

Optimizing Finance for Development

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Abstract

The World Bank Group recently adopted the “cascade framework” to “maximize finance for development.” The cascade recommends that reforms be tried first, followed by subsidies, and then public investments. To understand the economics of the cascade, this paper presents a model where reforms, subsidies, and public investments can be used to fill the investment gap, and computes the welfare

associated with their different sequencing. The cascade is optimal when reforms increase efficiency at no cost. When they are costly, if policies can be project specific, their sequencing does not matter; if not, the cascade can be optimal if agents are myopic, but not if they are forward-looking. Tensions may thus arise between maximizing private financing and optimizing financing for development.

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Optimizing Finance for Development*

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1 Introduction

In September 2015, the UN general assembly adopted a resolution laying out an ambitious agenda for sustainable development (Agenda 2030).¹ Multilateral development banks readily embraced it, but they soon realized that existing official resources were not enough for the undertaking.² The challenge, to use World Bank President Jim Yong Kim’s words, became that of finding ways to “leverage the billions of dollars in official development assistance to trillions in investment of all kinds, whether public or private, national or global.”³

Against this backdrop, in March 2017, the World Bank Group (WBG) adopted⁴ the “cascade approach as a concept to guide the WBG’s effort to leverage the private sector for growth and sustainable development,” (World Bank, 2017b, p.1). If such language may sound a bit elusive, the guidelines on how to implement the cascade are very clear:

“When a project is presented ask - ‘Is there a sustainable private sector solution that limits public debt and contingent liabilities?’ If the answer is ‘Yes’ - promote such private solutions. If the answer is ‘No’ - ask whether it is because of: (i) Policy or regulatory gaps or weakness? If so, provide WBG support for policy and regulatory reforms. (ii) Risks? If so, assess the risks and see whether WBG instruments can address them. If you conclude that the project requires public funding, pursue that option.” (World Bank, 2017b, p.2)

The aim of this paper is to provide a simple framework that helps us understand better (i) how the adoption of the cascade may affect the allocation of finance across projects, and (ii) the conditions under which maximizing and optimizing finance for development are likely to coincide, and those under which they are not.

With these goals in mind, we present a very simple model where investment projects, which create positive externalities, can be financed by the private or by the public sector. We then identify the set of worthy projects that the private sector should finance (because of its efficiency advantage *vis-à-vis* the public sector), but that it does not (because they are not commercially viable). To fill such an investment gap, following the cascade, we consider three different government interventions: the first is an upstream policy reform that allows private investors to extract (part of) the externalities; the second is a public subsidy (for instance in the form of public sector cofinancing of projects or of subsidized guarantees/insurance instruments) that induces the private sector to invest; the third intervention is the direct funding of the entire project by the government.

If upstream reforms are able to crowd private investments in at no cost for the society then, by themselves, they can address the investment gap. In such a case, the cascade is clearly optimal. When instead, there are costs associated with reforms (e.g., higher private returns are the outcome

¹ Available at http://www.un.org/ga/search/view_doc.asp?symbol=A/RES/70/1&Lang=E

² See <https://sustainabledevelopment.un.org/>

³ Address at the Third International Conference on Financing for Development Addis Ababa, Ethiopia, June 13, 2015. Available at <http://documents.worldbank.org/curated/en/144851468190446079/pdf/101985-WP-Box393267B-PUBLIC-2015-07-13-JK-Billions-To-Trillions-Ideas-to-Actions.pdf>

⁴ World Bank (2017a).

of efficiency gains but also of higher fees that restrict access) trade-offs between the three different policy instruments arise. To get a better understanding of such trade-offs, we consider first the case in which government interventions can be perfectly targeted to specific projects. When this is the case, the government has the ability to offer subsidies that are project specific and to decide the set of projects to which a particular reform applies or not. We then consider the case in which the subsidy should be the same for all projects and, if reforms are implemented, they affect all (unfinanced) projects. In both cases, we compute the allocation and the welfare levels associated with any of the possible sequences of policies: reforms first, subsidy second, public project third; subsidy first, reforms second, public project third, and so forth.

Our main findings are the following: (i) when reforms and subsidies can be perfectly targeted to projects, the sequencing of policies does not matter; (ii) when they cannot, the sequencing does matter. If the agents/agencies in charge of a specific policy are “myopic,” that is, they do not anticipate the policies implemented by other agencies, there are situations in which the cascade sequencing can be optimal; (iii) when agencies are forward-looking and anticipate the effects of their own policies on downstream policy interventions (and reforms are part of the optimal solution) then the cascade is never optimal and the optimal sequencing requires subsidies/guarantees to be offered first.

While the paper focuses on a very specific problem: the economic underpinnings of the cascade framework recently adopted by the WBG, the issues it touches relate to a wide body of literature. With respect to the Agenda 2030, Schmidt-Traub and Sachs (2015) provide an assessment of public and private investment needs to achieve the SDGs, while Samans (2016) analyzes the investment barriers that the private sector faces in emerging markets and looks at the role that blended finance can play in overcoming them.

Of course, the discussion on the role that public and private investment play in the process of economic development predates the Agenda 2030. Khan and Reinhart (1990), and Khan and Kumar (1997) study the contribution of investments, public and private, on long term growth in developing countries and conclude that the contribution of private investments is larger. The problem is that, in such countries, private investments often fail to take off. The reasons can be many, ranging from property rights and institutional quality (Banerjee et al., 2006) to political uncertainty (Rodrik, 1991).

Whatever the structural reasons that deter private investments in developing countries are, public policies can be part of the solution. OECD (2004) provides an overview of the existing sources of finance for developing countries and discusses how to mobilize new ones, including for the provision of global public goods. OECD (2015) looks at the different financing instruments that can support infrastructure investments in the sectors or regions that need them the most. Klemm, (2010) provides a comprehensive overview of existing tax incentives for business investments,⁵ while

⁵The fact that tax incentives are very common instruments used to attract investments does not mean that they are uncontroversial. According to Pennings (2000) and Yu et al. (2007) investment subsidies are a better instrument than tax cuts to foster private investments. However, Danielova and Sarkar (2011) show that when debt financing is possible, a combination of tax reduction and investment subsidy is optimal.

Engel et al. (2014) analyze the economics and finance of public-private partnerships. Finally, ADB (2015) discusses how public and private sources should coexist and reinforce each other in the new sustainable development agenda. While this paper builds on the financial incentives literature, the model we present is designed with the cascade framework in mind. This explains some of its non-standard features.

The paper is organized as follows: Section 2 describes the basic model and explains why a financing gap arises; Section 3 discusses the different instruments that can be used to fill such a gap, and how they should be allocated when they can perfectly target the different projects. Section 4 deals with the more realistic case in which subsidies and reforms cannot be project specific, so that the sequencing of policy actions matters; it then solves for the allocations and the welfare levels associated with different policy sequencing. It also distinguishes between the case in which agents/agencies are myopic and forward looking. Finally, Section 5 discusses the critical assumptions of the model and concludes.

2 The Basic Model

We assume that there is a continuum of investment projects, $k = \{x_i, \gamma_j\} \in K$. All projects are of the same size, which we normalize to 1, they are indivisible, and they can be undertaken either by the private, P , or by the public sector, G . Investment projects differ with respect to the returns they generate. The returns associated with project k , are $R_k^l = r_{ka}^l + r_{ke}^l$, where r_{ka}^l denotes appropriable returns (e.g., profits), and r_{ke}^l non-appropriable returns (e.g., externalities). Superscript l , $l \in [P, G]$, denotes whether a project is undertaken by the private or by the public sector.

The existence of a difference in the returns associated with privately and publicly financed investment projects is what motivates the cascade approach. The private sector's appropriable returns are denoted by $r_{ia}^P = 1 + x_i$, and the public sector's by $r_{ia}^G = 1$. To focus on the policy relevant trade-offs, most of the analysis features the case in which the private sector has an efficiency advantage *vis-à-vis* the public sector, that is, when $x_i \geq 0$.

We further assume that the private sector's efficiency advantage is project specific, and that it does not extend to externalities. Non-appropriable returns are thus the same, independently of whether the project is undertaken by the public or the private sector. We thus set $r_{je}^G = r_{je}^P = \gamma_j$, with $\gamma_j \geq 0$. Hence, a project k is characterized by an x_i and a γ_j that we assume to be independently and uniformly distributed on the interval $[0, 1]$, so that $K = [0, 1]^2$. As per the costs associated with the different projects, we assume that one unit of capital is needed to finance a project. The cost of such unit of capital, $c - 1$, the cost of funding hereinafter, is the same for the public and the private sector;⁶ therefore, the total cost of the project, c , is also the same. Without great loss of generality, we assume that $c \in (1, 2)$.

⁶Of course, the private sector's cost of borrowing can differ from the public sector's. However, in our simple framework, this would be equivalent to a horizontal shift in the support of x_i , towards the left when the public sector has a cost advantage, towards the right when the private sector has it.

