

Motivations, Monitoring Technologies, and Pay for Performance

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WORLD BANK GROUP

Development Economics Vice Presidency

Operations and Strategy Unit

December 2014

Abstract

Monitoring technologies and pay for performance contracts are becoming popular solutions to improve public services delivery. Their track record is however mixed. To show why this may be the case, this paper develops a principal agent model where agents' motivations vary and so does the effectiveness of monitoring technologies. In such a set-up the model shows that: (i) monitoring technologies should be introduced only if agents' motivations are poor; (ii) optimal pay for performance contracts are nonlinear/

non-monotonic in agents' motivations and monitoring effectiveness; (iii) investments aimed at improving agents' motivations and monitoring quality are substitutes when agents are motivated, complements otherwise; and (iv) if the agents' "type" is private information, the more and less motivated agents could be separated through a menu of pay for performance/non pay for performance contracts, such that only the less motivated choose the pay for performance ones.

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Motivations, Monitoring Technologies, and Pay for Performance*

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JEL Classification Numbers: D82, J33, J45, M52.

Keywords: Pay for Performance, Public Sector Management, Information and Communication Technologies, Asymmetric Information, Agents' Motivation, Optimal Contracts.

*We would like to thank David Rosenblatt and James Trevino for helpful comments and suggestions. The usual disclaimers apply.

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1 Introduction

In the last two decades, governments across the world have invested massively in monitoring and reporting technologies to improve the quality of public service delivery. The idea that such technologies promote efficiency gained increasing consensus in managerial circles, and it quickly spread to private companies and multilateral organizations.¹

But what are the channels through which monitoring and reporting technologies contribute to an improvement in public sector performance and to the provision of better services? According to the New Public Management school (NPM hereinafter), the road to efficiency is paved by the three “Ms”: markets, managers and measurement (Ferlie et al., 1996), and measurement is what markets and managers have to rely upon to be able to exert control and enforce pay for performance contracts² (PFP hereinafter). While a lot has been written on the effects of the introduction of PFP on the productivity of public sector organizations (Frey et al., 2013; Moynihan and Pandey, 2010; Weibel et al., 2010), much less has been written on the impact of investments made to increase measurability in public sector PFP schemes. This is quite surprising when many large ICT investments have been justified on the premise that enhanced monitoring and reporting technologies are key elements to improve organizational performances (Brynjolfsson et al., 2000; Dunleavy and Carrera, 2013; Garicano and Heaton, 2010).

To better understand the trade-offs associated with measurability, this paper provides a simple theoretical framework to analyze the channels through which monitoring and reporting technologies may (or may not) increase the effectiveness of PFP schemes. More precisely, we consider a stylized framework in which the measurement of outcomes is costly, and the alignment between the objectives of the agents and those of the principal is only partial. In such a set up, we show that (i) it is optimal for the principal to introduce a monitoring system only if the latter does not impose a too high burden on the agents, and/or if the agents are not sufficiently motivated; (ii) the design of an effective PFP is complicated, and the optimal contract is highly non linear and/or non monotonic both in agents’ motivations and in the “cost” of the monitoring system; (iii) investments aimed at improving agents’ motivations and the quality of the monitoring are complements when agents are highly motivated and substitutes when they are not; (iv) if the agents’ “type” is private information, an effective way for the principal to separate the more motivated from the less motivated agents is to offer a menu of contracts designed in a way that only the latter choose the PFP.

The above findings may shed a new light on the fierce debate on public administration reforms and on the role played by e-government investments aimed at increasing the measurability and hence transparency and accountability of public sector organization (Barzelay, 2001; Bertot et al., 2010; Dunleavy et al., 2005; Pina et al., 2007). On one side, NPM advocates argue that investments in technologies that increase measurability boost organizations’ productivity by facilitating the alignment of public servants’ motivations with predefined organizational objectives (Aral et al., 2012; Ba et al., 2001). NPM advocates also point at the increasing popularity of PFP and e-government projects around the world as a measure of their success.³ On the opposite side, NPM critics argue that the increasing reliance of government programs on PFP reforms is a fad driven by consulting firms, which by no means is justified by the actual record of PFP or of e-government solutions.⁴

Our own reading of the literature is that, overall, the adoption of PFP and the diffusion of e-government programs in the public sector has delivered mixed outcomes. Our model, suggesting that no one-size-fits-all solution exists, may thus provide a clear rationale for why this may be the case.

Of course, we are not the first who have looked at measurability in a principal agent framework; our model builds upon Holmstrom and Milgrom (1991),⁵ which first suggested that if agents have to perform

¹For instance, at the World Bank increased attentions is being paid on “deliverology,” that is, on how to maximize the developmental impact of the different programs by taking into account the incentives of the different stakeholders.

²See Picot et al. (1996).

³See, for instance, OECD (2005).

⁴See, among others, Francois (2000), Moynihan and Pandey (2007), and especially Perry et al. (2009) who offer a comprehensive review of the effects of PFP schemes on public sector organizations and come to the conclusions that performance-related pay consistently fails to deliver on its promises.

⁵For a comprehensive survey to the theoretical and empirical work on the provision of incentives in firms, see Prendergast

multiple tasks, some monitorable and some not, incentive based contracts, which (necessarily) focus on the latter, may induce agents to reallocate effort in an inefficient way. Given that most of the goals associated with the actions of public sector organizations are by nature not univocal and cannot not always be planned and defined before their executions (Moore, 2005; Alford and Huges, 2008), it is difficult to map them in performance indicators (Proper et al. 2003; Behn 2003). Baker (2002), Langbein (2010), and Grand (2010) provide comprehensive discussions of the costs and benefits of using PFP when goals are not univocal and/or quantifiable and performance indicators are difficult to establish. However, to our knowledge, there is no contribution that discusses how investments in monitoring technologies affect the enforcement of PFP schemes in such an environment.

Our main contribution to this literature is in explicitly modeling the costs associated with the introduction of monitoring and reporting technologies—the costs of managerial attention, according to Halac and Prat (2014)—and in studying how the interaction between such costs and agents’ motivations affects the optimal PFP scheme. Agents’ motivations, in our view, are indeed a critical factor to take into consideration when discussing PFP. In this dimension, we build upon Dixit (2002) who emphasizes that many public sector employees (judges, teachers, doctors, social workers) may share some “idealistic or ethic purpose served by the agency” (p. 715). Starting from such a premise, Delfgaauw and Dur (2008) show that a PFP system, offering steep incentives to the more dedicated workers, may help attract them to the public sector. Our model shares some of Delfgaauw and Dur’s (2008) features. However, in our set-up, performance assessment schemes detract resources from the ultimate goals of the agency, and this leads us to reach the opposite conclusions, that is, PFP may end up inducing the more motivated agents to leave the organization. This phenomenon may also be seen as a reflection of tensions between intrinsic and extrinsic motivations, as in Kreps (1997), and Benabou and Tirole (2003).

Another insight of this paper, namely the fact that optimal PFP and the associated investments in monitoring and reporting technologies have to be tailored according to output measurability, is also in line with empirical evidence. Hasnain and Pierskalla (2012), in an up to date and comprehensive survey of the (empirical) literature, find evidence that, when tasks are simple and outcomes observable, the use of PFP is more effective; at the same time, monitoring and reporting technologies are also more valuable in supporting PFP schemes (Ciborra, 1996) both because of moral hazard (i.e., incentive) and adverse selection (i.e., sorting more able workers, Lazear, 2000) considerations. Instead, when tasks are complex and outputs difficult to measure, the introduction of PFP schemes and the use of monitoring and reporting technologies can create distortions on incentives (e.g., discouraging the most motivated workers) or foster the wrong type of sorting. This means that the decision of whether to adopt PFP schemes, their optimal design, and the decision of how much to invest in monitoring and reporting technologies should take all these effects into consideration.

A less abstract description of the kind of problems we address, and a spicier flavor of our main results, may be derived from the following two examples, one in health care and the other in education. Consider first a hospital director who wants to improve the quality of patients’ care that, simplifying, depends on the number of hours doctors work and on the quality of the care they provide. Assume that hours are observable but quality not. In order to improve doctors’ incentives, the director may consider linking their compensation not only to the hours they spend in the hospital, but also to the quality of the care they provide. Since the latter is not directly measurable, the hospital can set up a costly monitoring system based on the doctors’ record of how they take care of each patient, and make part of the doctors’ pay linked to the quality of their respective records. Of course, filling a detailed record detracts precious time from actual patients’ care, so that the optimal PFP should carefully weigh the monitoring system’s costs and benefits.

Consider now the case of a school principal who cares about students’ learning that, simplifying again, depends on the number of hours kids are taught and on the quality of teaching. As before, assume that hours are observable, but quality (of teaching) is not. In order to improve teachers’ incentives to teach well, the principal may consider linking teachers’ compensation to the results of a proficiency test that students are asked to take. Since the results of such a test are an imperfect measure of what kids have

(1999).

actually learned at school, and the preparation of such tests is costly (in terms of hours subtracted from actual teaching), we are again in the presence of trade-offs. Clearly, the more committed doctors are to patients’ care, the more committed teachers are to education, the costlier the monitoring schemes—both in terms of set-up expenses and administrative effort—the larger is the deadweight loss associated with the PFP schemes. An additional cost to be taken into consideration is the one associated with the possibility that the more committed professionals may consider leaving workplaces where too much effort is devoted to costly performance measurement.⁶ These are the kinds of real world issue that our stylized model tries to address. The remaining of the paper is organized as follows: the next section presents the basic model, solves it when “quality” is observable and unobservable, and then derives the main results for the case in which it is observable if a costly (for the agent) performance assessment system is adopted. Section 3, relaxes the assumptions that the principal cannot invest to improve the assessment technology, and that the introduction of the performance assessment scheme does not affect the intrinsic motivations of the agents. In section 4, the analysis is extended to discuss the case where agents’ motivations are heterogeneous, and they are not observable by the principal. Finally, section 5 concludes.

