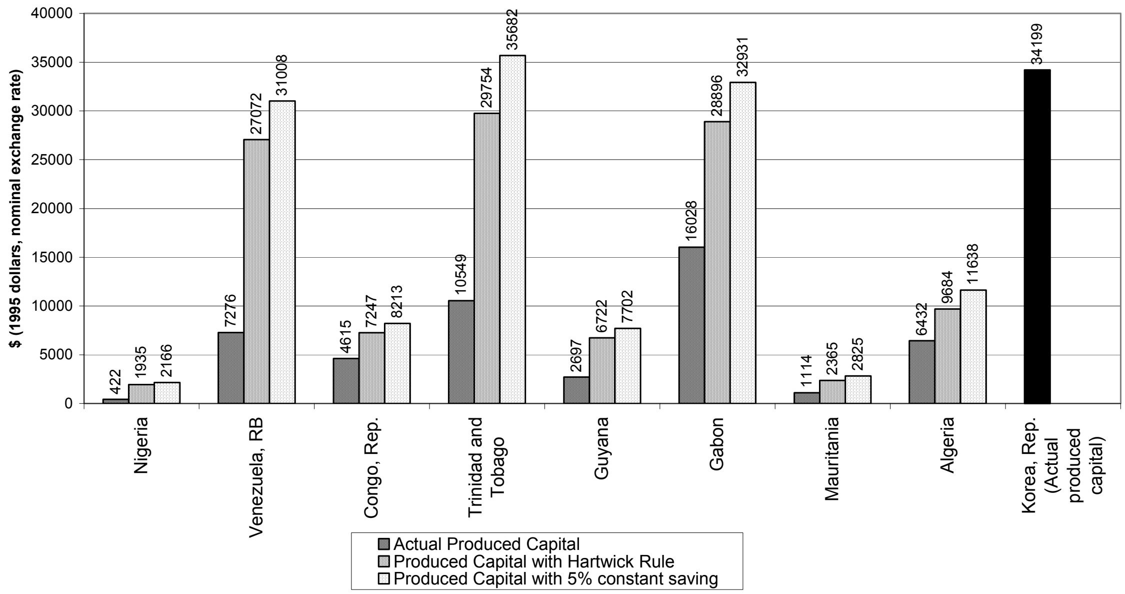
Figure 1 shows that no country with resource rents higher than 15% of GDP has followed the Hartwick rule. In many cases the differences are huge. Nigeria, a major oil exporter, could have had a year 2000 stock of produced capital five times higher than the actual stock. Moreover, if these investments had taken place, oil would play a much smaller role in the Nigerian economy today, with likely beneficial impacts on policies affecting other sectors of the economy[5]. Venezuela could have four times as much produced capital. In per capita terms, the economies of Venezuela, Trinidad and Tobago and Gabon, all rich in petroleum, could today have a stock of produced capital of roughly $30,000 per person, comparable to South Korea (see Figure 2).

Consumption rather than investment of resources rents is common in resource rich countries, but there are exceptions to the trend. In the bottom-right quadrant of Figure 1 are high resource dependence countries which have invested more than the level of exhaustible resource rents. Indonesia, China, Egypt, and Malaysia stand out in this group, while Chile and Mexico have effectively followed the Hartwick Rule – growth in produced capital is completely offset by resource depletion.

---

[5] We are grateful to Alan Gelb for pointing this out.