In the simple set-up we just described, it is easy to identify the set of welfare improving projects that the private sector should, but does not find it profitable to finance. Indeed, the private sector, which maximizes private returns, $R_i^P - c$, finds it profitable to invest in a project iff $1 + x_i \geq c$, or

$$x_i \geq c - 1 \equiv \underline{x}_P. \quad (1)$$

However, since the private sector does not internalize the externality, γ_j , its participation constraint (1) is stricter than the condition that insures that the project is welfare improving. Such a condition can be written as $1 + x_i + \gamma_j \geq c$, or

$$\gamma_j \geq c - 1 - x_i \equiv \underline{\gamma}_P. \quad (2)$$

Our basic set-up is summarized in Figure 1a, where, setting $c = \frac{3}{2}$, we order the projects' space according to the associated private sector's efficiency advantage and externalities, that is, in the (x_i, γ_j) space. Region **D**, is the locus of the projects that are quintessentially private. They are characterized by high private sector efficiency advantage ($x_i > \underline{x}_P$), and they always satisfy the private sector participation constraint (1). We will disregard such projects in the remaining of the paper, as we will disregard the projects in region **C**, with $x_i < 0$, where the public sector has an efficiency advantage and, thus, it should always undertake them. Hence, the area of interest for policy makers, and the focus of this paper, is Region **B**, where $x_i \in [0, \underline{x}_P]$, and $\gamma_j \geq \underline{\gamma}_P$. Projects that belong to such a region should, on welfare grounds, be undertaken by the private sector. Having disregarded regions **C** and **D**, the parameter space that is pertinent to the analysis is the one depicted in Figure 1b. In the figure, we split region **B** in two subregions, **B'** and **B''**. In Region **B'**, we have that

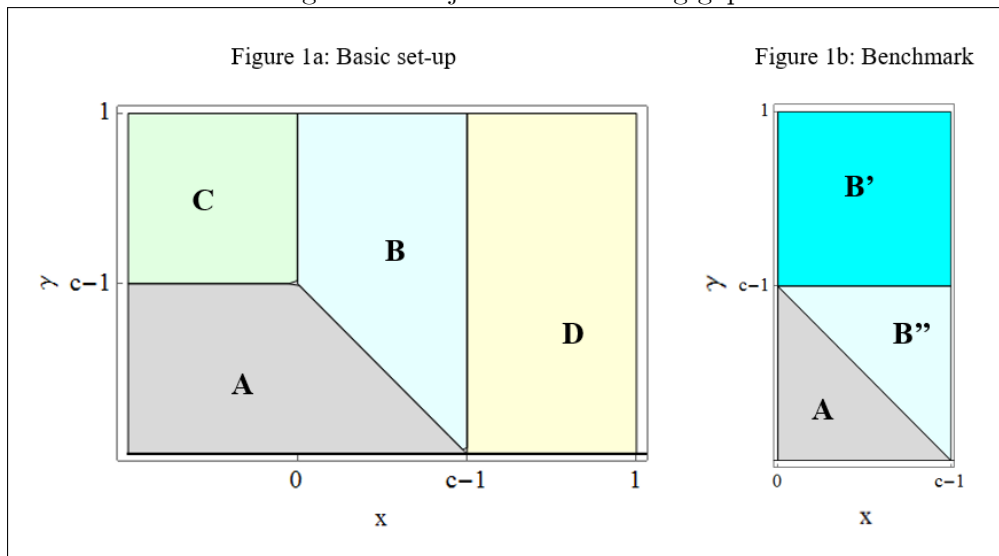
$$\gamma > c - 1 \equiv \underline{\gamma}_G, \quad (3)$$

and hence public investments are welfare improving. However, on efficiency grounds, it would be preferable if projects were undertaken by the private sector. Instead, in region **B''**, condition (3) does not hold so that the only projects that are welfare improving are those undertaken by the private sector. This being said, since the private sector does not internalize the positive externality, and projects do not generate positive net returns—condition (1) is not met—, the private sector is not willing to invest in region **B**. Finally, projects in region **A** are not worth financing by either the private or the public sector. However, we cannot ignore them altogether because, as we will see later, some of them could end up being financed if the government is not able to target subsidies to specific projects.

3 How to fill the investment gap?

Before discussing issues related with the sequencing of interventions, it is useful to introduce the three different policies that, in our framework, can fill the investment gap: reforms, subsidies, and public investments, separately. We then compute the conditions under which each of them is

Figure 1: Projects and financing gap



optimal, provided that the government can perfectly tailor policies according to the characteristics of every specific project.

3.1 Reforms

We start by assuming that the government can undertake policy reforms that allow firms to appropriate part of the (previously) non-appropriable returns. More precisely, we assume that the government can increase private appropriable returns by $\alpha\gamma_j$, with $\alpha \in [0, \bar{\alpha}]$, at a cost $(1 - \alpha)\gamma_j$ in terms of non-appropriable returns. This may sound quite abstract. So, what is the kind of reforms we have in mind? A good example would be a change in the regulatory framework that allows or facilitates the outsourcing of infrastructure investments to the private sector, which then can charge a usage fee. Through the fee, investors could extract some of the externalities/users' surplus generated by the infrastructure; however, when $\alpha < 1$, this entails a net efficiency loss, think, for instance, of the classical Harberger's triangle. Other examples of "costly" reforms could be a strengthening of property rights, a change in the regulatory framework of utilities, a weakening of antitrust enforcement, etc.

Notice that, while the analysis focuses on the more interesting case of costly reforms, there is no reason to rule out efficiency generating reforms, where $\alpha \geq 1$. As examples of such reforms, consider measures aiming at reducing corruption, at cutting red tape, or at simplifying bureaucracy. More generally all the measures that increase the "size of the pie" where the pie, here, includes all externalities belong to this category. Notice, however, that, when they exist, efficiency generating reforms would, by themselves alone, fill the investment gap, and the cascade algorithm, which in this case collapses to "just do reforms," is clearly optimal.

While, in our framework, we could allow the government to choose any reform α in the $[0, \bar{\alpha}]$

interval, in all possible scenarios, the welfare associated with reforms increases monotonically with α . Hence, without loss of generality, we only consider the case $\alpha = \bar{\alpha}$. That is, denoting the reform scenario by tilde, we assume that if reforms crowd in private investments the associated returns are given by

$$\begin{aligned}\tilde{r}_{ka}^P &= 1 + x_i + \bar{\alpha}\gamma_j, \\ \tilde{r}_{ke}^P &= 0.\end{aligned}\tag{4}$$

In such a case, the private sector finds it profitable to invest if $1 + x_i + \bar{\alpha}\gamma \geq c$, or

$$x_i \geq c - 1 - \bar{\alpha}\gamma_j \equiv \underline{x}_R.\tag{5}$$

Notice that, under assumption (4), private and total welfare⁷ associated with reform induced private sector's investments are exactly the same. This means that the private sector participation constraint, and the condition that insures that the private sector's projects are welfare improving, coincide. This condition can be written as

$$\gamma_j \geq \frac{c - 1 - x_i}{\bar{\alpha}} \equiv \underline{\gamma}_R.\tag{6}$$

Notice that, when $\bar{\alpha} = 1$, by the sole use of reforms the government is able to induce the private sector to finance all welfare improving projects at no cost for the society as a whole. Moreover, if $\bar{\alpha} > 1$, reforms increase the set of welfare improving projects B . For smaller values of $\bar{\alpha}$, a set of valuable projects (those below $\underline{\gamma}_R$ in Figure 2a) that are not financed by the private sector remains; in the special case of $\bar{\alpha} = 0$, no additional project is financed. Figure 2a illustrates how the introduction of reforms changes the private sector's incentives to invest for different values of $\bar{\alpha}$.

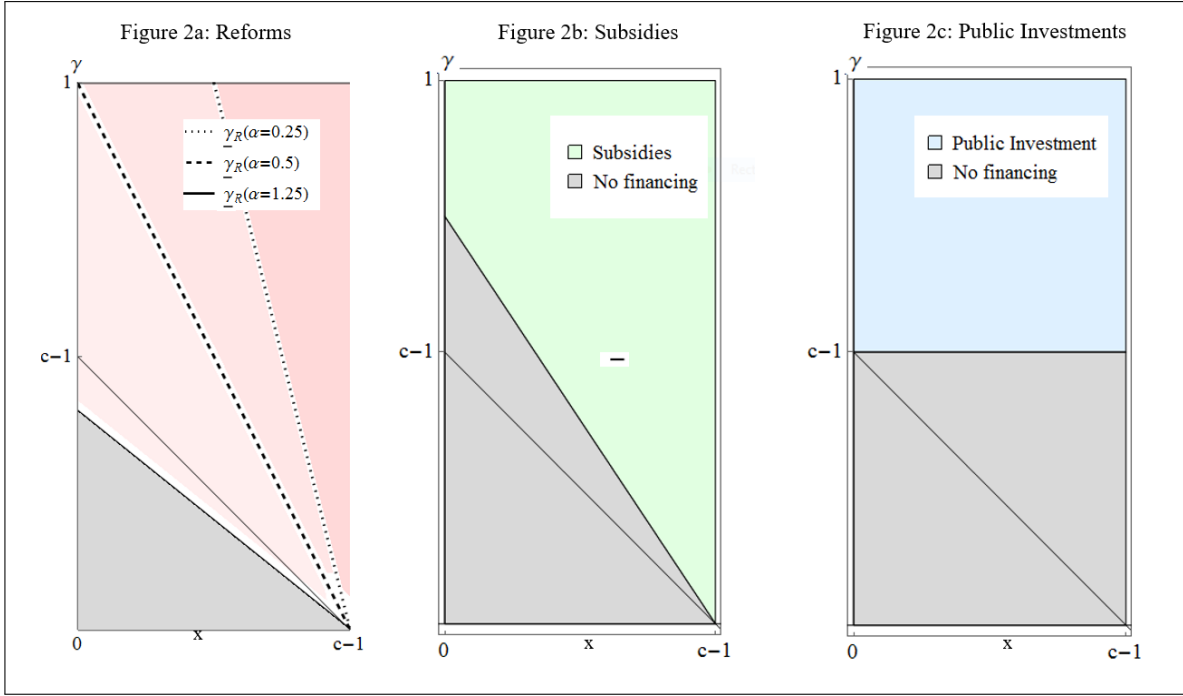
3.2 Public sector intervention: Subsidies and public investments

Let us now consider the situation in which the government is willing to use subsidies (guarantees) or public projects to fill the financing gap. We start with the subsidies, s_i , that the government can offer to the private sector at a cost cs_i . Since the government looks at total welfare, and s_i is a pure transfer from the government to the private sector, the welfare cost of a subsidy s_i , is $s_i(c - 1)$, which is equal to the government's cost of funding. Under such a policy, the private sector finds it profitable to invest if $1 + x_i + s_i \geq c$, or

$$x_i \geq c - 1 - s_i \equiv \underline{x}_s.\tag{7}$$

⁷Throughout the paper, we assume that the public sector maximizes total returns, that is, the profits and externalities associated with the projects.

