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Fair Inheritance Taxation

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Abstract

This paper studies the optimal taxation of bequests in a model in which individuals have heterogeneous preferences over their consumption and the net-of-tax bequest received by their heir. The bequest left by an individual depends on both her degree of altruism and the bequest received from her parents. First, the paper studies two principles that are at the heart of the debates on taxing inheritances: (1) children should not be penalized by the lack of altruism of their parents, and (2) parents should be free to choose their bequests. Only one social welfare function satisfies these two principles, together with Pareto efficiency and a separability principle. Second, the paper studies the shape of the inheritance tax scheme that maximizes this social welfare function. It shows that in the aggregate, the inheritance tax must collect money (redistributed through a non-negative demogrant). Moreover, small bequests cannot be taxed (they can potentially be subsidized), while bequests that are larger than those of the most altruistic individuals who did not receive bequests from their parents should be taxed as much as efficiency permits.

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1 Introduction

The theoretical literature on how to tax bequests has provided a long list of contradicting arguments. Some authors reach the conclusion that bequests should be subsidized, some others that they should be taxed, yet others prove that they should not be taxed nor subsidized. This lack of consensus comes from differences in, first, the modeling of the interactions between parents and children, and second, the objective that the egalitarian planner is supposed to follow. In many papers, individuals are assumed to have the same preferences but different abilities to earn income, with the consequence that the planner tries to redistribute from high to low-wage individuals. Because inheritance inequality does not reveal new information about individuals' wages, Farhi and Werning (2010) and Kaplow (2001), following a Atkinson and Stiglitz (1976) type of argument, prove that taxing labor income is more efficient than taxing inheritance. In a similar model, Kopczuk (2013a) makes the point that a countervailing force pushing in favor of bequests taxation is that receiving a large inheritance disincentivizes labor supply. In Cremer and Pestieau (2001), parents have different wealth levels and taxing bequests reduces inequality. Kaplow (1995), Farhi and Werning (2010) and Kopczuk (2013a) all make the point that subsidizing bequests is a way to incentivize parents to internalize the positive externality of giving.

Piketty and Saez (2013) have introduced two features that have fundamentally changed the picture. First, differences in bequests do not necessarily come from differences in parents' ability to earn income. They can come from parents' altruism, which differs across parents. As a result, taxing inheritance can be the most efficient way to redistribute from lucky to unlucky children. Second, they do not divide individuals into parents and children. They rather consider the entire lifetime, so that all individuals are children and parents in turn. Fiscal policies then affect both the resources that individuals receive early in life and the tax they pay at the end of their life. As a result, all previous arguments based on the combined effects of giving on parents' and children's utilities disappear. The positive externality is now reflected in the level of sustainable tax and transfer policies. As a consequence, Piketty and Saez (2013) show that taxing bequests may end up being optimal. Optimality here is measured with respect to the maximization of some social welfare function that, following the approach developed by Saez and Stantcheva (2016), assigns social welfare weights to individuals as a function of their income. The optimality of taxing bequests is proven for some distribution of these weights.

In this paper, we solve two problems that are left unsolved in Piketty and Saez (2013). First, we show how fairness principles can be used to endogenize social welfare weights. In Piketty and Saez (2013), indeed, the formula of the optimal tax is a function of how society values the utility of different individuals, but in a world in which individuals differ in many dimensions, all of which likely to create inequality, it is not clear who should be given priority. We solve this problem by resorting to two fairness principles, which are at the heart of the debates on taxing inheritance. The first principle is that parents should be free to choose their bequest (McCaffery, 1994). This is consistent with the principle of responsibility for one's preferences, which has inspired recent developments in optimal taxation theory (like, among others, Fleurbaey and Maniquet (2018), Lockwood and Weinzierl (2016), Piketty and Saez (2013)). The second fairness principle is that children should not be penalized by the lack of altruism of their parents.

These two fairness principles may sound in conflict, but we show that they are not. This comes from the fact that each individual is both a child, who may wish to receive a part of the bequest allocated to children of other families, and a parent, who wishes to be free to bequeath the amount she prefers. As a result, there exists one social welfare function (SWF) that reconciles both principles. This is the social welfare function that we maximize to identify the shape of the optimal tax function. By identifying one and only one SWF that satisfies the two relevant fairness principles, we complement Saez and Stantcheva (2016)'s general social marginal welfare weights approach by identifying the weights that should be used with this particular SWF. These weights depend on the amount of received and left bequests. As we show in Section 4, the egalitarian nature of our SWF implies that the weights of all those who inherit something from their parents is zero, and the weights of those who did not inherit anything depends on how much they leave to their children. This distribution of social weights does not belong to the set of distributions that Piketty and Saez (2013) use to derive their results.

The two fairness principles that are central to our undertaking have already appeared in both the theoretical and the empirical literature. Interestingly, Farhi and Werning (2013) embrace the same principles. However, by sticking to a static model in which individuals are either parents or children, they reach a trade-off between these values, whereas we obtain a single social welfare function. By adopting a weighted utilitarian social welfare function with a large variety of possible weights, they justify a large range of policies, including the ones that we prove optimal for our social welfare function.

In the empirical literature, we don't know of any experiment or questionnaire survey that directly elicits adherence to the values that our axioms capture, but many other studies indirectly justify our axioms. The view, driving our responsibility axiom, that inequalities are not unfair when they come from differences in choices has since long been proven to be backed by the public in the empirical social choice literature (see Schokkaert and Overlaet (1989) and Konow (1996) for early evidence, and Gaertner and Schokkaert (2012) for a survey). Gross et al. (2017), by showing a larger public support for inheritance taxation when the children are richer, and Bastani and Waldenström (2021), by showing a larger public support for inheritance taxation when individuals are informed about the link between inequality in inherited wealth and inequality of opportunity, suggest a public support in favor of using inheritance taxation to compensate poorer children.[1]

Second, and most importantly, we identify the shape of the optimal inheritance tax on low bequests, that is bequests left by parents who did not receive anything from their own parents. In Piketty and Saez (2013), indeed, the tax is assumed to be either linear, or linear after an interval of exemption. Starting

[1] In a series of contributions, Weinzierl elicit reasons why people might support taxing the rich more than the poor. The main reason he finds is the view that those who benefit more from a functioning society should also contribute more to its cost (see, for instance, Weinzierl (2014) and Weinzierl (2018)). In the labor income taxation problem, identifying those who benefit more from society as those who have a larger income is natural. In the inheritance taxation problem, we find it harder to claim that those who receive more from their parents benefit more from a functioning society. Therefore, we do not refer to Weinzierl's work to motivate our axioms.

with a much broader set of admissible tax functions, we prove that the optimal function has one of the following two shapes (Proposition 2). Either it exempts low bequests, or it first subsidizes and then taxes them.[2]

Exempting low bequests and taxing larger ones is also a candidate to be optimal in Farhi and Werning (2010). As we mention above, the authors study a static model in which individuals are either altruistic parents or their children. Their main result is that if parents transfer to their children an amount that the planner finds insufficient because the average utility of children is too low, then bequests should be subsidized to incentivize parents to give more. They also show that poorer parents should be incentivized more than richer parents because their children are also poorer so that their marginal utility is larger and they are the ones that the utilitarian planner wishes to help most, with the consequence that the tax should be progressive. Farhi and Werning (2010)'s argument in favor of exempting low bequests and taxing high ones comes from an additional argument in favor of progressivity. This argument is based on two assumptions: First, for a reason that is exogenous to the model, bequests cannot be subsidized. Second, the saving technology is concave. Under these assumptions, it may be optimal to tax large and only large bequests, because disincentivizing rich individuals from saving has a positive effect on the interest rate, which is equivalent to decreasing the cost of saving for the poor, which incentivizes them to leave larger bequests to their children, so that the utilitarian social welfare increases. Our exemption/positive tax result, on the contrary, does not follow from any a priori assumption on the sign of the possible tax. Moreover, our dynamical model à la Piketty and Saez (2013) does not allow us to consider that children necessarily consume less than their parents. On the other hand, the argument that a decrease in total saving could make bequest less costly and help increase lowest bequests could still hold in a dynamical model, but provided it does not prevent the total stock of capital to grow sufficiently. We do not address this question here, and we assume that the interest rate is constant, which removes a possible argument in favor of taxing large bequests.

Interestingly, exemption up to some threshold and taxation above the threshold corresponds to the most frequent tax scheme among OECD countries, suggesting that the SWF we study rationalizes the social preferences that have led to this scheme. We come back to this remark in the conclusion.

The second possible shape of the optimal tax in our model is a subsidy on low bequests followed by a positive tax. In this case, the largest tax amount paid by individuals who did not receive anything from their parents is capped (it cannot exceed the maximum of the bequests that are subsidized). Optimality of a subsidy in this case has a completely different rationale than in the literature reviewed above. Low bequests are subsidized in our case because it allows us to increase the marginal tax rates on bequests left by poor but altruistic individuals, so that they are incentivized to leave lower bequests without being worse off. The larger marginal rates on these low yet not-too-low bequests are desirable only to the extend that they lead to larger average tax rates on higher bequests. We also give sufficient conditions under which subsidizing bequests (and, thereby, increasing marginal tax rates on bequests left by poor altruistic individuals) do not lead to significant increase in average tax rates on high

[2] We prove that tax functions that first subsidize low bequests and then tax larger bequests can be optimal. These functions are not studied by Piketty and Saez (2013), but Kopczuk (2013a) conjectures that they might be optimal.

bequests, yielding the exemption scheme to be optimal (Proposition 3).

An intermediary result in our analysis of the optimal tax function is that bequests should, on average, be taxed rather than subsidized (Proposition 1). This coincides with a result of Piketty and Saez (2013), except that they reach this conclusion only for some distribution of normative weights. This further illustrates that introducing fairness principles allows us to restrict the set of admissible social weights so as to reach sharper conclusions.

The model we use to reach these results has three key assumptions. First, like in Piketty and Saez (2013), all individuals are children and parents in turn. This requires that we look at a dynamic model in which parents save during their life and get some capital income. A consequence of bequests is, therefore, that the stock of wealth increases in the economy, and we focus on the long run effects of taxation. Second, all individuals have the same labor income. This strong assumption allows us to single out the effects on the tax scheme of the heterogeneity in both parent's altruism and how much children receive when they are young. Alternatively, our results stay silent about the effects of differences in earning ability on bequest taxes. This is consistent with the assumption that bequests do not teach us anything about differences in earning ability. More generally, this is consistent with the assumption that the optimal labor income tax does not require to tax bequests. These two key assumptions allow us to focus on the trade-off between subsidizing bequests or transferring a demogrant to all. We come back in the conclusion on how to combine our results with other results on optimal labor income taxation. Third, we assume a joy-of-giving bequest motive. We discuss this assumption below.

Three policy recommendations are consistent with our results. First, taxing bequests should be viewed as a way to redistribute from individuals who inherited from their parents to those who were less lucky. That means that the tax/transfer scheme of bequests should bring some strictly positive surplus to the government, money that should be allocated to all individuals independently of how much they receive from their parents and how much they leave to their children.

Second, the trade-off between increasing the transfers to all and decreasing inheritance taxes should be solved by looking at how they affect the well-being of those who did not receive inheritance from their parents. Indeed, the most altruistic among them will prefer a decrease in inheritance tax (or an increase in inheritance subsidy) whereas the self-centered among them will prefer an increase in transfer.

Third, in spite of a positive average tax on bequests, there can be two reasons to subsidize low bequests. The first reason, reminiscent of results from the literature that we review below, is efficiency, because subsidizing may be a way to obtain a dominating distribution of bequests. The second reason is fairness, because it is a way to redistribute among the poorest individuals (the ones who did not get any bequests from their parents) from self-centered to altruistic individuals. This, however, goes with a precise condition: fairness can only justify subsidies in an interval that goes up to the bequests left by the poorest and most altruistic individuals.

As discussed in reviews by Cremer and Pestieau (2006) and Kopczuk (2013b), the efficiency and fairness implications of inheritance taxation may depend on the bequest motive. For instance, accidental bequests, which exist in the absence of a perfect annuity market when parents die before consuming all their savings, can be taxed without any efficiency costs. We study inheritance taxation under a joy-of-giving motive, which explicitly acknowledges the desire that parents may hold to leave a bequest. The legitimacy of such desire is central in discussions surrounding the taxation of bequests (McCaffery, 1994). An alternative bequest motive consistent with this desire is altruism, whereby parents care for the *utility* of their child (whereas parents care only about the net-of-tax inheritance received by their child under a joy-of-giving motive). The altruistic motive is at the center of the Barro-Becker dynastic model, which has been widely studied in the literature on optimal capital/inheritance taxation. Most centrally, Chamley (1986) and Judd (1985) conclude that the tax on inheritances should be zero in the long run. More recently, Straub and Werning (2020) overturn this early result by showing that it only holds for high values of intertemporal elasticity of substitution, but otherwise such tax is positive and significant. We do not consider altruism and it remains an open question whether our results also hold in this alternative setting. One result that is very likely to carry through is that the inheritance tax should globally collect a non-negative amount. The reason is that any tax violating this would be dominated by Laissez-Faire, because such tax would hurt self-centered individuals whose parents are self-centered. We also note that the altruistic model of bequests has been empirically tested and rejected by Wilhelm (1996).

Some authors study the effect on the optimal tax of the fact that the number of children may differ across families and parents may decide not to give equal bequests to all their children, like Cremer et al. (2001). We could take that into account at least to some extent: the worst-off would remain the same under more general assumptions on the number of children.

Some other aspects of inheritance taxation are completely ignored in our analysis. Cremer et al. (2003) study capital income taxation as subsidiary to inheritance taxation in a world in which bequests may not be observable. Nordblom and Ohlsson (2006) study the possibilities to escape bequest taxation through inter-vivos gifts. Stantcheva (2015) studies inheritance taxation in its relationship to investments in human capital. Mirrlees et al. (2010) discuss the administrative cost implied by the collection of an inheritance tax. Golosov et al. (2003) and Kocherlakota (2005) study tax instruments that are allowed to vary over time, leaving more room for improving welfare. Fleurbaey et al. (2018) study the implications of ex-post egalitarianism for the taxation of accidental bequests resulting from premature mortality.

In Section 2, we illustrate our main results in a simple model. In Section 3, we describe the model, by insisting on the similarities and differences with the pioneer model of Piketty and Saez (2013). In Section 4, we discuss our social welfare function and the axioms that justify it. In Section 5, we study the optimal tax function and we state our main results. In Section 6, we provide some concluding comments. In Section 7, we develop the proofs of the results.

2 A Simple, Illustrative, Model

In this section, we illustrate our main results in a simple model. We assume preferences are of two types: selfish or altruistic. Selfish individuals are only interested in their own consumption. Their utility function is defined as $u_s(c, h) = c$, where c is own consumption and h is the money inherited by one's child. Altruistic individuals have Cobb-Douglas preferences defined over own consumption and the money their children obtain from them. Their utility function is defined as $u_a(c, h) = c^{1/2}h^{1/2}$. We assume there is a fraction α of altruistic individuals in each generation. The type of children is independent of that of their parents, so that α is also the fraction of altruistic individuals from selfish parents in each generation.

Let us assume that bequests are not taxed. We call it a laissez-faire tax system. Each individual's budget is given by

$$w + g = c^* + \frac{h^*}{R}.$$

where w is the (inelastic) labor income, g is the money inherited from one's parents and R is the exogeneous interest rate. For an alloction to be feasible, there needs to be consistency between the g's, which are inherited by a generation, and the h's, which are transferred to the next generation. We define this consistency condition below, but we can skip it in this section. Selfish individuals have

$$\begin{aligned} c^* &= w + g \\ h^* &= 0, \end{aligned}$$

whereas altruistic individuals have

$$\begin{aligned} c^* &= \frac{w + g}{2} \\ h^* &= \frac{R(w + g)}{2} \end{aligned}$$

In the long run, a fraction $(1 - \alpha)$ of individuals do not receive bequests from their parents, a fraction $\alpha(1 - \alpha)$ of individuals receive bequest $\frac{Rw}{2}$, a fraction $\alpha^2(1 - \alpha)$ of individuals receive bequest $\frac{R(w+\frac{Rw}{2})}{2}$, etc.

The resulting laissez-faire allocation has a good and a bad property. The good property is that two individuals receiving the same amount of bequest from their parents are free to choose how to allocate their wealth between consumption and bequest. To say it differently, the government treats individuals who differ only in terms of their preferences identically. The bad property is that bequests are heterogeneous, so that two individuals having the same preferences, that is two selfish or two altruistic individuals, do not end up with the same satisfaction level. The government fails to correct for the inequality arising from the birth lottery.

Our first main result is that there exists a social welfare function satisfying these two properties (same treatment for same received bequests and same satisfaction level for same preferences) together with Pareto efficiency and separability conditions. This social welfare function works by applying an extremely egalitarian aggregator (the maximin aggregator) to numbers that we could call well-being indices. The well-being index of a selfish individual consuming c is equal to c. The well-being index of an altruistic individual is a calibration of their utility level. The calibrated utility level associated to consuming a generic bundle (c, h) is the number u^c such that this individual is indifferent between their actual bundle and freely allocating wealth u^c between consumption and bequest. For an altruistic individual we have

$$u_a(c,h) = c^{1/2}h^{1/2} = \left(\frac{u^c}{2}\right)^{1/2}\left(\frac{Ru^c}{2}\right)^{1/2}$$

so that their calibrated utility is

$$u^c\big((c,h),u_a\big) = \frac{2}{R^{1/2}}u_a(c,h).$$

Observe that if an altruistic individual freely allocates a wealth of u_c between consumption and bequest, their calibrated utility is exactly u_c. Therefore, this calibration has the property that if a government wishes to equalize calibrated utility between selfish and altruistic individuals who receive the same bequest (and, therefore, have the same total wealth), this government should treat selfish and altruistic in the same way, that is there should not be any redistribution among them.

Our second main result consists in proving that the laisser-faire allocation is necessarily dominated, given this social welfare function, by an allocation in which bequests are taxed, on average, and the tax returns are redistributed equally among all individuals as a demogrant. Indeed, the individuals who do not receive any bequests from their parents are the worst-offs in such an economy (that is, they have the lowest calibrated utility), and increasing their (calibrated) utility increases social welfare. This implies that the optimal bequest tax scheme is one in which bequests are taxed, on average, at a positive rate, so that those who do not receive any bequests from their parents receive a transfer from the government.

Our next main result has to do with the non-linear shape of the optimal tax scheme. We show that a strong candidate is the tax scheme that applies a zero tax rate on bequests that are equal or lower than the bequest level of the poorest altruistic individuals, the ones who do not receive bequests themselves. Let us assume that the optimal demogrant is D^*. Then bequests should be exempted of tax up to $e^* = \frac{D^*+w}{2}$.[3] Above that level, the tax should maximize the amount of collected taxes.

We can illustrate the difficulty of identifying the optimal tax in the case of a linear taxation above e^*. Let t denote the tax rate for bequests above this threshold. The budget constraint is now

$$w + D + g = c + e^* + \frac{h - Re^*}{R(1-t)}.$$

When the tax rate t is small enough not to generate complete bunching at $h = Re^*$, the optimal choice of altruistic individuals reads[4]

$$c^* = \frac{1}{2}\left(w + D + g + \frac{te^*}{(1-t)}\right)$$
$$h^* = \frac{R}{2}\big((1-t)(w + D + g) + te^*\big)$$

[3] We assume bequests are taxed before being capitalized, as explained in footnote 10.

[4] Letting $\bar{h}$ denote the maximal feasible inheritance, we have $\bar{h} = R(1-t)\,(w + D + g) + Rte^*$. As we assume $h^* > Re^*$ when $g > 0$ (no bunching), we get $h^* = \bar{h}/2$ and $c^* = \bar{h}/(2R(1-t))$.

from which we deduce that the tax collected, $w+D+g-c^*-h^*/R$, is equal to

$$T = \frac{R}{2}\left(t(w+D+g) - e^*\left(\frac{2t-t^2}{1-t}\right)\right)$$

This shows the dilemma of taxing bequests. On the one hand, the derivative of T with respect to t, which is equal to

$$\frac{dT}{dt} = \frac{R}{2}\left((w+D+g) - e^*\left(\frac{2-2t+t^2}{(1-t)^2}\right)\right),$$

is positive for small values of t, because $w+D+g > 2e^*$. On the other hand, h^* is decreasing in t, and g of a generation is determined by h^* of the preceding generation, so that increasing t has a detrimental effect on T through g. Taxing bequests does not only deter bequests today, but it also brings about a dynamic effect in which wealth, and therefore bequests of the altruistic individuals, decrease over time. We do not give an analytical formula of the optimal tax below, but we are able to identify some properties of its general shape.

We claim that the exemption/positive tax scheme is a candidate to be optimal. Again, we can illustrate why such an exemption may not be optimal with the model of this section. This corresponds to the last result of the paper. Let D^* be the largest demogrant that can be sustained with an exemption/positive tax scheme. That means that the bundle of the poorest altruistic individual is $\left(\frac{w+D^*}{2}, \frac{R(w+D^*)}{2}\right)$. The key observation is that this individual can reach the same utility with any other bundle $\left(\frac{k(w+D^*)}{2}, \frac{R(w+D^*)}{2k}\right)$. If $k > 1$, then this bundle can only be reached if bequests are subsidized, because the income of the selfish poor remains $w+D^*$. So, offering $\left(\frac{k(w+D^*)}{2}, \frac{R(w+D^*)}{2k}\right)$ to the altruistic poor has a cost, but it has the following advantages: the budget of the other, richer, altuistic individuals will be lower, which can be deduced from the fact that if a poor altruistic individual consumes $\left(\frac{k^*(w+D^*)}{2}, \frac{R(w+D^*)}{2k^*}\right)$ for some $k^* > 1$, then no other bundle $\left(\frac{k(w+D^*)}{2}, \frac{R(w+D^*)}{2k}\right)$ is affordable anymore for $k < k^*$. Therefore, richer individuals can be taxed more, so that a larger D may be sustainable. To sum up, the optimal tax scheme can consist in subsidizing the altruistic poor individuals, not to incentivize them to let larger bequests but, on the contrary to incentivize them to leave lower bequests because such a subsidy is a way to increase the marginal tax rates without affecting the utility of these individuals.

The model we define in the next sections is more general than the one of this section, as there is a continuum of types of agents and not 2, preferences are general and not Cobb-Douglas, and the tax we look at is not necessarily linear. On the other hand, we keep the assumption that all individuals have the same income, which implies that we do not study how income and bequest taxation can complement each other, and we keep the assumption of a fixed interest rate.

3 The model

We consider an economy with a discrete set of successive generations, $0, 1, \ldots$. Each generation contains a set $[0,1]$ of individuals of measure 1. We use λ to denote the probability measure on $[0,1]$, that is the mass of individuals whose names are between i and j ($i,j \in [0,1]$, $i<j$) is equal to $j-i$. We let $M[0,1]$ denote the set of Lebesgue-measurable subsets of $[0,1]$ and $\mu(J)$ denotes the measure of $J \in M[0,1]$.

We assume that all individuals earn an identical lifetime income of w.[5] Therefore, differences in lifetime budgets only come from the bequests individuals get (or not) at the beginning of their life. The assumption of equal w among individuals, however, is far from necessary for our results. We come back to this issue in the conclusion.