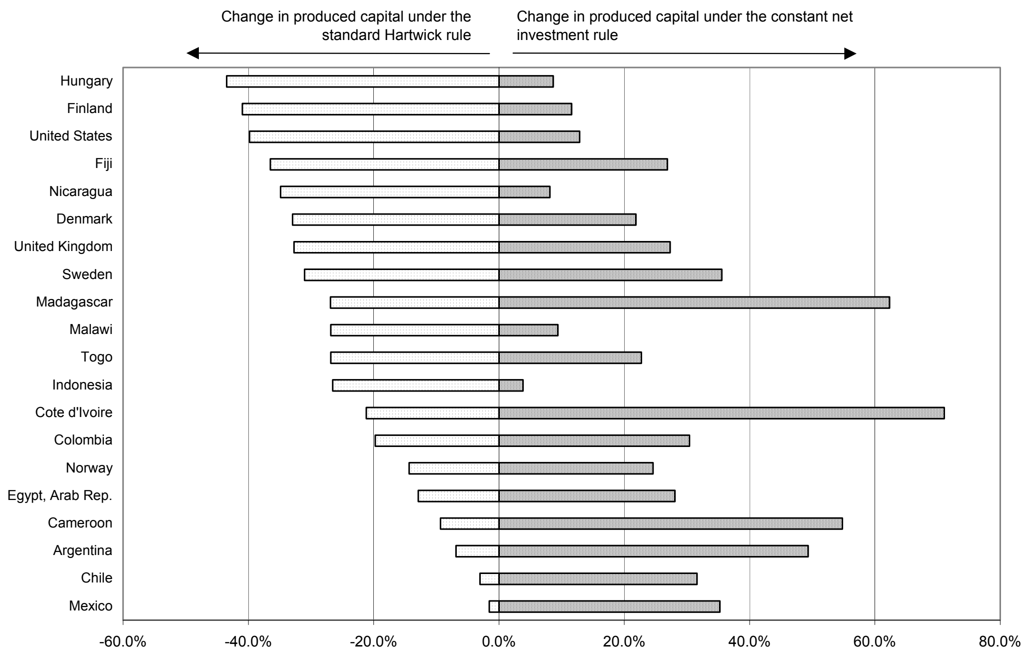
Figure 2 – Actual and counterfactual produced capital (per capita) – 2000



Among the countries with relatively low natural resource dependence and higher counterfactual capital, we find Ghana (gold, bauxite) and Zimbabwe (gold). This is indicative of very low levels of capital accumulation in these economies.

Figure 3 highlights countries which have invested more than their resource rents (as shown by the negative entries on the left side of the figure) but have failed to maintain constant genuine investment levels of at least 5% of 1987 GDP (as shown by the entries on the right). Developing countries in this group include Cote d'Ivoire, Madagascar, Cameroon and Argentina. A number of high income countries also appear in the figure. Sweden could have a stock of capital 36% higher if it had maintained constant genuine investment levels at the specified target. The corresponding difference for the UK is 27%, for Norway 25% and for Denmark 22%. The generally low level of genuine investment levels in the Nordic countries is particularly surprising. Are these countries trading off inter-generational equity against intra-generational equity? Further research would be required to clarify this, a question that is beyond the scope of this paper.

Figure 3 – Capital accumulation under the Hartwick and constant net investment rules



Finally, the next-to-last column in Table A1 shows the change in produced assets for countries if they had genuine investments of at least 5% of 1987 GDP. The positive figures indicate that, with the exception of Singapore, all countries experienced at least one year over 1970-2000 where genuine investments were less than the prescribed constant level.

Conclusions

As suggested in Hamilton and Withagen (2004), applying the standard Hartwick rule as development policy would be extreme – it implies a commitment to zero net saving for all time. Conversely, the constant genuine saving rule embodies a commitment to building wealth at each point in time. In a risky world this may be a more palatable development policy.

The Hartwick rule counterfactual calculations show how even a moderate saving effort, equivalent to the average saving effort of the poorest countries in the world, could have substantially increased the wealth of resource-dependent economies. Of course, for the most resource-dependent countries such as Nigeria there is nothing moderate about the implied rate of investment – a Nigerian genuine investment rate of 36.1% of GDP in 1987 is what our calculations suggest under the constant genuine investment rule.

The savings rules presented here are appealing in their simplicity. Maintaining a constant level of genuine saving will yield a development path where consumption grows monotonically, even as exhaustible resource stocks are run down. The real world is more complex. Poor countries place a premium on maintaining consumption levels, with negative effects on saving – the alternative may be starvation. At the same time financial crises, social instability and natural disasters all have deleterious effects on saving. Holding to a simple policy rule in such circumstances would be no small feat.

Saving effort is of course not the whole story in sustaining development. Savings must be channeled into *productive* investments that can underpin future welfare, rather than 'white elephant' projects. As Sarraf and Jiwanji (2002) document, Botswana's successful bid to avoid the resource curse was built upon a whole range of sound macroeconomic and sectoral policies, underpinned by generally positive political economy. Botswana's absorptive capacity for public investment was a real concern to policy makers, who were prepared to hold resource revenues offshore rather than engage in wasteful investments.