2 The Model

Assume that an organization (the principal hereinafter) wants to maximize the success of a specific activity, the quality of education in our last example, which depends upon the contribution of two distinct but complementary components y and q . Let y be the component which is easy to verify/contract upon, and q the one which is not. One can think at y as a quantitative component, the hours taught in the same example, and q as a more qualitative one, the quality of teaching. We further assume that the success of the project is given by qy , and that the principal, who cares about the success of the project: (i) has a limited budget T , and (ii) he has to delegate the implementation of the activity to a “partially motivated” agent. By this, we mean that the agent does care about the principal’s objective, but she also cares about her remuneration, and she receives negative utility from the effort she devotes to the different components of the project.

In what follows, we first present the two extreme cases, in which q is either perfectly observable or totally unobservable, and discuss the optimal remuneration schemes that the principal can offer in either case. This allows us to get a clear understanding of the distortions associated with incomplete contracting, and it facilitates the discussion of the third, most interesting case, in which q is observable, but only at a cost.

As standard in the literature, we restrict our attention to linear contracts; in addition, to spare the reader the tedious algebra, in the main text we present the results in an intuitive way, and we refer the reader to the Appendix for the technical details.

2.1 Observable “quality”

To set an efficiency yardstick, we assume that both y and q , are monitorable/contractible. When this is the case, the principal,⁷ who is interested in maximizing qy , offers a compensation package $\{w, k\}$, where w is the compensation⁸ per unit of y , and k the compensation per unit of q . Denoting by subscript O the observable case, the problem of the agent can be written as:

$$\underset{y,q}{Max} U_O = \alpha qy + (wy + kq) - \frac{\gamma}{2}(q + y)^2, \quad (1)$$

where $\alpha > 0$ is a measure of the alignments between the objectives of the principal and those of the agent, $wy + kq$ the agent’s remuneration, which enters linearly in the objective function, and $c(q, y) = \frac{\gamma(q+y)^2}{2}$ is

⁶This may explain why US private schools, where there is no testing, may not only attract highly qualified teachers, but can also pay them lower wages than in the public schools where test preparation is becoming an increasing burden.

⁷Throughout the model, we assume that the principal moves first and offers a non-renegotiable contract to the agent.

⁸Here we remain vague about what compensation exactly means. In a market environment, it can be the price/salary paid for each of the activities delivered. In a non market environment, it can be the budget allocated to different teams.

the cost of effort. We further assume that:⁹

$$\gamma \in \left(\frac{\alpha}{2}, \alpha\right). \quad (\text{A.1})$$

Solving for (1), we obtain:

$$y_O^*(w, k) = \frac{k(\alpha - \gamma) + w\gamma}{\alpha(2\gamma - a)}, \quad (2)$$

$$q_O^*(w, k) = \frac{w(\alpha - \gamma) + k\gamma}{\alpha(2\gamma - \alpha)}, \quad (3)$$

where $y_O^*(w, k)$, and $q_O^*(w, k)$ denote the optimal choice of the agent for any given compensation package $\{w, q\}$. The principal's problem can now be written as:

$$\underset{w, k}{Max} V_O = q_O^*(w, k)y_O^*(w, k), \text{ such that: } q_O^*(w, k)k + y_O^*(w, k)w \leq T, \quad (4)$$

the latter expression denoting the principal's budget constraint. The solution of the problem is given by:

$$w_O^* = k_O^* = \sqrt{\frac{T(2\gamma - \alpha)}{2}}, \quad (5)$$

$$y_O^* = q_O^* = \sqrt{\frac{T}{2(2\gamma - \alpha)}}, \quad (6)$$

$$V_O^* = \frac{T}{2(2\gamma - \alpha)}. \quad (7)$$

Since the objective functions (of the principal and of the agent) are symmetric in y and q , when both activities are contractible, the optimal compensation scheme rewards them equally, so that the effort devoted to each of them is also equalized at equilibrium.

2.2 Unobservable "quality"

We now move to the situation in which only y is observable/contractible. Designating by subscript U the unobservable case, the problem of the agent can now be written as:

$$\underset{y, q}{Max} U_U = \alpha qy + wy - \frac{\gamma}{2}(q + y)^2. \quad (8)$$

Solving for (8), one obtains

$$y_U^*(w) = \frac{\gamma w}{\alpha(2\gamma - a)}, \quad (9)$$

$$q_U^*(w) = \frac{w(\alpha - \gamma)}{\alpha(2\gamma - \alpha)}, \quad (10)$$

where $q_U^*(w)$ and $y_U^*(w)$ denote the optimal choice of the agent for any given w . The problem of the principal can now be written as:

$$\underset{w}{Max} V_U = q_U^*(w)y_U^*(w), \text{ such that } wy_U^*(w) \leq T, \quad (11)$$

but, since the budget is given, and only y is contractible, the only option for the principal is to set

$$w_U^* = T/E[y_U(w)], \quad (12)$$

⁹ $\gamma > \alpha/2$ insures that the objective function is concave and that a maximum exists. $\alpha > \gamma$, instead, insures the existence of an interior maximum, with $q^* > 0$. The last assumption is not necessary, but it allows us to reduce the number of cases, making the analysis simpler, without great loss of generality.

where E denotes the expectation operator. Assuming rational expectations, the solution of the problem is given by:

$$w_U^* = \sqrt{\frac{T\alpha(2\gamma - \alpha)}{\gamma}}, \quad (13)$$

$$y_U^* = \frac{\sqrt{T\gamma}}{\sqrt{\alpha(2\gamma - \alpha)}}, \quad (14)$$

$$q_U^* = \frac{(\alpha - \gamma)\sqrt{T}}{\sqrt{\alpha\gamma(2\gamma - \alpha)}}, \quad (15)$$

$$V_U^* = \frac{T(\alpha - \gamma)}{\alpha(2\gamma - \alpha)}. \quad (16)$$

Comparing these results with the ones in the previous section, it is immediate to verify that $y_U^* > y_O^*$, and $q_U^* < q_O^*$. When compared with the situation in which both activities are contractible, the agent now overdelivers on the measurable activity y , and underdelivers on the non measurable one q . Of course, the utility of the principal is lower than when both activities are contractible, and the cost of contractual incompleteness decreases with α . In fact, when α increases, the agent puts additional effort on the non measurable activity even if the latter is not remunerated; this results in an increase in q_U^* , relative to y_U^* , and thus in a reduction in the distortions. This is the reason why, when α increases, given (12), the remuneration of the observable activity also increases (its “supply” decreases).

2.3 Costly observable “quality”

After having briefly discussed the cases in which “quality” is either perfectly observable or totally unobservable, we are now in a position to analyze the interesting case in which, while the principal cannot observe the agent’s choice of q , he can nonetheless introduce a costly (for the agent) performance assessment scheme, which allows him to extract information about her effort. More precisely, we assume that the principal relies on a performance assessment scheme, $s = s(q, e)$, that requires some additional effort, e , on the part of the agent. To keep the analysis simple, we posit

$$s(q) = \min\{q, \beta e\}. \quad (17)$$

This means that the effort that the agent devotes to q is “scored” in the performance assessment scheme only if she devotes an additional effort equal to βe , with $\beta > 0$. β is thus a measure of the efficacy of the assessment scheme, the larger is β , the more efficient the scheme is. Notice that if e/β can be interpreted literally at the additional cost that the agent should bear to have its performance properly assessed, it can be more generally thought of as a measure of how effectual the performance assessment system is. More precisely, e/β can be the cost of inducing the agent to invest in a (suboptimal) but contractible technology rather than in the optimal but not contractible one. We further assume that, in this case, the cost function becomes $c(q, y, e) = \gamma(q + y + e)^2$. Denoting by subscript C the costly observable quality case, the problem of the agent can be written as:

$$\underset{y, q, e}{Max} U_C = \alpha q y + (w y + k \min\{q, \beta e\}) - \frac{\gamma}{2}(q + y + e)^2, \quad (18)$$

which yields

$$y_C^*(w, k) = \frac{w(1 + \beta)^2\gamma - k\beta\xi}{\alpha\beta\zeta}, \quad (19)$$

$$q_C^*(w, k) = \frac{\beta(k\gamma - w\xi)}{\alpha^2\beta\zeta}, \quad (20)$$

$$e_C^*(w, k) = \frac{\beta(k\gamma - w\xi)}{\alpha^2\beta^2\zeta}, \quad (21)$$

with $\xi \equiv (1 + \beta)\gamma - \alpha\beta$, and $\zeta \equiv 2(1 + \beta)\gamma - \alpha\beta$. As before, $y_C^*(w, k)$, $q_C^*(w, k)$, and $e_C^*(w, k)$ denote the optimal choice of the agent given a compensation package $\{w, k\}$. The problem of the principal can now be written as:

$$\text{Max}_{w,k} V_C = q_C^*(w, k)y_C^*(w, k), \text{ such that: } wy_C^*(w, k) + kq_C^*(w, k) \leq T \quad (22)$$

which has the following solutions:

$$\begin{aligned} w_C^* &= \sqrt{\frac{T\zeta}{2(1+\beta)}}, & w_C^* = w_U^* &= \sqrt{\frac{T\alpha(2\gamma-\alpha)}{\gamma}}, \\ k_C^* &= \sqrt{\frac{T(1+\beta)\zeta}{2\beta^2}}, & k_C^* &= 0, \\ y_C^* &= T\sqrt{\frac{(1+\beta)}{2\zeta}}, & y_C^* = y_U^* &= \frac{\sqrt{T\gamma}}{\sqrt{\alpha(2\gamma-\alpha)}}, & \text{if } \beta < \bar{\beta}, \\ q_C^* &= \frac{\beta T}{\sqrt{2(1+\beta)\zeta}}, & q_C^* = q_U^* &= \frac{(\alpha-\gamma)\sqrt{T}}{\sqrt{\alpha\gamma(2\gamma-\alpha)}}, & \text{or } \alpha > \bar{\alpha}_C; \\ e_C^* &= \frac{T}{\sqrt{2(1+\beta)\zeta}}, & e_C^* &= 0, \\ V_C^* &= \frac{T\beta}{2\zeta}, & V_C^{*P} = V_U^* &= \frac{T(\alpha-\gamma)}{\alpha(2\gamma-\alpha)}, \end{aligned} \quad (23)$$

where $\bar{\alpha}_C \equiv \frac{2\gamma((1+\beta)-\sqrt{1+\beta})}{\beta}$, and $\bar{\beta} \equiv \frac{4(\alpha-\gamma)\gamma}{(2\gamma-\alpha)^2}$.