Figure 2: Filling the gap



Since subsidies are costly, the set of welfare improving private sector projects shrinks when compared to the benchmark case; hence condition (2) becomes

$$\gamma_j \geq c - 1 - x_i + s_i(c - 1). \quad (8)$$

Expressing the minimum subsidy that makes project x_i profitable as

$$\widehat{s}_i = c - 1 - x_i, \quad (9)$$

and substituting it into (8), the latter becomes

$$\gamma_j \geq (c - 1 - x_i)c \equiv \underline{\gamma}_S. \quad (10)$$

Notice that, in this framework, the government offers a different subsidy to each individual firm. The alternative policy that the government can put in place is that of directly financing the project. As we already discussed, such a policy is welfare improving as long as:

$$\gamma_j \geq c - 1 \equiv \underline{\gamma}_G.$$

Figure 2b-c describes the projects that are worth financing with well targeted subsidies and with public investments.

3.3 Optimal sequencing of interventions

From the previous analysis, it is clear that a government can rely on different instruments to fill the investment gap. The question that follows is which instrument has to be preferred and under what circumstances. This is what we analyze next, under the assumption that the government can perfectly tailor policies according to the characteristics $\{x_i, \gamma_j\}$ of any specific project k . In other words, the government can implement reforms that only apply to projects with given values of x_i and γ_j , and the same is true for subsidies and public investments.⁸ In what follows, we characterize the allocation of instruments that maximizes total welfare. We denote such allocation as the optimal policy allocation, *OPA*. Lemma 1 below fully characterizes such an allocation as a function of the degree of private sector advantage (x_i) and the externalities (γ_j) associated with each specific project.

Lemma 1 (1): *If reforms are not efficient enough ($\bar{\alpha} < \frac{1}{c}$): (i) if γ_j is sufficiently low, no investment inducing policy is welfare improving; (ii) for sufficiently high values of γ_j , and low values of x_i , it is optimal for the government to implement public projects; (iii) for sufficiently high values of γ_j and x_i , subsidies are instead optimal.*

(2): *If reforms are efficient enough ($\bar{\alpha} \geq \frac{1}{c}$): (i) if γ_j is sufficiently low, no investment inducing policy is welfare improving; (ii) for high γ_j and low x_i , it is optimal for the government to implement public projects; (iii) for projects with intermediate γ_j and sufficiently high x_i , reforms are optimal; while (iv) for sufficiently large values of γ_j and x_i , subsidies are optimal.*

Proof: See Appendix.

Figure 3a summarizes our results for different values of $\bar{\alpha}$.⁹ The first, easy, takeaway from the figure is that, no matter how efficient reforms are, when dealing with high externalities/low private sector advantage-type of projects, the best option is public financing. When private sector advantage is high, instead, the best option is a subsidy. The reason for the optimality of a subsidy is that, when x_i is large, a very small subsidy is sufficient to crowd in private investors. Hence, the costs of the subsidy are of second order (\hat{s}_i tends to zero when x_i tends to $c - 1$), while those associated with public investments and reforms are always of first order (except in the limit cases in which $x_i = 0$, or $\bar{\alpha} = 1$). Thus, while there are always situations in which public investments or subsidies are optimal, for reforms to have a place in an *OPA*, they should be efficient enough, that is, we need that $\bar{\alpha} > \frac{1}{c}$. Efficiency here is measured against the costs c associated with the subsidy. In addition, the appeal of reforms weakens in the presence of large externalities because of the deadweight loss associated.

A pretty trivial, but important point, often forgotten in the cascade debate, is that

⁸For public investments this is always the case since the government decides which projects to finance.

⁹Also in this figure, we set $c = 3/2$. Hence, in the first two panels of Figure 3a, we have that $\bar{\alpha} < \frac{1}{c}$, and $\bar{\alpha} \geq \frac{1}{c}$ in the last one.

Remark 1 *If the government is able to perfectly target the policy instruments, the sequencing of interventions does not matter.*

Indeed, if the government can associate an instrument to any $\{x_i, \gamma_j\}$, it will perfectly define the scope of each policy so that the sequencing of the interventions does not matter. In other words, the fact that, in order to fill the investment gap, the government starts with reforms, subsidies, or public investments, is irrelevant; what is done first does not pose limits to what can be done next. In such a world, there is no need for a cascade, or for any other sequencing algorithm.

4 Imperfect targeting

Let us now remove the assumption that the government can offer subsidies or reforms that are project specific. Consequently, we assume that x_i , and γ_j are observable but not contractible. This implies that the government can decide which specific projects to finance directly, but it can only offer one subsidy, the same one to all investors. With respect to reforms, they apply to all projects that have not yet been financed, and only to those ones. This means that reforms do not affect the returns of the projects that they are unable to crowd in and that are later financed through a subsidy.¹⁰

We also assume that, once a project is financed, the source of financing cannot be modified. This means that if an investor decides to finance a project applying for particular subsidy, it cannot later on renounce the subsidy and take advantage of a reform; similarly, if an investor decides to finance a project taking advantage of a particular reform, then it cannot “renounce the reform” and apply for a subsidy.

It is clear that, now, the order of interventions does matter. If the government starts with reforms, all projects that have been financed through reforms cannot later be financed by subsidies or public investment. The same argument applies to public investment and subsidies. Of course, the optimal allocation of policies is not the same under perfect or imperfect targeting. The constrained optimal policy allocation (*COPA*)—that is, the policy allocation that is optimal when the government is not able to target policies according to the characteristics of the specific projects—should equalize the marginal returns associated with each policy (at least when all policy instruments are utilized in equilibrium). Asking the reader to be patient, and to wait until Section 4.4 for a characterization of the *COPA*, we illustrate such an allocation in Figure 3b. Unsurprisingly, given that subsidies cannot be properly targeted, and that they end up financing a number of projects that are not worth financing (those under the 45 degree $\underline{\gamma}_P$ line), the government will find it optimal to offer smaller subsidies under the *COPA* than under the *OPA*.

¹⁰This is the case when, for instance, a reform allows the private sector to invest in an infrastructure project with a cost plus fee contract. If there is no interest for the private sector to invest under such conditions, but there is with a subsidy, then the fact that the reform occurred does not affect the returns when the subsidy is offered. In the concluding section, we discuss how the relaxation of such assumption may affect our results.