Table A1. Change in produced assets under varying rules for genuine investment (I^G).

	Produced capital in 2000, $bn (1995 dollars)	$I^G = 0$ % difference	I^G = 5% of 1987 GDP % difference	I^G >= 5% of 1987 GDP % difference	Rent / GDP Average (1970-2000)
Nigeria	53.5	358.9%	413.6%	413.6%	32.6%
Venezuela, RB	175.9	272.1%	326.1%	326.1%	27.7%
Congo, Rep.	13.9	57.0%	78.0%	116.9%	25.2%
Mauritania	3.0	112.3%	153.7%	154.0%	25.0%
Gabon	19.7	80.3%	105.5%	130.4%	24.1%
Trinidad and Tobago	13.7	182.1%	238.3%	239.1%	23.6%
Algeria	195.4	50.6%	80.9%	83.9%	23.3%
Bolivia	13.7	116.1%	169.8%	177.5%	12.8%
Indonesia	540.6	-26.5%	3.8%	32.1%	12.5%
Ecuador	37.7	95.3%	158.0%	158.3%	11.6%
Zambia	7.5	312.3%	383.4%	388.0%	11.5%
Guyana	2.1	149.3%	185.6%	191.2%	11.4%
China	2899.4	-62.1%	-45.0%	5.1%	10.8%
Egypt, Arab Rep.	159.7	-12.9%	28.1%	36.2%	9.5%
Chile	151.4	-3.0%	31.6%	54.0%	9.5%
Malaysia	305.2	-52.7%	-31.4%	6.6%	8.3%
Mexico	975.5	-1.5%	35.3%	42.2%	8.2%
Peru	132.3	37.2%	98.1%	103.9%	7.5%
Cameroon	24.1	-9.3%	54.8%	67.6%	6.5%
South Africa	349.5	50.7%	109.3%	115.8%	6.5%
Jamaica	13.4	39.9%	87.8%	99.6%	5.7%
Colombia	198.0	-19.7%	30.4%	39.3%	5.3%
Norway	456.6	-14.3%	24.6%	33.0%	4.3%
India	965.4	-52.9%	-18.3%	8.6%	3.4%
Zimbabwe	14.9	9.1%	64.8%	89.1%	3.3%
United States	16926.7	-39.8%	12.9%	26.1%	2.7%
Argentina	569.6	-6.9%	49.4%	53.9%	2.6%
Togo	3.6	-26.8%	22.7%	55.1%	2.6%
Pakistan	125.6	-50.7%	-1.7%	11.1%	2.2%
Hungary	149.1	-43.5%	8.7%	22.3%	2.2%
Morocco	93.8	-59.1%	-16.3%	7.8%	2.0%
Brazil	1750.5	-59.0%	-6.6%	9.1%	1.9%
United Kingdom	2400.1	-32.7%	27.3%	32.8%	1.6%
Dominican Republic	33.8	-73.0%	-27.9%	1.2%	1.6%
Philippines	195.0	-58.4%	-14.5%	10.6%	1.5%
Honduras	12.3	-66.9%	-29.7%	8.9%	1.5%
Ghana	16.1	30.6%	73.2%	76.7%	1.0%
Fiji	3.6	-36.5%	26.9%	59.3%	0.9%
Benin	4.6	-72.7%	-21.7%	10.6%	0.8%
Senegal	10.0	-44.0%	14.2%	27.5%	0.7%
Thailand	520.6	-86.3%	-63.6%	3.0%	0.7%
Haiti	2.8	-62.7%	109.2%	109.5%	0.6%
Korea, Rep.	1607.6	-93.5%	-68.6%	0.9%	0.6%
Israel	215.8	-72.8%	-31.3%	4.2%	0.5%
Cote d'Ivoire	16.1	-21.2%	71.1%	108.7%	0.5%