From a simple inspection of (23) it follows that

Result 1 *It is optimal for the principal to introduce a pay for performance scheme if, and only if: (i) the monitoring scheme is efficient enough ($\beta > \bar{\beta}$), or (ii) the agent is not sufficiently motivated ($\alpha < \bar{\alpha}_C$).*

To get a clear intuition of the results, it may be worth noticing that the observable and unobservable scenarios are the limit cases for the costly observable one, when β tends to infinity and zero, respectively. When β is large, the distortions associated with performance assessment schemes tend to vanish, “quality” becomes easy to observe, and this is reflected in the optimal contract. On the opposite, for sufficiently low values of β , the cost of using the assessment system is so high that it is in the interest of the principal to treat quality as non contractible. In other words, effort is costly for the agent and, thus, the introduction of an assessment scheme for incentive purposes is justified only when it does not detract an excessive amount of resources from the other, productive, activity. Similarly, the distortions induced by the assessment scheme are justified only if the agent is not already sufficiently committed to the goals of the organization. If she is, she will end up subtracting effort, which she would have otherwise devoted to the productive activity, to score better in the performance assessment scheme. If these results are very intuitive, the characteristics of the optimal compensation contract are less so. In particular, using the expressions in (23), we have¹⁰ that:

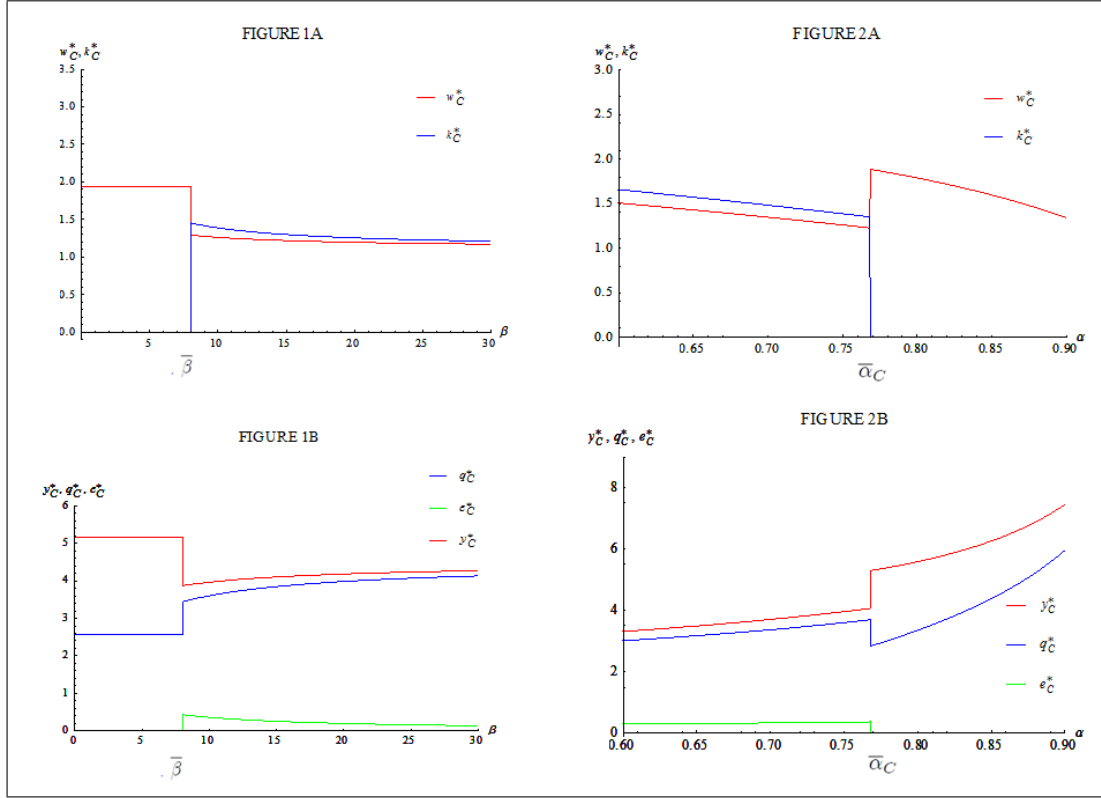
Result 2 *The optimal compensation scheme has the following characteristics: (i) the remuneration w , associated with the quantitative component y , is non linear in α and β , and non monotonic in α ; (ii) the remuneration k , associated with the quantitative component q , is non linear in α and β , and non monotonic in β .*

The characteristics of the optimal compensation scheme are illustrated in Figures 1 and 2 where, for arbitrary but reasonable values of the parameters,¹¹ we plot the equilibrium value of compensation packages and the agent’s activities for different values of the effectiveness of the assessment system, β , and of the agent’s motivation α .

Looking at figure 1A, for a given value of α , when the performance assessment scheme is highly distortionary (i.e, for small values of β , $\beta < \bar{\beta}$) it is better not to rely on it and to set $k^* = 0$, as if effort were not contractible. When the threshold level $\bar{\beta}$ is reached, then it is optimal for the principal to adopt a performance assessment scheme. At this point, w_C^* decreases sharply and k_C^* jumps from zero to its maximum

¹⁰See Appendix.

¹¹More precisely we assumed: $T = 10$; $\gamma = .5$; $\alpha = .75$ in Figure 1, and $\beta = 10$ in Figure 2.



to then decrease for higher values of β . The reason for this behavior is that when the assessment scheme is relatively costly (but not so much to discourage its use) substantial monetary incentives are needed to convince the agent to bear the burden of the scheme, a burden that decreases when the performance assessment system becomes more precise. Finally, when β becomes large enough, the distortionary effects of the performance assessment scheme tend to vanish, and the two inputs tend to be compensated in the same way.

The corresponding levels of effort devoted to the different types of inputs are plotted in Figure 1B. When the performance assessment system is highly distortionary, and only quantity is remunerated, the agents respond to the compensation scheme by overinvesting in the observable inputs y . When we reach $\bar{\beta}$, y_C^* makes a discrete downward jump, q_C^* and e_C^* a discrete upward one. Notice that, for large values of β , e_C^* decreases and q_C^* increases reflecting the less distortionary nature of the performance assessment scheme.

Moving now to Figure 2A, for any given value of β , if α is small, k_C^* is large, and it decreases continuously with α , until $\bar{\alpha}$, when it drops to zero and only the quantitative component is remunerated. The reason, which we already mentioned, is that since the introduction of the assessment scheme is distortionary, such distortions are worth bearing only if the agent is not sufficiently motivated; in the case of highly motivated agents the assessment scheme creates an unnecessary burden. The fact that k_C^* decreases with α (for $\alpha < \bar{\alpha}$) just reflects the fact that lower values of α are associated with lower output levels and higher (per unit) remuneration (w_C^* follows the same behavior). To get a better understanding of why this is the case, notice that:

$$\begin{aligned} k_C^*/w_C^* &= 0, & \text{if } \beta > \bar{\beta}, \\ y_C^*/q_C^* &= k_C^*/w_C^* = \frac{1+\beta}{\beta}, & \text{if } \beta > \bar{\beta}, \end{aligned}$$

which in turns implies that *if the performance assessment scheme is introduced, its relative weight in the compensation package (k_C^*/w_C^*) is independent of the motivation of the agent, α , and decreases with its effectiveness (β).*

The reason is that, when the performance assessment system is used, as in standard production theory,

the best the principal can do is to equalize the marginal rate of technical substitution between q and y (y/q) with the (constant) economic rate of substitution between the two factors, which is equal to $1 + 1/\beta$ for the agent. To do so in our set up (where the different inputs have the exact same cost in terms of effort), the principal will also have to fix the remuneration of the two activities according to the same proportions, which do not depend on α . Instead, what does depend on α is $\bar{\beta}$, the threshold value of β for which it is worth to adopt the performance assessment system. Since $\partial\bar{\beta}/\partial\alpha > 0$, when the agent is more motivated, the less distortionary the assessment scheme should be for it to be worth implementing; this is, of course, because the more motivated the agent is, the higher is the output she produces when her effort is not contractible. Finally, it is worth remarking that we find threshold levels of β and α below (or above) which the principal is better off not using a performance assessment scheme in a model *in which the principal bears no costs in introducing the performance assessment system*. Of course, had we assumed a fixed cost associated with the introduction of the performance system our results would *a fortiori* hold true.

3 Extensions

In order to verify the robustness of our results, we now relax two key assumptions in our model, namely the fact that the assessment technology is given, and that the introduction of the performance assessment scheme does not affect the intrinsic motivations of the agents.