Figure 3: Alternative policy sequences

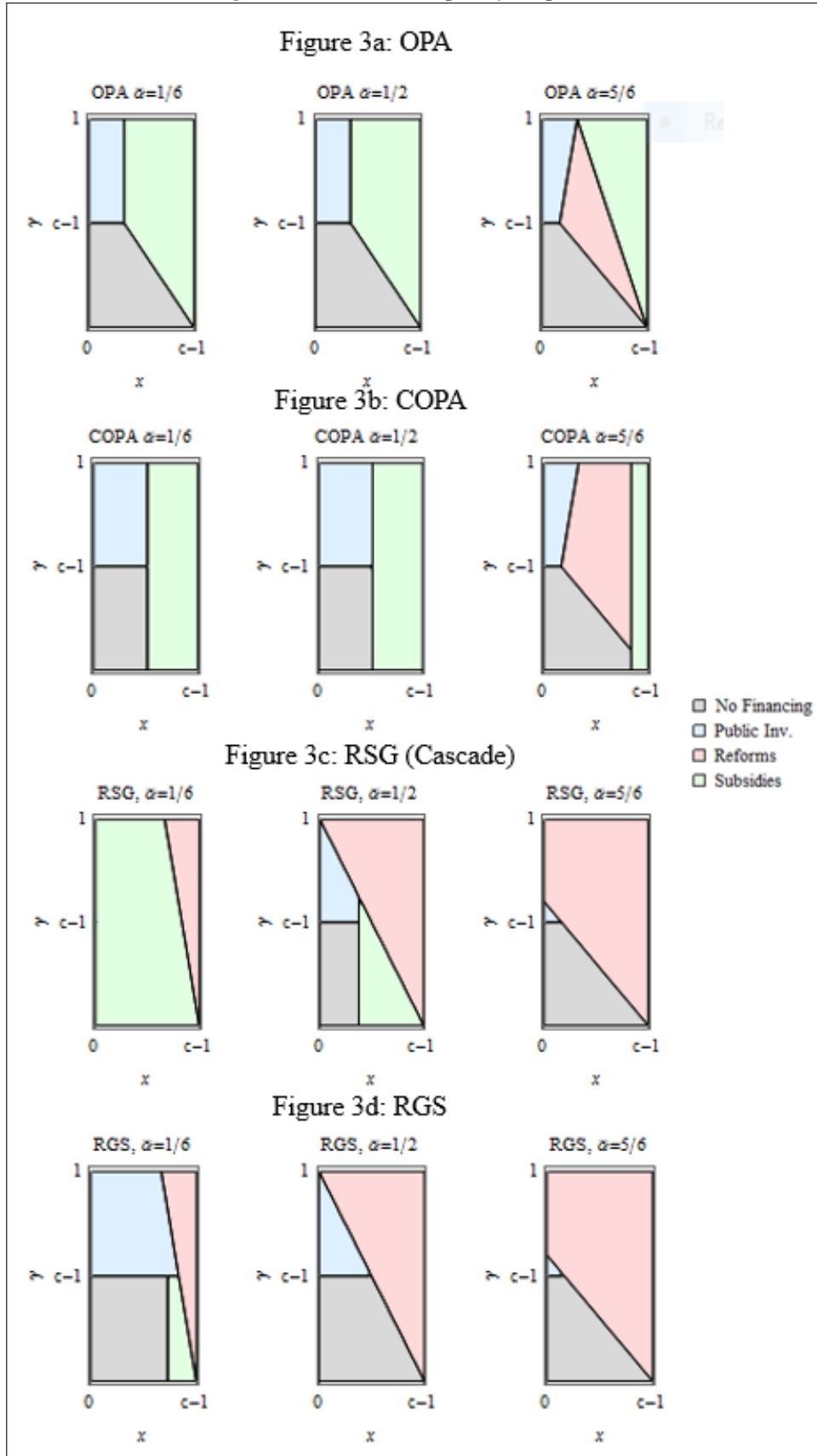


Figure 3e: SRG

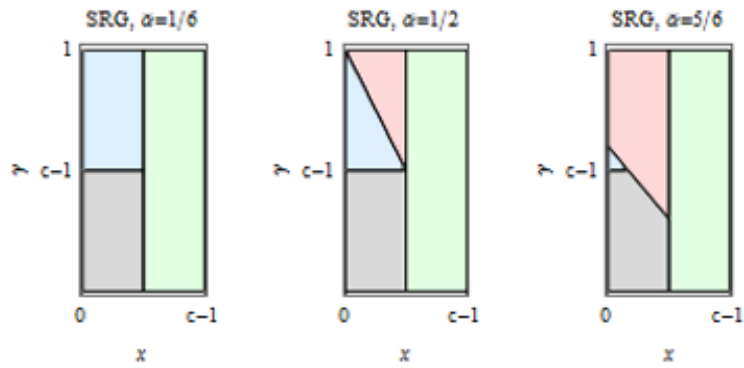


Figure 3f: SGR

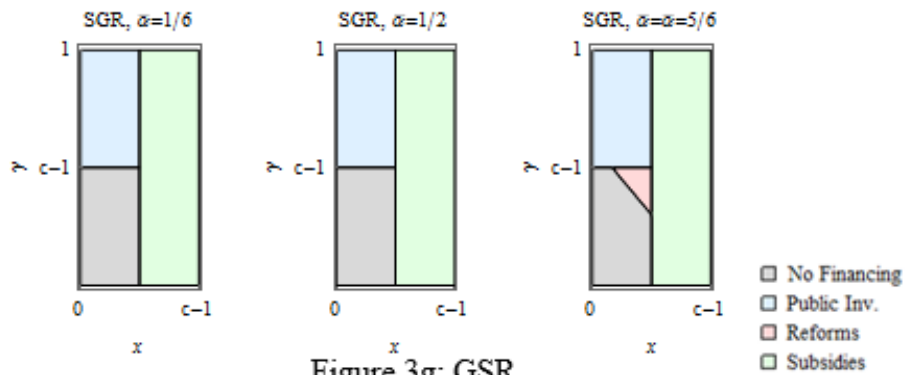


Figure 3g: GSR

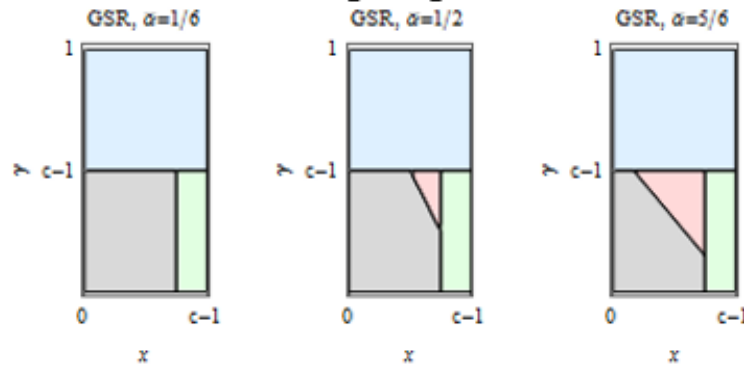
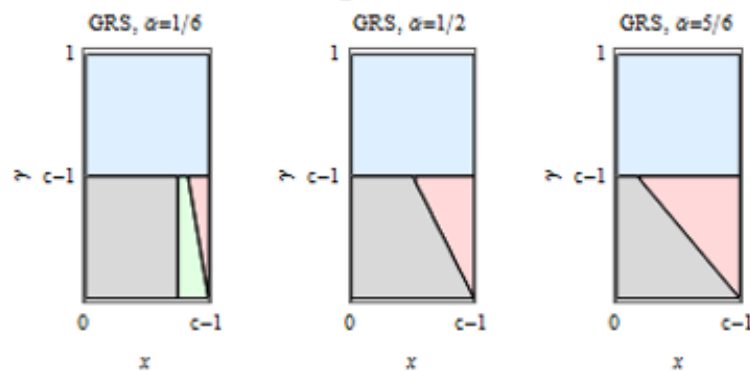


Figure 3h: GRS



4.1 Myopic beliefs

Having set the optimal benchmark as a yardstick to compare different policies, we now assume that, when implementing a policy, whatever it be (reform, subsidies, public investments), the agent, or better the governmental agency in charge, does not anticipate that other agencies will adopt other policies at a later stage. When this is the case, each agency maximizes social returns assuming no other policy will ever be put in place.

The reason we focus on such myopic beliefs is that, in our reading, these are the ones implicitly assumed by the cascade approach. Indeed, the algorithm we discussed in the introduction requires that upstream policy reforms be tried first, public coinvestment or risk-sharing next, and finally, only if both reforms and subsidies are insufficient to close the financing gap, public investments should be pursued. The assumption of myopic beliefs will be removed in Section 4.5, where we allow the different agencies to be forward looking and implement each single policy anticipating how it will affect the policies that will be put in place at a later stage by other agencies.

Starting with the case of myopic beliefs, we analyze the policy allocations and the welfare associated with the different sequencing of the policies. Having three policies, R, S, G , where R stands for reforms, S for subsidies, and G for public (government funded) investments, we have $3!=6$ different sequences. Following the order of the interventions, we denote the cascade approach by RSG , the “anticascade” scenario (that with the opposite sequencing as the cascade) by GSR , and so forth.

In the main text, we provide a sketch on how to compute policy assignments and welfare under the cascade approach, and we compare the cascade with the other scenarios. We refer the reader to the Appendix for the formal derivation of all the different cases.

4.2 The Cascade Approach (RSG)

If the government follows the cascade approach, it starts by implementing reforms, it then moves to subsidies and, if worthy projects remain unfinanced, it fills such a gap with public investments. The welfare gains W_C^R associated with reforms (R) under the cascade approach (C) are given by:

$$W_C^R = \int_{\text{Max}\{0, \tilde{x}\}}^{\underline{x}_P} \int_{\underline{\gamma}_R}^1 (1 + x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i, \quad (11)$$

where $\tilde{x} \equiv \{x : \underline{\gamma}_R = 1\} = c - 1 - \bar{\alpha}$.

Let us now introduce subsidies and compute the additional associated welfare W_C^S . We should distinguish between two cases. In the first, $\alpha \geq c - 1$, and thus $\underline{\gamma}_R < 1$ for all $x_i \in [0, 1]$. When this condition is verified, the problem of the government is that of finding the optimal subsidy s (it can be zero) that maximizes

$$W_C^{Sa} = \int_{\text{Max}\{0, c-1-s\}}^{\underline{x}_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (12)$$

When, instead, $\bar{\alpha} < c - 1$, the problem of the government is that of finding the optimal subsidy s that maximizes

$$W_C^{Sb} \equiv \int_{c-1-\bar{\alpha}}^{\underline{x}_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i + \int_{\text{Max}\{0, c-1-s\}}^{c-1-\bar{\alpha}} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (13)$$

Finally, the government directly finances through public investments all projects in the domain $\int_0^{\underline{x}_P} \int_{c-1}^1 d\gamma_j dx_i$ that have not been crowded in by the reforms and/or the subsidies put in place. We refer the reader to the Appendix for the full derivation of the results. The findings of our analysis are summarized in Figure 3c. When we compare the allocations obtained using the cascade algorithm with the constrained optimal ones, unsurprisingly, we find that the cascade framework pushes reforms far beyond, and reduces public investments far below, what is optimal from a welfare perspective. Interestingly, since the relative cost of subsidies increases with the degree of efficiency of the reforms,¹¹ it is when reforms are less efficient that subsidies become increasingly generous and, thus, they are more likely to crowd out more efficient public sector projects.

4.3 Alternative sequencing

In the same way as we did for the cascade, we can compute the allocations that are associated with the five other possible sequences of policies (see Appendix). The results are summarized in Figure 3d-h. If we compare the cascade, *RSG*, with the *RGS* sequence, reforms necessarily lead to the financing of the same set of projects. This, in turn, implies that results are identical when reforms are so efficient that subsidies are never implemented (high values of $\bar{\alpha}$). However, they differ substantially for low values of $\bar{\alpha}$. When this is the case, public investments replace subsidies.

Let us now consider the *SRG* sequence. Of course, in this case, subsidies play a substantial role. However, differently from the cascade, they are implemented when they are comparatively more efficient, namely when the private sector efficiency advantage is substantial (high values x_i). In addition, while for low values of $\bar{\alpha}$ subsidies completely crowd out reforms, for intermediate values of $\bar{\alpha}$ both policies coexist at equilibrium. When we move to the *SGR* sequencing, the scope for subsidies remains the same. However, now, public investments replace reforms completely, for intermediate values, and partially for high values of $\bar{\alpha}$.

When we move to the “anticascade” sequence *GSR*, public investments are implemented when-

¹¹When reforms cover most of the high- γ projects, the share of “bad” projects, that is, those that lie below $\gamma_P = c - 1 - x$, crowded in by subsidies necessarily increases.

ever they yield positive returns (including the externality), while subsidies crowd in the remaining projects where the private sector has a substantial efficiency advantage; reforms, when sufficiently efficient, play the residual role. Lastly, in the *GRS* scenario, reforms completely crowd out subsidies for intermediate and high values of $\bar{\alpha}$, but not for lower ones.