Preferences are defined over lifetime consumption, c, and the inheritance received by their heir, h. In addition to goods c and h, it is convenient to consider the quantity of money that an individual receives at the beginning of her life, g, and the bequest left by an individual, b, which is the quantity of money that she does not consume at the benefit of her heir. Quantities g, c, b and h will be related to each other when we model taxation below, but we don't need to introduce these relations in the first step of our analysis, when we discuss the social welfare function.

Preferences are heterogeneous in the population and preferences of parents and children are independent of each other, even within a dynasty: a generous parent can be followed by a selfish one, whose child can be generous again, etc. For the sake of exposition, we call preferences leading to positive bequests altruistic, even if they differ from pure altruism as it is used in the literature, in which utility of the parents depends on the utility of the children.

We make three assumptions on the *joy-of-giving* utility functions representing these preferences.

1. We assume preferences to be normal on both goods, consumption and inheritance, at all prices.

2. We assume that at each period there exist some selfish individuals, that is their utility function is
$$u^{\mathbf{s}}(c,h)=c.$$
A consequence of this assumption is that at each period there exist some individuals who do not receive any inheritance.

3. Finally, we assume that at each period there exist individuals exhibiting the largest level of altruism. That is, some individuals have utility function $u^{\mathbf{a}}$ and for all compact opportunity set $O \subset \mathbb{R}^2_+$, if bundle (c^a,h^a) is the best bundle in O according to $u^{\mathbf{a}}$ and (c_i,h_i) the best bundle according to any utility function u_i of another individual then $h^a \geq h_i$. Observe that this assumption would be a consequence of imposing the classical single-crossing property (which amounts to assume that individuals can be ranked according to their level of altruism) and a kind of compactness of the domain of preferences. We don't need these assumptions and limit ourselves at imposing the existence of most altruistic individuals.

[5] Remember that, contrary to Piketty and Saez (2013), we are not interested into the trade-off between labor income taxation and bequest taxation. We only raise the subquestions of whether total lifetime labor income should be taxed (resp., subsidized) so as to subsidize (resp., tax) (at least some) bequest leavers.

To sum up, the consequence of the last two assumptions is that some individuals have the lowest level of altruism, and they will never leave any bequest to their children, some other individuals have the largest level of altruism, and they will always be the ones with largest bequests among those with the same wealth, and all the other individuals with the same wealth leave bequests between these two extremes, even if there is no complete ranking of them in terms of their altruism.

Individuals live one period, after which they are replaced by the individual of the same dynasty and the next generation. Hence, any individual has only one child.

We begin by focusing on what happens to one generation. Anticipating that we will later restrict our attention to long-run equilibrium allocations, we restrict our attention to allocations in which the distribution of money received by this generation from the previous one, g, is identical to the distribution of money received by the following one, h. Formally, individual $i \in [0,1]$ consumes a bundle

$$z_i := (g_i, c_i, h_i) \in X = \mathbb{R}^3_+.$$

An *allocation* $z \in Z := X^{[0,1]}$ is a function $z : [0,1] \to X$. An allocation $z = (g_i, c_i, h_i)_{i\in[0,1]} \in Z$ is a *steady-state* allocation if the distribution of the g_i's is equal to the distribution of h_i's, that is, if $(\hat{g}_i)_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$, where $(\hat{x}_i)_{i\in[0,1]}$ denotes the permutation of $(x_i)_{i\in[0,1]}$ in which elements are ranked in increasing order. We let S denote the set of steady-state allocations. A (one generation) *economy* is a profile of utility functions $u = (u_i)_{i\in[0,1]} \in \mathcal{U} = U^{[0,1]}$, where U is the set of acceptable utility functions. In particular, $u^{\mathbf{s}}, u^{\mathbf{a}} \in U$.

4 Social welfare

In this section, we define the social welfare function (SWF) that we use in the next section to study the optimal tax and we discuss its axiomatic foundation. As it is customary in taxation theory, we do so without reference to any institutional context or constraint. As a result, a social welfare function is required to rank all allocations, independently of how they can be implemented. In the next section, on the contrary, we will take account of the fact that preferences are not observable and thus only consider allocations that can be implemented by a bequest tax function and a demogrant.

The SWF works by applying the lexicographic aggregator to individual well-being indices that capture the fairness principles of the planner. Any individual's well-being index represents her preferences, which implies that the SWF is not paternalistic.[6] Let us define this index first. It is illustrated in Fig. 1. Individual i is consuming (c_i, h_i). The indifference curve through (c_i, h_i) shows that individual i is indifferent between (c_i, h_i) and maximizing her utility over a budget set of slope $-R$ starting at $(\tilde{c}, 0)$, where R is the exogenous rate of return on savings per generation. Budget sets of slope $-R$ are non-distortionary, first-best, budget sets. They have the slope of the individual budget sets in the Laissez-Faire allocation, that is, in the absence of taxation. To say it differently,

[6] A property of this index representing preferences is that it does not depend on the *distribution* of preferences in the economy, but only on the individual's own preferences.

this individual is indifferent between her actual consumption, (c_i, h_i), and being free to allocate a wealth of $\tilde{c}$ between own consumption today and children's inheritance tomorrow in the absence of taxation. We state that this individual has a current well-being of $\tilde{c}$. The objective of the planner is to maximize the lowest well-being, and in case of a tie, to maximize the second lowest well-being, etc.

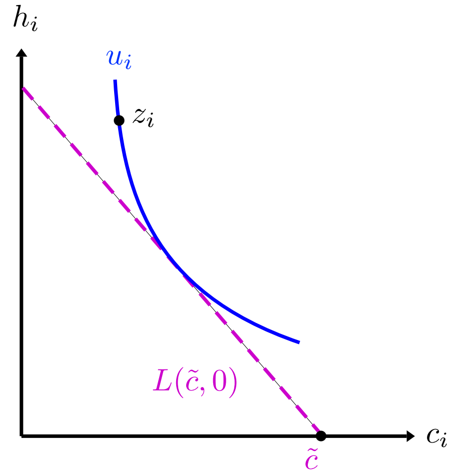

Figure 1: The c-equivalent utility. Individual i is indifferent between the bundle she consumes, z_i, and her favorite bundle in the non-distortionary budget set $L(\tilde{c}, 0)$. As a result, her c-equivalent utility is equal to $\tilde{c}$.

To define this SWF formally, we need the following notation. A *social ordering* is a complete ordering on steady-state allocations. A *Social Welfare Function (SWF)* is a function $\boldsymbol{R}$ associating each economy $u \in \mathcal{U}$ with a social ordering $\boldsymbol{R}(u)$.

We define the non-distortionary budget set of individual i associated with consumption bundle (c_i, h_i) as the set of all bundles (c'_i, h'_i) that individual i can afford with her wealth at (c_i, h_i), that is with a wealth of $c_i + \frac{h_i}{R}$[7]

$$L(c_i, h_i) := \left\{ (c'_i, h'_i) \in \mathbb{R}^2_+ \,\middle|\, c'_i + \frac{h'_i}{R} \leq c_i + \frac{h_i}{R} \right\}.$$

These budgets are linear and of slope $-R$.

The well-being index we are interested in, which we denote as u^c and we call c-equivalent utility, can be defined as follows.

[7]As mentioned above, g_i, c_i and h_i will be related to each other when we model taxation. Thus, even if g_i does not appear in the inequality defining L, c_i and h_i will depend on g_i when we model taxation.

Definition 1 (c-equivalent utility)**.**
For all $i \in [0,1]$*,* $z_i = (g_i, c_i, h_i) \in X$ *and* $u_i \in U$*,*

$$u^c(z_i, u_i) = \tilde{c} \Leftrightarrow u_i(c_i, h_i) = u_i \left(\arg\max_{u_i} L(\tilde{c}, 0) \right).$$

As shown in Fig. 1, individual i is indifferent between consuming z_i and her favorite bundle in the non-distortionary budget set $L(\tilde{c}, 0)$, where i's favorite bundle in $L(\tilde{c}, 0)$ is different from bundle $(\tilde{c}, 0)$.

The SWF $\boldsymbol{R}^{c\text{–}lex}$ compares two allocations by applying the leximin aggregator to lists of c-equivalent utilities associated to the allocations.

SOF 1 ($\boldsymbol{R}^{c\text{–}lex}$)**.** *For all* $u \in \mathcal{U}$ *and any two allocations* $z = (g_i, c_i, h_i)_{i \in [0,1]}, z' = (g'_i, c'_i, h'_i)_{i \in [0,1]} \in S$

$$z' \ \boldsymbol{R}^{c-lex}(u) \ z \quad \Leftrightarrow \quad (u^c(z'_i, u_i))_{i \in [0,1]} \geq_{lex} (u^c(z_i, u_i))_{i \in [0,1]}.$$

Maximizing this SWF gives a very simple formula for the Piketty and Saez (2013) social welfare weights. These weights depend on the allocation at which social welfare is measured. Therefore, at any precise allocation, the welfare weights of the worst-off individuals in terms of their c-equivalent utilities are positive and all equals, and the weights of all other individuals are equal to zero. Of course, which individuals are the worst-off depends on the allocation. As we will see in the next section, when we take incentive constraints into account, the worst-off individuals in c-equivalent utilities will be found among those who did not inherit anything from their parents, but, depending on the allocation, they may be found among selfish or altruistic individuals.

c-equivalent utilities are equalized between two individuals if they freely allocate the same total quantity of money (arising from their labor, the demogrant and their inheritance) between own consumption and bequests in the absence of distortive taxation. This explains why exempting low bequests from taxation will play a prominent role when we study second-best taxation in the next sections.

It may be useful to contrast our SWF with a classical Rawlsian SWF. When there is only one variable of heterogeneity and preferences are identical among individuals, the Rawlsian SWF consists in maximizing the minimal utility, which is the utility of the individuals with the lowest level of the heterogeneous variable (typically, wage rate). These individuals and only them have a positive welfare weight. When there are several dimensions of heterogeneity, including preferences, the choice of the utility function representing the preferences is key to define the Rawlsian objective. This is exactly what our axiomatic analysis achieves: c-equivalent utilities are a calibration of individual utility functions that makes sense of interpersonal welfare comparison. Without this axiomatic foundation, there would be as many Rawlsian SWFs as there are possible utility representations of the different preferences that we allow for. As a consequence, who are the worst-off individuals not only would depend on the policy but also on this utility representation.

c-equivalent utilities have two key properties. First, they do not directly depend on g_i, that is the quantity of money one individual received as inheritance does not matter per se. The only quantities that matter are the money consumed by an individual and the money inherited by her child. Second, how precisely an individual allocates her wealth between own consumption and bequest does not matter, provided this individual allocates it freely. As a result, more altruistic or more selfish individuals have the same c-equivalent utility when they allocate the same quantity of money (that is, the same sum of own consumption and bequest) in the absence of taxation.

The combination of these two properties is the basis on which the axiomatization of this SWF is grounded. Indeed, it satisfies the following three important axioms. The first one is the classical Pareto axiom. It requires weak social preference when all individuals weakly prefer one allocation over another. In addition, it requires strict social preference when one set (of positive measure) of individuals strictly prefer the former allocation.

Axiom 1 (Pareto)**.**
For all economy $u \in \mathcal{U}$ and steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, if for all $i \in [0,1]$

$$u_i(c'_i, h'_i) \geq u_i(c_i, h_i)$$

then z' $\boldsymbol{R}(u)$ z, and if, in addition, there exists a subset of individuals $J \in M[0,1]$ such that $\mu(J) > 0$ and for all $j \in J$

$$u_j(c'_i, h'_i) > u_j(c_j, h_j)$$

then z' $\boldsymbol{P}(u)$ z.

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Pareto comes from c-equivalent utility being a recalibration of the utility function.

The second axiom, compensation for children's lack of luck, in short Compensation, encapsulates the idea that individuals should not be held responsible for the lack of altruism of their parents, that is they should be compensated for receiving low inheritance. Formally, it requires that if two individuals with identical preferences consume bundles that dominate one another (that is, one individual has both a larger consumption and a larger inheritance received by her child) then a transfer from the richer to the poorer of these individuals is a strict social improvement. The restriction that the axiom applies only when individuals have identical preferences is added to avoid a classical impossibility with Pareto. This makes this axiom a rather weak requirement, as there is no inequality restriction between the bundles of two parents as soon as they have different preferences (remember that all preferences that are normal on both goods are admissible).

Axiom 2 (Compensation)**.**
For all economy $u \in \mathcal{U}$, steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, subsets of individuals $J, K \in M[0,1]$ such that $\mu(J) = \mu(K) > 0$, and $\delta \in (0, \frac{1}{2}]$, if for all $j, q \in J$ and $k, \ell \in K$,

- $u_j = u_q = u_k = u_\ell$, $c_j = c_q, c_k = c_\ell$, $h_j = h_q, h_k = h_\ell$,
- $c_j + \delta(c_k - c_j) = c'_j = c'_q \leq c'_\ell = c'_k = c_k - \delta(c_k - c_j)$,
- $h_j + \delta(h_k - h_j) = h'_j = h'_q \leq h'_\ell = h'_k = h_k - \delta(h_k - h_j)$,

and $z_i = z'_i$ for all $i \notin J \cup K$ then z' $\boldsymbol{P}(u)$ z.

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Compensation comes from c-equivalent utility being independent of the inheritance received g. As a consequence, equalizing consumption and bequest among individuals with the same preferences is a way to make individual well-being independent of how much one inherited from their parents.

The third axiom, responsibility for parents' choices, in short Responsibility, encapsulates the idea that individuals should be considered responsible for their preferences, that is they should be free to allocate their wealth the way they wish. It requires that two individuals with the same inheritance from their parents should ideally be free to choose their preferred bundle in the same non-distortionary budget set. Rather than requiring that they should choose in the same budget set, the axiom requires that budget inequality between two such individuals should be reduced.

Axiom 3 (Responsibility)**.**
For all economy $u \in \mathcal{U}$, steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g_i', c_i', h_i')_{i\in[0,1]} \in S$, subsets of individuals $J, K \in M[0,1]$ such that $\mu(J) = \mu(K) > 0$, if there exists $\delta > 0$ such that for all $j, q \in J$ and $k, \ell \in K$,

- $(c_i, h_i) \in \max_{|u_i} L(c_i, h_i),\ \forall\, i \in \{j, q, k, \ell\}$,
  $(c_i', h_i') \in \max_{|u_i} L(c_i', h_i'),\ \forall\, i \in \{j, q, k, \ell\}$
- $y_j + \delta = y_q + \delta = y_j' = y_q' < y_k' = y_\ell' = y_k - \delta = y_\ell - \delta$,

where

$$y_i = c_i + \frac{h_i}{R}, y_i' = c_i' + \frac{h_i'}{R}, \forall\, i \in \{j, q, k, \ell\},$$

and $z_i = z_i'$ for all $i \notin J \cup K$ then z' $\boldsymbol{P}(u)$ z.

The fact that $\boldsymbol{R}^{c\text{–}lex}$ satisfies Responsibility comes from c-equivalent utility assigning the same well-being to two individuals as soon as they both freely allocate their wealth in the same non-distortionary budget set.

In Online Appendix S4, we formally prove that $\boldsymbol{R}^{c\text{–}lex}$ is the only SWF that satisfies these three axioms together with a consistency requirement of the SWF across economies (Separability). A number of important remarks have to be made at this stage.

1. Both Compensation and Responsibility are transfer axioms, that is, they are satisfied if a transfer of goods or of budget is implemented from the richer to the poorer individual, that is to say that they display a positive degree of inequality aversion. The SWF we come at, though, exhibits an infinite inequality aversion. The fact that the combination of Pareto, transfer and consistency axioms leads to a SWF exhibiting infinite inequality aversion is common in the literature and the underlying logics is now well understood. That $\boldsymbol{R}^{c\text{–}lex}$ uses the leximin aggregator of utilities may look extreme, but the proof in Online Appendix S4 implies that any less extreme aggregator (such as Gini) would violate one of the axioms.

2. There is something less common, though, in the axiomatic foundation of $\boldsymbol{R}^{c\text{–}lex}$. Compensation and responsibility axioms, indeed, are typically incompatible with each other. It is therefore a surprise that they turn out to be compatible in this model. This comes from the fact that individuals in this model are both parents and children and each individual's utility is influenced both when she receives inheritance, so that her wealth increases,

and when she is prevented from bequeathing/incentivized to bequeath money to her child. It can be illustrated through the following simple example. Assume an altruistic parent plans to bequeath some amount of money to her child, whereas a selfish parent with the same wealth does not wish to bequeath anything to her own child. As a result of the bequest, the child of the altruistic parent would turn out better-off than that of the selfish parent. On the other hand, if the altruistic parent is prevented from bequeathing, she will end up worse-off than the selfish parent. The first-best solution to this paradox, assuming preferences are observable and non-distortionary transfers can take place, would be to withdraw some wealth from both parents and to allocate it to the child of the selfish parent in compensation for the lack of inheritance, taking into account that the two children themselves can be required to give a part of their wealth in case other individuals are worse-off. That is, the ideal non-distortionary allocation would be to equalize wealth of all individuals of all generations, independently of whether this wealth comes from bequest or redistribution.[8] Useless to say, such an allocation is impossible to achieve through bequest taxation.

5 Optimal tax

In this section, we study the allocations that maximize our SWF among those that can be implemented by a bequest tax function and a demogrant, that is, we stick to the typical assumptions in the literature that preferences are not observable whereas bequests are.

Contrary to what we did in the previous section, we now take account of the influence of the tax scheme on the transmission of bequests, as the behavior of members of one generation influences the well-being of members of the next generation. To take account of these long-run effects of the tax, we restrict our attention to tax functions and demogrants that do not depend on time and we look at the corresponding long-run allocations, that is the allocations obtained when the distribution of bequests is stabilized across generations.

More precisely, we assume that at time t, each individual it (from dynasty i living in generation t) receives inheritance $g_{it} \geq 0$ (which is a function of the bequest left from individual $it-1$) and demogrant D, so that their total resources are $w+D+g_{it}$. Individual it chooses consumption $c_{it} \geq 0$ and bequest $b_{it+1} \geq 0$ under the budget constraint

$$c_{it} + b_{it+1} = w + D + g_{it}.$$

Bequests are taxed according to tax function τ, so that amount $b_{it+1} - \tau(b_{it+1})$ is transferred to individual $it+1$, who receives

$$h_{it+1} = R(b_{it+1} - \tau(b_{it+1})),$$

where R is the interest rate. We assume that $\tau(0) = 0$, because any other value would amount to transferring the same (negative or positive) amount to all, which is exactly what D achieves.

[8] Formally, using the terminology that will be introduced in the next section, if T_{it} stands for the first-best tax or subsidy of the member of dynasty i living at time t, the optimal first-best taxation requires that for all individuals it, $g_{it} = b_{it}$ (children get exactly what parents leave them) and $w + T_{it} + g_{it}$ is equalized across all it.

The same process takes place at $t+1$, with inheritance $g_{it+1} = h_{it+1} \geq 0$. Starting conditions at time $t+1$ may, therefore, differ from those at time t. Note that, in this model, it is the money collected at time t through τ that is used to fund demogrant D. This captures the fact that, had we considered a model in which individuals live for many periods, with a fraction of them born and dead at each period, what an individual gets out of the redistribution system at each period of her life, D, is funded by the taxes on bequests of the individuals that lived (and died) during this individual's own life. This modeling is particularly appropriate to our objective to study the trade-off between modifying the budgets of individuals through a demogrant, funded by taxes on bequests, or subsidies to bequests, at the price of a lower or even negative demogrant.

For a given *tax-demogrant scheme* (τ, D), an *equilibrium* allocation at time t is an allocation $z_t = (z_{it})_{i\in[0,1]} = (g_{it}, c_{it}, h_{it+1})_{i\in[0,1]} \in Z$ for which all $i \in [0,1]$ choose in the budget set defined by this scheme and their inheritance g_{it}, i.e.

$$B^{\tau}(w+D+g_{it}, 0) := \left\{ (c_{it}, h_{it+1}) \in \mathbb{R}^2_+ \,\middle|\, h_{it+1} \leq R\left(w+D+g_{it}-c_{it}-\tau\left(w+D+g_{it}-c_{it}\right)\right) \right\},$$

implying for all $i \in [0,1]$ that

$$h_{it+1} = R\left(w+D+g_{it}-c_{it}-\tau\left(w+D+g_{it}-c_{it}\right)\right). \tag{1}$$

A *long-run equilibrium* allocation for a tax scheme (τ, D) is a steady-state equilibrium allocation $z = (g_i, c_i, h_i)_{i\in[0,1]} \in S$ to which this sequence of equilibrium allocations at time t may converge. That is, at a long-run equilibrium allocation, Eq. (1) holds and the profile of inheritances received, $(\hat{g}_i)_{i\in[0,1]}$, is equal to the profile of inheritances left, $(\hat{h}_i)_{i\in[0,1]}$.

We need some further assumptions to guarantee that long-run equilibrium allocations exist and that we are able to apply our SWF to them.

As Piketty and Saez (2013), we assume that the stochastic transmission of preferences across generations is such that the distribution of preferences remains constant through time. In the terms of the previous section, that means that economy $u \in \mathcal{U}$ is constant through time, up to some (measure preserving) permutation of $i \in [0,1]$.

We also assume that the economy converges over time to a unique long-run equilibrium independent on the initial distribution of inheritances $(g_{i0})_{i\in[0,1]}$. Piketty and Saez (2012) show that this assumption is met in their framework under reasonable conditions. In particular, the average taste for bequest cannot be too strong and the stochastic transmission of preferences across generations must satisfy an ergodicity property.[9] Importantly, this property does NOT imply that all members of one dynasty have the same preferences. Some altruistic parents have selfish children and the converse is true as well.

Observe that (τ, D) may yield a long-run equilibrium allocation in which not enough money is collected through tax τ to fund demogrant D. We need to further restrict our attention to tax schemes that meet the government budget constraint

$$D \leq \int_i \tau\left(g_i + w + D - c_i\right) di. \tag{2}$$

[9]See Piketty and Saez (2013), page 1854.

A demogrant D is *sustainable* for the tax τ if the long-run equilibrium allocation associated to (τ, D) satisfies Eq. (2).[10] When this is the case, we say that the tax-demogrant scheme (τ, D) is sustainable. Observe that a given tax may admit several sustainable demogrants. For instance, both a zero demogrant and a negative demogrant are sustainable under a linear tax with rate zero.

A sustainable tax-demogrant scheme (τ, D) is *optimal* if there is no other sustainable tax scheme whose associated long-run equilibrium allocation is preferred by SWF $\boldsymbol{R}^{c\text{–}lex}$ to that associated to (τ, D). A tax τ is *optimal* in some domain if there is no alternative tax τ' in that domain for which the long-run equilibrium allocation associated to a sustainable scheme (τ', D') is preferred by SWF $\boldsymbol{R}^{c\text{–}lex}$ to the long-run equilibrium allocation associated to all sustainable schemes (τ, D).

An individual is among the *worst-offs* if all other individuals (in the long-run equilibrium generation) have a c-equivalent utility at least as large as this individual.

Laissez-Faire is a tax-demogrant scheme defined by a zero tax and a zero demogrant: $\tau^{LF}(b) = 0 \, \forall b \geq 0$ and $D^{LF} = 0$. The long-run equilibrium allocation associated to Laissez-Faire is sustainable. At this allocation individuals freely choose to allocation their wealth $w + g_i$ between consumption and bequest. As a result,

$$B^{\tau}(w + g_i, 0) = L(w + g_i, 0),$$

and each individual i maximizes her utility over $L(w + g_i, 0)$. This implies that the c-equivalent utility of individual i is equal to $w + g_i$. As a result, 1) the worst-off individuals are those with $g_i = 0$, and 2) they all have a well-being level equal to w, that is, all of them are to be considered as worst-off, independently of their preferences.