3.1 Investing in technology

Until now, we have assumed that the efficiency of the performance assessment system is given, so that the only choice the principal has to undertake is whether to adopt it or not. Of course, these are quite strong assumptions, and it is interesting to see what happens in the more realistic scenario in which the principal can tailor the performance assessment scheme to his needs, by deciding how much to invest in its efficiency. To keep the analysis simple, we assume a linear investment technology so that the cost $g(\beta)$ of a performance assessment technology with precision β is $\mu\beta$, where μ is the (constant) marginal cost of improving precision. Since the principal moves first, the problem of the agent is the same as in the previous section, and the best response functions are still given by (19)-(20). The budget constraint of the principal now becomes:

$$T = wy_C^*(w, k) + kq_C^*(w, k) - \mu\beta, \quad (24)$$

so that his problem can be written as:

$$\underset{w, k, \beta}{Max} V_{CT} = q_C^*(w, k)y_C^*(w, k), \text{ such that: } (wy_C^*(w, k) + kq_C^*(w, k) - \mu\beta - T). \quad (25)$$

Solving for (25), we obtain that if, in the previous case, the principal found it profitable to use a performance assessment system only if the latter was precise enough, what now makes the difference is how costly it is to have a precise enough performance assessment scheme. Accordingly,¹² we found a threshold $\bar{\mu}(T, \alpha, \gamma)$, with

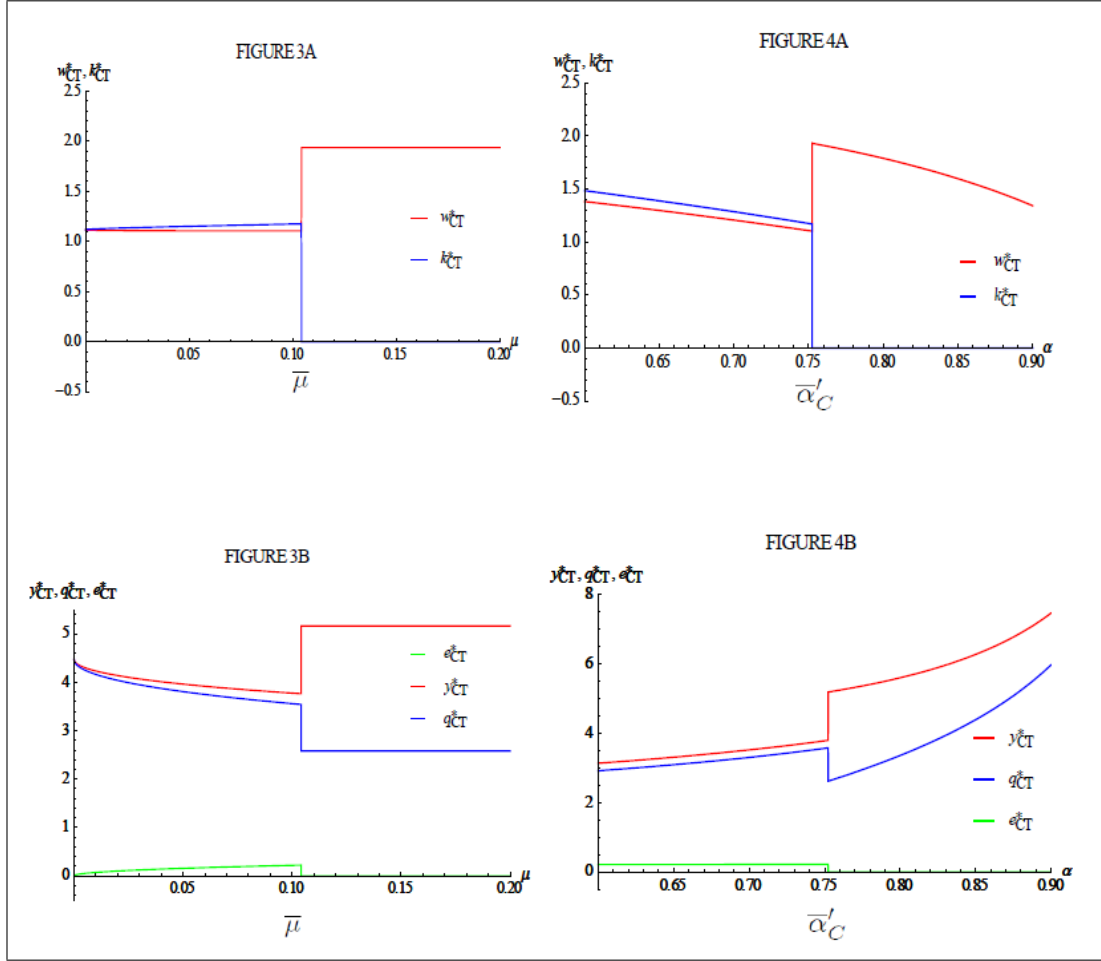
$$\bar{\mu} \equiv \frac{T(2\gamma - \alpha)^3}{16\alpha(\alpha - \gamma)\gamma}, \quad (26)$$

such that if, and only if, $\mu < \bar{\mu}$ the principal finds it optimal to use the performance assessment scheme. It is worth remarking that

$$\alpha > \frac{\gamma}{2} \implies \frac{\partial\bar{\mu}}{\partial\alpha} = \frac{T(2\gamma - \alpha)^2(2\gamma^2 - 2\alpha\gamma - \alpha^2)}{16\alpha^2(\alpha - \gamma)^2\gamma} < 0, \quad (27)$$

so that, also in this case, the more motivated the agent is, the more likely it is optimal for the principal not to adopt the performance assessment system.

¹²See Appendix.



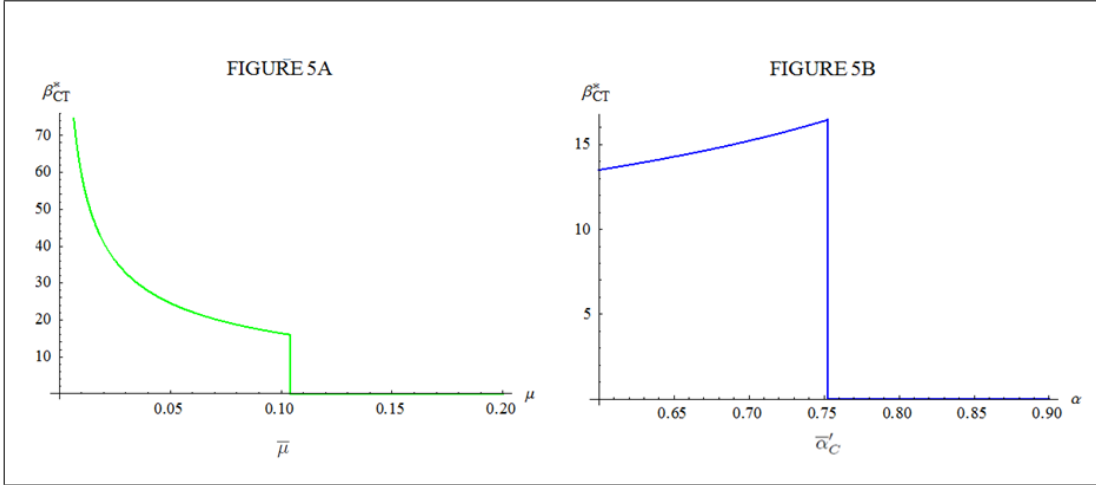
To illustrate the main results, we rely on Figures 3–5, which are plotted using the same parametrization as in the previous section.¹³ The first thing that is worth noticing is that the comparative statics with respect to μ (see Figure 3) is just the mirror image of the one with respect to β (see Figure 1). This is not surprising at all; low marginal costs of improving the precision of the performance assessment scheme lead to a less distortionary scheme. The comparative statics with respect to α (see Figures 2, and 4) is also not affected by the introduction of the investment decision.

We now turn our attention to the determinants of the investment in the performance assessment system. If, unsurprisingly, β_C^* decreases monotonically with μ and drops to 0, when μ reaches $\bar{\mu}$ (see Figure 5A), the relation between the investment in technology and the agent's motivations may deserve a more thorough analysis. Indeed, as we prove in the Appendix and illustrate in Figure 5A,

Result 3 *The optimal investment in the precision of the performance assessment technology, β , is non monotonic in the motivations α of the agent: β increases with α until it reaches a point $\bar{\alpha}'_C$ (corresponding to $\bar{\mu}$) where it drops to zero.*

The reason for such a behavior is that, as we know from the previous analysis, the more motivated the agent is, the less distortionary the performance assessment scheme should be for it to be worth implementing. This means that the more motivated an agent is the more the additional investment in the precision of the

¹³In addition, we assumed $\mu = .1$ in Figure 3A and 3B.



assessment scheme is worth. This is, of course, until we reach a strong enough level of motivation for which, no matter how precise, any feasible assessment system will just distract the agent from her duties. An interesting corollary of the result above is that

Corollary 3.1 *Investments aimed at improving agents' motivations and the quality of the performance assessment system are substitutes when agents are highly motivated and complements when the opposite is true.*

3.2 Intrinsic and extrinsic motivations

A large psychology based literature (Kreps, 1997 and Perry et al., 2009, among others) argues that the deployment of PFP schemes can harm individual performance when agents are committed to the goals of the organization as extrinsic motivations may undermine intrinsic ones. This can be easily incorporated in our framework, for instance by assuming that should a performance based contract be offered the problem of the agent becomes:

$$\text{Max}_{y,q,e} U_M = \frac{\alpha}{\nu} qy + (wy + k \min\{q, \beta e\}) - \frac{\gamma}{2} (q + y + e)^2, \quad (28)$$

where $\nu \in [1, \infty)$ is a measure of the loss of intrinsic motivations due to the introduction of a PFP scheme, and subscript M stands for motivations. The solution of this problem¹⁴ in this case is very similar to (23). The only difference (assuming that an interior maximum exists) is that $\zeta_M \equiv 2(1 + \beta)\gamma - \frac{\alpha}{\nu}\beta$ and that $\beta_M > \bar{\beta}$, and $\alpha_M < \bar{\alpha}_C$. As expected, the general appeal of the pay for performance scheme decreases but the main trade-offs remain.