4.4 Optimal myopic sequencing

In the previous section, we illustrated the policy allocations associated with the different possible policy sequences. The obvious question that remains to be answered is which sequence is superior on welfare grounds, and under which circumstances. By a simple comparison of the different policy allocations with the *COPA*, it is evident that the latter coincides with *SRG* and *SGR* for low values of $\bar{\alpha}$ and with *SGR* alone for intermediate values. Instead, for high values of $\bar{\alpha}$, the cascade, or the identical *RGS* scenario, are the ones that resemble more to the *COPA*. To find a definitive answer to the question we compute, for all values of $\bar{\alpha}$ and c , the welfare functions derived in the Appendix (integrating over x_i and γ_j), we compare them all and, for each combination of the parameters, we compute the policy allocation(s) that leads to higher welfare levels. We find that:

Proposition 1 *(i) When reform efficiency is low (low values of $\bar{\alpha}$) the optimal myopic sequencing is SRG or SGR (which are identical). (ii) When reform efficiency is high, the optimal myopic sequencing is RSG (the cascade) or RGS (which are identical). (iii) For intermediate values of $\bar{\alpha}$, the optimal myopic sequencing is SRG or SGR (which are identical), when the cost of resources c is low, SGR for intermediate values of c , and SRG when c is large.*

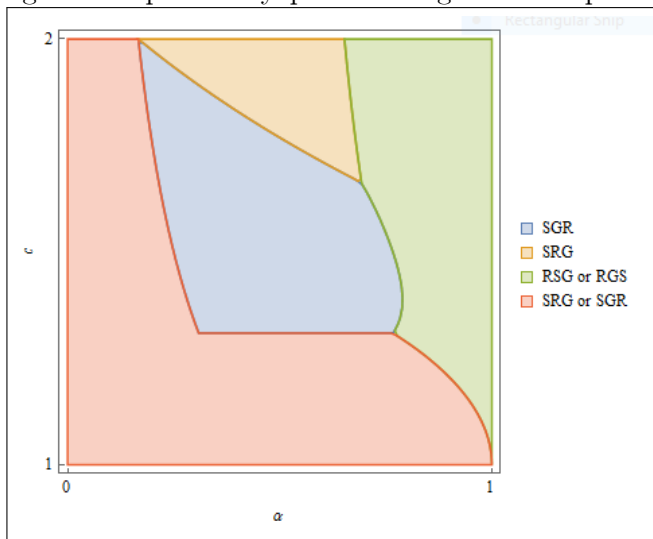
Proof: In Appendix

The findings of our analysis are illustrated in Figure 4, which fully characterizes the optimal myopic policy allocations for any value of the parameters, that is in the $(\bar{\alpha}, c)$ space. From the discussion in the previous section, and from an even cursory inspection of the figure, it is evident that, for low values of $\bar{\alpha}$, the optimal (myopic) policy sequencing are *SRG* and *SGR*—which in this case coincide, since no reform is undertaken—that mimic the *COPA*. When reforms are very efficient, the cascade (*RSG*) or *RGS*—which coincide since reforms completely crowd out subsidies—are instead the optimal allocations. For intermediate values of $\bar{\alpha}$, the optimal allocation depends on the value of c . If c is small, so that subsidy costs are limited, reforms will never be implemented and, again, *SRG* and *SGR* will be identical, and constitute the preferred option. For intermediate values of c , reforms are implemented and the larger is c , the more likely it is that reforms are a better instrument to fill the financing gap than public investments.

4.5 Optimal forward-looking sequencing

To understand the results derived in the previous section, it is useful to think of a government with three agencies dealing with the investment gap, one in charge of reforms, one of subsidies, and one of public investments. Each of the agencies is given the mandate of maximizing total

Figure 4: Optimal myopic financing for development



welfare using the policy it can implement, without looking at what other agencies do. The problem of the government is that of deciding which agency moves first, which second and which third. The setback (and additional source of inefficiency) here is that no agency anticipates that its own decisions affect the decisions of the other agencies. This is the reason why we called such behavior myopic.

We now relax such an assumption and let each agency maximize total welfare (that is, the returns from investments triggered by the policies of all three agencies) fully anticipating how its own policies affect the behavior of the other agencies. Again, the order of the “moves” does matter because reforms and subsidies cannot be perfectly targeted. When this is the case, the optimal forward-looking sequencing is the constrained optimal policy allocation (COPA), we depicted in Figure 3c. More precisely, we have that

Proposition 2 *The COPA requires that subsidies be offered first and reforms and public investment later (the order does not matter). When agents are forward looking, the cascade sequencing coincides with the COPA, only when reforms do not belong to it.*

Proof: In Appendix

In the Appendix, together with the proof, we provide the full derivation of the COPA. Here, we will spare the reader the technical details and just focus on the intuition behind Proposition 2. As we already mentioned, when the efficiency advantage of the private sector is substantial, the costs associated with the implementations of subsidies are of second order. Since the marginal gains associated with reforms or public investment do not depend on x_i , it is always better to implement subsidies first, to avoid that the most efficient subsidies be crowded out by reforms or public investments. Since the government can perfectly target public investments, the order in

which it implements public investments and reforms really does not matter (as the choice will only be based on their relative effectiveness). Using the cascade sequencing, if reforms are part of the policy menu, they will necessarily crowd out more efficient subsidies. Hence, it is only when no reforms are part of the COPA that the cascade is as good as any other sequencing, since, in this case, it does not matter whether to implement reforms or public investments first, since reforms will not be implemented in either case.

5 Concluding Remarks

According to a recent McKinsey report (McKinsey Global Institute, 2016), to keep current growth trajectories, the world needs to invest about \$3.3 trillion, 3.8 percent of GDP a year, in economic infrastructure, about 60 percent of which in developing and emerging economies. The estimated financing gap for the world as a whole is about \$350 billion a year, a figure that should be multiplied by three, if one budgets in new global commitments. The available public and official resources alone clearly do not suffice, so that the success of the Agenda 2030 critically depends on private sector participation; this is not only because of the financial resources but also because of the expertise the private sector can bring to the table.

The cascade framework, adopted by the WBG, is an attempt to crowd in private resources by using public funds as a last resort. In adopting the framework, the WBG wants to signal that the timing of “using public finance till there is money and only then consider private sector options” had to be replaced by that of “trying everything and only if nothing works use public funds.”¹² While the motivations and the political economy behind the cascade framework are easy to understand, the economic underpinnings of the algorithm are less clear. This is what drove us to “write a model.”

The model we presented is the simplest one—at least the simplest one we could come up with—that can describe the main trade-offs associated with the cascade framework. We like simple models; however, it is important to discuss how reasonable our simplifying assumptions, and more importantly how robust our results, are. This is what we try to do next.

First, discussing the sequencing of policies, we assumed that when investors are offered a subsidy they do not wait for a more profitable reform (at a later stage) to invest, or if offered a subsidy they do not wait for a more profitable reform. While such behavior could be seen as a consequence of myopic behavior (similar to the one of the government agencies), it can also be seen as the outcome of a competitive environment: if one investor passes on a profitable opportunity, another will take advantage of it.

Second, the main rationale behind the cascade framework is that public resources are scarce, and this is why they should be used only as a last resort. In our model, we do not have a public

¹²Actually, the IBRD article of agreements 1.(ii) already stated that the purpose of the Bank is “To promote private foreign investment by means of guarantees or participations in loans and other investments made by private investors; and when private capital is not available on reasonable terms, to supplement private investment by providing, on suitable conditions, finance for productive purposes out of its own capital, funds raised by it and its other resources,” something that echoes the cascade framework. I thank Aart Kraay for pointing this out.

resource constraint, and we further assume that the marginal cost of public funds is constant. However, such assumptions are not critical. Consider the case in which the government has a hard budget constraint or faces an increasing marginal cost of funding. This does affect the amount of subsidies it can offer or the number of projects it can finance, but not the optimal sequence of policies, both when agencies are myopic and forward looking.

Third, we assumed that the government cannot perfectly target subsidies and reforms. Such an assumption, as we already mentioned, is critical for the sequencing of policies to matter, and thus should be part of any analysis of the cascade. Our modeling of subsidies is pretty standard and, were the government able to offer project specific subsidies, this would make the case for “subsidies first” even stronger. As per the reforms, we assumed that if a reform does not attract a particular investment project, it does not affect the returns of the same project if the project is financed through subsidies. This is a reasonable assumption if one thinks of regulatory reforms such as those that allow the private sector to bid for concessions. However, it is a less reasonable one if reforms have a cost in terms of externalities. Think, for instance, of a reform that reduces some environmental standards, or one that strengthens patent rights. Once such reforms are undertaken, they apply to all projects, already, and yet to be financed. When this is the case, and agents are myopic, the appeal of subsidies *vis-à-vis* reforms decreases and the cascade sequencing is optimal for a larger set of parameters. When, instead, agents are forward-looking, then the sequencing becomes irrelevant; the incentives of both subsidies and reforms coexist in any single project; however, the overall appeal of reforms decreases as the associated costs in term of lost externalities now affect all projects that end up being financed by the private sector.

While the previous discussion suggests that our main conclusions should hold true in a more general framework, we do not claim that the subsidy first policy should replace the cascade algorithm under all possible circumstances. For instance, one could add government failures to the model, and this can tilt the decision tree in different directions.

There is, however, one general lesson that can be learned from our analysis. The lesson is that the objective of maximizing private finance for development may conflict with the objective of optimizing finance for development; this means that policy-makers should carefully weigh the different trade-offs if they want to use the scarce existing resources, which are vital to achieve the Sustainable Development Goals, in the most effective way.