5.1 A positive average tax on bequests

Our first result answers the following question: at the optimal tax, should individuals' incomes be taxed so that bequests can be, on average, subsidized, or should bequests be taxed, on average, so as to subsidize individuals' incomes (through a demogrant)? The answer is that bequests should be taxed: the amount globally collected by an optimal inheritance tax cannot be negative. If subsidies are provided for some bequest levels, they must be paid for by taxes collected at other bequest levels. This answer is the opposite to that of Atkinson and Stiglitz (1976), Kaplow (2001) and Farhi and Werning (2010). Our result that bequests should be, on average, taxed and not subsidized crucially depends on the fact that we take account in our model of the influence of bequest taxes on the inheritance distribution, and, therefore, the wealth of the parents. The optimal formula of Piketty and Saez (2013) is consistent with taxing bequests on average, but this happens only for some distributions of normative weights. Our result shows that the distribution of normative weights that follows from

[10] This sustainability constraint allows us to link the government and individuals budget constraints in the following way. A sustainable (τ, D) is optimal only if $D \leq \int_i \tau(g_i + w + D - c_i)\, di$ and $h_i = R(w + D + g_i - c_i - \tau(w + D + g_i - c_i))$. These two equations give us the following sustainability constraint: $D \leq \int_i g_i + w + D - c_i - \frac{h_i}{R} di \Leftrightarrow \int_i c_i + \frac{h_i}{R} di \leq \int_i g_i + w di$.

imposing the axioms we propose unambiguously leads to a positive average tax rate on bequests.

The proof goes by comparing the optimal tax scheme with Laissez-Faire. Under any tax-demogrant scheme (τ, D), the c-equivalent utility of any *self-centered* individual is also equal to her consumption $(w+D+g_i)$. Provided that at least one individual who inherits nothing is self-centered,[11] an assumption that we impose (see assumption A1 below), her c-equivalent utility is equal to $w+D$. If the demogrant is negative, then her well-being is smaller than the well-being of the worst-off under Laissez-Faire. The result follows from the fact that our SWF ranks tax-demogrant schemes by comparing the long-run equilibrium well-being of the worst-off. To sum up, redistribution cannot take place from the general population towards those who leave some bequests, because such a redistribution hurts those who did not receive anything from their parents and do not plan to leave anything to their children either, and these are among the worst-offs.

The property of Laissez-Faire that all individuals for whom $g_i = 0$ are among the worst-offs and they all have the same c-equivalent utility is shared by all tax schemes (τ^*, D^*) in which τ^* exempts the bequests left by those who inherit nothing. This suggests that if τ^* maximizes the sustainable demogrant D^* under the constraint that τ^* exempts the bequests left by those who inherit nothing, (τ^*, D^*) is a strong candidate to be optimal.

Indeed, our second result identifies a necessary condition on the optimal (τ', D') to be different from (τ^*, D^*): it needs to be the case that $D' \geq D^*$. It is an easy consequence of what we already said. If (τ', D') has $D' < D^*$, then the c-equivalent utility of a self-centered individual who did not inherit anything is lower at (τ', D'), where it is equal to $w + D'$, than at (τ^*, D^*), where it is equal to $w + D^*$ and where this individual is among the worst-offs.

To define the largest bequest left by a zero-inheritor precisely, we impose the assumption that there are zero-inheritors with the most altruistic preferences. The needed assumption for the following proposition is, therefore:

Assumption **A1**: In any long-run equilibrium allocation, there are two disjoint subsets $I^s, I^a \subset [0,1]$ with $\mu(I^s) > 0$ and $\mu(I^a) > 0$ such that for all $s \in I^s$ and all $a \in I^a$ we have $g_a = g_s = 0$, $u_a = u^{\mathbf{a}}$ and $u_s = u^{\mathbf{s}}$.

Our first proposition summarizes the discussion above.

Proposition 1 makes use of the following definition. Let $b_a^{LF}(w+D)$ denote the optimal bequest left by individual $a \in I^a$ under a scheme (τ, D) that provides an exemption strictly larger than $b_a^{LF}(w+D)$, i.e.

$$b_a^{LF}(w+D) = \arg\max_{\tilde{b}_a \geq 0} u_a(w + D - \tilde{b}_a, R\tilde{b}_a).$$

Proposition 1. *(i) Under A1, a tax-demogrant scheme (τ, D) is optimal only if $D \geq 0$. (ii) Under A1, a tax-demogrant scheme (τ^*, D^*) that provides an exemption up to $b_a^{LF}(w + D^*)$ is optimal if there is no other sustainable tax-demogrant scheme (τ', D') such that $D' \geq D^*$.*

Proof. The proof is relegated in Online Appendix S1. ■

[11] Piketty and Saez (2013) document that about half the population in France and the US receives negligible bequests in 2010.

The proof of (i) relies on the following argument. At the laisser-faire allocation, individuals who did not receive anything from their parents have the lowest c-equivalent utility. As a result, taxing large bequests and offering a positive demogrant is a way to increase their utility, which is socially desirable given the leximin nature of the SWF. This argument, interestingly, does not depend on the specific assumption that we make on the type of altruism of the parents. If parents are interested in the total lifetime income of their offsprings or if they are interested in the utility of their children, it remains true that those who did not inherit anything from their own parents will be the worst-off, so that taxing large bequests to increase the demogrant increases their utility, with the consequence that only $D \geq 0$ can be optimal.

The condition $D' \geq D^*$ in (ii) is necessary for (τ', D') to be optimal, it is of course not sufficient. In the long-run equilibrium allocation associated to (τ^*, D^*), all zero-inheritors are equally well-off, and the optimal tax scheme needs to make all of them at least as well-off as at (τ^*, D^*).

Note that the long-run equilibrium allocation associated with the optimal tax scheme depends on the preference profile considered, preferences are heterogeneous and the long-run equilibrium profile of inheritances received is *endogenous* to the shape of the tax. Even without being able to characterize this allocation exactly, we derive in the next section some constraints on the shape of an optimal tax.

5.2 Tax exemption on low bequests, or limited subsidies and taxes

In the previous section, we proved that D, the demogrant, cannot be too low, and, in particular, cannot be negative. The demogrant should be thought of as the average amount transferred from those who leave a bequest for their children to the general population. In this section, we prove that D cannot be too *large*, either. The intuition for this result is the following one. Worst-off individuals are to be found among the zero-inheritors. Among them, the well-being of the self-centered individuals is entirely determined by D. It is not the case for the other zero inheritors. In particular, the most altruistic among them, individuals a, receive D but they pay $\tau(b_a)$, in which b_a stands for their bequest. Proposition 1 implies that $D \geq \tau(b_a)$: individuals a cannot be strict contributors to the tax system. It suggests that even if a end up with a lower well-being than self-centered individuals, the difference in well-being should be limited.

This has the following implication for the optimal tax scheme. Let b_a denote the bequest of these individuals a at (τ, D). Let β be a positive bequest level smaller than b_a and let Δ be an amount of money smaller than β. Consider the alternative tax scheme (τ', D') satisfying

$$\begin{aligned} \tau'(b) &= \tau(b + \Delta) - \Delta, \quad \forall b \geq \beta - \Delta \\ D' &= D - \Delta, \end{aligned}$$

which is illustrated in Figure 2. Facing (τ', D'), individuals a do not see any difference between (τ', D') and (τ, D): the decrease in their lifetime income is perfectly compensated by the decrease in the tax they pay on their bequest. Zero-inheritor-self-centered individuals, on the contrary, are affected by the change, as their well-being decreases by Δ. If, moreover, (τ', D') leaves money on the table, which is quite likely because D decreases, this money can be redistributed to the entire population, thereby making individuals a better-off. That illustrates that decreasing D and decreasing the tax below some threshold is a policy tool to increase the well-being of a at the expense of s (the zero-inheritor-self-centered individuals).

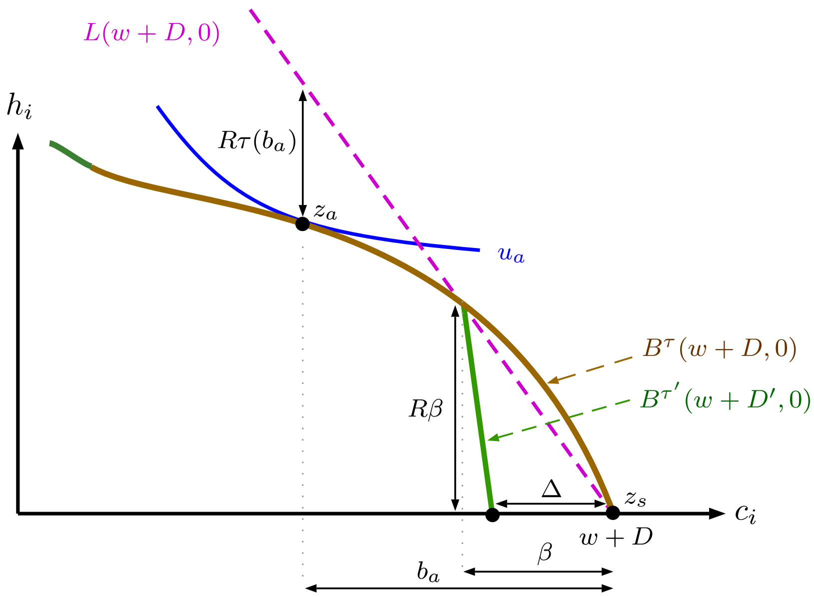

Figure 2: Tax-demogrant schemes (τ, D) and (τ', D') provide equal c-equivalent utility to the worst-off individuals a even if the latter has a smaller demogrant.

This illustration is too simple, however, as (τ', D') is typically non-sustainable. To prove Proposition 2, we do identify a truncation of (τ, D) that is sustainable and that increases social welfare as soon as D, or, equivalently, $\tau(b_a)$, is too large.[12] As a consequence, we show in the next proposition that

1. tax functions with positive taxes on small bequest amounts are not optimal,
2. monotonically increasing tax functions are not optimal unless they provide an exemption up to the amount of bequest that would be chosen under Laissez-Faire by individuals a (who inherit nothing and have the most-altruistic preference),
3. a positive tax on the amount of bequest left by a is not excluded though, at least when subsidies are provided on smaller bequest amounts, and

[12] When defining this truncation below, we give a precise value to β and to Δ.

4. even when the optimal tax function subsidizes smaller bequest amounts, the tax on the bequest amount left by a must be limited (see Eq. (3) in the proposition below).

To proves these claims, we restrict our attention to tax functions τ for which $-\tau$ is *single-peaked*. This domain contains all tax functions that are policy relevant. In particular, this domain is the union of the two most relevant subdomains. The first subdomain contains all (weakly) monotonically increasing tax functions τ (bequests are taxed at an increasing non-negative rate), in which 0 is a peak for $-\tau$ (there may be an entire interval of peaks, in which case τ exempts bequests on this interval). The second subdomain contains all tax functions τ that are first monotonically (weakly) decreasing (small bequests are subsidized) and then monotonically (weakly) increasing. In this subdomain the peaks are all positive. In order to simplify the exposition, we exclude tax functions that provide an exemption on small bequests and are not (weakly) monotonically increasing.[13] We refer to the latter subdomain as that of *positive peak* tax functions, and to the former as that of (weakly) monotonically increasing tax functions. Observe that, by Proposition 1, any monotonically decreasing tax function is dominated because such tax cannot sustain a non-negative demogrant. We do not comment on these functions anymore.

In this domain, the worst-off individuals are either individuals a or s. As illustrated in Figure 3, the reason is that the consumption of any zero-inheritor is at least as large as the consumption of a, but not larger than the consumption of s. As a result, their c-equivalent utility cannot be smaller than both the c-equivalent utilities of s and a. Since either s or a are among the worst-offs, we can restrict the normative analysis to these two individuals. When a tax function τ implies a tax on the amount of bequest left by a, this individual is among the worst-offs whereas s is not. Welfare would be improved if it were possible to increase the well-being of a while keeping the well-being of s above the well-being of a. The difficulty here is to make sure that such improvement materializes in the new long-run equilibrium allocation, which depends on the maximal demogrant that can be sustained.

Two additional mild assumptions are required for these results. First, when they consume at least as much as their labor income, the utility of individuals who are *not* self-centered must be strictly increasing in the inheritance received by their child. This assumption will guarantee that there exists a sufficiently large subsidy rate that induces these individuals to leave at least a threshold amount to their children.

Assumption **A2**: For all $i \in [0,1]$ with $u_i \neq u^{\mathbf{s}}$ we have that $u_i(c_i, h_i)$ is strictly monotonic in h_i when $c_i \geq w$.

Second, the long-run equilibrium amount collected by a tax-demogrant scheme is continuous in the demogrant.

Assumption **A3**: For all tax τ, the amount of tax collected, i.e.

$$\int_i \tau(b_i)\, di,$$

[13] Hence, we exclude tax functions that first provide an exemption on small bequests, then provide subsidies on intermediate bequests and finally tax positively large bequests. All our results would remain valid if these tax functions remain included in our domain. We nonetheless exclude them in order to avoid having to treat their un-interesting case in Proposition 3.

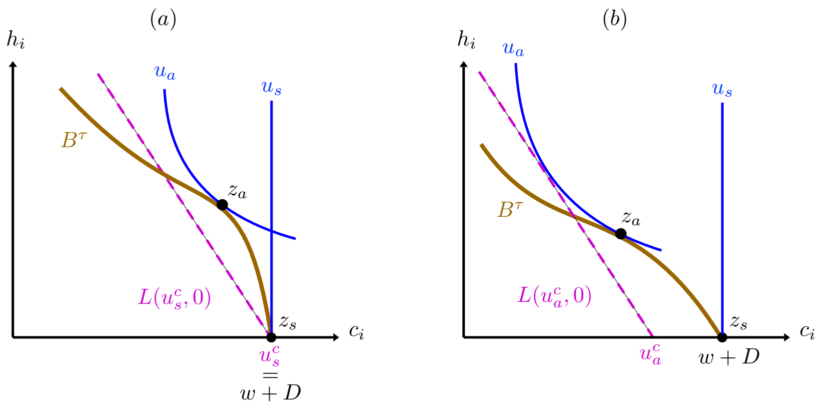

Figure 3: (a) Individuals s with $g_s = 0$ and $u_s = u^{\mathbf{s}}$ are among the worst-offs, where u_s^c denotes the c-equivalent utility $u_s^c = u^c(z_s, u_s)$. (b) Individuals a with $g_a = 0$ and $u_a = u^{\mathbf{a}}$ are among the worst-offs, where u_a^c denotes the c-equivalent utility $u_a^c = u^c(z_a, u_a)$.

where b_i is the long-run equilibrium bequest left under (τ, D), is continuous in D.

Assumption A3 implies that, if (τ, D) is sustainable and leaves money on the table, then for some $D' > D$ the tax-demogrant scheme (τ, D') is also sustainable. Observe that assumption A3 is not necessary in the proposition below for the constraint derived on monotonically increasing tax functions.

We introduce the following notation. Let $\underline{b} \geq 0$ be the minimal bequest amount above which the tax τ provides zero subsidy, i.e.

$$\underline{b} = \min \tilde{x} \in \mathbb{R}_+ \text{ such that } \tau(x) \geq 0 \text{ for all } x \geq \tilde{x}.$$

Obviously, amount $\underline{b}$ is endogenous to the tax considered. For any positive peak tax, we have $\underline{b} > 0$ and we construct the alternative tax scheme (τ', D') using $\beta = \underline{b}$. For any monotonically increasing tax, we have $\underline{b} = 0$ and we construct the alternative tax scheme (τ', D') using some $\beta > \underline{b}$.

Proposition 2, our main result, provides us with the formal statements from which the four claims above are deduced.

Proposition 2. *Consider any tax τ for which $-\tau$ is single-peaked. Let $\underline{b}$ be the minimal bequest amount above which no subsidies are provided under τ. Let D^{max} be the maximal sustainable demogrant under τ. Let a be an individual who inherits nothing and holds the most altruistic preference. Let b_a be the equilibrium bequest left by a under (τ, D^{max}). Let $b_a^{LF}(w)$ be the equilibrium bequest left by a under Laissez-Faire.*

(i) Under A1 and A2, if τ is monotonically increasing, then τ is optimal only if τ provides an exemption up to $b_a^{LF}(w)$.

(ii) Under A1, A2 and A3, τ is optimal only if

$$\tau(b_a) \leq \underline{b}. \tag{3}$$

Proof. The proof is relegated in Online Appendix S2. ■

The proof of Proposition 2 is quite long. This follows from the inherent challenges summarized in the last paragraph of section 5.1. One key difficulty is that, unlike Piketty and Saez (2013), we consider non-linear taxes.

An important feature of Proposition 2 is that the shape of monotonically increasing taxes is completely characterized up to the minimal amount exempted. Importantly, this amount is *exogenous* to the optimal tax. This amount only depends on the preference $u^{\mathbf{a}}$, the interest rate R and the wage rate w. This implies this shape is valid regardless of the exact preferences profile defining the economy. This contrasts with characteristics of optimal tax as derived in the literature (Piketty and Saez, 2013), which typically depend on statistics endogenous to the optimal tax. On the contrary, the bequest amount b_a in claim (ii) of Proposition 2 is also endogenous to the tax.

A consequence of Part (ii) of the proposition is that c-equivalent utilities may not be equalized at the optimal allocation. Given the leximin nature of our SWF, this possibility only arises when individuals who did not inherit anything from their parents are all better-off than at the best allocation that provides an exemption up to $b_a^{LF}(w)$.

To conclude this section, we note that our two propositions have an interesting corollary, namely that linear taxes are never optimal.

Corollary 1. *Under A1 et A2, no positive or negative linear tax is optimal.*

Proof. Linear tax with negative rates cannot sustain non-negative demogrants. By Proposition 1, any tax scheme based on a negative demogrant is not optimal. Therefore, linear tax with negative rates are not optimal. Linear tax with positive rates are monotonically increasing and do not exempt bequests up to $b_a^{LF}(w)$. By Proposition 2, these tax functions are not optimal. ■

5.3 The case for a tax exemption

A consequence of Proposition 2 is that the optimal tax either exempts inheritance up to the bequest left by the poorest most altruistic individuals under Laissez-Faire, or it subsidizes small bequests. We need to underline that the reason why subsidies can be optimal is drastically different from the arguments offered in the literature. In Kaplow (1995), Farhi and Werning (2010) and Kopczuk (2013a), subsidies are desirable because they increase the bequests left, which increases the utility of both parents and children. On the contrary, subsidies are desirable here because they incentivize poor altruistic individuals to *decrease* their bequest below what they would leave under a non-distortionary budget set defined by the same demogrant. The advantage for society of incentivizing poor altruistic individuals to decrease their bequests comes from the fact that 1) it does not decrease the utility of these individuals, while 2) it allows the tax system to increase the tax paid by other, non-poor, altruistic individuals, the objective being to sustain a larger demogrant. This immediately suggests the drawback of these subsidies: by decreasing bequests, they impoverish future generations, so that the sustainable demogrant may fail to increase, in which case exemption is optimal.

This trade-off is illustrated in Figure 4. There is a tax function τ that exempts bequests below b_{a1}, the bequest left by individual $a1$ who is the most altruistic individual having not received anything from her parents. Beyond b_{a1}, bequests are taxed, so that a demogrant of D is funded. Can we do better by subsidizing small bequests? Figure 4 also illustrates tax function τ', which subsidizes bequests below b'_{a1} while leaving the utility of both the selfish and the poor most altruistic individuals unchanged. In particular, the poor most altruistic individual now leaves a bequest $b'_{a1} < b_{a1}$ and her child receives an inheritance $h'_{a1} < h_{a1}$. Indeed, subsidizing bequests while leaving the utility of the poor most altruistic individual unchanged implies that this individual faces a *positive* marginal bequest tax rate, as illustrated in the figure.

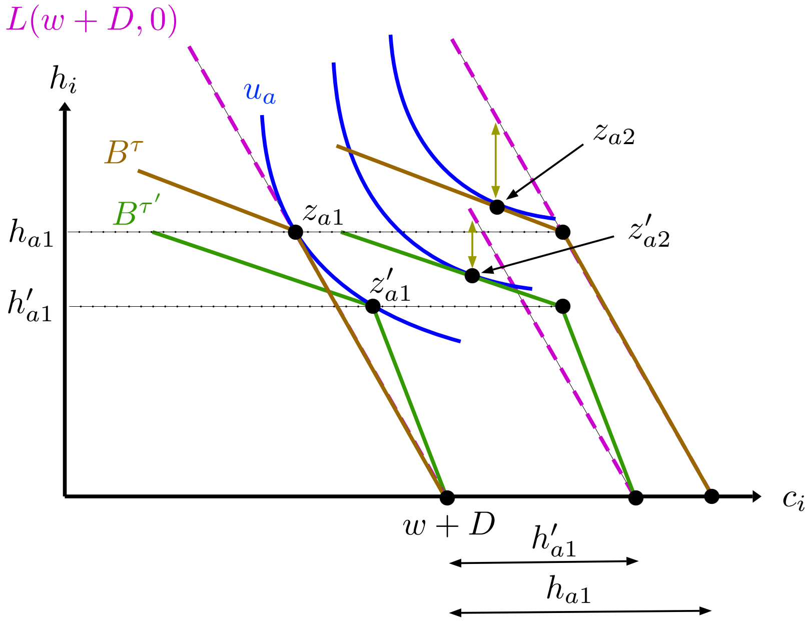

Figure 4: Subsidizing small bequests allows increasing average tax rates but impoverishes future generations. Individual $a2$ is the altruistic child of $a1$ who is an altruistic individual who receives zero inheritance.

The figure also illustrates the budget facing individual $a2$, who is the altruistic child of the poor most altruistic individual. Following τ', the child faces higher average tax rates than with τ, but she is also poorer (because she inherited less). Here are then the consequences of shifting from a tax function with exemption on bequests left by poor individuals to a tax function that subsidizes low bequests without affecting the lowest c-utility in the population, that is the c-utility of the poor selfish and the poor most altruistic individuals: 1) it may increase the bequests left by poor, moderately altruistic, individuals, 2) it decreases the bequests left by poor, most altruistic, individuals, 3) it is costly to the government budget regarding the treatment of poor individuals, 4) it allows the government to tax richer individuals more, but 5) richer individuals (those who inherit something) are themselves poorer. As a result, whether subsidizing small bequests can help support a larger demogrant depends on the preferences of the altruistic individuals (if the elasticity of bequest is high, a small subsidy may decrease bequests a lot, keeping utility unchanged), and the relative fraction in the population of poor and rich altruistic individuals.

We complete this section by developing an example in which no subsidy tax policy can outperform the exemption tax policy. This requires to be able to identify the long run equilibrium associated with tax functions, which can only be done under very restrictive assumptions. We indeed assume that there are only two types of individuals, selfish and altruistic. We further assume, and this is the most restrictive assumption, that preferences of the altruistic individuals are locally inelastic at their preferred bundles in all non-distortionary budgets, which goes against the result we want to prove, but also that preferences are linear over bundles involving more consumption than would be optimal under non-distortionary budgets.

Linearity yields two important simplifications. First, it allows us to precisely build incentive compatible allocations, which is a rather technical simplification. Second, and more importantly, it restricts so much the marginal rates of taxation at incentive compatible allocations (altruistic individuals having inherited less than a given amount should all face the same marginal tax rate) that we can prove that at least some individuals pay taxes *and* leave a larger bequest than what the poorest most altruistic individuals would have left with her demogrant but in the absence of taxation.