4 Heterogeneous agents

From the previous analysis, we concluded that the design of optimal performance schemes should be tailored according to agents' motivations which we assumed to be known by the principal. However, usually principals do not know how motivated agents are, and, in addition, agents tend to differ with respect to motivations. To take this into account, in this section, we extend the analysis to discuss the case where agents' motivations are heterogeneous, and they are not observable by the principal. As standard in the literature, the latter, however, knows how motivations are distributed and can offer a menu of contracts to induce agents to

¹⁴Available upon request.

reveal their type and compensate them accordingly. The kind of environment we describe here could be an organization where a fraction p of agents is of type α^H (high motivated), and a fraction $(1-p)$ is of type α^L (low motivated) with $\alpha^H > \alpha^L$. We further assume that $\alpha^L < \bar{\alpha}_C < \alpha^H$ so that a performance based contract is optimal for the low motivated agents but not for the high motivated ones. We then ask ourselves if there exists a pair of contracts $\{w^H, k^H\}$ and $\{w^L, k^L\}$, such that type α^H prefers $\{w^H, k^H\}$, type α^L prefers $\{w^L, k^L\}$, and the principal prefers this menu of contract to any pooling contract $\{k^P, w^P\}$ that does not lead to a separation between types.

Before moving to the discussion of the separating contract, it is useful to discuss how the introduction of two different types of agents may affect the preferences of the principal for the different contracts. Assuming that the principal has a budget T , and he is forced to offer the same type of contract to all agents, we investigate the conditions under which he prefers to offer a performance based contract rather than a wage only (standard) one. If he offers a standard contract, from (9) and (10) the agents optimal response, for any given w , is given by:¹⁵

$$y_{\tilde{U}}^{i*}(w) = \frac{\gamma w}{\alpha^i(2\gamma - \alpha^i)}, \quad (29)$$

$$q_{\tilde{U}}^{i*}(w) = \frac{w(\alpha^i - \gamma)}{\alpha^i(2\gamma - \alpha^i)}, \quad (30)$$

with $i \in \{L, H\}$. The problem of the principal can now be written as:

$$\text{Max}_w V_{\tilde{U}} = (1-p)q_{\tilde{U}}^{L*}(w)y_{\tilde{U}}^{L*}(w) + pq_{\tilde{U}}^{H*}(w)y_{\tilde{U}}^{H*}(w), \text{ such that: } (1-p)y_{\tilde{U}}^{L*}(w) + py_{\tilde{U}}^{H*}(w) \leq T. \quad (31)$$

As in the unobservable case, since the budget is given, and only y is contractible, the only option for the principal is to set

$$w_{\tilde{U}}^* = T/E[(1-p)y_{\tilde{U}}^L(w) + py_{\tilde{U}}^H(w)], \quad (32)$$

where E denotes the expectation operator. Assuming rational expectations, from (29) and (32) we have that:

$$w_{\tilde{U}}^* = \sqrt{\frac{T}{\gamma} \frac{\alpha^L \alpha^H (2\gamma - \alpha^L)(2\gamma - \alpha^H)}{(1-p)\alpha^H(2\gamma - \alpha^H) + p\alpha^L(2\gamma - \alpha^L)}}; \quad (33)$$

substituting this expression in (9) and (10) we can solve for $V_{\tilde{U}}^*$.

Let now consider the performance based contract, for any given $\{w, k\}$, from (19)-(21), the agents' best responses are given by:

$$y_{\tilde{C}}^{i*}(w, k) = \frac{w(1+\beta)^2\gamma - k\beta((1+\beta)\gamma - \alpha^i\beta)}{(\alpha^i\beta(2\gamma(1+\beta) - \alpha^i\beta))}, \quad (34)$$

$$q_{\tilde{C}}^{i*}(w, k) = \frac{k\beta\gamma - w\beta((1+\beta)\gamma - \alpha^i\beta)}{\alpha^i(\alpha^i\beta(2\gamma(1+\beta) - \alpha^i\beta))}, \quad (35)$$

$$e_{\tilde{C}}^{i*}(w, k) = \frac{k\beta\gamma - w\beta((1+\beta)\gamma - \alpha^i\beta)}{\beta\alpha^i(\alpha^i\beta(2\gamma(1+\beta) - \alpha^i\beta))}. \quad (36)$$

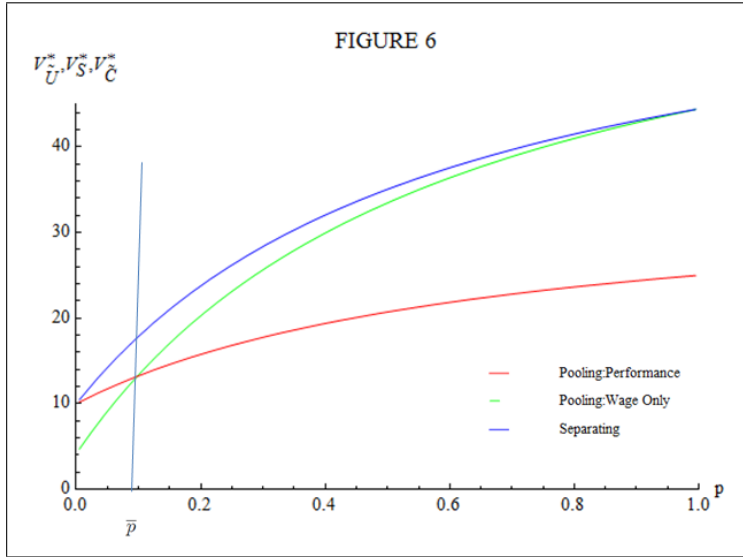
The problem of the principal can now be written as:

$$\text{Max}_{w,k} V_C = (1-p)q_{\tilde{C}}^{L*}(w, k)y_{\tilde{C}}^{L*}(w, k) + pq_{\tilde{C}}^{H*}(w, k)y_{\tilde{C}}^{H*}(w, k), \quad (37)$$

such that:

$$T \geq w((1-p)y_{\tilde{C}}^{L*}(w, k) + py_{\tilde{C}}^{H*}(w, k)) + k((1-p)q_{\tilde{C}}^{L*}(w, k) + pq_{\tilde{C}}^{H*}(w, k)). \quad (38)$$

¹⁵With a slight abuse of notation, in this section, we use subscript \tilde{U} , to denote the wage only pooling contract (which is similar to the unobservable case discussed above), and subscript \tilde{C} , for the performance based pooling contract (which is similar to the costly observable case discussed above).



Since the problem is very complex, and closed form solutions are difficult to find, we decided to carry out our analysis numerically, using the same parametrization as in the previous sections, and assuming that $\alpha^L = .6$ and $\alpha^H = .9$. In Figure 6, we compare the utility of the principal when he offers a standard and a performance based pooling contract—as well as a separating contract that we will discuss next—as a function of the share p of motivated agents. For the very same reasons why, in our previous discussion, we found that a performance based contract is preferred by the principal if agents are not motivated enough, now we find that this is the case if the distribution of types is “bad enough.” More precisely we show that there is a threshold level of p , \bar{p} , such that if $p < \bar{p}$ the principal prefers the performance based contract and the standard one otherwise.

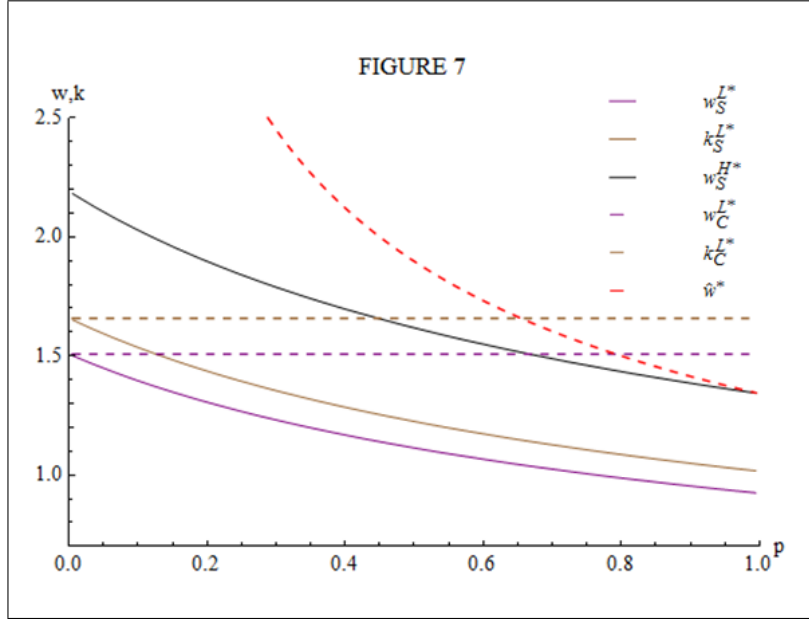
A pay for performance contract can however be costly—especially when there are enough motivated agents—and the principal may thus consider offering separating contracts. Among these, the most natural choice would be a menu of two contracts, a performance based $\{w^L, k^L\}$, and a wage only one $\{w^H\}$, designed in a way that the low motivated agents choose the former and the more motivated ones the latter. Before proceeding, it is important to notice that, in our simple set up—see (16) and (23)—the utility of the principal is linear homogeneous in T . This means that, if the principal knew the agents’ type, he would confer the entire production to the the motivated agents, and the final outcome would not depend on their number.¹⁶ From (13), in this case,¹⁷ the “optimal” contract would be:

$$\hat{w}^* = \sqrt{\frac{T\alpha^H(2\gamma - \alpha^H)}{p\gamma}}. \quad (39)$$

We now move to the separating contract that we denote by subscript S . Focusing our attention on linear compensation contracts, the optimal separating contract $\{w_S^{L*}, k_S^{L*}, w_S^{H*}\}$ is the solution of the following

¹⁶Such a result, of course, depends on the very specific functional forms we selected, but the intuition we derive here is quite general. Indeed, the smaller is the number of committed agents, the more the principal is willing to pay them so that they would be in charge of a larger share of the production.