	Produced capital in 2000, $bn (1995 dollars)	$I^G = 0$ % difference	I^G = 5% of 1987 GDP % difference	I^G >= 5% of 1987 GDP % difference	Rent / GDP Average (1970-2000)
Bangladesh	89.7	-59.0%	-12.9%	15.5%	0.5%
Rwanda	3.9	-83.2%	-6.9%	24.6%	0.4%
Sweden	508.0	-31.1%	35.6%	36.1%	0.4%
Nicaragua	6.9	-34.9%	8.1%	44.8%	0.3%
Spain	1623.6	-58.9%	-15.1%	6.1%	0.3%
Denmark	437.2	-33.0%	21.9%	28.7%	0.2%
France	3724.7	-55.0%	-1.9%	6.9%	0.1%
Italy	2711.2	-44.8%	7.5%	10.2%	0.1%
Finland	347.6	-40.9%	11.6%	23.3%	0.1%
Belgium	681.9	-48.0%	2.3%	10.4%	0.1%
Niger	3.0	9.7%	95.2%	136.1%	0.1%
Burundi	1.6	-87.3%	10.1%	30.2%	0.1%
Portugal	308.8	-71.0%	-30.8%	5.7%	0.0%
Costa Rica	24.1	-80.0%	-30.6%	3.6%	0.0%
El Salvador	17.1	-59.7%	-2.5%	24.6%	0.0%
Hong Kong, China	445.9	-88.6%	-56.4%	0.9%	0.0%
Kenya	20.1	-51.9%	2.0%	20.8%	0.0%
Madagascar	4.9	-26.9%	62.4%	65.5%	0.0%
Sri Lanka	41.2	-88.1%	-55.4%	1.0%	0.0%
Malawi	4.6	-26.8%	9.4%	68.2%	0.0%
Uruguay	29.9	-55.5%	22.1%	37.2%	0.0%
Luxembourg	43.3	-63.2%	-22.0%	15.7%	0.0%
Paraguay	23.7	-88.6%	-46.6%	3.0%	0.0%
Lesotho	5.7	-95.7%	-79.9%	0.1%	0.0%
Singapore	314.8	-92.7%	-73.2%	0.0%	0.0%

Note: negative entries indicate that hypothetical produced assets would be lower than observed assets under the specified rule.

Annex I. Proof of Proposition 3

Expressions (A1)-(A3) establish some basic properties of the path defined by the constant net saving rule:

$$\dot{K} - F_R R = \overline{G} \Rightarrow \ddot{K} = \dot{F}_R R + F_R \dot{R} \tag{A1}$$

so that

$$\dot{C} = F_K \dot{K} + F_R \dot{R} - \ddot{K} = F_K \dot{K} - \dot{F}_R R = F_K \overline{G} \ . \tag{A2}$$

Constant returns to scale implies that,

$$C = F - \dot{K} = F_K K + F_R R - \dot{K} = F_K K - \overline{G} \tag{A3}$$

The Hotelling rule is used to derive the following expression for the path of R:

$$\dot{F}_R = F_{RR} \dot{R} + F_{RK} \dot{K} = F_R F_K \quad \Rightarrow \quad \frac{\dot{R}}{R} = -F_K + \frac{\overline{G}}{K} . \tag{A4}$$

The growth rates of K and F are derived as follows:

$$\dot{K} = F_R R + \overline{G} = \beta F + \overline{G} \quad \Rightarrow \quad \frac{\dot{K}}{K} = \frac{\beta}{\alpha} F_K + \frac{\overline{G}}{K} \tag{A5}$$

$$\frac{\dot{F}}{F} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{R}}{R} = \frac{\overline{G}}{K} . \tag{A6}$$

Subtracting (A5) from (A4) we have,

$$\frac{d}{dt}\left(\frac{K}{R}\right) = \frac{1}{\alpha} F_K \frac{K}{R} = \left(\frac{K}{R}\right)^{\alpha} \text{, which has solution,}$$

$$\frac{K}{R} = \left[(1-\alpha)t + \left(K_0/R(0)\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} . \tag{A7}$$

It will be useful in what follows to derive the integral of the discount factor $\int_0^\infty e^{-\int_0^s F_K d\tau} ds$. We begin by subtracting (A5) from (A6),

$$\frac{\frac{d}{dt}(F/K)}{F/K} = -\frac{\beta}{\alpha} F_K \quad \Rightarrow \quad e^{-\int_0^t F_K d\tau} = \left(\frac{F}{K}\right)^{\frac{\alpha}{\beta}} \left(\frac{K_0}{F(0)}\right)^{\frac{\alpha}{\beta}} .$$