Under these assumptions, subsidizing bequests is dominated by exempting bequests up to the bequest level of the poorest most altruistic individuals with her demogrant but in the absence of taxation while keeping the same tax scheme for bequests above that level. This comes from the fact that 1) all altruistic individuals who would benefit from the subsidy in the former policy leave larger bequests under the latter, in addition to being less costly to the government budget, whereas 2) all altruistic individuals who pay taxes in the subsidy policy pay the same amount of taxes under the latter policy while still leaving the same bequest. As a result, the exemption policy must be associated in the long run to a larger demogrant.

We consider a simplified economy with only two preference types $u^{\mathbf{s}}$ and u^*_a. Preferences of the altruistic individuals have the following properties. First, indifference curves admit a kink at bundles (c, h) for which $h = \alpha + \sigma c$, $\sigma > 0$. The typical case would be that bequests are a luxury good, which implies $\alpha < 0$, but we don't need this restriction. Second, preferences are linear below the kinks, that is

$$u^*_a(c, h) = \frac{\sigma}{\sigma + r} h + \frac{\sigma r}{\sigma + r} c \qquad \text{if } h < \alpha + \sigma c,$$

with $0 < r < R$ is the marginal rate of substitution of u^*_a at any bundle below this increasing path of kinks. Preferences u^*_a are illustrated in Figure 5. This two-types economy satisfies A2.

In the two-types economy, the inheritance g_i received by individual i only depends on the minimal number $X \in \{1, 2, \dots\}$ of generations one needs to go back in her dynasty in order to find a self-centered individual. For instance, if i's parent is self-centered, then $X = 1$. In that case her inheritance g_i is simply zero. If i's parent is altruistic but her grand-parent is self-centered, then $X = 2$. In that case, her inheritance g_i correspond to that left by an altruist who inherited nothing. And so on.

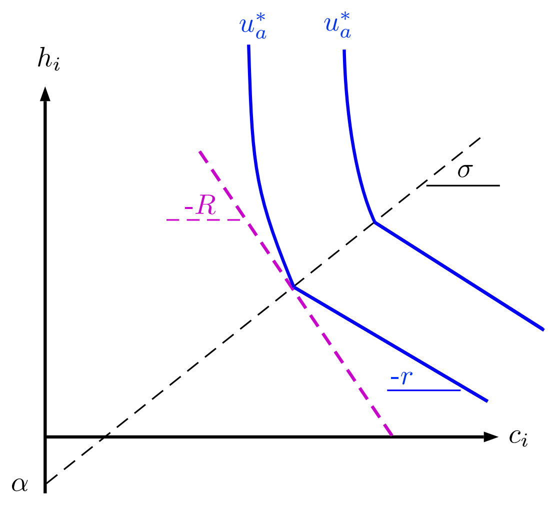

Figure 5: Indifference map of preference u_a^*.

Let z be a long-run equilibrium allocation. It can be characterized by $(z_{sX}, z_{aX})_{X\in\{1,2,\dots\}}$ where sX (resp. aX) is a self-centered (resp. altruistic) individual whose number is X.

The **incentive compatibility constraint** (IC) requires that for all $X \in \{1, 2\dots\}$, individual aX does not prefer the bequest level selected for $aX+1$, which is

$$u_a^*(c_{aX}, h_{aX}) \geq u_a^*(c_{aX+1} - (g_{aX+1} - g_{aX}), h_{aX+1}), \tag{4}$$

and individual $aX+1$ does not prefer the bequest level selected for aX, which is

$$u_a^*(c_{aX+1}, h_{aX+1}) \geq u_a^*(c_{aX} + (g_{aX+1} - g_{aX}), h_{aX}), \tag{5}$$

where in the two-types economy we have $g_{aX+1} = h_{aX}$ for all $X \in \{1, 2\dots\}$.

Consequently, a tax scheme (τ, D) is entirely characterized by the countable sequence of bundles $(z_{sX}, z_{aX})_{X\in\{1,2,\dots\}}$ it generates, and, simultaneously, any sequence $(z_{sX}, z_{aX})_{X\in\{1,2,\dots\}}$ satisfying the above incentive compatibility constraints can be generated by a set of tax systems. We restrict our attention to allocations z that can be long-run equilibrium allocations generated by a tax scheme (τ, D) such that $-\tau$ is single-peaked.

Proposition 3. *Consider the two-types economy described above. Consider any long-run equilibrium allocation z generated by a tax scheme (τ, D) such that (i) $-\tau$ is single-peaked and (ii) such that both self-centered and altruistic individuals who inherit nothing are among the worst-offs. Let $b_a^{LF}(w)$ be the equilibrium bequest left under Laissez-Faire by an altruistic individual who inherits nothing. Under A1 and A3, z is optimal only if it can be generated by a tax system that provides an exemption up to $b_a^{LF}(w)$.*

Proof. The proof is relegated in Online Appendix S3. ■

6 Conclusion

The model that we study in this paper is designed to focus on the trade-off between subsidizing bequests or transferring a demogrant to all. A number of simplifying assumptions have been needed, to which we now come back.

We assumed away the issue of taxing labor incomes. We assume that all individuals have the same labor time and the same lifetime income, so that fairness does not require to redistribute labor income. Our result that bequests should, on average, be taxed so as to transfer a demogrant to all does not, therefore, come from the need to alleviate income inequalities, but only from the need to compensate children of selfish parents while preserving the parents' freedom to allocate their lifetime income the way they wish. As a consequence, our conclusions are compatible with heterogeneity of wages and labor times and the existence of a labor income tax system maximizing social welfare. This claim, however, calls for two qualifications.

First, our formal analysis can be replicated in a more general model only under the assumption that individuals' lifetime incomes are not influenced by the design of our bequest taxation system. However, this assumption is unlikely to be valid, in which case, a complete framework should study income redistribution and bequest taxation systems simultaneously. We suspect that deriving analytical results in such a complicated model is out of reach. Intuitively, though, the result of such an exercise is likely to remain that bequests are, on average, taxed, because it is the way to compensate the children of self-centered parents while maintaining a sufficiently high utility level to self-centered individuals who did not receive anything from their own parents. Identifying who are the worst-off individuals and dealing with sustainability issues, however, would become much harder.

The second qualification has to do with the identification of the worst-off individuals in the case in which lifetime incomes are not influenced by the bequest taxation system. Given that the labor income taxation system aims at redistributing from higher wage individuals to lower wage individuals, it is extremely likely that the worst-off have to be found among the minimal-wage individuals. Consequently, our assumption A1 has to be strengthened into the existence, in any long-run equilibrium allocation, of individuals who did not inherit anything from their parents, who have the minimum wage and who have either the most altruistic or self-centered preferences. As a result, Proposition 2, part (i), for instance, would become that bequests should be exempted from taxes up to the amount left by the most altruistic individuals who did not receive any bequest from their parents and worked all their life at the minimum wage.

Our main results do not give us a formula that can be calibrated, but yet they can be used to qualitatively assess current tax systems. According to a recent report (see OCDE (2021)) 12 of the 36 OECD countries do not tax bequests. Our Proposition 1, part (i) implies that this can not be optimal given our social welfare function. All the 24 countries that do tax bequests to children have a system consistent with the optimal tax system of Proposition 1, Part (ii) and Proposition 2, part (i): exemption for small bequests and a positive tax on larger ones. The interval of exemptions considerably varies across countries, from \$17,133 in Belgium to \$11,580,000 in the United States (numbers in 2020 USD). We note that those who work full-time (40 hours a week, 48 weeks a year) for an hourly wage of \$15 and invest 15% of their income at a yearly rate of 3% have accumulated more than $325,000 after 40 years, suggesting that the exemption amount in Belgium (as well as the median exemption level of $171 329, Greece) is likely to be smaller than the bequest left by the most altruistic parents whereas the exemption amount in the United States is clearly above this threshold. The money collected through bequest taxes is below 2% of the total fiscal revenues in all countries, and even below 1% in most countries, suggesting that the corresponding demogrant is not maximized. More research is needed, however, to compute the optimal tax systems in these countries.

In the model, we also assumed that the number of children is identical across households. Allowing heterogeneity among the number of children would not change the fact that the worst-off individuals have to be found among those who did not inherit anything from their parents. A new question would emerge, however, regarding the amount of exempted bequest. It would still be defined with reference to the amount left by the most altruistic individuals who did not receive any bequest from their parents, but the choice is between considering the bequests of parents of the largest number of children or with only one child. The former choice is appropriate if parents are modeled as caring about the per capita bequest received by their children. The latter choice is appropriate if parents are modeled as caring about the total bequest left.

The interest rate, R, is exogenous in our model. This is typical of a small open economy. If it is endogenous, but further assumptions make it depend only on the distribution of preferences in the economy, then our analysis carries over with R being replaced with the endogenous rate. If, on the contrary, the interest rate may vary across time, then our results change. The intuition is that our optimal tax scheme needs to be amended so as to redistribute further from the lucky ones who face higher interest rates towards those who face lower interested rates. Moreover the leximin nature of our SWF implies that the worst-offs belong to those who did not inherit anything from their parents. Therefore, whether an individual is lucky or not only depends on the future interest rates. So, an optimal tax system should redistribute from those who can save for their children at a high rate towards those who save at a low rate. How precisely this should be done requires additional research.

Other assumptions would be much more difficult to relax. They would require to redefine the social welfare function or the policy tools. It would be the case, for instance, if individuals are interested in the entire lifetime of their children and not only how much their children inherit, in which case an increase of the demogrant benefits the altruistic parents more than the self-centered ones, if they have unequal life expectancy, in which case the social planner may wish to subsidize the bequest of short-lived individuals, if fertility choices are constrained, in which case the social planner may wish to favor those who wanted to have children but could not, and, therefore, do not leave any bequest, if children can inherit from different adults, raising the question of whether the tax should be donor-based or recipient-based, if bequests are only partially observable, etc.

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ONLINE APPENDIX: Fair Inheritance taxation

Benoit Decerf and Francois Maniquet

S1 Proof of Proposition 1

First, we prove claim (i). We show that any (τ, D) with $D < 0$ is dominated by Laissez-Faire.

We start by showing for the long-run equilibrium allocation $z^{LF} \in S$ associated to Laissez-Faire that $u^c(z_i^{LF}, u_i) \geq w$ for all $i \in [0, 1]$. Under Laissez-Faire, any $i \in [0, 1]$ choses in the budget set

$$L(w + g_i^{LF}, 0),$$

implying that $u^c(z_i^{LF}, u_i) = w + g_i^{LF}$.

We then show for the long-run equilibrium allocation $z \in S$ associated to (τ, D) that some subset $J \subset [0, 1]$ with $\mu(J) > 0$ is such that $u^c(z_j, u_j) = w + D$ for all $j \in J$. By assumption A1, there is a subset $J \subset [0, 1]$ with $\mu(J) > 0$ such that $g_j = 0$ and $u_j = u^{\mathbf{s}}$ for all $j \in J$. Since these individuals are self-centered, we have that $z_j = (0, w + D, 0)$ and $u^c(z_j, u_j) = w + D$.

As $D < 0$, this shows that $u^c(z_j, u_j) < w$ for all $j \in J$, showing that Laissez-Faire is preferred to (τ, D) by $\boldsymbol{R}^{c\text{–}lex}$.

Second, we prove claim (ii). We show that any (τ'', D'') with $D'' < D^*$ is dominated by (τ^*, D^*).

We start by showing for the long-run equilibrium allocation $z^* \in S$ associated to (τ^*, D^*) that $u^c(z_i^*, u_i) \geq w + D^*$ for all $i \in [0, 1]$. In equilibrium, any $i \in [0, 1]$ choses in the budget set $B^{\tau^*}(w + D^* + g_i^*, 0)$. For all $i, j \in [0, 1]$ with $u_i = u_j$ and $g_j^* = 0$ we must have $u_i(z_i^*) \geq u_j(z_j^*)$ because $B^{\tau^*}(w + D^*, 0) \subseteq B^{\tau^*}(w + D^* + g_i^*, 0)$. As $u_i = u_j$, this implies that $u^c(z_i^*, u_i) \geq u^c(z_j^*, u_j)$. There remains to show that $u^c(z_j^*, u_j) = w + D^*$. As τ^* provides an exemption up to $b_a^{LF}(w + D^*)$, any $j \in [0, 1]$ with $g_j^* = 0$ choses the same bundle z_j^* in her budget set $B^{\tau^*}(w + D^*, 0)$, as she would chose in the non-distortionary budget set

$$L(w + D^*, 0),$$

implying that $u^c(z_j^*, u_j) = w + D^*$.

We then show for the long-run equilibrium allocation $z'' \in S$ associated to (τ'', D'') that some subset $J \subset [0, 1]$ with $\mu(J) > 0$ is such that $u^c(z_j'', u_j) = w + D''$ for all $j \in J$. By assumption A1, there is a subset $J \subset [0, 1]$ with $\mu(J) > 0$ such that $g_j'' = 0$ and $u_j = u^{\mathbf{s}}$ for all $j \in J$. Since these individuals are self-centered, we have under the long-run equilibrium allocation $z'' \in S$ associated to (τ'', D'') that $z_j'' = (0, w + D'', 0)$ and so $u^c(z_j'', u_j) = w + D''$.

As $D'' < D^*$, this shows that $u^c(z_j'', u_j) < w + D^*$ for all $j \in J$, showing that (τ^*, D^*) is preferred to (τ'', D'') by $\boldsymbol{R}^{c\text{–}lex}$.

S2 Proof of Proposition 2

Proposition 2 provides a limit on the tax paid on the amount of bequest left by a. The proof constructs another sustainable tax-demogrant scheme that dominates (τ, D). The construction is based on a sustainable tax-demogrant scheme

$(\tau^\Delta, D-\Delta)$ illustrated in Figure 6. As shown in the figure, this second scheme linearly truncates the budget set faced by individuals under (τ, D). This truncation reduces the demogrant by an amount Δ, but the new tax τ^Δ provides large subsidies on small bequests. Provided the inheritance they receive is unchanged, individuals chose the same bundle under both schemes, at least if this bundle does not lie in the truncated part of their budget set. This implies that the well-being of a is the same under both schemes. The smaller demogrant reduces the well-being of s, but s still has a larger well-being than a. In the proof, we show that when (τ, D) is sustainable, scheme $(\tau^\Delta, D-\Delta)$ leaves money on the table. Money is left on the table because (i) the new scheme saves on self-centered individuals who receive a smaller demogrant and (ii) the new scheme does not spend more on altruistic individuals because its subsidies on small bequests are financed by the reduction in the demogrant. Then, the money saved by the new scheme can finance a small increase in the demogrant, such that $(\tau^\Delta, D-\Delta+\epsilon)$ is sustainable for some $\epsilon > 0$. The larger demogrant increases the well-being of all individuals, including that of the worst-off.

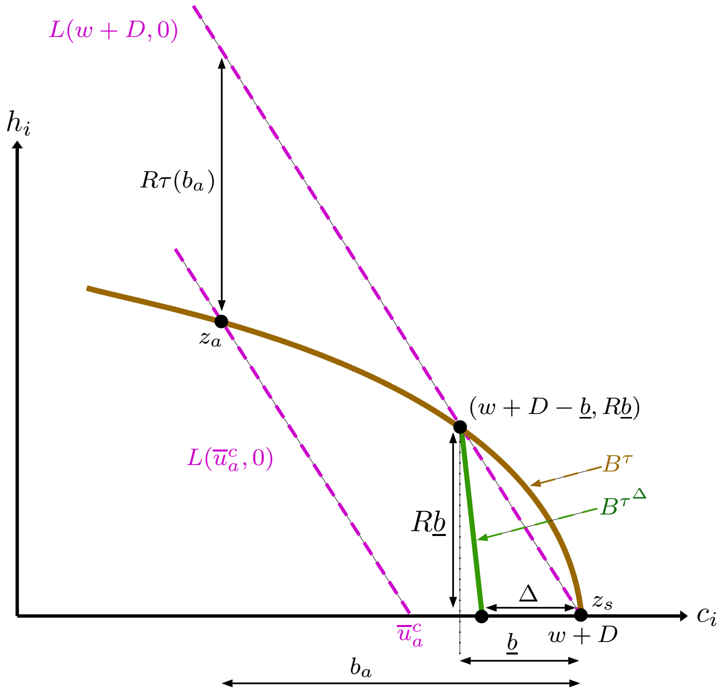

Figure 6: The tax-demogrant scheme (τ, D) is dominated because the tax function τ taxes too much the bequest b_a left by a. Individual a is the worst-off because $u^c(z_a, u_a) \le \overline{u}^c_a$. The sustainable scheme $(\tau^\Delta, D-\Delta)$ has a smaller demogrant, does not affect $u^c(z_a, u_a)$ and leaves money on the table.

Here is the intuition why scheme $(\tau^\Delta, D-\Delta)$ is sustainable. The truncation creates a kink in the budet set faced by individuals under (τ, D). Importantly, this kink is located at an amount of bequest $\underline{b}$ above which the tax τ is non-negative and monotonically increasing. The larger the rate of subsidies on small bequests associated to τ^{Δ}, the more numerous the altruistic individuals who "bunch" at the kink, at least for those who used to leave a bequest smaller than $\underline{b}$ under (τ, D). If all of these individuals "bunch" at the kink, then the long-run equilibrium profile of inheritances under (τ, D) would be first-order stochastically dominated by the long-run equilibrium profile of inheritances under $(\tau^{\Delta}, D - \Delta)$. We can then show that $(\tau^{\Delta}, D - \Delta)$ is sustainable if (τ, D) is sustainable because, above the kink, the tax paid is increasing in the bequest left.

Both claims (i) and (ii) in Proposition 2 rely on the following two lemmas. Recall that we denote by a an individual for whom the long-run equilibrium $g_a = 0$ and for whom $u_a = u^{\mathbf{a}}$, and by s an individual for whom the long-run equilibrium $g_s = 0$ and for whom $u_s = u^{\mathbf{s}}$.

Lemma 1. *Under A1, for any tax-demogrant scheme (τ, D) for which $-\tau$ is single-peaked, either a or s are among the worst-offs.*

Proof. Let $z \in S$ denote the long-run equilibrium allocation associated to (τ, D). By assumption A1, there exist two individuals a and s with $g_a = 0$, $u_a = u^{\mathbf{a}}$, $g_s = 0$ and $u_s = u^{\mathbf{s}}$. We derive a contradiction when assuming that for some $k \in [0, 1]$ we have $u^c(z_k, u_k) < u^c(z_a, u_a)$ and $u^c(z_k, u_k) < u^c(z_s, u_s)$.

Under z, any $i \in [0, 1]$ choses in her budget set $B^{\tau}(w + D + g_i, 0)$. This implies for all $j \in [0, 1]$ with $u_j = u_k$ and $g_j = 0$ that $u_j(z_j) \le u_k(z_k)$, and hence $u^c(z_j, u_j) \le u^c(z_k, u_k)$. Thus, we can assume without loss of generality that $g_k = 0$.

By definition of a and s, the fact that $g_a = g_k = g_s = 0$ implies that $c_a \le c_k \le c_s = w+D$. Letting $u_k^c = u^c(z_k, u_k)$, we show that $(c_k, h_k) \notin L(u_k^c, 0)$. There are two cases for z_s, which are illustrated in Figure 7.

- Case 1: $(c_s, 0) \in L(u_k^c, 0)$.

  Let $z_s' = (g_s', c_s', 0)$ be such that $(c_s', 0) = \arg\max_{u_s} L(u_k^c, 0)$. This case is such that $u_s(z_s) \le u_s(z_s')$. As by definition $u^c(z_s', u_s) = u_k^c$, we have that $u^c(z_s, u_s) \le u_k^c$ and thus $u^c(z_s, u_s) \le u^c(z_k, u_k)$, a contradiction.

- Case 2: $(c_s, 0) \notin L(u_k^c, 0)$.

  As $(w + D, 0) \notin L(u_k^c, 0)$ and $g_k = g_s = 0$, we have $(c_k, h_k) \notin L(u_k^c, 0)$ unless $\tau(w + D - c_k) \ge 0$.[14] As $-\tau$ is single-peaked and $c_a \le c_k$, this implies in turn that $\tau(w + D - c_a) \ge \tau(w + D - c_k)$. Since $g_k = g_a = 0$, the fact that $\tau(w + D - c_a) \ge \tau(w + D - c_k) \ge 0$ and $(c_k, h_k) \in L(u_k^c, 0)$ together imply that $(c_a, h_a) \in L(u_k^c, 0)$. Then, $u^c(z_a, u_a) \le u_k^c$ and thus $u^c(z_a, u_a) \le u^c(z_k, u_k)$, a contradiction.

Since $(c_k, h_k) \notin L(u_k^c, 0)$ but $u^c(z_k, u_k) = u_k^c$, this implies that for some bundle $\hat{z}_k = (0, \hat{c}_k, \hat{h}_k)$ such that $(\hat{c}_k, \hat{h}_k) \in L(u_k^c, 0)$ we have $u_k(\hat{z}_k) = u_k(z_k)$

[14] By definition, $L(u_k^c, 0) = \{(c_i, h_i) \mid c_i + h_i/R \le u_k^c\}$. When $(c_s, 0) = (w+D, 0) \notin L(u_k^c, 0)$, we have $u_k^c < w + D$. Since in equilibrium, $h_k = R(w + D - c_k - \tau(w + D - c_k))$, we have $c_k + h_k/R < w + D$ only if $\tau(w + D - c_k) > 0$.

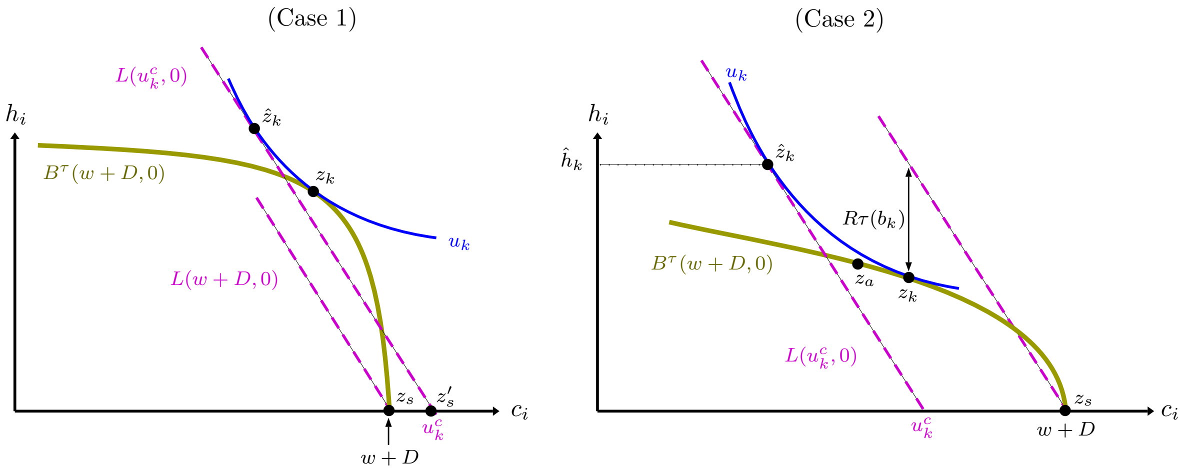

Figure 7: Constructions used in the proof of Lemma 1.