¹⁷Using (29) and (32) and assuming $y_L^L(w) = 0$.



problem:

$$\underset{w_S^L, k_S^L; w_S^H}{Max} V_S = (1-p)y_C^{L*}(w_S^L, k_S^L)q_S^{L*}(w_S^L, k_S^L) + py_U^{H*}(w_S^H)q_U^{H*}(w_S^H), \quad (40)$$

such that :

$$T \geq (1-p)(w_S^L y_C^{L*}(w_S^L, k_S^L) + k_S^L q_C^{L*}(w_S^L, k_S^L)) + pw_S^H y_U^{H*}, \quad (41)$$

$$U^H(w_S^H) > U^H(w_S^L, k_S^L), \quad (42)$$

$$U^L(w_S^L, k_S^L) > U^L(w_S^H), \quad (43)$$

where (42) and (43) represent incentive compatibility constraints of the more and less motivated agents, respectively. Of course, in this set-up, where the principal would like to pay the more motivated agents more (more so if their number is lower), the only binding incentive compatibility constraint is the one of the less motivated agent, and it is more binding the larger their number is. The solution of the problem is illustrated in Figure 7—while the utility level of the principal offering a separating contract (*vis-à-vis* the pooling one) in Figure 6 above.

From a simple inspection of the figure, is it evident that, in order to satisfy the less motivated agents' participation constraint, the best the principal can do is to reduce the wage it offers to the motivated agent *vis-à-vis* the constrained optimal one given by (39). This distortion, which is equal to the difference between the red dotted and the black line in Figure 7, decreases with p and tends to zero when the share of motivated agents tends to one. This also implies that the utility of the principal increases with p , but it is always higher than in the case of a pooling contract, see Figure 6. At the same time, the compensation of the less motivated agents increases with their number, as they have to make up for the lower production levels of the high motivated agents.

5 Conclusions

Tight fiscal constraints and increased awareness about “citizens’ rights” are pressing governments to find innovative solutions to reform the public administration, cutting on its costs and increasing its “value proposition.” Among these, the most popular one, advocated by NPM scholars—and NPM oriented consulting

firms—is perhaps that of increasing investments in managerial and technological solutions and rationalize public sector organizations by increasing the accountability and transparency of their activities.

The main driver of such an agenda is the belief that an increase in the measurability of public administration activities allows one to address those incentive problems that contribute significantly to its poor performance. To get a better understanding of whether (and under which conditions) this is indeed the case, this paper develops a simple model that looks at the cost and benefits of using monitoring technologies to design PFP in the presence of principal agents problems.

Our analysis, establishing that the decision of whether to adopt, and how to design, PFP depends on the interaction between agents' motivations and the quality of the available monitoring technology, warns against one-size-fits-all solutions. More precisely, we show that managerial and technological solutions that allow measuring the effort of poorly motivated agents at a reasonable cost, and paying them accordingly, are definitively part of the solution, as NPM advocates argue. However, we also show that these solutions become part of the problem, when the contribution of the different tasks to the creation of value is difficult to measure and/or when agents are committed to the goals of the organization. In addition, our analysis contributes to the current debate by providing a framework that allows discussion of the impact that investments in monitoring technologies—such as those that drive many e-government projects—have on the effectiveness of the PFP schemes and the associated trade-offs.

These findings can help explaining the mixed results associated with NPM reforms, and they call for a more critical approach to the adoption of monitoring technologies which pays at least as much attention to agents' commitment to public service delivery as to the “measurability” of their daily activities.

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6 Appendix

6.1 Observable “quality”

The problem of the agent is:

$$\underset{y,q}{Max} U_O = \alpha q y + (w y - k q) - \frac{\gamma}{2} (q + y)^2, \quad (44)$$

with first order conditions:

$$\frac{\partial U_O}{\partial y} = (\alpha - \gamma)q - \gamma y + w = 0, \quad (45)$$

$$\frac{\partial U_O}{\partial q} = (\alpha - \gamma)y - \gamma q + k = 0. \quad (46)$$

It is immediate to verify that

$$y_O^*(w, k) = \frac{k(\alpha - \gamma) + w\gamma}{\alpha(2\gamma - \alpha)}, \quad (47)$$

$$q_O^*(w, k) = \frac{w(\alpha - \gamma) + k\gamma}{\alpha(2\gamma - \alpha)}, \quad (48)$$

solve this system. Finally, if (A.1) holds, the Hessian matrix $\begin{vmatrix} \frac{\partial^2 U_O}{\partial y^2} & \frac{\partial^2 U_O}{\partial y \partial q} \\ \frac{\partial^2 U_O}{\partial q \partial y} & \frac{\partial^2 U_O}{\partial q^2} \end{vmatrix} = \begin{vmatrix} -\gamma & \alpha - \gamma \\ \alpha - \gamma & -\gamma \end{vmatrix}$ is negative definite, so that the second order conditions for a maximum are verified.

The principal’s problem can now be written as:

$$\underset{w,k,\lambda}{Max} V_O = q_O^*(w, k) y_O^*(w, k), \text{ such that: } q_O^*(w, k) + y_O^*(w, k) - T \leq 0. \quad (49)$$

Substituting (47) and (48) into (49), the associated Lagrangian problem can be written as:

$$\underset{w,k,\lambda}{Max} \mathcal{L}_O = \frac{(w(\alpha - \gamma) + k\gamma)(k(\alpha - \gamma) + w\gamma)}{(\alpha^2 - 2\alpha\gamma)^2} + \frac{(a(2kw + T\alpha) + ((k - w)^2 - 2T\alpha)\gamma)\lambda}{\alpha(\alpha - 2\gamma)}. \quad (50)$$

where λ denotes the multiplier associated with the budget constraint. The first order conditions of the problem are:

$$\frac{\partial \mathcal{L}_O}{\partial w} = \frac{2w\gamma(\alpha - \gamma - \alpha(2\gamma - \alpha)\lambda) + k(\alpha^2 - 2\alpha\gamma + 2\gamma^2 - 2\alpha(2\gamma - \alpha)(\alpha - \gamma)\lambda)}{\alpha^2(\alpha - 2\gamma)} = 0, \quad (51)$$

$$\frac{\partial \mathcal{L}_O}{\partial k} = \frac{2k\gamma(\alpha - \gamma - \alpha(2\gamma - \alpha)\lambda) + w(\alpha^2 - 2\alpha\gamma + 2\gamma^2 - 2\alpha(2\gamma - \alpha)(\alpha - \gamma)\lambda)}{\alpha^2(\alpha - 2\gamma)} = 0, \quad (52)$$

$$\frac{\partial \mathcal{L}_O}{\partial \lambda} = \frac{\alpha(T\alpha + 2k\omega) + ((k - w)^2 - 2T\alpha)\gamma}{\alpha(\alpha - 2\gamma)} = 0. \quad (53)$$

It is easy to verify that:

$$w_O^* = k_O^* = \sqrt{\frac{T(2\gamma - \alpha)}{2}}, \quad (54)$$

$$\lambda_O^* = \frac{1}{2(2\gamma - \alpha)}, \quad (55)$$

solve this system. Substituting these values in (47) and (48), we have:

$$y_O^* = q_O^* = \sqrt{\frac{T}{2(2\gamma - \alpha)}}. \quad (56)$$

Now, defining by g_O , the budget constraint,

$$g_O \equiv (q_O^*(w, k) + y_O^*(w, k) - T) = \frac{\alpha(T\alpha + 2k\omega) + ((k - w)^2 - 2T\alpha)\gamma}{\alpha(2\gamma - \alpha)}, \quad (57)$$

we can check that the determinant of the bordered Hessian matrix $\begin{vmatrix} 0 & \frac{\partial g_O}{\partial k} & \frac{\partial g_O}{\partial w} \\ \frac{\partial g_O}{\partial k} & \frac{\partial^2 V_O}{\partial k^2} & \frac{\partial^2 V_O}{\partial k \partial w} \\ \frac{\partial g_O}{\partial w} & \frac{\partial^2 V_O}{\partial w \partial k} & \frac{\partial^2 V_O}{\partial w^2} \end{vmatrix}$ is equal to $\frac{8kw}{\alpha^2(2\gamma - \alpha)^2}$ and is positive, which is a sufficient condition for an interior maximum.