6 Appendix

6.1 Proof of Lemma 1

(1):(i) $\bar{\alpha} < \frac{1}{c}$ implies that $\underline{\gamma}_S < \underline{\gamma}_R$. Hence, $\gamma < \text{Min}\{\underline{\gamma}_G, \underline{\gamma}_S\}$ is a necessary and sufficient condition for no investment inducing policy to be welfare improving.

(iii) We further have that $\bar{\alpha} < \frac{1}{c}$ implies that, for all $x < \underline{x}_P$, $\underline{\gamma}_R > \underline{\gamma}_S > \underline{\gamma}_{RS}$, so that reforms are never optimal. Subsidies are, instead, welfare improving if $\gamma > \underline{\gamma}_S$, and they are preferred to public investment if

$$1 + x_i + \gamma + \widehat{s}(c - 1) - c > 1 + \gamma - c, \quad (14)$$

which can be re-written as

$$x_i > \frac{(1 - c)^2}{c} \equiv \underline{x}_{SG}. \quad (15)$$

(ii) Public investments are thus optimal if $\gamma > \underline{\gamma}_G$, and $x < \underline{x}_{SG}$.

(2):(i) $\bar{\alpha} > \frac{1}{c}$ implies that $\underline{\gamma}_R < \underline{\gamma}_S$. Hence, $\gamma < \text{Min}\{\underline{\gamma}_G, \underline{\gamma}_R\}$ is a necessary and sufficient condition for no investment inducing policy to be welfare improving.

(iii) Notice that for reforms to be preferred to subsidies we need that

$$1 + x + \bar{\alpha}\gamma - c > 1 + x + \gamma - c - \widehat{s}(c - 1), \quad (16)$$

or

$$\gamma < \frac{(c - 1)(c - 1 - x)}{1 - \bar{\alpha}} \equiv \underline{\gamma}_{RS}. \quad (17)$$

We further have that $\bar{\alpha} > \frac{1}{c}$ implies that for all $x < \underline{x}_P$, $\underline{\gamma}_{RS} > \underline{\gamma}_R$, so that there is a non empty set of values in which reforms are preferred to subsidies. Finally, reforms are preferred to public investments iff

$$1 + x + \bar{\alpha}\gamma - c > 1 + \gamma - c, \quad (18)$$

or

$$\gamma < \frac{x}{1 - \bar{\alpha}} \equiv \underline{\gamma}_{RG}. \quad (19)$$

Notice that

$$\underline{\gamma}_{RG} > \underline{\gamma}_R \iff x_i > (c - 1)(1 - \bar{\alpha}) \equiv \underline{x}_{RG}. \quad (20)$$

Hence, for reforms to be optimal we need that $\gamma < \text{Min}\{\underline{\gamma}_{RS}, \underline{\gamma}_{RG}\}$, and $x_i > \underline{x}_{RG}$.

(iv) For subsidies to be preferred to reforms we need that $\gamma > \underline{\gamma}_{RS}$. Notice that, since $\underline{\gamma}_{RS} > \underline{\gamma}_S$, such a condition also insures that subsidies are welfare improving. Finally, subsidies are preferred to public investments if $x_i > \underline{x}_{SG}$.

(ii) For public investments to be welfare improving, we need that $\gamma > \underline{\gamma}_G$, for public investment to be preferred to reforms that $\gamma > \underline{\gamma}_{RG}$, and $x < \underline{x}_{SG}$ for public investments to be preferred to subsidies.

6.2 Alternative sequencing (myopic beliefs)

In this section, we compute the allocations corresponding to each of the different policy sequences z , $z = \{1, \dots, 6\}$, and the associated welfare W_z . We start with the cascade, *RSG*.

6.2.1 RSG ($z = 1$).

The welfare gains W_1^R associated with reforms (R) under the cascade approach ($z = 1$) are given by

$$W_1^R = \int_{\text{Max}\{0; \tilde{x}\}}^{\underline{x}_P} \int_{\underline{\gamma}_R}^1 (1 + x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i, \quad (21)$$

where $\tilde{x} \equiv \{x : \underline{\gamma}_R = 1\} = c - 1 - \alpha$. Solving for (21) we have that

$$W_1^R = \begin{cases} \frac{\bar{\alpha}^2}{6}, & \text{if } \bar{\alpha} < c - 1; \\ \frac{(3\bar{\alpha}^2 + (c-1)(c-1-3\bar{\alpha}))(c-1)}{6\bar{\alpha}}, & \text{if } \bar{\alpha} \geq c - 1. \end{cases} \quad (22)$$

Let us now introduce subsidies, s ; we first consider the case $\bar{\alpha} \leq c - 1$, so that $\underline{\gamma}_R < 1$ for all $x_i \in [0, 1]$. If the optimal subsidy s_1^* (we denote the optimal subsidy by “*” hereinafter) is greater than $\bar{\alpha}$, it is the one that maximizes

$$\begin{aligned} W_1^{Sa_1} = & \int_{c-1-\alpha}^{\underline{x}_P} \int_0^{\underline{\gamma}_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i + \\ & + \int_{c-1-s}^{c-1-\alpha} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \end{aligned} \quad (23)$$

Differentiating (23) with respect to s , we have that

$$\frac{\partial W_1^{Sa_1}}{\partial s} = \frac{1 + \alpha(c-1) - (4c-2)s}{2}, \quad (24)$$

so that a necessary condition for an internal maximum is

$$\frac{\partial W_1^{Sa_1}}{\partial s} = 0 \iff s = \frac{1 + \bar{\alpha}(c-1)}{4c-2} \equiv \bar{s}_1. \quad (25)$$

Substituting this value into (23) we have that

$$W_1^{Sa_1} = \frac{\bar{\alpha} + 3c(c-1)(2c-3) + \bar{\alpha}(1-3(2-c)c)}{6}. \quad (26)$$

Notice that, for \bar{s}_1 to be the optimal subsidy, we need that $\bar{s}_1 \geq \alpha$, that is, $\bar{\alpha} \leq \frac{1}{3c-1}$, and $c-1-\bar{s} \geq 0$, that is, $\bar{\alpha} \leq 2(2c-1) - \frac{1}{c-1} \equiv \bar{\alpha}_a$.

Assume now that $\bar{\alpha} > \frac{1}{3c-1}$. When this is the case, the problem of the government is that of

maximizing

$$W_1^{Sa_2} = \int_{c-1-s}^{\bar{x}_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (27)$$

Differentiating (27) with respect to s , we have that

$$\frac{\partial W_1^{Sa_2}}{\partial s} = \frac{(1 + \alpha - 3\alpha c)s^2}{2\bar{\alpha}^2} < 0 \iff \bar{\alpha} > \frac{1}{3c-1}. \quad (28)$$

Hence, $\bar{\alpha} > \frac{1}{3c-1} \implies s^* = 0$.

Consider now the case $\bar{\alpha} > \bar{\alpha}_a$, then the government would set $s_1^* = c-1$ iff

$$\begin{aligned} W_1^{Sa_3} |_{\bar{s}=c-1} &= \int_{c-1-\alpha}^{\bar{x}_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - (c-1)^2 - c) d\gamma_j dx_i + \\ &+ \int_0^{c-1-a} \int_0^{\gamma_R} (1 + x_i + \gamma_j - (c-1)^2 - c) d\gamma_j dx_i = \\ &\frac{\bar{\alpha}^2 - 3c(c-1)(2c-3) + \bar{\alpha}(1-3(2-c)c)}{6} > 0 \iff \bar{\alpha} < \bar{\alpha}_b, \end{aligned} \quad (29)$$

with $\bar{\alpha}_b \equiv \frac{3(2-c)c-1-\sqrt{1+3c(3c-4)(c^2-2)}}{2}$, and $s_1^* = 0$, otherwise.

We now consider the case $\bar{\alpha} > c-1$. In this case, the the welfare associated with subsidies is given by:

$$W_1^{Sb} = \int_0^{\bar{x}_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i = \frac{(1 - \bar{\alpha}(1-3c))s^2}{2\bar{\alpha}^2}. \quad (30)$$

Iff $\bar{\alpha} \leq \frac{1}{3c-1}$, (30) is increasing in s , and it is positive at $\bar{s} = c-1$. Hence, we have $s^* = c-1$, if $\bar{\alpha} \leq \frac{1}{3c-1}$, and $s^* = 0$ otherwise. Summarizing our findings about the optimal subsidy, we have that

$$s_1^* = \left\{ \begin{array}{ll} \bar{s}_1 & \text{if } \bar{\alpha} < \bar{\alpha}_a, \\ c-1, & \text{if } \bar{\alpha}_a \leq \bar{\alpha} < \bar{\alpha}_b, \\ 0, & \bar{\alpha} \geq \bar{\alpha}_b, \end{array} \right\} \quad \left. \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{3c-1}; \\ \text{if } \bar{\alpha} \geq \frac{1}{3c-1}; \end{array} \right\} \quad \text{if } \bar{\alpha} < c-1; \quad (31)$$

$$\left. \begin{array}{l} c-1, \\ 0, \end{array} \right\} \quad \text{if } \bar{\alpha} \geq c-1.$$

Putting all the pieces together, using (26), (29), and (30), we have that:

$$W_1^{S^*} = \left\{ \begin{array}{l} \left. \begin{array}{l} \frac{3-\bar{\alpha}(c+1)(1-\bar{\alpha}(1-3c))}{24(2c-1)}, \quad \text{if } \bar{\alpha} < \bar{\alpha}_a, \\ \frac{\bar{\alpha}+3c(c-1)(2c-3)+\bar{\alpha}(1-3(2-c)c)}{6}, \quad \text{if } \bar{\alpha}_a \leq \bar{\alpha} < \bar{\alpha}_b, \\ 0, \quad \bar{\alpha} \geq \bar{\alpha}_b \end{array} \right\} \quad \left. \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{3c-1}; \\ \\ \text{if } \bar{\alpha} \geq \frac{1}{3c-1} \end{array} \right\} \quad \left. \begin{array}{l} \text{if } \bar{\alpha} < c-1; \\ \\ \text{if } \bar{\alpha} \geq c-1. \end{array} \right\} \quad (32)$$