(see Figure 7 Case 2). This implies that

$$\hat{z}_k \in \arg \max_{\tilde{z}_k \in B^\tau(w+D,0) \cup L(u_k^c,0)} u_k(\tilde{z}_k).$$

But by definition of the most-altruistic preferences $u^{\mathbf{a}}$, it means that a would chose in $B^\tau(w+D,0) \cup L(u_k^c,0)$ a bundle $\hat{z}_a = (0, \hat{c}_a, \hat{h}_a)$ such that $\hat{h}_a \geq \hat{h}_k$. By construction, we have $(\hat{c}_a, \hat{h}_a) \in L(u_k^c, 0)$ and $(\hat{c}_a, \hat{h}_a) \notin B^\tau(w+D,0)$. This shows that $u^c(z_a, u_a) \leq u^c(\hat{z}_a, u_a) = u_k^c$ and thus $u^c(z_a, u_a) \leq u^c(z_k, u_k)$, a contradiction. This concludes the proof of Lemma 1. ■

Let $\mu_a^c = u^c(z_a, u_a)$ denote the long-run equilibrium c-equivalent utility of individual a with $g_a = 0$ and $u_a = u^{\mathbf{a}}$ under scheme (τ, D). Let b_a denote the equilibrium bequest left by individual a under scheme (τ, D).

Lemma 2. *Consider any sustainable tax-demogrant scheme (τ, D) such that $D \geq 0$ and $-\tau$ is single-peaked. Under A1 and A2, if τ is monotonically increasing, then (τ, D) is dominated if $\mu_a^c < w + D$ and $b_a > 0$. Under A1, A2 and A3, (τ, D) is dominated if $\mu_a^c < w + D - \underline{b}$ and $b_a > \underline{b}$.*

Proof. Let $z = (g_i, c_i, h_i)_{i \in [0,1]} \in S$ denote the long-run equilibrium allocation associated to (τ, D) and let $(b_i)_{i \in [0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i = w + D + g_i - c_i$ for all $i \in [0,1]$. Let $A \subset [0,1]$ be the subset of altruistic individuals, i.e. $u_i \neq u^{\mathbf{s}}$ for all $i \in A$.

There are two cases to consider. For each case, the proof proceeds in three steps. In Step 1, we construct a particular tax-demogrant scheme (τ', D'). In Step 2, we show that (τ', D') is sustainable if (τ, D) is sustainable. In Step 3, we show that the long-run equilibrium allocation $z' \in S$ associated to (τ', D') is preferred by $\boldsymbol{R}^{c\text{-}lex}$ over z.

CASE 1: for all bequest amount $b > 0$ there is a subset $J \subseteq A$ with $\mu(J) > 0$ such that $b_j < b$ for all $j \in J$.

Step 1. We construct a particular tax-demogrant scheme (τ', D'). The construction of τ' is based on a particular bequest amount $\beta > 0$, whose definition depends on the type of τ.

- If τ is *monotonically increasing*, then $\underline{b} = 0$ and we take any β such that $0 < \beta < \min(b_a, w + D - \mu_a^c)$.
- If $-\tau$ is *positive peak*, then $\underline{b} > 0$ and we take $\beta = \underline{b}$.

For both types, we have $0 < \beta < \min(b_a, w + D - \mu_a^c)$.

Given β, we construct (τ', D') from a specific member of a parametric family of "truncated" tax-demogrant schemes $(\tau^\Delta, D - \Delta)$ with parameter $\Delta \in (0, \beta)$. The construction of $(\tau^\Delta, D - \Delta)$ is illustrated in Figure 8 for the case $\underline{b} = 0$ and in Figure 6 for the case $\underline{b} > 0$ (where $\beta = \underline{b}$). All members of this family linearly truncate the budget set $B^\tau(w + D + g_i, 0)$ for bequests smaller than β and differ by their associated demogrant $D - \Delta$.[15] Formally, we define τ^Δ as

$$\tau^\Delta(x) := \begin{cases} \tau(x + \Delta) - \Delta & \text{for all } x \geq \beta - \Delta \\ \frac{\tau(\beta) - \Delta}{\beta - \Delta} x & \text{for all } x \in [0, \beta - \Delta]. \end{cases} \tag{6}$$

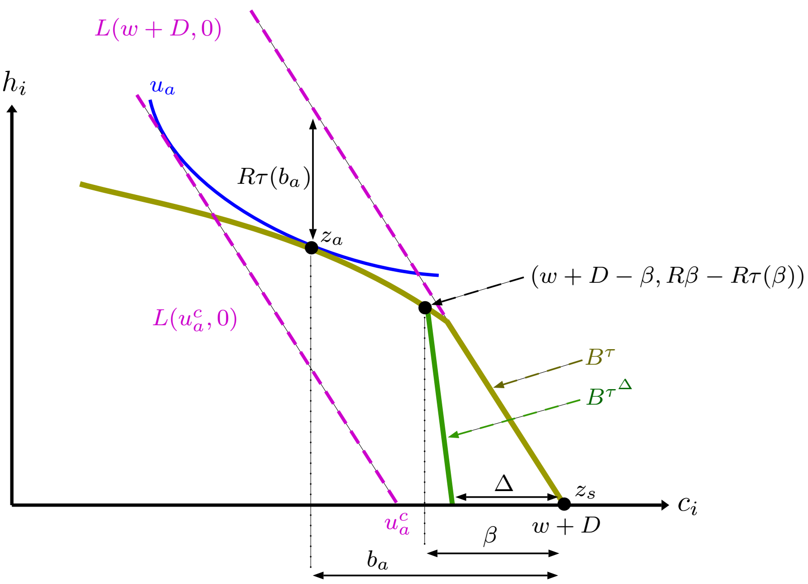

Figure 8: Construction of scheme $(\tau^\Delta, D - \Delta)$ for the case $\underline{b} = 0$.

The particularity of scheme $(\tau^\Delta, D - \Delta)$ is that any individual $i \in [0, 1]$ for whom $b_i \geq \beta$ choses the same bundle under both $(\tau^\Delta, D - \Delta)$ and (τ, D), i.e.

$$\arg \max_{\tilde{z}_i \in B^\tau(w+D+g_i,0)} u_i(\tilde{z}_i) = \arg \max_{\tilde{z}_i \in B^{\tau^\Delta}(w+D-\Delta+g_i,0)} u_i(\tilde{z}_i).$$

[15] Demogrant $D - \Delta$ need not be the maximal sustainable demogrant under tax τ^Δ.

Let $z^{\Delta} = (g_i^{\Delta}, c_i^{\Delta}, h_i^{\Delta})_{i\in[0,1]} \in S$ denote the long-run equilibrium allocation associated to $(\tau^{\Delta}, D - \Delta)$ and let $(b_i^{\Delta})_{i\in[0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i^{\Delta} = w + D - \Delta + g_i^{\Delta} - c_i^{\Delta}$ for all $i \in [0,1]$. Recall that $R\beta - R\tau(\beta)$ is the amount inherited by the child of any $i \in [0,1]$ for whom $b_i = \beta$. Consider the subset $J^{\Delta} \subseteq A$ for whom $h_j^{\Delta} < R\beta - R\tau(\beta)$ for all $j \in J^{\Delta}$.

We show that $\mu(J^{\Delta}) \to 0$ when $\Delta \to \beta$. The intuition for this statement is that the "truncated" budget set

$$B^{\tau^{\Delta}}(w + D - \Delta, 0)$$

has a kink at bundle $(0, w + D - \beta, R\beta - R\tau(\beta))$. Therefore, the larger is Δ, the steeper is the slope of this truncated budget sets for small bequests (this slope tends to $-\infty$ when $\Delta \to \beta$), and the greater is the incentive to "bunch" at the kink for the "moderately" altruistic individuals who inherit nothing. More formally, for any altruistic preference $u \in U \backslash \{u^{\mathrm{s}}\}$, there is a $\Delta < \beta$ such that for any $i \in [0,1]$ with $u_i = u$ and $g_i = 0$ we have $h_i^{\Delta} \geq R\beta - R\tau(\beta)$. This follows from A2, which requires that the altruistic preference u_i is strictly monotonic in h_i when $c_i \geq w$. The latter is guaranteed because the kink is located at bundle $(0, w + D - \beta, R\beta - R\tau(\beta))$ where $w + D - \beta \geq w$.[16] By the binormality of preferences, we also have $h_i^{\Delta} \geq R\beta - R\tau(\beta)$ for any $i \in [0,1]$ with an altruistic preference u_i and $g_i > 0$, which yields the result.

We are now equipped for the definition of scheme (τ', D'). This definition is based on the per-capita money amount $\beta\lambda_s$, which is saved by the government on the mass λ_s of self-centered individuals when reducing the demogrant by an amount β. For some $\frac{\beta\lambda_s}{2} > 0$, consider the subset of altruistic individuals $J^{\frac{\beta\lambda_s}{2}} \subseteq A$ for whom $h_j < \frac{\beta\lambda_s}{2}$ for all $j \in J^{\frac{\beta\lambda_s}{2}}$. In words, all altruistic individuals in $J^{\frac{\beta\lambda_s}{2}}$ leave an inheritance smaller than half the per-capita amount saved on self-centered individuals. Under Case 1, we have $\mu(J^{\frac{\beta\lambda_s}{2}}) > 0$. Since $\mu(J^{\Delta}) \to 0$ when $\Delta \to \beta$, there is a value Δ^* with $\Delta^* > \frac{\beta}{2}$ such that[17]

- $\mu(J^{\Delta^*}) < \mu(J^{\frac{\beta\lambda_s}{2}})$, and
- τ^{Δ^*} *subsidizes* bequests smaller than $\beta - \Delta^*$.[18]

In fact any $\Delta > \Delta^*$ also satisfies these two properties. We define scheme (τ', D') as

$$(\tau', D') = \left(\tau^{\Delta *}, D - \Delta^* + \frac{\beta\lambda_s}{2}\right),$$

whose construction is illustrated in Figure 9.

Step 2. We show that (τ', D') is sustainable if (τ, D) is sustainable. Let $z' = (g_i', c_i', h_i')_{i\in[0,1]} \in S$ denote the long-run equilibrium allocation associated to (τ', D') and let $(b_i')_{i\in[0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i' = w + D' + g_i' - c_i'$ for all $i \in [0,1]$. Let $(\hat{h}_i')_{i\in[0,1]}$ be the profile obtained

[16]Indeed, we assume that $\beta \leq w + D - u_a^c$ and we have $u_a^c \geq w$ (otherwise by A1 τ is dominated by Laissez-Faire).

[17]Recall that by definition of τ^{Δ} we have $\Delta < \beta$.

[18]If the tax τ is positive peak, then τ^{Δ^*} *subsidizes* bequests smaller than $\beta - \Delta^*$ for all $\Delta^* > 0$. In contrast, when the tax τ is monotonically increasing, τ^{Δ^*} *subsidizes* small bequests when $\tau(\beta) < \Delta^*$.

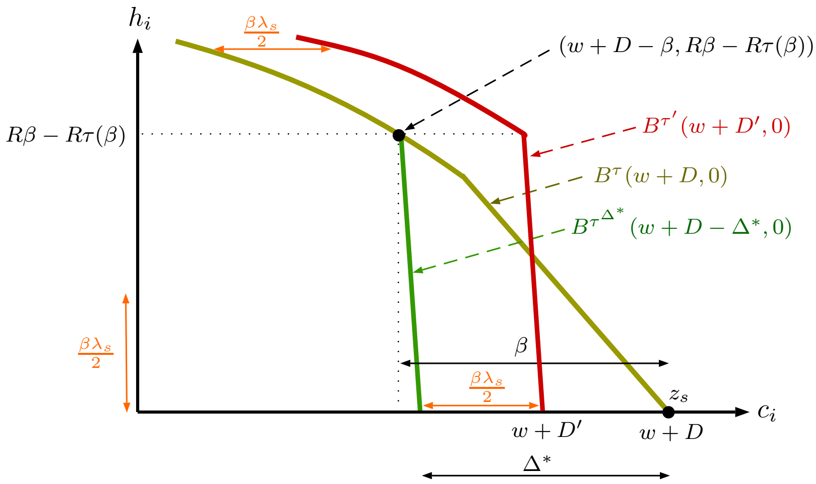

Figure 9: Construction of scheme (τ', D') for the case $\underline{b} = 0$.

from $(h'_i)_{i\in[0,1]}$ when sorting dynasties by increasing order of h'_i, i.e. $\hat{h}'_j \leq \hat{h}'_k$ for any $j,k \in [0,1]$ with $j < k$. Similarly, we use symbol "$\hat{}$" to denote other bequest or inheritance profiles sorted in increasing order.

We show that (τ', D') is sustainable if (τ, D) is sustainable. If (τ, D) is sustainable, then we have from the government's budget constraint that

$$0 \leq \int_{i\in[0,1]} (\tau(b_i) - D)\, di,$$

where $(b_i)_{i\in[0,1]}$ is the long-run equilibrium profile of bequests left under (τ, D). In order to show that (τ', D') is sustainable, it is sufficient that[19]

$$\int_{i\in[0,1]} (\tau(b_i) - D)\, di \leq \int_{i\in[0,1]} (\tau'(b'_i) - D')\, di$$

where $(b'_i)_{i\in[0,1]}$ is long-run equilibrium profile of bequests left under (τ', D').

Recalling that the mass of self-centered individuals is λ_s, we have for all $i \in [0, \lambda_s]$ that $\hat{b}_i = \hat{b}'_i = 0$ and thus $\tau(\hat{b}_i) = \tau(\hat{b}'_i) = 0$. Last inequality holds if the money saved by reducing the demogrant from D to D' is sufficient to cover the reduction in tax collected on altruistic individuals, i.e.

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\hat{b}'_i)di \leq D - D'. \tag{7}$$

In the remainder of Step 2, we first show that Eq. (7) holds in a special case. Then, we build on this special case in order to show that Eq. (7) holds

[19]Recall that the sustainability of a scheme (τ, D) relates only to the government budget constraint under its associated long-run equilibrium allocation. Indeed, a long-run equilibrium allocation is by definition a steady-state allocation, implying that the profile of inheritances left corresponds to the profile of inheritances received.

in general. To do that, we show that the special case is in fact the worst-case scenario.

We now show that Eq. (7) holds for the special case for which $\hat{h}'_i = \hat{h}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$. When $\hat{h}'_i \geq R\beta - R\tau(\beta)$, because $\tau' = \tau^{\Delta *}$ we have by the definition of τ^{Δ} in Eq. (6) that $\tau'(\hat{b}'_i) = \tau\left(\hat{b}'_i + \Delta^*\right) - \Delta^*$. If $\hat{h}'_i = \hat{h}_i$, which is equivalent to $\hat{b}'_i - \tau'(\hat{b}'_i) = \hat{b}_i - \tau(\hat{b}_i)$, the definition of τ' implies that $\hat{b}'_i = \hat{b}_i - \Delta^*$ for all $i \in (\lambda_s, 1]$. In turn, this implies that $\tau'(\hat{b}'_i) = \tau(\hat{b}_i) - \Delta^*$. Replacing this expression in Eq. (7) yields

$$(1 - \lambda_s)\Delta^* \leq D - D'.$$

Since $D - D' = \Delta^* - \frac{\beta\lambda_s}{2}$, last inequality becomes $\Delta^* \geq \frac{\beta}{2}$, which holds as the construction of Δ^* is such that $\frac{\beta}{2} < \Delta^* < \beta$.

There remains to show that Eq. (7) holds in general if Eq. (7) holds for the special case for which $\hat{h}'_i = \hat{h}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$. We use the following *Technical Claim* (proved at the end of Step 2): we have $\hat{h}'_i \geq \hat{h}_i$ and $\hat{h}'_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$. From profiles $(\hat{h}_i)_{i\in(\lambda_s,1]}$ and $(\hat{h}'_i)_{i\in(\lambda_s,1]}$, we show it is possible to construct two alternative profiles $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ that correspond to the special case, i.e. $\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$, and for which

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\hat{b}'_i)di \leq \int_{i\in(\lambda_s,1]} \tau(\tilde{\hat{b}}_i)di - \int_{i\in(\lambda_s,1]} \tau'(\tilde{\hat{b}}'_i)di, \quad (8)$$

where $\tilde{\hat{h}}_i = R\tilde{\hat{b}}_i - R\tau(\tilde{\hat{b}}_i)$ and $\tilde{\hat{h}}'_i = R\tilde{\hat{b}}'_i - R\tau'(\tilde{\hat{b}}'_i)$ for all $i \in (\lambda_s, 1]$. If Eq. (8) holds, then Eq. (7) holds for $(\hat{b}_i)_{i\in(\lambda_s,1]}$ and $(\hat{b}'_i)_{i\in(\lambda_s,1]}$ because the inheritance profiles associated to $(\tilde{\hat{b}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{b}}'_i)_{i\in(\lambda_s,1]}$ correspond to the special case as $\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ for all $i \in (\lambda_s, 1]$.

We construct $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ from $(\hat{h}_i)_{i\in(\lambda_s,1]}$ as follows (see Figure 10.a for an illustration for the case of a positive-peak tax τ, for which $\tau(\beta) = 0$):

$$\tilde{\hat{h}}_j = R\beta - R\tau(\beta) \qquad \text{for all } j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})],$$
$$\tilde{\hat{h}}_i = \hat{h}_i \qquad \text{for all } i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1],$$

and construct $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ from $(\hat{h}_i)_{i\in(\lambda_s,1]}$ and $(\hat{h}'_i)_{i\in(\lambda_s,1]}$ as follows (see Figure 10.b for an illustration for the case of a positive-peak tax τ, for which $\tau(\beta) = 0$):

$$\tilde{\hat{h}}'_j = R\beta - R\tau(\beta) \qquad \text{for all } j \in (\lambda_s, \lambda_s + \mu'],$$
$$\tilde{\hat{h}}'_i = \tilde{\hat{h}}_i \qquad \text{for all } i \in (\lambda_s + \mu', 1].$$

In words, profiles $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$ and $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$ are constructed by replacing the inheritances $\hat{h}_j$ and $\hat{h}'_j$ that are smaller than $R\beta - R\tau(\beta)$ by $R\beta - R\tau(\beta)$, and then replacing inheritances $\hat{h}'_i$ that are larger than $\hat{h}_i$ by $\hat{h}_i$.

There remains to show that Eq. (8) holds. First, we show that

$$\int_{i\in(\lambda_s,1]} \tau(\hat{b}_i)di \leq \int_{i\in(\lambda_s,1]} \tau(\tilde{\hat{b}}_i)di. \quad (9)$$

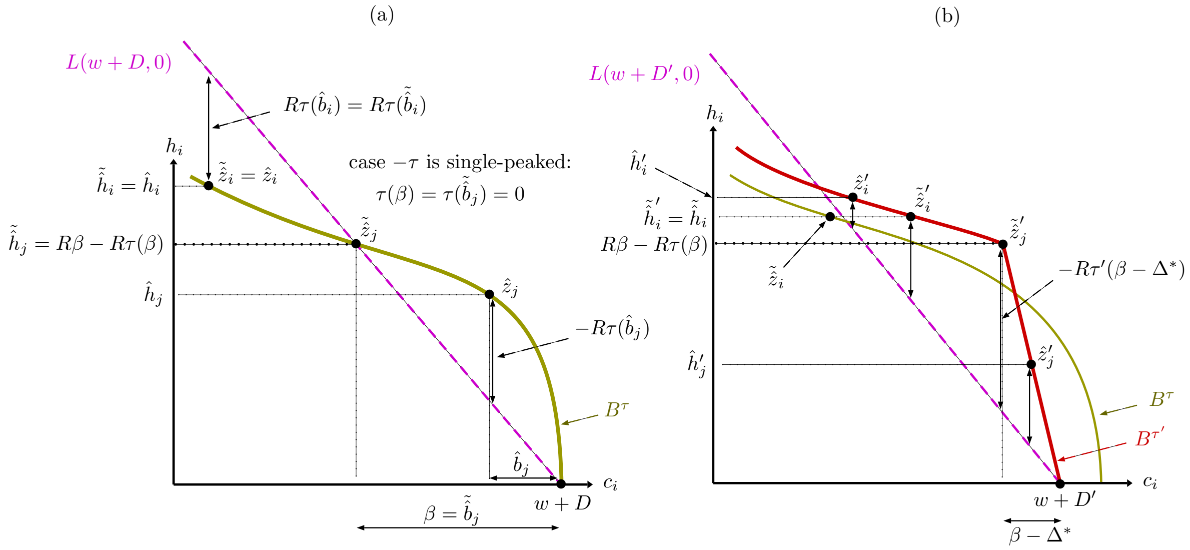

Figure 10: (a) Construction of profile $(\tilde{\hat{h}}_i)_{i\in(\lambda_s,1]}$, where $j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})]$ and $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$. (b) Construction of profile $(\tilde{\hat{h}}'_i)_{i\in(\lambda_s,1]}$, where $j \in (\lambda_s, \lambda_s + \mu']$ and $i \in (\lambda_s + \mu', 1]$. Case of a positive-peak tax τ, for which $\tau(\beta) = 0$.

For all $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$, we have $\hat{h}_i = \tilde{\hat{h}}_i \geq R\beta - R\tau(\beta)$ implying that $\tau(\hat{b}_i) = \tau(\tilde{\hat{b}}_i)$. For all $j \in (\lambda_s, \lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}})]$, we have $\hat{h}_j < R\beta - R\tau(\beta)$ and $\tilde{\hat{h}}_j = R\beta - R\tau(\beta)$, and we show that $\tau(\hat{b}_j) \leq \tau(\tilde{\hat{b}}_j)$. This is obvious if τ is monotonically increasing because $\hat{b}_j < \tilde{\hat{b}}_j = \beta$. If τ is positive peak (the case illustrated in Figure 10.a), then our construction is such that $\beta = \underline{b}$. As $-\tau$ is single-peaked, this implies that $\tau(\hat{b}_j) \leq 0$ whereas $\tau(\beta) = 0$. Therefore Eq. (9) holds.

Second, we show that

$$\int_{i\in(\lambda_s,1]} \tau'(\hat{b}'_i)di \geq \int_{i\in(\lambda_s,1]} \tau'(\tilde{\hat{b}}'_i)di. \tag{10}$$

For all $i \in (\lambda_s + \mu', 1]$, we have $\hat{h}'_i \geq \tilde{\hat{h}}'_i \geq R\beta - R\tau(\beta)$ and we show that $\tau'(\hat{b}'_i) \geq \tau'(\tilde{\hat{b}}'_i)$ (see illustration in Figure 10.b, where $-\tau'(\hat{b}'_i) \leq -\tau'(\tilde{\hat{b}}'_i)$). The construction of τ' from τ is such that $-\tau'$ is single-peaked when $-\tau$ is single-peaked. Also, as $\hat{h}'_i \geq \tilde{\hat{h}}'_i \geq R\beta - R\tau(\beta)$ we have that $\hat{b}'_i \geq \tilde{\hat{b}}'_i \geq \beta - \Delta^*$. For bequest amounts larger than $\beta - \Delta^*$, τ' is monotonically increasing in bequest, which implies that $\tau'(\hat{b}'_i) \geq \tau'(\tilde{\hat{b}}'_i)$. For all $j \in (\lambda_s, \lambda_s + \mu']$, we have $\hat{h}'_j < R\beta - R\tau(\beta)$ and $\tilde{\hat{h}}'_j = R\beta - R\tau(\beta)$, and we show that $\tau'(\hat{b}'_j) \geq \tau'(\tilde{\hat{b}}'_j)$. For such j, we have thus $\hat{b}'_j < \tilde{\hat{b}}'_j = \beta - \Delta^*$. As $\tau' = \tau^{\Delta^*}$ and $\hat{b}'_j < \beta - \Delta^*$, we have $\tau'(\hat{b}'_j) < 0$. What is more, the construction of τ' is such that $\tau'(x') < \tau'(x)$ for all $0 \leq x < x' \leq \beta - \Delta^*$, i.e. the subsidy received under τ' is increasing in the bequest left, when the bequest is smaller than $\beta - \Delta^*$. Therefore Eq. (10) holds.