6.2 Unobservable “quality”

The problem of the agent can be written as:

$$\underset{y, q}{Max} U_U = \alpha q y + w y - \frac{\gamma}{2}(q + y)^2, \quad (58)$$

with first order conditions:

$$\frac{\partial U_U}{\partial y} = (1 - \alpha)q + \gamma y - w = 0, \quad (59)$$

$$\frac{\partial U_U}{\partial q} = (1 - \alpha)y + \gamma q = 0. \quad (60)$$

It is easy to verify that

$$y_U^*(w) = \frac{\gamma w}{\alpha(2\gamma - \alpha)}, \quad (61)$$

$$q_U^*(w) = \frac{w(\alpha - \gamma)}{\alpha(2\gamma - \alpha)}, \quad (62)$$

solve this system. The Hessian matrix is the same as in the previous case and is negative definite, so that the second order conditions for a maximum are verified. The problem of the principal can now be written as:

$$\underset{w}{Max} V_U = q_U^*(w)y_U^*(w), \text{ such that: } w y_U^*(w) \leq T, \quad (63)$$

but, since the budget is given, and only y is contractible, the only option for the principal is to set

$$w_N^* = T/E[y_U(w)]. \quad (64)$$

Assuming rational expectations, that is, $E[y_U(w)] = y_U^*(w)$, substituting (64) in (61) and solving we have that

$$w_U^* = \sqrt{\frac{T\alpha(2\gamma - \alpha)}{\gamma}}. \quad (65)$$

Substituting now this expression into using (61) and (62), we have that:

$$y_U^* = \frac{\sqrt{T\gamma}}{\sqrt{\alpha(2\gamma - \alpha)}}, \quad (66)$$

$$q_U^* = \frac{(\alpha - \gamma)\sqrt{T}}{\sqrt{\alpha\gamma(2\gamma - \alpha)}}, \quad (67)$$

$$V_U^* = \frac{T(\alpha - \gamma)}{\alpha(2\gamma - \alpha)}. \quad (68)$$

6.3 Costly observable “quality”

The problem of the agent is:

$$\underset{y,q,e}{Max} U_C = \alpha qy + (1 - \alpha)(wy + k \min\{q, \beta e\}) - \frac{\gamma}{2}(q + y)^2. \quad (69)$$

We should distinguish two cases: in the first $\beta e = q$, in the second $\beta e < q$.

Case 1): $\beta e = q$

The problem of the agent can be written as:

$$\underset{y,q,e}{Max} \tilde{U}_C = \alpha qy + (1 - \alpha)(wy + kq) - \frac{\gamma}{2}(q + y + \frac{q}{\beta})^2, \quad (70)$$

with first order conditions:

$$\frac{\partial \tilde{U}_C^P}{\partial y} = (\alpha - \frac{(1 + \beta)\gamma}{\beta})q - \gamma y + w = 0, \quad (71)$$

$$\frac{\partial \tilde{U}_C^P}{\partial q} = (\alpha - \frac{(1 + \beta)\gamma}{\beta})y + \frac{(1 + \beta)^2\gamma}{\beta^2}q + k = 0, \quad (72)$$

which yield

$$\tilde{y}_C^*(w, k) = \frac{w(1 + \beta)^2\gamma - k\beta((1 + \beta)\gamma - \alpha\beta)}{(\alpha\beta(2\gamma(1 + \beta) - \alpha\beta))}, \quad (73)$$

$$\tilde{q}_C^*(w, k) = \frac{k\beta\gamma - w\beta((1 + \beta)\gamma - \alpha\beta)}{\alpha(\alpha\beta(2\gamma(1 + \beta) - \alpha\beta))}. \quad (74)$$

Finally, the Hessian matrix $\begin{vmatrix} \frac{\partial^2 U_A^P}{\partial y^2} & \frac{\partial^2 U_A^P}{\partial y \partial q} \\ \frac{\partial^2 U_A^P}{\partial q \partial y} & \frac{\partial^2 U_A^P}{\partial q^2} \end{vmatrix} = \begin{vmatrix} -\gamma & \alpha - \frac{(1 + \beta)\gamma}{\beta} \\ \alpha - \frac{(1 + \beta)\gamma}{\beta} & -\frac{(1 + \beta)^2\gamma}{\beta^2} \end{vmatrix}$ is negative definite if its determinant $\alpha(\frac{2(1 + \beta)\gamma}{\beta} - \alpha)$ is positive, which is always the case if condition (A.1) holds, so that the second order conditions for a maximum are verified.

Substituting (73) and (74) in (70), we obtain:

$$\tilde{U}_C^*(w, k) = \frac{2kw\alpha\beta^2 + \gamma(w - \beta(w - k))^2}{2\alpha\beta(2(1 + \beta)\gamma - \alpha\beta)}. \quad (75)$$

The principal’s problem can now be written as:

$$\underset{w,k,\lambda}{Max} \tilde{V}_C = \tilde{q}_C^*(w, k)\tilde{y}_C^*(w, k), \text{ such that: } (\tilde{q}_C^*(w, k) + \tilde{y}_C^*(w, k) - T). \quad (76)$$

Substituting (73) in (74), into (76), the associated Lagrangian problem can be written as:

$$\underset{w,k,\lambda}{Max} \mathcal{L}_C = \frac{(\gamma(k\beta - w(1 + \beta) - w\alpha\beta)(k(\alpha\beta^2 + (1 + \beta)(w - kb + wb)\gamma))}{\alpha\beta\Omega} + \frac{\Omega(k^2\beta^2\gamma + \gamma w^2(1 + \beta)^2 + T\beta\Omega + 2kw\beta(\alpha\beta - (1 + \beta)\gamma)\lambda)}{\alpha\beta\Omega},$$

with first order conditions:

$$\frac{\partial \tilde{\mathcal{L}}_C}{\partial w} = \frac{w(1+\beta)^2\Psi + k\beta\Phi}{\alpha\beta\Omega^2} = 0, \quad (77)$$

$$\frac{\partial \tilde{\mathcal{L}}_C}{\partial k} = \frac{k\beta\Psi + w\beta\Phi}{\alpha\Omega^2} = 0, \quad (78)$$

$$\frac{\partial \tilde{\mathcal{L}}_C}{\partial \lambda} = \frac{k^2\beta^2\gamma + w^2(1+\beta)^2\gamma + T\alpha\beta(\alpha\beta - 2(1+\beta)\gamma) + 2kw\beta(\alpha\beta - (1+\beta)\gamma)}{\beta\Omega}, \quad (79)$$

where λ is the Lagrange multiplier associated with the budget constraint and

$$\begin{aligned} \Psi &\equiv -2\gamma((2(1+\beta)\gamma - \alpha\beta)\alpha\lambda + (1+\beta)\gamma - \alpha\beta), \\ \Phi &\equiv 2(1+\beta)^2\gamma^2 + 2\alpha^3\beta^3\lambda + \alpha^2\beta(\beta - 6(1+\beta)\gamma\lambda) + 2\alpha(1+\beta)\gamma(2(1+\beta)\gamma\lambda - \beta), \\ \Omega &\equiv \alpha(\alpha\beta - 2(1+\beta)\gamma)^2. \end{aligned}$$

It is then possible to verify that

$$\tilde{w}_C^* = \sqrt{\frac{T(2(1+\beta)\gamma - \alpha\beta)}{2(1+\beta)}}, \quad (80)$$

$$\tilde{k}_C^* = \sqrt{\frac{T(1+\beta)(2(1+\beta)\gamma - \alpha\beta)}{2\beta^2}}, \quad (81)$$

$$\tilde{\lambda}_C^* = \frac{\beta}{4(1+\beta)\gamma - 2\alpha\beta}; \quad (82)$$

solve the system (77)-(79). Substituting these values into (73), (74) and (76) we obtain:

$$\tilde{y}_C^* = \sqrt{\frac{(1+\beta)T}{2(2(1+\beta)\gamma - \alpha\beta)}}, \quad (83)$$

$$\tilde{q}_C^* = \beta\sqrt{\frac{T(2(1+\beta)\gamma - \alpha\beta)}{2(1+\beta)(2(1+\beta)\gamma - \alpha\beta)}}, \quad (84)$$

$$\tilde{V}_C^* = \frac{T\beta}{2(2(1+\beta)\gamma - \alpha\beta)}. \quad (85)$$

Now, defining by g_C , the budget constraint,

$$g_C \equiv (q_C^*(w, k) + y_C^*(w, k) - T) = \frac{\gamma(w^2(1+\beta)^2 + k^2\beta^2 - 2kw\beta((1+\beta)\gamma - \alpha\beta) - \alpha\beta T(2(1+\beta)\gamma - \alpha\beta))}{\alpha\beta(2(1+\beta)\gamma - \alpha\beta)}, \quad (86)$$

we can check that the determinant of the bordered Hessian matrix $\begin{vmatrix} 0 & \frac{\partial g_C}{\partial k} & \frac{\partial g_C}{\partial w} \\ \frac{\partial g_C}{\partial k} & \frac{\partial^2 V_C}{\partial w^2} & \frac{\partial^2 V_C}{\partial w \partial k} \\ \frac{\partial g_C}{\partial w} & \frac{\partial^2 V_C}{\partial k \partial w} & \frac{\partial V_C}{\partial w^2} \end{vmatrix}$ is equal to $\frac{8kw\beta^2}{\alpha^2(2(1+\beta)\gamma - \alpha\beta)^2}$ and is positive, which is a sufficient condition for an interior maximum.