Let us now look at the welfare associated with public investments. Using (31), it is easy to verify that

$$W_1^G = \left\{ \begin{array}{l} \left. \begin{array}{l} \int_0^{c-1-\bar{s}_1} \int_{c-1}^1 (1+\gamma_j-c) d\gamma_j dx_i \quad \text{if } \bar{\alpha} < \frac{1}{3c-1} \ \&\& \ \bar{\alpha} < \bar{\alpha}_a, \\ 0, \quad \text{if } \bar{\alpha}_a \leq \bar{\alpha} < \bar{\alpha}_b, \\ (1-\bar{\alpha})(c-1) \int_{c-1}^{\gamma_R} (1+\gamma_j-c) d\gamma_j dx_i + \\ \frac{c-1-a}{c-1-\bar{\alpha}} \int_0^1 \int_{c-1}^1 (1+\gamma_j-c) d\gamma_j dx_i, \quad \text{otherwise,} \end{array} \right\} \quad \bar{\alpha} < c-1; \\ \\ \left. \begin{array}{l} 0, \quad \text{if } \bar{\alpha} < \frac{1}{3c-1}, \\ (1-\bar{\alpha})(c-1) \int_0^{\gamma_R} \int_{c-1}^1 (1+\gamma_j-c) d\gamma_j dx_i \quad \text{if } \bar{\alpha} \geq \frac{1}{3c-1}, \end{array} \right\} \quad \bar{\alpha} \geq c-1; \end{array} \right. \quad (33)$$

where $(1-\bar{\alpha})(c-1) = \gamma : \{\gamma_R = c-1\}$. Now, working through the algebra we have that

$$W_1^G = \left\{ \begin{array}{l} \left. \begin{array}{l} \frac{(2-c)^2(4c^2+1+\bar{\alpha}-(6+\bar{\alpha})c)}{4(2c-1)}, \quad \text{if } \bar{\alpha} < \frac{1}{3c-1} \ \&\& \ \bar{\alpha} < \bar{\alpha}_a, \\ 0, \quad \text{if } \bar{\alpha}_a \leq \bar{\alpha} < \bar{\alpha}_b, \\ \frac{(c-1)^3(1-\bar{\alpha})^3}{6\bar{\alpha}^2}, \quad \text{otherwise,} \end{array} \right\} \quad \bar{\alpha} < c-1; \\ \\ \left. \begin{array}{l} 0, \quad \text{if } \bar{\alpha} < \frac{1}{3c-1}, \\ \frac{(c-1)^3(1-\bar{\alpha})^3}{6\bar{\alpha}^2}, \quad \text{if } \bar{\alpha} \geq \frac{1}{3c-1}, \end{array} \right\} \quad \bar{\alpha} \geq c-1. \end{array} \right. \quad (34)$$

Finally, we have that, $W_1^* = W_1^R + W_1^{S^*} + W_1^G$.

6.2.2 RGS ($z = 2$)

As in the cascade, we have that the welfare associated with reforms is given by:

$$W_2^R = \left\{ \begin{array}{l} \frac{\bar{\alpha}^2}{6}, \quad \text{if } \bar{\alpha} < c-1; \\ \frac{(3\bar{\alpha}^2+(c-1)(c-1-3\bar{\alpha}))(c-1)}{6a}, \quad \text{if } \bar{\alpha} \geq c-1. \end{array} \right. \quad (35)$$

The welfare associated with public investments, G , which are implemented after, is instead given by

$$W_2^G = \begin{cases} \int_{c-1-\bar{\alpha}}^{(c-1)(1-\bar{\alpha})} \int_{c-1}^{\gamma_R} (1 + \gamma_j - c) d\gamma_j dx_i + \int_0^{c-1-\bar{\alpha}} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i & \text{if } \bar{\alpha} < c - 1; \\ \int_0^{(c-1)(1-\bar{\alpha})} \int_{c-1}^{\gamma_R} (1 + \gamma_j - c) d\gamma_j dx_i, & \text{if } \bar{\alpha} \geq c - 1; \end{cases} \quad (36)$$

or, solving,

$$W_2^G = \begin{cases} \frac{(2-c)^2(3+\bar{\alpha}-(3-\bar{\alpha})c)}{6} & \text{if } \bar{\alpha} < c - 1; \\ \frac{(c-1)^3(1-\bar{\alpha})^3}{6\bar{\alpha}^2} & \text{if } \bar{\alpha} \geq c - 1. \end{cases} \quad (37)$$

Let us now consider subsidies. If the optimal subsidy s_2^* is such that $c - 1 - s_2^* < (1 - \bar{\alpha})(c - 1)$, it is the one that maximizes

$$W_2^{Sa} = \int_{(c-1)(1-\bar{\alpha})}^{x_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i + \int_{c-1-s}^{(c-1)(1-\bar{\alpha})} \int_0^{c-1} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (38)$$

Differentiating (38) with respect to s , we have that

$$\frac{\partial W_2^{Sa}}{\partial s} = 0 \iff s = \frac{((c-1)\bar{\alpha} + 1)(c-1)}{2(2c-1)} \equiv \bar{s}_2. \quad (39)$$

It is easy to verify that $c - 1 - \bar{s}_2 > 0$. In addition, we have that

$$c - 1 - \bar{s}_2 < (1 - \bar{\alpha})(c - 1) \iff \bar{\alpha} < \frac{1}{3c-1}, \quad (40)$$

so that $s_2^* = \bar{s}_2$, if $\bar{\alpha} < \frac{1}{3c-1}$. Substituting \bar{s}_2 into (38), we have that

$$W_2^{Sa} = \frac{(c-1)^3(3 - \bar{\alpha}(1+c))(1 - \bar{\alpha}(1-3c))}{24(2c-1)}. \quad (41)$$

Assume now that the optimal subsidy s_2^* is such that $c - 1 - s_2^* > (1 - \bar{\alpha})(c - 1)$. Then, it should be the one that maximizes

$$W_2^{Sb} = \int_{c-1-s}^{x_P} \int_0^{\gamma_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (42)$$

Differentiating (42) with respect to s , we have that

$$\frac{\partial W_2^{Sb}}{\partial s} = \frac{(1 + \bar{\alpha} - 3\bar{\alpha}c)s^2}{2\bar{\alpha}^2} \leq 0 \iff \bar{\alpha} \geq \frac{1}{3c-1}, \quad (43)$$

which implies that $s_2^* = 0$ if $\bar{\alpha} \geq \frac{1}{3c-1}$. Hence, we have that

$$W_2^{S^*} \begin{cases} \frac{(c-1)^3(3-\bar{\alpha}(1+c))(1-\bar{\alpha}(1-3c))}{24(2c-1)}, & \text{if } \bar{\alpha} < \frac{1}{3c-1}, \\ 0, & \text{otherwise.} \end{cases} \quad (44)$$

Finally, we have that, $W_2^* = W_2^R + W_2^G + W_2^{S^*}$.

6.2.3 SRG ($z = 3$)

Starting with subsidies, the government has to maximize

$$W_3^{Sa} = \int_{c-1-s}^{\underline{x}_P} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (45)$$

Differentiating (45) with respect to s , we have that a necessary and sufficient condition for an internal maximum is

$$\frac{\partial W_3^S}{\partial s} = 0 \iff s = \frac{1}{2(2c-1)} \equiv \bar{s}_3. \quad (46a)$$

Notice, further, that $c - 1 - \bar{s}_3 > 0 \iff c > \frac{3+\sqrt{5}}{4} \equiv c_a \approx 1.3$. If $c < c_a$, the government will put subsidies in place if it derives positive utility with $s = 1 - c$, that is, if

$$W_3^{Sb} = \int_0^{\underline{x}_P} \int_0^1 (1 + x_i + \gamma_j - (c-1)^2 - c) d\gamma_j dx_i = \frac{-2c^2 + 5c - 3}{2} > 0 \iff c < \frac{3}{2}, \quad (47)$$

which is always the case. Hence, we have that

$$s_3^* = \begin{cases} 1 - c, & \text{if } c < c_a; \\ \bar{s}_3, & \text{if } c \geq c_a; \end{cases} \quad (48)$$

and

$$W_3^{S^*} = \begin{cases} \frac{(c-1)(3-2c)c}{2}, & \text{if } c < c_a, \\ \frac{1}{8(2c-1)}, & \text{if } c \geq c_a. \end{cases} \quad (49)$$

Let us now move to reforms. When $c < c_a$, there is no space for reforms as subsidies completely crowd out any other policy. When, instead, $c \geq c_a$, as always, we have to distinguish between the case in which $\bar{\alpha}$ is smaller or larger than $c - 1$. When $\bar{\alpha} < c - 1$, reforms occur if $\bar{s}_3 < \bar{\alpha}$, that is, $\bar{\alpha} > \frac{1}{2(2c-1)}$. Consequently, the utility associated with reforms is given by

$$W_3^R = \left\{ \begin{array}{l} 0, \\ 0, \\ \int_{c-1-\bar{\alpha}}^{c-1-\bar{s}_3} \int_{\underline{\gamma}_R}^1 (1+x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i, \\ \int_0^{c-1-\bar{s}_3} \int_{\underline{\gamma}_R}^1 (1+x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{2(2c-1)}, \\ \text{if } \bar{\alpha} \geq \frac{1}{2(2c-1)}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < c-1; \\ \text{if } \bar{\alpha} \geq c-1; \end{array} \right\} \left\{ \begin{array}{l} \text{if } c < c_a; \\ \text{if } c \geq c_a; \end{array} \right\} \quad (50)$$

or, doing the algebra,

$$W_3^R = \left\{ \begin{array}{l} 0, \\ 0 \\ \frac{(4c-2)\bar{\alpha}-1}{48\bar{\alpha}(2c-1)^3}, \\ \frac{\Psi}{48\bar{\alpha}(2c-1)^3}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{2(2c-1)}, \\ \text{if } \bar{\alpha} \geq \frac{1}{2(2c-1)}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < c-1; \\ \text{if } \bar{\alpha} \geq c-1; \end{array} \right\} \left\{ \begin{array}{l} \text{if } c < c_a; \\ \text{if } c \geq c_a; \end{array} \right\} \quad (51)$$

with $\Psi \equiv (1 - 6c + 4c^2)(7 + 12\bar{\alpha}^2(1 - 2c)^2 - 30c + 8c^2(7 - 2(3 - c)c) + 6\bar{\alpha}(3 - 4c(3 - 2(2 - c)c)))$.