Eq. (9) and Eq. (10) together imply that (8) holds. To concludes Step 2, there only remains to prove the Technical Claim.

Technical Claim: For any $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$ we have $\hat{h}'_i \geq \hat{h}_i$ and $\hat{h}'_i \geq R\beta - R\tau(\beta)$.[20]

Proof of Technical Claim: For $t \in \{0, 1, \dots\}$, we consider successive equilibrium allocations $z'_t = (g'_{it}, c'_{it}, h'_{it+1})_{i\in[0,1]}$ for which all $i \in [0, 1]$ chose in $B^{\tau'}(w + D' + g'_{it}, 0)$ and $(\hat{g}'_{it+1})_{i\in[0,1]} = (\hat{h}'_{it+1})_{i\in[0,1]}$, under an initial profile of inheritances $(g'_{i0})_{i\in[0,1]} = (h_i)_{i\in[0,1]}$, which corresponds to the long-run equilibrium profile associated to (τ, D). We show for all $t \in \{0, 1, \dots\}$ that

(1) $\hat{h}'_{it+1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_i$ for all $i \in [0, 1]$,

(2) $\hat{h}'_{it+1} \geq \hat{h}_i$ and $\hat{h}'_{it+1} \geq R\beta - R\tau(\beta)$ for any $i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]$.

Observe that, at all t, (1) and (2) compare the sorted profile of inheritances left in t to the sorted profile of inheritances left under the long-run equilibrium allocation z.

If (1) and (2) hold for all $t \geq 0$, then (2) holds as well for the profile $(h'_i)_{i\in[0,1]}$ associated to the long-run equilibrium allocation z', because we assume that for any given tax-demogrant scheme, the economy converges over time to a unique long-run equilibrium allocation independent on the initial distribution of inheritances.

Consider first $t = 0$. In order to show that claims (1) and (2) hold for $t = 0$ when the tax-demogrant scheme is (τ', D'), it is sufficient to show that claims (1) and (2) hold for $t = 0$ when the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$ instead of (τ', D'). Indeed, let $(h^{\Delta*}_{i1})_{i\in[0,1]}$ be the profile of inheritances obtained in period $t = 0$ if the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$. Since $\tau' = \tau^{\Delta*}$ and $D' > D - \Delta^*$, the binormality of preferences implies that $h'_{i1} \geq h^{\Delta*}_{i1}$ for all $i \in [0, 1]$, which shows that it is indeed sufficient to prove these claims when the tax-demogrant scheme is $(\tau^{\Delta*}, D - \Delta^*)$.

Consider first claim (1) in $t = 0$, i.e.

$$\hat{h}^{\Delta*}_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_i \qquad \text{for all} \qquad i \in [0, 1]. \tag{11}$$

Consider the profile $(h_{i1})_{i\in[0,1]}$ of inheritances obtained in period $t = 0$ if the tax-demogrant scheme is (τ, D) instead of $(\tau^{\Delta*}, D - \Delta^*)$. As the initial profile $(g_{i0})_{i\in[0,1]} = (h_i)_{i\in[0,1]}$, we have $(\hat{h}_{i1})_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$ because z is the long-run equilibrium allocation associated to (τ, D). It is thus sufficient to show that

$$\hat{h}^{\Delta*}_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{h}_{i1} \qquad \text{for all} \qquad i \in [0, 1] \tag{12}$$

[20] Recall that index i need not refer to the same dynasty in the two sorted distributions.

for Eq. (11) to hold.

Since we have $h_{i1}^{\Delta *} = h_{i1} = 0$ for all $i \in [0,1]$ for whom $u_i = u^{\mathbf{s}}$, we can focus on the subset $A = \{i \in [0,1] | u_i \neq u^{\mathbf{s}}\}$ of altruistic individuals. We partition A into three subgroups A^1, A^2 and A^3, respectively defined as

- $A^1 = \{i \in A | h_{i1} \geq R\beta - R\tau(\beta)\}$.
  By construction of $(\tau^{\Delta *}, D - \Delta^*)$, any $i \in A^1$ choses the *same* bundle in $B^{\tau}(w + D + g_{i0}, 0)$ and in $B^{\tau^{\Delta *}}(w + D - \Delta^* + g_{i0}, 0)$. This implies that $h_{i1}^{\Delta *} = h_{i1}$ for all $i \in A^1$.
- $A^2 = \{i \in A | h_{i1} < R\beta - R\tau(\beta) \text{ and } h_{i1}^{\Delta *} \geq R\beta - R\tau(\beta)\}$.
  By definition, we have $h_{i1}^{\Delta *} > h_{i1}$ for all $i \in A^2$.
- $A^3 = \{i \in A | h_{i1} < R\beta - R\tau(\beta) \text{ and } h_{i1}^{\Delta *} < R\beta - R\tau(\beta)\}$.
  By definition, any altruistic $j \in A^3$ leaves a smaller inheritance than $R\beta - R\tau(\beta)$, and thus a smaller inheritance than any $i \in A^1 \cup A^2$, i.e. $h_{j1}^{\Delta *} \leq h_{i1}^{\Delta *}$.

  We can assume without loss of generality that $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$. If it is not the case, consider for the construction of (τ', D') a larger $\Delta \in (\Delta^*, \beta)$ for which we have $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$. Such larger value exists by assumption A2.

By the definition of the above partition of $[0,1]$, we have

$$h_{i1}^{\Delta *} \geq h_{i1} \tag{13}$$

for all $i \in [0,1] \backslash A^3$,[21] but Eq. (13) may not hold for some $i \in A^3$. However, we have $\mu(A^3) \leq \mu(J^{\frac{\beta\lambda_s}{2}})$ and there is a subset of individuals of mass $\mu(J^{\frac{\beta\lambda_s}{2}})$ for whom $h_{i1} < \frac{\beta\lambda_s}{2}$. Therefore, even if $h_{i1}^{\Delta *} = 0$ for all $i \in A^3$, we have $h_{i1}^{\Delta *} + \frac{\beta\lambda_s}{2} \geq \frac{\beta\lambda_s}{2}$ for all $i \in A^3$,[22] which shows that (12) holds, and thus (11) holds.

We now turn to claim (2) for $t = 0$ if the tax-demogrant scheme is $(\tau^{\Delta *}, D - \Delta^*)$ instead of (τ', D'), i.e.

$$\hat{h}_{i1}^{\Delta *} \geq \hat{h}_{i1} \quad \text{and} \quad \hat{h}_{i1}^{\Delta *} \geq R\beta - R\tau(\beta) \text{ for all } \; i \in (\lambda_s + \mu(J^{\frac{\beta\lambda_s}{2}}), 1]. \tag{14}$$

By definition, if $i \in A^1 \cup A^2$ and $j \notin A^1 \cup A^2$ we have $h_{i1}^{\Delta *} \geq h_{j1}^{\Delta *}$. Also, as $A_2 \cap J^{\frac{\beta\lambda_s}{2}}$ may be non-empty, the mass of individuals in $A^1 \cup A^2$ is such that $\mu(A^1 \cup A^2) \geq 1 - \lambda_s - \mu(J^{\frac{\beta\lambda_s}{2}})$. Hence, it is sufficient that Eq. (14) holds for individuals in $A^1 \cup A^2$. As shown when defining these two subgroups, we have $h_{i1}^{\Delta *} = h_{i1} \geq R\beta - R\tau(\beta)$ for all $i \in A^1$ and $h_{i1}^{\Delta *} = R\beta - R\tau(\beta) > h_{i1}$ for all $i \in A^2$, which proves Eq. (14).

Together, we have shown claims (1) and (2) for $t = 0$. We next prove these claims for $t = 1$.

[21] We have shown that $h_{i1}^{\Delta *} = h_{i1}$ for all $i \in [0,1] \backslash A$, $h_{i1}^{\Delta *} = h_{i1}$ for all $i \in A^1$ and $h_{i1}^{\Delta *} > h_{i1}$ for all $i \in A^2$.

[22] In words, even if all individuals in A^3 leave no inheritances, there are enough altruistic individuals who, in the long-run equilibrium allocation z, leave inheritances smaller than the additional amount $\frac{\beta\lambda_s}{2}$ considered in claim (1).

In $t = 1$, the profile of inheritances received under (τ', D') is such that $(\hat{g}'_{i1})_{i\in[0,1]} = (\hat{h}'_{i1})_{i\in[0,1]}$. By construction, since $\tau' = \tau^{\Delta*}$ and $D' = D - \Delta^* + \frac{\beta\lambda_s}{2}$, we have for all $i \in [0,1]$ that

$$B^{\tau^{\Delta^*}}\left(w + D - \Delta^* + g'_{i1} + \frac{\beta\lambda_s}{2}, 0\right) = B^{\tau'}\left(w + D' + g'_{i1}, 0\right),$$

as can be seen in Figure 9 (for the case $g'_{i1} = 0$). Therefore, the profile $(\hat{h}'_{i2})_{i\in[0,1]}$ obtained under (τ', D') for $(\hat{g}'_{i1})_{i\in[0,1]} = (\hat{h}'_{i1})_{i\in[0,1]}$ is the same as the profile $(\hat{h}^{\Delta^*}_{i2})_{i\in[0,1]}$ that would be obtained under $(\tau^{\Delta*}, D - \Delta^*)$ if the profile of inheritances received in $t = 1$ was instead $(\hat{h}'_{i1} + \frac{\beta\lambda_s}{2})_{i\in[0,1]}$. From claim (1) for $t = 0$, we have that $\hat{g}'_{i1} + \frac{\beta\lambda_s}{2} \geq \hat{g}'_{i0}$ for all $i \in [0,1]$. Therefore, by the binormality of preferences, the same reasoning implies again (1) and (2) for $t = 1$.

The same reasoning extends (1) and (2) to any $t \geq 2$, which concludes the proof of the Technical Claim.

Step 3. We show that allocation z' is preferred by $\boldsymbol{R}^{c\text{-}lex}$ to allocation z.

By assumption A1, there is a positive mass of individuals a and a positive mass of individuals s. The construction of τ' from τ is such that $-\tau'$ is single-peaked when $-\tau$ is single-peaked. By Lemma 1, under both z' and z, either a or s are among the worst-offs. Individual a is among the worst-offs under z because $\mu^c_a = u^c(z_a, u_a) < w + D = u^c(z_s, u_s)$, as illustrated in Figure 8.

There remains to show that $\mu^c_a < u^c(z'_a, u_a)$ and $\mu^c_a < u^c(z'_s, u_s)$. We have $\mu^c_a < u^c(z'_s, u_s)$ because $u^c(z'_s, u_s) = w + D' > w + D - \beta$ and $\beta \leq w + D - u^c_a$.[23] Finally, we show that $\mu^c_a < u^c(z'_a, u_a)$. We have selected β such that $\beta < b_a$.[24] When $\beta < b_a$, the construction of τ^{Δ^*} implies that $z^{\Delta^*}_a = z_a$, where $z^{\Delta^*}_a$ is the equilibrium bundle of a under scheme $(\tau^{\Delta^*}, D - \Delta^*)$. Since $\tau' = \tau^{\Delta^*}$ but $D' > D - \Delta^*$, bundle z_a lies in the *interior* of[25]

$$B^{\tau'}\left(w + D', 0\right),$$

which is a's budget set under (τ', D'), as illustrated in Figure 9. This implies that $u_a(z_a) < u_a(z'_a)$, hence $\mu^c_a < u^c(z'_a, u_a)$.

CASE 2: there exists a bequest amount b^* with $0 < b^* < \min(b_a, w + D - \mu^c_a)$ such that for all $J \subseteq A$ with $b_j < b^*$ for all $j \in J$ we have $\mu(J) = 0$.

Step 1. We construct (τ', D') from a particular tax-demogrant scheme (τ'', D''). Take $D'' = D - b^*/2$. The tax τ'' is constructed from τ by linearly truncating the budget set $B^\tau(w + D + g_i, 0)$ for bequests smaller than b^* (as illustrated in Figure 11). Formally, we define τ'' as

$$\tau''(x) := \begin{cases} \tau\left(x + \frac{b^*}{2}\right) - \frac{b^*}{2} & \text{for all } x \geq b^*/2 \\ \frac{\tau(b^*) - b^*/2}{b^*/2} x & \text{for all } x \in [0, b^*/2]. \end{cases}$$

[23] We have $w + D' > w + D - \beta$ because $D' = D - \Delta^* + \frac{\beta\lambda_s}{2}$ and $\Delta^* \in (\beta/2, \beta)$ and $\lambda_s \in (0,1)$.

[24] If τ is monotonically increasing, then $\underline{b} = 0$ and this case is such that $\beta < b_a$. If τ is positive peak, then $\underline{b} > 0$ and this case is such that $\beta = \underline{b}$ and $b_a > \underline{b}$.

[25] As $D' > D - \Delta^*$, all bundles in $B^{\tau^{\Delta^*}}(w + D - \Delta^*, 0)$ lie in the interior of $B^{\tau'}(w + D', 0)$.

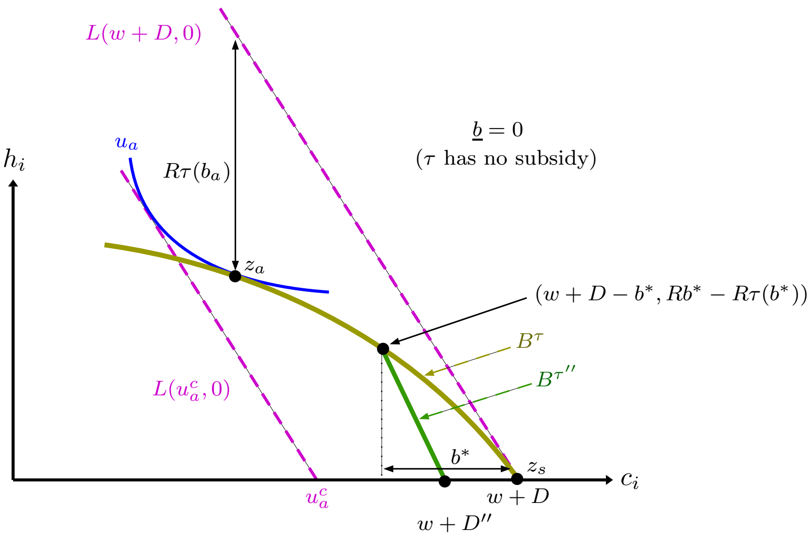

Figure 11: The tax-demogrant scheme (τ, D) is dominated because the tax function τ taxes small bequests. Individual a is the worst-off because $u^c(z_a, u_a) = u_a^c$. The sustainable scheme (τ'', D'') has a smaller demogrant, does not affect $u^c(z_a, u_a)$ and leaves money on the table.

The particularity of scheme (τ'', D'') is that any individual $i \in [0,1]$ for whom $b_i \geq b^*$ choses the same bundle under both (τ'', D'') and (τ, D), i.e.

$$\arg \max_{\tilde{z}_i \in B^\tau(w+D+g_i,0)} u_i(\tilde{z}_i) = \arg \max_{\tilde{z}_i \in B^{\tau''}(w+D''+g_i,0)} u_i(\tilde{z}_i).$$

Let $z'' = (g_i'', c_i'', h_i'')_{i\in[0,1]} \in S$ denote the long-run equilibrium allocation associated to (τ'', D'') and let $(b_i'')_{i\in[0,1]}$ be the long-run equilibrium profile of bequests left, i.e. $b_i'' = w + D'' + g_i'' - c_i''$ for all $i \in [0,1]$. Case 2 is such that the profile $(\hat{h}_i'')_{i\in[0,1]} = (\hat{h}_i)_{i\in[0,1]}$ because for all $J \subseteq A$ with $b_j < b^*$ for all $j \in J$ we have $\mu(J) = 0$. This implies for all $i \in (\lambda_s, 1]$ that $\hat{b}_i'' = \hat{b}_i - b^*/2$ and thus $\tau''(\hat{b}_i'') = \tau(\hat{b}_i) - b^*/2$. As (τ, D) is sustainable, we have that (τ'', D'') leaves on the table an amount at least $\lambda_s \frac{b^*}{2}$, i.e.

$$\frac{\lambda_s b^*}{2} \leq \int_{i\in[0,1]} (\tau''(b_i'') - D'')\, di.$$

If τ is positive peak, then by assumption A3 there exists a sustainable scheme (τ', D') with $\tau' = \tau''$ and $D' > D''$. If τ is monotonically increasing, then A3 is not assumed and we define scheme (τ', D') as

$$(\tau', D') = \left(\tau'', D'' + \frac{b^*\lambda_s}{4}\right).$$

Step 2. We show that (τ', D') is sustainable if (τ, D) is sustainable. We have already shown it using A3 in Step 1 in the case for which τ is positive peak. In the case for which τ is monotonically increasing, then a simplified version of the argument used in Step 2 of Case 1 shows that (τ', D') is sustainable. The argument can be simplified because all altruistic individuals leave a bequest larger than b^*, implying that the partition of A as $A = A^1 \cup A^2 \cup A^3$ is such that $A^2 = A^3 = \varnothing$. We do not repeat this argument.

Step 3. The long-run equilibrium allocation z' associated to (τ', D') is preferred by $\boldsymbol{R}^{c\text{–}lex}$ to allocation z. The argument is the same as the argument used in Step 3 of Case 1. We do not repeat this argument. This concludes the proof of Lemma 2. ■

First, we prove claim (i) of Proposition 2. Assume to the contrary that τ is monotonically increasing but τ does not provide an exemption up to $b_a^{LF}(w)$, i.e. $\tau(b_a^{LF}(w)) > 0$. Under this contradiction assumption, we show that scheme (τ, D) is not optimal whatever the value of D. As any scheme (τ, D) with $D < 0$ is not optimal (Proposition 1), we consider any (τ, D) with $D \geq 0$.

As τ is monotonically increasing, we have $\underline{b} = 0$ and either $\tau(b_a) > 0$ or $\tau(b_a) = 0$. (Recall that b_a denotes the equilibrium bequest left by individual a with $g_a = 0$ and $u_a = u^{\mathbf{a}}$.) If $\tau(b_a) > 0$, then Eq. (3) is violated and (τ, D) is not optimal by Proposition 2 (ii), whose proof is given below. So assume that $\tau(b_a) = 0$, which implies that $\tau(x) = 0$ for all $x \in [0, b_a]$ because τ is monotonically increasing. If $b_a \geq b_a^{LF}(w)$, then we have $\tau(b_a) > 0$ because $\tau(b_a^{LF}(w)) > 0$ and τ is monotonically increasing, a contradiction to our assumption that $\tau(b_a) = 0$.

There remains the case for which $b_a < b_a^{LF}(w)$ and $\tau(b_a) = 0$. First, we show that any optimal (τ, D) has $b_a > 0$. If $b_a = 0$ under (τ, D) with $D \geq 0$, then we can show that the economy converges to a long-run equilibrium allocation for which all inheritances are zero, i.e. $D = 0$. Indeed, if $b_a = 0$ under a scheme (τ, D) with $D \geq 0$, the binormality of preferences implies that a leaves no bequest under scheme $(\tau, 0)$. Therefore, any dynasty i with a member it' such that $u_{it'} = u^{\mathbf{s}}$ leaves no bequest for all it with $t \geq t'$. As there is in each generation a mass λ_s of individuals $i \in [0,1]$ with $u_i = u^{\mathbf{s}}$, and preferences are drawn at random in each generation, all dynasties have a member it' such that $u_{it'} = u^{\mathbf{s}}$ for some $t' \leq t$ when t is sufficiently large. Therefore all inheritances are zero in the long-run equilibrium allocation, which implies that the largest sustainable demogrant is $D = 0$ when $b_a = 0$. We show that $(\tau, D = 0)$ is dominated by Laissez-Faire when $b_a = 0$. Under Laissez-Faire, the sustainable demogrant is also zero. The equilibrium bundle of a under $(\tau, D = 0)$ is $z_a = (0, w, 0)$ because $b_a = 0$, whereas it is $z_a^{LF} = (0, w - b_a^{LF}(w), Rb_a^{LF}(w))$ with $b_a^{LF}(w) > 0$ under Laissez-Faire. As illustrated in Figure 12.a, we have $u_a(z_a) < u_a(z_a^{LF})$ because $(c_a, h_a) \in L(w, 0)$ but $z_a \neq z_a^{LF}$ where

$$(c_a^{LF}, h_a^{LF}) = \arg \max_{(\tilde{c}_a, \tilde{h}_a) \in L(w,0)} u_a((0, \tilde{c}_a, \tilde{h}_a)).$$

Under Laissez-Faire, any individual $i \in [0,1]$ allocates her lifetime resources freely in the non-distortionary budget $L(w + g_i, 0)$, thus we have $u^c(z_i^{LF}, u_i) = w + g_i$. This shows that $u^c(z_i^{LF}, u_i) \geq u^c(z_a^{LF}, u_a)$ for all $i \in [0,1]$ because $u^c(z_a^{LF}, u_a) = w$ as $g_a = 0$. Now, since $u_a(z_a^{LF}) > u_a(z_a)$, we have $u^c(z_a^{LF}, u_a) > u^c(z_a, u_a)$, implying that $u^c(z_i^{LF}, u_i) > u^c(z_a, u_a)$ for all $i \in [0,1]$. By A1, there is mass of individuals a with $g_a = 0$ and $u_a = u^{\mathbf{a}}$, showing that $(\tau, D = 0)$ is dominated by Laissez-Faire, i.e. not optimal.

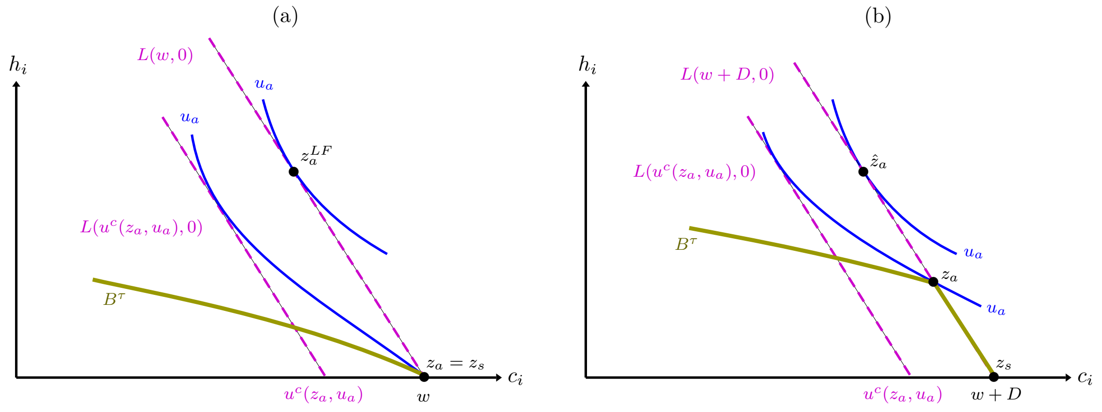

Figure 12: (a) $(\tau, D = 0)$ is dominated by Laissez-Faire when $b_a = 0$. (b) $u^c(z_a, u_a) < w + D$ when $0 < b_a < b_a^{LF}(w)$.