Since $\left(\frac{F}{K}\right)^{\frac{\alpha}{\beta}}=\left(\frac{K}{R}\right)^{(\alpha-1)\frac{\alpha}{\beta}}$, (A7) implies that $e^{-\int_0^t F_K d\tau}=\left(\frac{K_0}{F(0)}\right)^{\frac{\alpha}{\beta}}\left[(1-\alpha)t+\left(K_0/R(0)\right)^{1-\alpha}\right]^{-\frac{\alpha}{\beta}}$. We can therefore derive,

$$\int_0^\infty e^{-\int_0^s F_K d\tau}ds=\frac{\alpha-\beta}{\beta^2}\left(\frac{K_0}{R(0)}\right)^{\beta}=\frac{\alpha(\alpha-\beta)}{\beta^2}\cdot\frac{1}{F_K(0)}. \qquad \text{(A8)}$$

Expression (A4) implies that $R=R(0)e^{-\int_0^t F_K-\frac{\overline{G}}{K}d\tau}$, while (A6) implies that $F=F(0)e^{\int_0^t \frac{\overline{G}}{K}d\tau}$. Now Proposition 2 and expression (A3) can be used along with the preceding expressions for R and F to derive the following expression for initial wealth:

$$\begin{aligned}W(0)=K_0+F_R(0)S_0&=\int_0^\infty Ce^{-\int_0^s F_K d\tau}ds=\int_0^\infty F_K K e^{-\int_0^s F_K d\tau}ds-\int_0^\infty \overline{G}e^{-\int_0^s F_K d\tau}ds\\&=\frac{\alpha F(0)}{R(0)}\int_0^\infty R(0)e^{-\int_0^s F_K-\frac{\overline{G}}{K}d\tau}ds-\frac{\alpha(\alpha-\beta)}{\beta^2}\cdot\frac{1}{F_K(0)}\overline{G}\\&=\frac{\alpha F(0)S_0}{R(0)}-\frac{\alpha(\alpha-\beta)}{\beta^2}\cdot\frac{1}{F_K(0)}\overline{G}\end{aligned}$$

Since $F_R(0)S_0=\beta K_0^{\alpha}R(0)^{\beta-1}S_0$, this expression can be solved for $R(0)$ to yield,

$$R^{\overline{G}}(0)=(\alpha-\beta)^{\frac{1}{\alpha}}S_0^{\frac{1}{\alpha}}K_0\left(K_0+\frac{\alpha(\alpha-\beta)}{\beta^2}\cdot\frac{1}{F_K(0)}\overline{G}\right)^{-\frac{1}{\alpha}}=R^H(0)\left(1+\frac{\alpha-\beta}{\beta^2}\cdot\frac{1}{F(0)}\overline{G}\right)^{-\frac{1}{\alpha}} \qquad \text{(A9)}$$

Here superscript $\overline{G}$ denotes values on the path for the constant savings rule, while superscript H denotes values on the Hartwick rule ($\overline{G}=0$) path[6]. Feasibility (positive initial period resource extraction) requires that $\alpha>\beta$. Since $C^{\overline{G}}(0)=F^{\overline{G}}(0)-F_{R^{\overline{G}}}(0)R^{\overline{G}}(0)-\overline{G}=\alpha F^{\overline{G}}(0)-\overline{G}$, it follows (i) that $C^{\overline{G}}(0)<C^H(0)$, and (ii) that $\overline{G}<\alpha F^{\overline{G}}(0)$ is necessary for feasibility (positive initial period consumption). This implies that

$$R^H(0)\left(1+\frac{\alpha(\alpha-\beta)}{\beta^2}\right)^{-\frac{1}{\alpha}}<R^{\overline{G}}(0)<R^H(0). \qquad \text{(A10)}$$

Initial resource extraction is lower on the constant genuine saving path than on the Hartwick rule path, and feasibility ensures a strict lower limit for this value.

[6] Note that, since $F(0)=F(K_0,R(0))$, we do not have an analytic solution for $R(0)$ in expression (A9).