Case 2): $\beta e < q$

The agent's problem is:

$$\underset{y, q, e}{Max} \hat{U}_C = \alpha q y + (1 - \alpha)(w y + k \beta e) - \frac{\gamma}{2}(q + y + e)^2, \quad (87)$$

and the necessary and sufficient first order conditions for a maximum are:

$$\frac{\partial \widehat{U}_C}{\partial y} = (\alpha - \gamma)q - \gamma(y + e) + w = 0, \quad (88)$$

$$\frac{\partial \widehat{U}_C}{\partial q} = (\alpha - \gamma)y - \gamma(q + e) + k = 0, \quad (89)$$

$$\frac{\partial \widehat{U}_C}{\partial e} = k\beta - \gamma(y + q + e) + k = 0, \quad (90)$$

which yield

$$\widehat{y}_C^*(w, k) = \frac{k\beta}{\alpha}, \quad (91)$$

$$\widehat{q}_C^*(w, k) = \frac{k\beta - w}{\alpha}, \quad (92)$$

$$\widehat{e}_C^*(w, k) = \frac{w\gamma - (2\gamma - \alpha)k}{\alpha\gamma}. \quad (93)$$

Substituting(91)-(93) into (87), we obtain

$$\widehat{U}_C^*(w, k) = \frac{k\beta(2w\gamma - (2\gamma - \alpha)k\beta)}{2\alpha\gamma}. \quad (94)$$

Using (75) and (94), we have that

$$\widetilde{U}_C^*(w, k) - \widehat{U}_C^*(w, k) = \frac{(w(1 + \beta)\gamma + k\beta(\alpha\beta - (1 + 2\beta)\gamma))^2}{2\alpha\beta\gamma(2(1 + \beta)\gamma - \alpha\beta)},$$

and thus that

$$\widetilde{U}_C^*(w, k) > \widehat{U}_C^*(w, k) \iff 2\left(\frac{1}{\beta} + 1\right)\gamma > \alpha,$$

which is always verified if (A.1) holds. This, in turns, implies that (83)-(85) is the solution of the problem.

It now remains to verify that the principal is better off by offering a compensation scheme $\{\widetilde{w}_C^*, \widetilde{k}_C^*\}$ than by linking the compensation only to y , as in the non observable case. Using (68) and (85), we have that:

$$\begin{aligned} \widetilde{V}_C^* - V_U^* &= T \left(\frac{(\alpha - \gamma)}{\alpha(2\gamma - \alpha)} - \frac{\beta}{2(2(1 + \beta)\gamma - \alpha\beta)} \right), \\ \widetilde{V}_C^* - V_U^* &> 0 \iff \beta > \frac{4(a - \gamma)\gamma}{(2\gamma - \alpha)^2}, \text{ or } \alpha > \frac{2\gamma((1 + \beta) - \sqrt{1 + \beta})}{\beta}. \end{aligned}$$

This, in turn implies that the solution of the problem is given by:

$$\begin{aligned} w_C^* &= \sqrt{\frac{T\zeta}{2(1 + \beta)}}, & w_C^* = w_U^* &= \sqrt{\frac{T\alpha(2\gamma - \alpha)}{\gamma}}, \\ k_C^* &= \sqrt{\frac{T(1 + \beta)\zeta}{2\beta^2}}, & k_C^* &= 0, \\ y_C^* &= T\sqrt{\frac{(1 + \beta)}{2\zeta}}, & y_C^* = y_U^* &= \frac{\sqrt{T\gamma}}{\sqrt{\alpha(2\gamma - \alpha)}}, & \text{if } \beta < \bar{\beta}, \\ q_C^* &= \frac{\beta T}{\sqrt{2(1 + \beta)\zeta}}, & q_C^* = q_U^* &= \frac{(\alpha - \gamma)\sqrt{T}}{\sqrt{\alpha\gamma(2\gamma - \alpha)}}, & \text{or } \alpha > \bar{\alpha}; \\ e_C^* &= \frac{T}{\sqrt{2(1 + \beta)\zeta}}, & e_C^* &= 0, \\ V_C^* &= \frac{T\beta}{2\zeta}, & V_C^{*P} = V_U^* &= \frac{T(\alpha - \gamma)}{\alpha(2\gamma - \alpha)}, \end{aligned}$$

where $\bar{\alpha}_C \equiv \frac{2\gamma((1+\beta)-\sqrt{1+\beta})}{\beta}$, $\bar{\beta} \equiv \frac{4(\alpha-\gamma)\gamma}{(2\gamma-\alpha)^2}$, and $\zeta \equiv 2(1+\beta)\gamma - \alpha\beta$. Finally, noticing that $\beta > \bar{\beta} \implies \zeta > 0$, it is immediate to verify that:

$$\begin{aligned}
\frac{\partial w_C^*}{\partial \alpha} &= -\frac{T\beta}{2\sqrt{2T(1+\beta)\zeta}} < 0, \\
\frac{\partial w_C^*}{\partial \beta} &= -\frac{T\alpha}{2(1+\beta)^{\frac{3}{2}}\sqrt{2T\zeta}} < 0, \\
\frac{\partial k_C^*}{\partial \alpha} &= -\frac{T\sqrt{(1+\beta)}}{2\sqrt{2T\zeta}} < 0, \\
\frac{\partial k_C^*}{\partial \beta} &= -\frac{T(4(1+\beta)\gamma - \alpha\beta)}{2\beta^2\sqrt{2T(1+\beta)\zeta}} < 0, \\
\frac{\partial y_C^*}{\partial \alpha} &= \frac{T^2\beta\sqrt{(1+\beta)}}{(2T\zeta)^{3/2}} > 0, & \text{if } \beta > \bar{\beta}, \\
\frac{\partial y_C^*}{\partial \beta} &= \frac{T\alpha}{2\zeta^{3/2}\sqrt{2(1+\beta)T}} > 0, & \text{or } \alpha > \bar{\alpha}, \\
\frac{\partial q_C^*}{\partial \alpha} &= \frac{T^2\beta^2}{2(T\zeta)^{3/2}\sqrt{2(1+\beta)}} > 0, \\
\frac{\partial q_C^*}{\partial \beta} &= \frac{T(4(1+\beta)\gamma - \alpha\beta)}{2\sqrt{2T}((1+\beta)\zeta)^{3/2}} > 0, \\
\frac{\partial e_C^*}{\partial \alpha} &= \frac{T^2\beta}{2(T\zeta)^{3/2}\sqrt{2(1+\beta)}} > 0, \\
\frac{\partial e_C^*}{\partial \beta} &= -\frac{T(4(1+\beta)\gamma - \alpha(1+2\beta))}{2\sqrt{2T}((1+\beta)\zeta)^{3/2}} < 0,
\end{aligned}
\quad , \quad
\begin{aligned}
\frac{\partial w_C^*}{\partial \alpha} &= -\frac{T(\alpha-\gamma)}{\sqrt{T\alpha\gamma(2\gamma-\alpha)}} < 0, \\
\frac{\partial w_C^*}{\partial \beta} &= 0, \\
\frac{\partial y_C^*}{\partial \alpha} &= \frac{T^2\gamma(\alpha-\gamma)}{(T\alpha\gamma(2\gamma-\alpha))^{3/2}} > 0, & \text{if } \beta < \bar{\beta}, \\
\frac{\partial y_C^*}{\partial \beta} &= 0 & \text{or } \alpha < \bar{\alpha}. \\
\frac{\partial q_C^*}{\partial \alpha} &= \frac{T^2\gamma^{\frac{3}{2}}}{(T\alpha\gamma(2\gamma-\alpha))^{3/2}} > 0, \\
\frac{\partial q_C^*}{\partial \beta} &= 0
\end{aligned}$$

6.4 Investing in technology

The problem of the principal is to

$$Max_{w,k,\lambda,\beta} V_T = q_C^*(w,k)y_C^*(w,k) - \lambda(wy_C^*(w,k) + kq_C^*(w,k) - \mu\beta - T), \quad (95)$$

where $q_C^*(w,k)$, $y_C^*(w,k)$ are given by (73)-(74). The first order conditions of the problem are quite cumbersome and available upon request (as the details for this section). Using Mathematica[®], we can show that:

$$w_T^* = \frac{\sqrt{(2(\mu+T)\gamma - T\alpha)(2(\mu+T)\gamma + \varphi)}}{\sqrt{2\mu\alpha + 4(\mu+T)\gamma}}, \quad (96)$$

$$k_T^* = \frac{((\mu+T)\gamma + \varphi)\sqrt{(\varphi - 2(\mu+T))(T\alpha - 2(\mu+T)\gamma)}}{2T\gamma\sqrt{2\mu\alpha + 4(\mu+T)\gamma}}, \quad (97)$$

$$\beta_T^* = \frac{2\mu\gamma - \varphi}{\mu(\alpha - 2\gamma)}, \quad (98)$$

solve the system of first order conditions.¹⁸ Substituting these values into the principal's objective function, we get:

$$V_T^* = \frac{4\mu\gamma - T(\alpha - 2\gamma) - \varphi}{2(\alpha - 2\gamma)^2}. \quad (99)$$

Comparing now this solution with the solution for the unobservable case, (68), we have that

$$V_{CT}^* - V_U^* = \frac{T(\alpha - 2\gamma)^2 + 4\mu\alpha\gamma - 2\alpha\phi}{2(\alpha - 2\gamma)^2}, \quad (100)$$

¹⁸While we have not been able to verify analytically that the second order conditions for a maximum are verified, through numerical simulations we confirmed that they are for the parametrization we use in Figures 3-5.

and

$$V_{CT}^* - V_U^* > 0 \iff \mu < \bar{\mu} \equiv \frac{T(2\gamma - \alpha)^3}{16\alpha(\alpha - \gamma)\gamma}. \quad (101)$$

In addition,

$$\frac{\partial \bar{\mu}}{\partial \alpha} = \frac{T(2\gamma - \alpha)^2(2\gamma^2 - 2\alpha\gamma - \alpha^2)}{16\alpha^2(\alpha - \gamma)^2\gamma} < 0 \iff \alpha > (\sqrt{3} - 1)\gamma$$

condition that is always verified since $\alpha > \gamma$.

Finally, to prove Result 4, it is enough to show that $\frac{\partial \beta_t^*}{\partial \alpha} = \frac{\gamma(\sqrt{2(2\mu+T)\gamma-T\alpha}-4\sqrt{2(\mu+T)\gamma-T\alpha})}{2(\alpha-2\gamma)^2\sqrt{\mu\gamma(2(\mu+T)\gamma-T\alpha)}} > 0$, which is always the case.