Now, for the case $0 < b_a < b_a^{LF}(w)$, the binormality of preferences implies that $b_a^{LF}(w) \leq b_a^{LF}(w + D)$ and thus $b_a < b_a^{LF}(w + D)$. In words, a would increase her bequest if the exemption proposed by τ was larger. As illustrated in Figure 12.b, because $(c_a, h_a) \in L(w + D, 0)$ but $z_a \neq \hat{z}_a$ where

$$(\hat{c}_a, \hat{h}_a) = \arg \max_{(\tilde{c}_a, \tilde{h}_a) \in L(w,0)} u_a((0, \tilde{c}_a, \tilde{h}_a)).$$

we have $u_a(z_a) < u_a(\hat{z}_a)$.

As $u^c(\hat{z}_a, u_a) = w + D$, this implies that $u^c(z_a, u_a) < w + D$. As $\underline{b} = 0$ and $b_a > 0$, we have $b_a > \underline{b}$, and Lemma 2 implies that (τ, D) is not optimal.

Second, we prove claim (ii) of Proposition 2. We show that, if $\tau(b_a) > \underline{b}$, then scheme (τ, D) is not optimal whatever the value of D. As any scheme (τ, D) with $D < 0$ is not optimal (Proposition 1), we consider any (τ, D) with $D \geq 0$. As any sustainable (τ, D) for which $D < D^{max}$ is dominated by (τ, D^{max}), and thus not optimal, we can focus on $D = D^{max}$. We show that the preconditions for Lemma 2 are all met, which implies that (τ, D) is not optimal.

By definition of $\underline{b}$ we have $\tau(\underline{b}) = 0$. Since $\tau(b_a) > 0$, we have $b_a > \underline{b}$ because $-\tau$ is single-peaked, implying that τ is monotonically increasing in x for all $x \geq \underline{b}$.

Any individual a with $g_a = 0$ and $u_a = u^{\mathbf{a}}$ choses the equilibrium bundle $z_a = (0, w + D - b_a, Rb_a - R\tau(b_a))$ in the budget set $B^\tau(w + D, 0)$. Bundle z_a is on the frontier of the non-distortionary budget set $L(w + D - \tau(b_a), 0)$, which implies that $u^c(z_a, u_a) \leq w + D - \tau(b_a)$. As $\tau(b_a) > \underline{b}$, this implies that $u^c(z_a, u_a) < w + D - \underline{b}$.

Together, we have $D \geq 0$ and we have shown $\mu_a^c < w + D - \underline{b}$ and $b_a > \underline{b}$. Therefore, Lemma 2 implies that (τ, D) is not optimal.

S3 Proof of Proposition 3

Let z be the long-run equilibrium allocation generated by a tax system (τ, D), so that D denote the largest demogrant for which the scheme (τ, D) is sustainable. Let us assume τ is optimal.

By assumption, both self-centered and altruistic individuals are among the worst-offs, which implies that $u^c(z_{s1}, u^{\mathbf{s}}) = u^c(z_{a1}, u^*_a)$. Graphically, bundle z_{a1} must be on the indifference curve of preference u^*_a that is tangent to the non-distortionary budget set $L(c_{s1}, 0)$, as shown in Figure 13. Let bundle $z^k := (c^k, h^k)$ be the bundle at the kink of this indifference curve, which is the bundle that $a1$ would select in the non-distortionary budget set $L(c_{s1}, 0)$. By definition, we have

$$(c^k, h^k) = (w + D - b_a^{LF}(w + D), Rb_a^{LF}(w + D)), \tag{15}$$

where $b_a^{LF}(w + D)$ is the equilibrium bequest left under Laissez-Faire by an altruistic individual who receives an inheritance equal to D.

Observe that the incentive compatibility constraint of $a1$ implies that z_{a1} lies on the segment of this indifference curve that is below the kink, that is, $h_{a1} \leq h^k$. Assume to the contrary that $a1$ selects a bundle z'_{a1} on the segment of this indifference curve that is above the kink, such that $h'_{a1} > h^k$ (see Figure 13). Individual $a1$ would then receive a subsidy $(\tau(b'_{a1}) < 0)$. This implies in turn that bundle z^k lies in the interior of her budget set $B^\tau(w + D, 0)$.[26] This is a contradiction to bundle z'_{a1} being incentive compatible for $a1$ because $u^*_a(z'_{a1}) = u^*_a(z^k)$ and z^k lies in the interior of her budget set.

If $h_{a1} = h^k$, then $z_{a1} = z^k$ because $u^c(z_{s1}, u^{\mathbf{s}}) = u^c(z_{a1}, u^*_a)$. We don't need to consider this case because it is such that τ admits an exemption up to bequest level $b_a^{LF}(w + D)$, where $b_a^{LF}(w + D) \geq b_a^{LF}(w)$.[27] We can thus concentrate on the case $h_{a1} < h^k$. This case is such that $\tau(b_{a1}) < 0$ because z_{a1} does not lie in the non-distortionary budget set $L(c_{s1}, 0)$.

We show that τ is not optimal when $h_{a1} < h^k$ and $u^c(z_{s1}, u^{\mathbf{s}}) = u^c(z_{a1}, u^*_a)$. To do this, we show there exists another sustainable scheme $(\overline{\tau}, \overline{D})$ whose long-run equilibrium allocation is strictly preferred by SWF $\boldsymbol{R}^{c\text{-}lex}$ to z.

Let $\overline{z} = (\overline{z}_{sX}, \overline{z}_{aX})_{X \in \{1,2...\}}$ denote the long-run allocation associated to scheme $(\overline{\tau}, D)$, where D is the maximal sustainable demogrant for tax τ. We will construct $\overline{\tau}$ in such a way that

(i) $\min_{i \in [0,1]} u^c(\overline{z}_i, u_i) = \min_{i \in [0,1]} u^c(z_i, u_i)$,

(ii) $\overline{\tau}(\overline{b}_{a1}) > \tau(b_{a1})$, and

(iii) $\overline{\tau}(\overline{b}_{aX}) \geq \tau(b_{aX})$ for all $X \in \{2, 3, \dots\}$.

[26] Bundle z^k lies in the interior of her budget set $B^\tau(w+D, 0)$ because $\tau(b_a^{LF}(w+D)) < 0$. We must have $\tau(b_a^{LF}(w+D)) < 0$ because $\tau(b'_{a1}) < 0$ and $b_a^{LF}(w+D) < b'_{a1}$. Indeed, $-\tau$ is single-peaked and our domain of tax function excludes that τ provides an exemption on small bequests, then provides subsidies on intermediate bequests and finally taxes positively large bequests. Hence, if $\tau(b_a^{LF}(w+D)) \geq 0$, then τ must provide subsidies for all $b \geq b'_{a1}$. But this implies that $D' < 0$, which contradicts that τ is optimal (Proposition 1).

[27] The exact value of $\tau(b)$ for $b \in (0, b_a^{LF}(w+D))$ does not matter because no individual selects $b < b_a^{LF}(w+D)$. In other words, as this economy only has a countable number of individuals, we only need to consider a countable number of points on the tax function, and the exact value of the tax beyond these points is irrelevant.

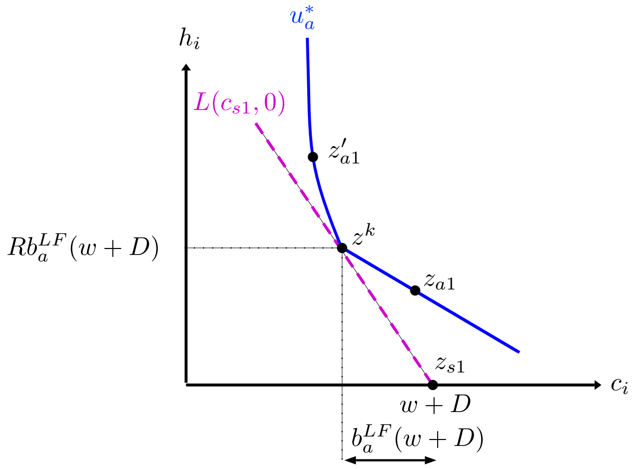

Figure 13: Illustration of bundle z_{a1}.

We show that conditions (i), (ii) and (iii) imply that there exists a sustainable scheme $(\overline{\tau}, \overline{D})$ whose long-run equilibrium allocation is strictly preferred by SWF $\boldsymbol{R}^{c\text{-}lex}$ to z. Conditions (ii) and (iii) together imply that $\int_i \overline{\tau}(\bar{b}_i)di > \int_i \tau(b_i)di$, because for any X, the measure of altruistic individuals with number X is the same under both schemes. As D is the maximal sutainable demogrant for τ, we have $D = \int_i \tau(b_i)di$. This implies that $\int_i \overline{\tau}(\bar{b}_i)di > D$, and thus Assumption **A3** implies there exists some $\overline{D} > D$ such that scheme $(\overline{\tau}, \overline{D})$ is sustainable. Let $\tilde{z}$ denote the long-run allocation associated to scheme $(\overline{\tau}, \overline{D})$. As $\overline{D} > D$, we have that $\min_{i\in[0,1]} u^c(\tilde{z}_i, u_i) > \min_{i\in[0,1]} u^c(\overline{z}_i, u_i)$. By (i), we have $\min_{i\in[0,1]} u^c(\tilde{z}_i, u_i) > \min_{i\in[0,1]} u^c(z_i, u_i)$, the desired result.

There remains to show that we can construct $\overline{\tau}$ in a way that meets conditions (i), (ii) and (iii). We consider two cases:

Case 1: $b_{a2} \geq b_a^{LF}(w+D)$.

The construction of $\overline{\tau}$ is based on bundle z_{a2}. We start by showing that z_{a2}, whose position is illustrated in Figure 14, must satisfy the following two conditions:

(1) $u_a^*(z_{a2}) \geq u_a^*(c_{a1} + h_{a1}, h_{a1})$ and

(2) $\tau(b_{a2}) \geq 0$.

First, by incentive compatibility, individual $a2$ must prefer to leave bequest b_{a2} rather than b_{a1}, which by Eq. (4) yields condition (1). As illustrated in Figure 14, condition (1) implies that bundle z_{a2} lies in the upper contour set of the indifference curve passing through bundle $(c_{a1}+h_{a1}, h_{a1})$. Second, by incentive compatibility again, individual $a1$ must prefer to leave bequest b_{a1} rather than any other bequest $b > b_{a1}$. This implies that bundle z^k does not lie in the interior of her budget set $B^\tau(w+D, 0)$, and therefore $\tau(b_a^{LF}(w+D)) \geq 0$. As $-\tau$ is single-peaked, the fact that we simultaneously have

- $\tau(b_{a1}) < 0$ and
- $\tau(b_a^{LF}(w + D)) \geq 0$,

implies that $\tau(b_{a2}) \geq 0$ because $b_{a1} < b_a^{LF}(w + D) \leq b_{a2}$. Graphically, bundle z_{a2} must be in the non-distortionary budget set $L(c_{s2}, 0)$. Hence, condition (2) is also met, as desired.

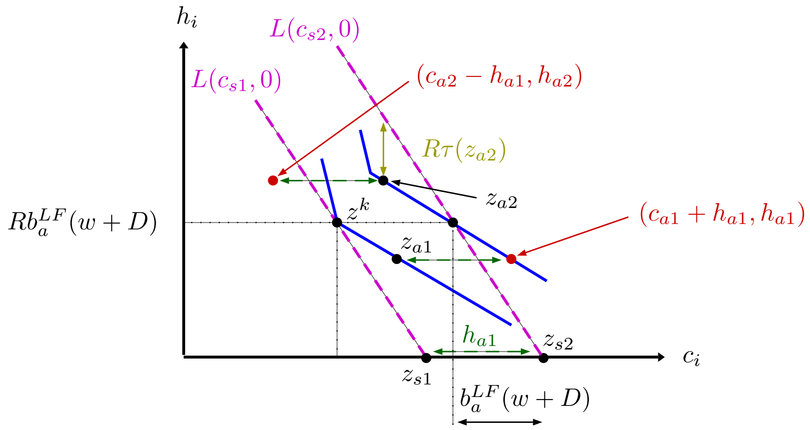

Figure 14: Position of bundle z_{a2} when $b_{a2} \geq b_a^{LF}(w + D)$ (Case 1).

Tax $\overline{\tau}$ is constructed from tax τ and bundle z_{a2}. In a nutshell, $\overline{\tau}$ provides an exemption up to $b_a^{LF}(w + D)$, then $\overline{\tau}$ is defined by a constant marginal tax rate such that $\overline{\tau}(b_{a2}) = \tau(b_{a2})$, and then $\overline{\tau}$ remains equal to τ for larger bequests. The implied budget sets $B^\tau(w+D, 0)$ and $B^{\overline{\tau}}(w+D, 0)$ are contrasted in Figure 15. Formally, tax $\overline{\tau}$ is defined as

- $\overline{\tau}(b) = 0$ for all $b \in [0, b_a^{LF}(w + D)]$.
- $\overline{\tau}(b) = \frac{b - b_a^{LF}(w+D)}{b_{a2} - b_a^{LF}(w+D)} \tau(b_{a2})$ for all $b \in [b_a^{LF}(w + D), b_{a2}]$,
- $\overline{\tau}(b) = \tau(b)$ for all $b \geq b_{a2}$.

Tax $\overline{\tau}$ is monotonically increasing up to bequest level b_{a2} by construction, because its constant marginal tax rate is non-negative because $\tau(b_{a2}) \geq 0$ (condition 2). This implies that $-\overline{\tau}$ is single-peaked because τ is monotonically increasing for $b \geq b_{a2}$.[28]

We now prove that tax $\overline{\tau}$ satisfies (i), (ii) and (iii). First, we prove (i), namely that $\min_{i \in [0,1]} u^c(\overline{z}_i, u_i) = \min_{i \in [0,1]} u^c(z_i, u_i)$. (Recall that $\overline{z}$ denotes the long-run allocation associated to scheme $(\overline{\tau}, D)$, where D is the maximal sustainable demogrant for tax τ.) Under tax τ we have $\min_{i \in [0,1]} u^c(z_i, u_i) = u^c(z_{s1}, u^{\mathrm{s}}) = u^c(z_{a1}, u_a^*) = w + D$. Under tax $\overline{\tau}$ we have that $\overline{z}_{a1} = z^k$, which

[28] Tax τ is monotonically increasing for $b \geq b_{a2}$ because $-\tau$ is single-peaked, $\tau(b_{a1}) < 0$ and $\tau(b_{a2}) \geq 0$.

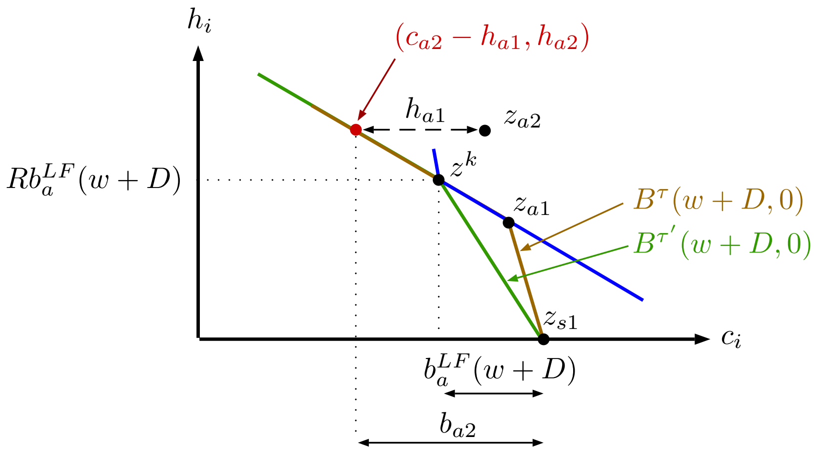

Figure 15: Budget sets for individuals who ineherit zero as implied by taxes τ and $\overline{\tau}$.

implies that $u^c(\overline{z}_{a1}, u_a^*) = w + D$. Given that we also have $u^c(\overline{z}_{s1}, u^{\mathbf{s}}) = w + D$, this yields again $\min_{i \in [0,1]} u^c(\overline{z}_i, u_i) = w + D$ (Lemma 1). This proves (i).

Second, we prove (ii), namely that $\overline{\tau}(\overline{b}_{a1}) > \tau(b_{a1})$. We have $\overline{\tau}(\overline{b}_{a1}) = 0$ by construction of $\overline{\tau}$ because $\overline{b}_{a1} = b_a^{LF}(w+D)$ as $\overline{z}_{a1} = z^k$. As a result, (ii) follows immediately from the fact that $\tau(b_{a1}) < 0$.

Third, we prove (iii), namely that $\overline{\tau}(\overline{b}_{aX}) \geq \tau(b_{aX})$ for all $X \in \{2, 3, \dots\}$. To do this, we show that $\overline{b}_{aX} \geq b_{aX}$ for all $X \in \{2, 3, \dots\}$. If this holds, then (iii) holds because $\overline{\tau}(b) = \tau(b)$ for all $b \geq b_{a2}$ and tax τ is monotonically increasing for $b \geq b_{a2}$. We prove by induction that $\overline{b}_{aX} \geq b_{aX}$ for all $X \in \{2, 3, \dots\}$. The induction basis requires us to show that $\overline{b}_{a2} \geq b_{a2}$. First, we show that $\overline{b}_{a2} \geq b_{a2}$ for the particular (counterfactual) case for which $\overline{h}_{a1} = h_{a1}$. This case implies that $a2$ receives the same inheritance under both schemes (τ, D) and $(\overline{\tau}, D)$. As a_2 selects z_{a2} under scheme (τ, D) rather than bundle $(c_{a1} + h_{a1}, h_{a1})$ (see Figure 14), this implies that $a2$ will prefer to leave a bequest of at least $b = b_{a2}$ under scheme $(\overline{\tau}, D)$ when $\overline{g}_{a2} = h_{a1}$ rather than another smaller bequest $b' < b_{a2}$.[29] As inheritance is a normal good under preference u_a^*, we also have $\overline{b}_{a2} \geq b_{a2}$ when $\overline{h}_{a1} > h_{a1}$, which is the case as $\overline{z}_{a1} = z^k$. This concludes the proof of the induction basis. There remains to prove the induction step, namely that $\overline{b}_{aX+1} \geq b_{aX+1}$ when $\overline{b}_{aX} \geq b_{aX}$. We have $\overline{g}_{aX+1} \geq g_{aX+1}$ because $\overline{b}_{aX} \geq b_{aX}$ and $\overline{\tau}(b) = \tau(b)$ for all $b \geq b_{a2}$. In words, individual $aX+1$ receives a (weakly) larger inheritance under scheme $(\overline{\tau}, D)$ than under scheme (τ, D). This implies that $aX+1$ will leave at least as much bequest under $(\overline{\tau}, D)$ than under (τ, D) because inheritance is a normal good under preference u_a^* and $\tau(b) = \overline{\tau}(b)$ for $b \geq b_{a2}$. This implies that $\overline{b}_{aX+1} \geq b_{aX+1}$, which proves the induction step. We

[29]Graphically, the marginal rate of substitution $-r$ of her indifference curves is (weakly) less steep than the frontier of her budget set for bequest levels in $[b_a^{LF}(w+D), b_{a2}]$, which has a constant slope because the marginal rate of taxation is constant on that interval. If the frontier of $a2$'s budget set under scheme $(\overline{\tau}, D)$ was strictly less steep than $-r$, then it would imply that $u_a^*(z_{a2}) < u_a^*(c_{a1} + h_{a1}, h_{a1})$, which would violate the incentive compatibility constraint of $a2$ under scheme (τ, D).

have thus shown that tax $\overline{\tau}$ satisfies (i), (ii) and (iii), as desired.

Case 2: $b_{a2} < b_a^{LF}(w + D)$.

The construction of $\overline{\tau}$ is the same as that provided for Case 1, except that here the construction is not based on bundle z_{a2}. Rather, it is based on bundle $z_{a\overline{X}}$, where $\overline{X}$ is the smallest element in $\{3, 4, \dots\}$ for which $b_{a\overline{X}} \geq b_a^{LF}(w+D)$. We show there must exist such $\overline{X} \in \{3, 4, \dots\}$.

First, we show that $u_a^*(z_{aX}) = u_a^*(c_{a1} + h_{aX-1}, h_{a1})$ for all $X \in \{1, \dots, \overline{X} - 1\}$, namely that the incentive compatibility constraints are binding for these individuals aX. By the incentive compatibility constraint of aX, which should prefer her bundle over $(c_{a1} + h_{aX-1}, h_{a1})$, we have

$$u_a^*(z_{aX}) \geq u_a^*(c_{a1} + h_{aX-1}, h_{a1}). \tag{16}$$

By the the incentive compatibility constraint of $a1$, which should prefer her bundle over $(c_{aX} - h_{aX-1}, h_{aX})$, we have

$$u_a^*(z_{a1}) \geq u_a^*(c_{aX} - h_{aX-1}, h_{aX}). \tag{17}$$

By definition of $\overline{X}$ we have $b_{aX} < b_a^{LF}(w + D)$ for all $X \in \{1, \dots, \overline{X} - 1\}$. Therefore, we only need to consider the linear part of u_a^* (the kinks of the relevant indifference curves lies at larger bequest levels). Then, Eqs. (16) and (17) together imply that $u_a^*(z_{aX}) = u_a^*(c_{a1}+h_{aX-1}, h_{a1})$ for all $X \in \{1, \dots, \overline{X}- 1\}$, as desired.

We can now show there must exist some $\overline{X} \in \{3, 4, \dots\}$ such that $b_{a\overline{X}} \geq b_a^{LF}(w+D)$ if τ is optimal. If there is no such $\overline{X}$, then we have $b_{aX} < b_a^{LF}(w+D)$ for all $X \in \{3, 4, \dots\}$. This implies that $\tau(b_{aX}) < 0$ for all $X \in \{1, 2, \dots\}$ because $u_a^*(z_{aX}) = u_a^*(c_{a1} + h_{aX-1}, h_{a1})$ for all $X \in \{1, \dots, \overline{X} - 1\}$, which implies that $(c_{aX} - h_{aX-1}, h_{aX})$ lies on the indifference curve passing through z_{a1}. The fact that $\tau(b_{aX}) < 0$ for all $X \in \{1, 2, \dots\}$ implies that τ is not optimal because then $D < 0$ (Proposition 1). This contradiction implies that such $\overline{X}$ exists.

The argument that $\overline{\tau}$ (when its construction is based on $z_{a\overline{X}}$ rather than z_{a2}) satisfies properties (i), (ii) and (iii) is a straightforward adaptation of the argument presented for Case 1, and is thus omitted.

S4 Proof of Proposition 4

The following axiom is the well-known Separability axiom, according to which individuals who are assigned identical bundles in two allocations should not matter for the social ranking between these two allocations. The idea that they should not matter is captured by the requirement that the social ranking remain the same if the preferences and bundles assigned to these individuals change in such a way that the bundles assigned to these individuals remain identical in the two allocations.