Expression (A9) implies that

$$W^{\overline{G}}(0) = \frac{\alpha}{\alpha-\beta}K_0 + \frac{1}{\beta}\left(\frac{K_0}{R(0)}\right)^{\beta}\overline{G}\,. \qquad \text{(A11)}$$

Total wealth is therefore greater under the constant savings rule than under the Hartwick Rule. Note that total wealth is independent of the initial resource endowment S_0 under the Hartwick Rule. Feasibility ($\overline{G} < \alpha F^{\overline{G}}(0)$) implies that,

$$\frac{\alpha}{\alpha-\beta}K_0 < W^{\overline{G}}(0) < \left(\frac{\alpha}{\alpha-\beta} + \frac{\alpha}{\beta}\right)K_0 \text{ or, } W^H(0) < W^{\overline{G}}(0) < W^H(0) + \frac{\alpha}{\beta}K_0\,.$$

Total wealth under the constant savings rule is therefore constrained by bounds that are independent of initial resource endowment.

Finally, (A2) implies that,

$$\dot{C} = F_K\overline{G} = \alpha\left(\frac{K}{R}\right)^{\alpha-1}\overline{G} = \alpha\left[(1-\alpha)t + (K_0/R(0))^{1-\alpha}\right]^{-1}\overline{G}$$

so that, by integrating and applying expression (3),

$$C = \frac{\alpha}{1-\alpha}\ln\left[(1-\alpha)(K_0/R(0))^{\alpha-1}t + 1\right]\overline{G} + \alpha F(0) - \overline{G}\,. \qquad \text{(A12)}$$

Expression (A12) implies that consumption increases without bound under the constant savings rule.

QED.

References

Asheim, G.B., W. Buchholz and C. Withagen, 2003. The Hartwick Rule: Myths and Facts. *Environmental and Resource Economics* 25: 129-150.

Auty, R., ed. 2001. *Resource Abundance and Economic Development*. WIDER Studies in Development Economics, Oxford: Oxford University Press.

Dixit, A., Hammond, P., and Hoel, M., 1980. On Hartwick's rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion, *Review of Economic Studies* (1980) XLVII, 551-556.

Hamilton, K., 2002. Saving effort and population growth: theory and measurement. Presented to the World Congress on Environmental and Resource Economics, Monterey. Washington: The World Bank.

Hamilton, K. and K. Bolt, 2004. Resource Price Trends and Development Prospects. *Portuguese Economic Journal* (2004) 3, 85-97.

Hamilton, K. and J.M. Hartwick, (forthcoming). Investing Exhaustible Resource Rents and the Path of Consumption. *Canadian Journal of Economics*.

Hamilton, K. and C. Withagen, 2004. Savings, Welfare and Rules for Sustainability. (mimeo) Washington: The World Bank.

Hartwick, John M. 1977, "Intergenerational Equity and the Investing of Rents from Exhaustible Resources", *American Economic Review*, 66, 972-974.

Lange, G-M and M. Wright, 2004. Sustainable development in mineral economies: the example of Botswana. *Environment and Development Economics* 9:4, 485-505.

Pezzey, J., 1989, Economic Analysis of Sustainable Growth and Sustainable Development, Environment Dept. Working Paper No. 15, Washington: The World Bank.

Pritchett, L., 2000. The Tyranny of Concepts: CUDIE (Cumulated, Depreciated, Investment Effort) is *not* Capital. *J. of Economic Growth* 5 (December), 361-84.

Sachs, J., and A. Warner. 1995. Natural Resource Abundance and Economic Growth. Development Discussion Paper No. 517a, Harvard Institute for International Development.

Sarraf, M., and M. Jiwanji 2001. Beating the Resource Curse: The Case of Botswana. Environment Department Working Papers, Environmental Economics Series No. 83, Washington: The World Bank.

Solow, R., 1986, On the Intergenerational Allocation of Natural Resources, *Scandinavian Journal of Economics*, 88(1), 141-49.

Vincent, J., Panayotou, T., and Hartwick, J., 1997, Resource Depletion and Sustainability in Small Open Economies, *J. Environmental Economics and Management*, 33, 274-86.

World Bank 2004. *World Development Indicators 2004.* Washington: The World Bank.