Axiom 4 (Separability)**.**
For all economy $u \in \mathcal{U}$*, steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}$*,* $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]}$*,* $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]}$*,* $z''' = (g'''_i, c'''_i, h'''_i)_{i\in[0,1]} \in S$*, subset of individuals* $J \in M[0,1]$*, if*

- *for all* $j \in J$*:* $z_j = z''_j$ *and* $z'_j = z'''_j$*,*
- *for all* $j \in [0,1] \setminus J$*:* $u_j = u'_j$*,* $z_j = z'_j$ *and* $z''_j = z'''_j$*,*

then $z\ \boldsymbol{R}(u)\ z''$ *if and only if* $z'\ \boldsymbol{R}(u')\ z'''$*.*

The bite of this axiom is that it allows us to modify the economy in such a way that sets of individuals of positive measure have the same preferences, which is unlikely in a generic economy, whereas it is crucial to allow us to use Compensation (see Step 1 in the proof below). We now state and prove the following result, which justifies using SWF $\boldsymbol{R}^{c\text{–}lex}$.

Proposition 4. *If a SWF* (R) *satisfies axioms Pareto, Compensation, Responsibility and Separability, then for all* $u \in \mathcal{U}$*,* $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$*, if there exists* $J \in M[0,1]$ *such that* $\mu(J) > 0$ *and*

$$\sup_{j\in J} u^c(z_j, u_j) < \inf_{i\in[0,1]} u^c(z'_i, u_i)$$

then

$$z'\ \boldsymbol{P}(u)\ z.$$

Proof. This proof is reminiscent of similar proofs developed in models of labor income taxation in Fleurbaey and Maniquet (2006) and Fleurbaey and Maniquet (2007). The main differences are, first, that we deal here with economies with a continuum of individuals, which makes some arguments longer, whereas all individuals face the same prices (that is, the price of h is equal to R), which allows us to simplify the proof.

The proof is divided in three steps. In the first step, we show that the combination of the four axioms implies a strengthening of the Responsibility axiom in which inequality aversion is infinite. In the second step, we show that the infinite inequality aversion is extended to $u^c(z_i, u_i)$. In the final step, we show that this allows us to derive the desired property.

Step 1. We begin by defining the following strengthening of Responsibility.

Axiom 5 (Responsibility*)**.**
For all economy $u \in \mathcal{U}$*, steady-state allocations* $z = (g_i, c_i, h_i)_{i\in[0,1]}$*,* $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$*, subsets of individuals* $J, K \in M[0,1]$ *such that* $\mu(J) = \mu(K) > 0$*, if there exists* $\delta, \Delta > 0$ *such that for all* $j, q \in J$ *and* $k, \ell \in K$*,*

- $(c_i, h_i) \in \max_{|u_i} L(c_i, h_i),\ \forall\, i \in \{j, q, k, \ell\}$*,*
  $(c'_i, h'_i) \in \max_{|u_i} L(c'_i, h'_i),\ \forall\, i \in \{j, q, k, \ell\}$
- $y_j + \delta = y_q + \delta = y'_j = y'_q \leq y'_k = y'_\ell = y_k - \Delta = y_\ell - \Delta$*,*

where

$$y_i = c_i + \frac{h_i}{R}, y'_i = c'_i + \frac{h'_i}{R}, \forall\, i \in \{j, q, k, \ell\}$$

and $z_i = z'_i$ *for all* $i \notin J \cup K$ *then* $z'\ \boldsymbol{P}(u)\ z$*.*

With Responsibility*, we require strict social preference as soon as all individuals in J gain, even if their budget gain is arbitrarily small and the budget loss of members of K is arbitrarily large. This is why Responsibility*, contrary to Responsibility, conveys an infinite inequality aversion.

We prove the following claim: If a SWF (R) satisfies Pareto, Compensation, Responsibility and Separability, then it satisfies Responsibility*.

Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, and $J, K \in M[0,1]$ satisfy the conditions of Responsibility*. Let us assume, contrary to the claim, that $z\ \mathbf{R}(u)\ z'$. We can assume, without loss of generality, that $\mu(J) = \mu(K) \leq \frac{1}{4}$. If it is not the case, then the claim is proven by repeating this proof twice. Let $y_j, y'_j, y_k, y'_k \in \mathbb{R}_+$ be defined by

$$y_i = c_i + \frac{h_i}{R}\ \forall\ i \in \{j,k\},$$

so that $y_j < y'_j \leq y'_k < y_k$. By Pareto, we can assume, w.l.o.g., that $y'_j < y'_k$. Indeed, if it is not the case, then we can create $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]} \in S$ by replacing $z'_k = (g'_k, c'_k, h'_k)$ with $z''_k = (g''_k, c''_k, h''_k)$ such that

$$y'_k < y''_k = c''_k + \frac{h''_k}{R} < y_k$$

for all $k \in K$, so that $z''\ \mathbf{P}(u)\ z'$ and, by transitivity, $z''\ \mathbf{P}(u)\ z$ and continue the proof. So, we assume $y'_j < y'_k$. We can even assume, w.l.o.g., that

$$(y'_j - y_j) + (y_k - y'_k) < \frac{y_k - y_j}{2}.$$

Indeed, if it is not the case, then the claim is proven by repeating this proof the required number of times.[30]

Let $u^* \in U$ and $\bar{z}_a = (\bar{g}_a, \bar{c}_a, \bar{h}_a), \bar{z}'_a = (\bar{g}'_a, \bar{c}'_a, \bar{h}'_a), \bar{z}''_a = (\bar{g}''_a, \bar{c}''_a, \bar{h}''_a), \bar{z}'''_a = (\bar{g}'''_a, \bar{c}'''_a, \bar{h}'''_a), \bar{z}_b = (\bar{g}_b, \bar{c}_b, \bar{h}_b), \bar{z}'_b = (\bar{g}'_b, \bar{c}'_b, \bar{h}'_b), \bar{z}''_b = (\bar{g}''_b, \bar{c}''_b, \bar{h}''_b), \bar{z}'''_b = (\bar{g}'''_b, \bar{c}'''_b, \bar{h}'''_b) \in X$, $\bar{y}_a, \bar{y}'_a, \bar{y}_b, \bar{y}'_b \in \mathbb{R}$ be such that

$$\bar{y}_i = \bar{c}_i + \frac{\bar{h}_i}{R}, \bar{y}'_i = \bar{c}'_i + \frac{\bar{h}'_i}{R}, \forall\ i \in \{a,b\}$$

$$\begin{aligned} \bar{y}_a &= \bar{y}_b \\ \bar{y}_a - \bar{y}'_a &= y'_j - y_j \\ \bar{y}'_b - \bar{y}_b &= y_k - y'_k \\ y'_j &\leq \bar{y}'_a \\ \bar{y}'_b &\leq y'_k \end{aligned}$$

$$(\bar{c}_a, \bar{h}_a) \in \max\nolimits_{|u^*} L(\bar{c}_a, \bar{h}_a),\ (\bar{c}'_a, \bar{h}'_a) \in \max\nolimits_{|u^*} L(\bar{c}'_a, \bar{h}'_a),$$
$$(\bar{c}_b, \bar{h}_b) \in \max\nolimits_{|u^*} L(\bar{c}_b, \bar{h}_b),\ (\bar{c}'_b, \bar{h}'_b) \in \max\nolimits_{|u^*} L(\bar{c}'_b, \bar{h}'_b),$$

$$\bar{c}''_a < \bar{c}''_b, \bar{h}''_a < \bar{h}''_b,$$

$$(\bar{c}'''_a, \bar{h}'''_a) = (\bar{c}'''_b, \bar{h}'''_b) = \frac{(\bar{c}''_b, \bar{h}''_b) + (\bar{c}''_a, \bar{h}''_a)}{2}$$

[30] The fact that $y'_k > y'_j$ always allows us to construct sets A and B with $\bar{y}'_a < \bar{y}_a = \bar{y}_b < \bar{y}'_b$ and the proof below has to be replicated a finite number of times at least as large as $\frac{y'_j - y_j}{\bar{y}'_a - \bar{y}_a}$.

$$
\begin{aligned}
u^*(z_a) &= u^*(z_a''') \\
u^*(z_a') &= u^*(z_a'') \\
u^*(z_b') &= u^*(z_b'').
\end{aligned}
$$

The construction of $u^*, \bar{z}_a, \bar{z}_a', \bar{z}_a'', \bar{z}_a''', \bar{z}_b, \bar{z}_b', \bar{z}_b'', \bar{z}_b''', \bar{y}_a, \bar{y}_a', \bar{y}_b, \bar{y}_b'$ is illustrated in Fig. 16.

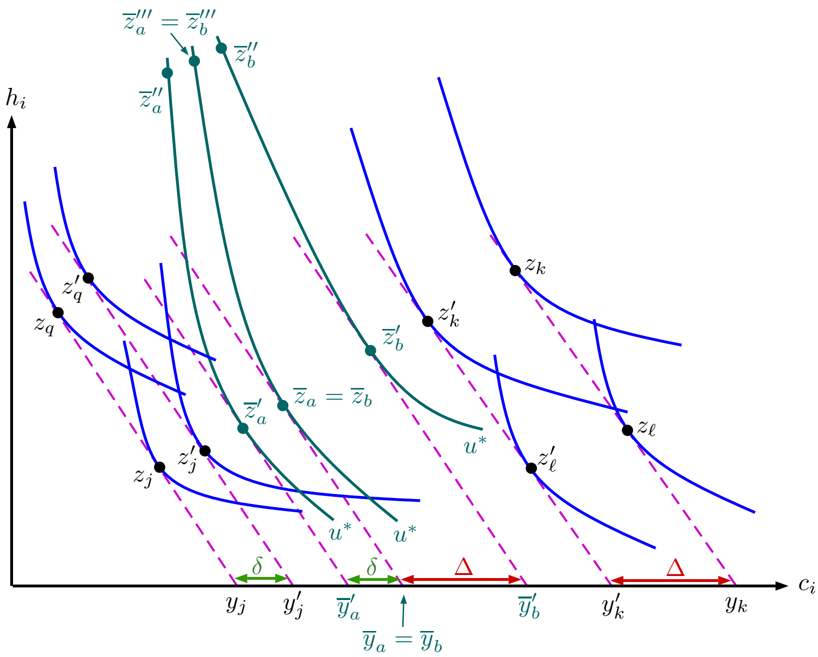

Figure 16: Construction of $u^*, \bar{z}_a, \bar{z}_a', \bar{z}_a'', \bar{z}_a''', \bar{z}_b, \bar{z}_b', \bar{z}_b'', \bar{z}_b''', \bar{y}_a, \bar{y}_a', \bar{y}_b, \bar{y}_b'$.

The intuition of the proof and the role of the axioms can be illustrated with the figure. We need to prove that the budget increase of an amount δ for individuals j and q at the expense of a budget decrease of an amount Δ for individuals k and ℓ, with Δ possibly much larger than δ, is a social improvement. Separability allows us to modify the preferences and bundles of a sufficiently large number of individuals and insert individuals of type a and b in the economy. The design of their preferences is key: they are indifferent between a transfer of resources, represented by bundles $\bar{z}_i''$ and $\bar{z}_i'''$, $i \in \{a, b\}$, in which the beneficiary gets an amount equal to that left by the contributor, and a transfer of resources in which the beneficiary gets a different amount of resources, possible much smaller, than the one lost by the contributor, represented by bundles $\bar{z}_i$ and $\bar{z}_i'$, $i \in \{a, b\}$. Pareto forces us to be indifferent between these two sets of transfers. Transfers are calibrated in such a way that a sequence of transfers between individuals j and a (using Responsibility), individuals a and b (using Compensation) and then individuals b and k (using Responsibility) allows us to reach the desired conclusion.

The following axiom, known in the literature as Pareto indifference, is a well-known consequence of Pareto.

Axiom 6 (Pareto Indifference)**.**
For all economy $u \in \mathcal{U}$ and steady-state allocations $z = (g_i, c_i, h_i)_{i\in[0,1]}, z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, if for all $i \in [0,1]$

$$u_i(c'_i, h'_i) = u_i(c_i, h_i)$$

then z' $\boldsymbol{I}(u)$ z.

Let $A, B \in M[0,1]$ be such that $\mu(A) = \mu(B) = \mu(J) = \mu(K)$, A, B, J and K are all disjoint. Since they are all disjoint, we have $(c_i, h_i) = (c'_i, h'_i)$ for all $i \in A \cup B$. Let $u' \in \mathcal{U}$ be defined by

$$\begin{aligned} u'_a &= u^*, \forall\, a \in A \\ u'_b &= u^*, \forall\, b \in B \\ u'_i &= u_i, \forall\, i \in [0,1] \setminus (A \cup B). \end{aligned}$$

Let allocations $z^1 = (g^1_i, c^1_i, h^1_i)_{i\in[0,1]}, z^2 = (g^2_i, c^2_i, h^2_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned} (c^1_a, h^1_a) = (c^2_a, h^2_a) &= (\bar{c}_a, \bar{h}_a), \forall\, a \in A \\ (c^1_b, h^1_b) = (c^2_b, h^2_b) &= (\bar{c}_b, \bar{h}_b), \forall\, b \in B, \end{aligned}$$

which implies $(c^1_a, h^1_a) = (c^1_b, h^1_b)$, and by

$$\begin{aligned} (c^1_i, h^1_i) &= (c_i, h_i), \forall\, i \in [0,1] \setminus (A \cup B), \\ (c^2_i, h^2_i) &= (c'_i, h'_i), \forall\, i \in [0,1] \setminus (A \cup B), \end{aligned}$$

and $g^1_i, g^2_i, i \in [0,1]$, are fixed so as to guarantee that $z^1, z^2 \in S$. By Separability,

$$z\ \mathbf{R}(u)\ z' \Leftrightarrow z^1\ \mathbf{R}(u')\ z^2,$$

so that, by the premise of the argument, $z^1\ \mathbf{R}(u')\ z^2$.

Let $z^3 = (g^3_i, c^3_i, h^3_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned} (c^3_a, h^3_a) &= (\bar{c}'_a, \bar{h}'_a), \forall\, a \in A \\ (c^3_j, h^3_j) &= (c'_j, h'_j), \forall\, j \in J \\ (c^3_i, h^3_i) &= (c^1_i, h^1_i), \forall\, i \in [0,1] \setminus (A \cup J), \end{aligned}$$

and $g^3_i, i \in [0,1]$, are fixed so as to guarantee that $z^3 \in S$. By Responsibility, $z^3\ \mathbf{P}(u')\ z^1$, so that, by transitivity, $z^3\ \mathbf{P}(u')\ z^2$.

Let $z^4 = (g^4_i, c^4_i, h^4_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned} (c^4_b, h^4_b) &= (\bar{c}'_b, \bar{h}'_b), \forall\, b \in B \\ (c^4_k, h^4_k) &= (c'_k, h'_k), \forall\, k \in K \\ (c^4_i, h^4_i) &= (c^3_i, h^3_i), \forall\, i \in [0,1] \setminus (B \cup K), \end{aligned}$$

and $g^4_i, i \in [0,1]$, are fixed so as to guarantee that $z^4 \in S$. By Responsibility, $z^4\ \mathbf{P}(u')\ z^3$, so that, by transitivity, $z^4\ \mathbf{P}(u')\ z^2$.

Let $z^5 = (g^5_i, c^5_i, h^5_i)_{i\in[0,1]} \in \mathbb{R}^{3[0,1]}_+$ be defined by

$$\begin{aligned} (c^5_a, h^5_a) &= (\bar{c}''_a, \bar{h}''_a), \forall\, a \in A \\ (c^5_b, h^5_b) &= (\bar{c}''_b, \bar{h}''_b), \forall\, b \in B \\ (c^5_i, h^5_i) &= (c^4_i, h^4_i), \forall\, i \in [0,1] \setminus (A \cup B), \end{aligned}$$

and $g_i^5, i \in [0,1]$, are fixed so as to guarantee that $z^5 \in S$. By Pareto Indifference, z^5 $\mathbf{I}(u')$ z^4, so that, by transitivity, z^5 $\mathbf{P}(u')$ z^2.

Let $z^6 = (g_i^6, c_i^6, h_i^6)_{i\in[0,1]} \in \mathbb{R}_+^{3[0,1]}$ be defined by

$$\begin{aligned}
(c_a^6, h_a^6) &= (\bar{c}_a''', \bar{h}_a'''), \forall\, a \in A \\
(c_b^6, h_b^6) &= (\bar{c}_b''', \bar{h}_b'''), \forall\, b \in B \\
(c_i^6, h_i^6) &= (c_i^5, h_i^5), \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and $g_i^6, i \in [0,1]$, are fixed so as to guarantee that $z^6 \in S$. By Compensation, z^6 $\mathbf{P}(u')$ z^5, so that, by transitivity, z^6 $\mathbf{P}(u')$ z^2.

Let $z^7 = (g_i^7, c_i^7, h_i^7)_{i\in[0,1]} \in \mathbb{R}_+^{3[0,1]}$ be defined by

$$\begin{aligned}
(c_a^7, h_a^7) &= (\bar{c}_a, \bar{h}_a), \forall\, a \in A \\
(c_b^7, h_b^7) &= (\bar{c}_b, \bar{h}_b), \forall\, b \in B \\
(c_i^7, h_i^7) &= (c_i^6, h_i^6), \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and $g_i^7, i \in [0,1]$, are fixed so as to guarantee that $z^7 \in S$. By Pareto Indifference, z^7 $\mathbf{I}(u')$ z^6, so that, by transitivity, z^7 $\mathbf{P}(u')$ z^2.

Let $z^8 = (g_i^8, c_i^8, h_i^8)_{i\in[0,1]}, z^9 = (g_i^9, c_i^9, h_i^9)_{i\in[0,1]} \in \mathbb{R}_+^{3[0,1]}$ be defined by

$$\begin{aligned}
z_a^8 &= z_a, \forall\, a \in B \\
z_b^8 &= z_b, \forall\, b \in B \\
z_i^8 &= z_i^7, \forall\, i \in [0,1] \setminus (A \cup B),
\end{aligned}$$

and

$$\begin{aligned}
z_a^9 &= z_a, \forall\, a \in B \\
z_b^9 &= z_b, \forall\, b \in B \\
z_i^9 &= z_i^2, \forall\, i \in [0,1] \setminus (A \cup B).
\end{aligned}$$

By Separability,

$$z^8 \ \mathbf{R}(u)\ z^9 \Leftrightarrow z^7 \ \mathbf{R}(u')\ z^2,$$

so that, by transitivity, z^8 $\mathbf{P}(u)$ z^9. Finally, observe that $z^8 = z^9 = z'$, so that z' $\mathbf{P}(u)$ z', the desired contradiction.

Step 2. We now prove the following claim: If a SWF (R) satisfies axioms Pareto, Compensation, Responsibility and Separability, then it satisfies the following property, which amounts to requiring an infinite aversion towards inequality in u^c: For all $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g_i', c_i', h_i')_{i\in[0,1]} \in S$, $J, K \in M[0,1]$ such that $\mu(J) = \mu(K) > 0$, if for all $j \in J$ and $k \in K$,

- $u^c(z_j, u_j) < u^c(z_j', u_j) < u^c(z_k', u_k) < u^c(z_k, u_k)$,
- $\sup_{i\in J} u^c(z_i, u_i) < \inf_{i\in J} u^c(z_i', u_i)$,

and $z_i = z_i'$ for all $i \notin J \cup K$ then z' $\mathbf{P}(u)$ z.

Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g_i', c_i', h_i')_{i\in[0,1]} \in S$, and $J, K \subseteq M[0,1]$ satisfy the conditions of this property. Let us assume, contrary to the claim, that z $\mathbf{R}(u)$ z'.

By Pareto Indifference, we can assume, without loss of generality, that

$$(c_j, h_j) \in \max_{|u_j} L(c_j, h_j),\ (c'_j, h'_j) \in \max_{|u_j} L(c'_j, h'_j),$$
$$(c_k, h_k) \in \max_{|u_k} L(c_k, h_k),\ (c'_k, h'_k) \in \max_{|u_k} L(c'_k, h'_k).$$

Indeed, if it is not the case, then, by Pareto Indifference, we can replace bundles z_j, z'_j and z_k, z'_k by a bundle on the same indifference curve that is optimal in the corresponding budget.

By Pareto, we can assume that for all $j, q \in J$, $c_j + \frac{h_j}{R} = c_q + \frac{h_q}{R}$ and for all $k, \ell \in J$, $c_k + \frac{h_k}{R} = c_\ell + \frac{h_\ell}{R}$. Indeed, if it is not the case, we can replace each z_j with z''_j such that $u^c(z''_j, u_j) = \sup_{i\in J} u^c(z_i, u_i)$, each z'_j with z'''_j such that $u^c(z'''_j, u_j) = \inf_{i\in J} u^c(z_i, u_i)$, each z_k with z''_k such that $u^c(z''_k, u_k) = \sup_{i\in K} u^c(z_i, u_i)$, each z'_k with z'''_k such that $u^c(z'''_k, u_k) = \inf_{i\in K} u^c(z_i, u_i)$. By Pareto, $z'' \ \mathbf{P}(u)\ z$ and $z' \ \mathbf{P}(u)\ z'''$, so that $z'' \ \mathbf{P}(u)\ z'''$, and the proof continues.

By Responsibility*, $z' \ \mathbf{P}(u)\ z$, so that, by transitivity, $z' \ \mathbf{P}(u)\ z'$, the desired contradiction.

Step 3. We now prove the claim presented in the statement of the Proposition. Let $u \in \mathcal{U}$, $z = (g_i, c_i, h_i)_{i\in[0,1]}$, $z' = (g'_i, c'_i, h'_i)_{i\in[0,1]} \in S$, $J \in M[0,1]$ such that $\mu(J) > 0$ and

$$\sup_{j\in J} u^c(z_j, u_j) < \inf_{i\in[0,1]} u^c(z'_i, u_i).$$

Let us assume, contrary to the claim, that $z \ \boldsymbol{R}(u)\ z'$. Let $z'' = (g''_i, c''_i, h''_i)_{i\in[0,1]} \in S$ be such that $u^c(z''_i, u_i) = u$ for all $i \in [0,1]$ and

$$\sup_{j\in J} u^c(z_j, u_j) < u < \inf_{i\in[0,1]} u^c(z'_i, u_i).$$

Let $N \in \mathcal{N}$ be an integer such that $N\mu(J) > 1$. Let $J' \subseteq J$ be such that $\mu(J') = \frac{1-\mu(J)}{N}$. We can create a sequence $z^0, \ldots, z^n, \ldots, z^N$ such that $z^0_i = z_i$ for all $i \in [0,1] \setminus (J \setminus J')$, $u^c(z^0_j, u_j) = u$ for all $j \in J \setminus J'$, $z^N = z''$ and for each $n \in \{1, \ldots, N\}$, there exists a set $K^n \in M[0,1]$ such that $\mu(K^n) = \mu(J')$, $\cup_{n\in\{1,\ldots,N\}} K^n \cup J = [0,1]$,

$$\begin{aligned}
u^c(z^n_k, u_k) &= u, \forall\, k \in K^n \\
u^c(z^n_j, u_j) &= u^c(z^{n-1}_j, u_j) + \frac{1}{N}\left(u - u^c(z_j, u_j)\right), \forall\, j \in J' \\
u^c(z^n_j, u_j) &= u, \forall\, k \in J \setminus J' \\
z^n_i &= z^{n-1}_i, \forall\, i \in [0,1] \setminus (K^n \cup J').
\end{aligned}$$

By Pareto, $z^0 \ \boldsymbol{P}(u)\ z$. By the property proven in Step 3, $z^n \ \boldsymbol{P}(u)\ z^{n-1}$. By transitivity, $z'' \ \boldsymbol{P}(u)\ z$. By Pareto, $z' \ \boldsymbol{P}(u)\ z''$, so that $z' \ \boldsymbol{P}(u)\ z$, the desired contradiction.

■