Finally, let us now consider public investments. The associated utility is given by

$$W_3^G = \left\{ \begin{array}{l} 0, \\ \int_0^{c-1-\bar{s}_3} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i \\ \int_{c-1-\bar{\alpha}}^{c-1-\bar{s}_3} \int_{c-1}^{\underline{\gamma}_R} (1 + \gamma_j - c) d\gamma_j dx_i + \\ \int_{c-1-\bar{\alpha}}^{c-1-\bar{\alpha}} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i, \\ \int_0^{c-1-\bar{s}_5} \int_{c-1}^{\underline{\gamma}_R} (1 + \gamma_j - c) d\gamma_j dx_i, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{2(2c-1)}, \\ \text{if } \bar{\alpha} \geq \frac{1}{2(2c-1)}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < c-1; \\ \text{if } \bar{\alpha} \geq c-1; \end{array} \right\} \left\{ \begin{array}{l} \text{if } c < c_a; \\ \text{if } c \geq c_a; \end{array} \right\} \quad (52)$$

or, doing the algebra,

$$W_3^G = \left\{ \begin{array}{l} 0, \\ \frac{(2-c)^2(c^2-6c+1)}{4(2c-1)}, \\ \frac{\Phi}{(48\bar{\alpha}^2(2c-1)^3)(-1+6\bar{\alpha}(-1+c)(-1+2c)}, \\ \frac{\Theta}{48\bar{\alpha}^2(2c-1)^3(-1+6\bar{\alpha}(-1+c)(-1+2c)}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < \frac{1}{2(2c-1)}, \\ \text{if } \bar{\alpha} \geq \frac{1}{2(2c-1)}, \end{array} \right\} \left\{ \begin{array}{l} \text{if } \bar{\alpha} < c-1; \\ \text{if } \bar{\alpha} \geq c-1; \end{array} \right\} \left\{ \begin{array}{l} \text{if } c < c_a, \\ \text{if } c \geq c_a; \end{array} \right\} \quad (53)$$

where,

$$\Phi \equiv -1 - 6\bar{\alpha}(c-1)(2c-1) + 8\bar{\alpha}^3(2c-1)^3(3c-5) + 12(c-1)(2\bar{\alpha}c - \bar{\alpha})^2(c(23 - 2c(9 - 2c) - 7),$$

$\Theta \equiv (1 - 6c + 4c^2)(7 - 30c + 8c^2(7 - 2(3 - c)c) + 12\bar{\alpha}^2(1 - 3c + 2c^2)^2 - 6\bar{\alpha}(1 - c)(2c - 1)(3 - 6c + 4c^2))$.
 Finally, we have that, $W_3^* = W_3^{S^*} + W_3^R + W_3^G$.

6.2.4 SGR ($z = 4$)

As in the previous case, the welfare associated with subsidies is given by

$$W_4^{S^*} = \begin{cases} \frac{(c-1)(3-2c)c}{2}, & \text{if } c < c_a, \\ \frac{1}{8(2c-1)}, & \text{if } c \geq c_a. \end{cases} \quad (54)$$

Let us now consider public investments, welfare is given by

$$W_4^G \begin{cases} 0, & \text{if } c < c_a, \\ \int_{c-1}^{c-1-\bar{s}_3} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i = \frac{(c-2)^2(4c^2-6c+1)}{4(2c-1)}, & \text{if } c \geq c_a. \end{cases} \quad (55)$$

After subsidies and (eventually) public investments have been implemented, there is a residual space for reforms iff,

$$x : \underline{\gamma}_R = (c - 1)(1 - \bar{\alpha}) < c - 1 - \bar{s}_3 \iff \bar{\alpha} > \frac{1}{2 - 6c + 4c^2} = \bar{\alpha}_d, \quad (56)$$

where $(c - 1)(1 - \bar{\alpha}) = x_i : \underline{\gamma}_R < (c - 1)$, and hence there is space for subsidies after public investments have been put in place. Notice that for $c \in [1, 2]$, $\bar{\alpha}_d > \bar{s}_3$, hence

$$W_4^R \begin{cases} 0 & \text{if } c < c_a; \\ \left. \begin{array}{l} 0, \quad \bar{\alpha} < \bar{\alpha}_d, \\ \int_{(c-1)(1-\bar{\alpha})}^{c-1-\bar{s}_5} \int_{\underline{\gamma}_R}^{c-1} (1 + x + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i, \quad \bar{\alpha} \geq \bar{\alpha}_d, \end{array} \right\} & \text{if } c \geq c_a; \end{cases} \quad (57a)$$

or, doing the algebra,

$$W_4^R \begin{cases} 0 & \text{if } c < c_a; \\ \left. \begin{array}{l} 0, \quad \bar{\alpha} < \bar{\alpha}_d, \\ \frac{(a(2-6c+4c^2)-1)^3}{48\bar{\alpha}(2c-1)^3}, \quad \bar{\alpha} \geq \bar{\alpha}_d, \end{array} \right\} & \text{if } c \geq c_a. \end{cases} \quad (58)$$

Finally, we have that, $W_4^* = W_4^{S^*} + W_4^G + W_4^R$.

6.2.5 GRS ($z = 5$)

Starting with public investments, the associated welfare gains are given by

$$W_5^G = \int_0^{\underline{x}_P} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i = \frac{(2-c)^2(c-1)}{2}. \quad (59)$$

If the government implements reforms after public investments, the associated utility is given by

$$W_5^R = \int_{(1-\bar{\alpha})(c-1)}^{\underline{x}_P} \int_{\underline{\gamma}_R}^{c-1} (1 + x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i = \frac{(c-1)^3 \bar{\alpha}^2}{6}. \quad (60)$$

Let us now consider subsidies. If the optimal subsidy s^* is such that $c - 1 - s^* < (1 - \bar{\alpha})(c - 1)$, it is the one that maximizes

$$W_5^{Sa} = \int_{(1-\bar{\alpha})(c-1)}^{\underline{x}_P} \int_0^{\underline{\gamma}_R} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i + \int_{c-1-s}^{(1-\bar{\alpha})(c-1)} \int_0^{c-1} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (61)$$

Differentiating (61) with respect to s , we have that

$$\frac{\partial W_5^{Sa}}{\partial s} = 0 \iff s = \frac{((c-1)\bar{\alpha} + 1)(c-1)}{2(2c-1)} \equiv \bar{s}_5. \quad (62)$$

It is easy to verify that $c - 1 - \bar{s}_5 > 0$; it remains to verify that $c - 1 - \bar{s}_5 < (1 - \bar{\alpha})(c - 1)$. It is easy to show that

$$c - 1 - \bar{s}_5 < (1 - \bar{\alpha})(c - 1) \iff \bar{\alpha} < \frac{1}{3c - 1}.$$

When, instead, $c - 1 - s > (1 - \bar{\alpha})(c - 1)$, the problem is the same as in (42) and, again, we obtain that $s_5^* = 0$ if $\bar{\alpha} \geq \frac{1}{3c-1}$. Hence, we have that

$$W_5^{S^*} \begin{cases} W_5^{Sa^*} = \frac{(c-1)^3(3-\bar{\alpha}(1+c))((1-\bar{\alpha}(1-3c)))}{24(2c-1)}, & \text{if } \bar{\alpha} < \frac{1}{3c-1}, \\ 0, & \text{if } \bar{\alpha} \geq \frac{1}{3c-1}. \end{cases} \quad (63)$$

Finally, we have that, $W_5^* = W_5^G + W_5^R + W_5^{S^*}$.

6.2.6 GSR ($z = 6$)

Starting with public investments, as in the previous case, the associated welfare gains are given by

$$W_6^G = \int_0^1 \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i = \frac{(2-c)^2(c-1)}{2}. \quad (64)$$

Let us now move to subsidies, the government has to maximize

$$W_6^S = \int_{c-1-s}^{\frac{x_P}{c-1-s}} \int_0^{c-1} (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i. \quad (65)$$

Differentiating (65) with respect to s , a necessary and sufficient condition for an internal maximum is

$$\frac{\partial W_6^S}{\partial s} = 0 \iff s = \frac{c-1}{2(2c-1)} \equiv s_6^*, \quad (66)$$

and substituting this expression in (64), we have that

$$W_6^{S^*} = \frac{(c-1)^3}{8(2c-1)}. \quad (67)$$

Let us now consider reforms. After public investment and subsidies, there is a residual space for reforms if, and only if,

$$x : \underline{\gamma}_R = (c-1)(1-\bar{\alpha}) < c-1-s_6^* \iff \bar{\alpha} \geq \frac{1}{2(2c-1)}. \quad (68)$$

Hence, we have that

$$W_6^R \begin{cases} 0, & \text{if } \bar{\alpha} < \frac{1}{2(2c-1)}, \\ \int_{(1-\bar{\alpha})(c-1)}^{c-1-s_6^*} \int_{\underline{\gamma}_R}^{c-1} (1 + x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i = \frac{(c-1)^3((4c-2)\bar{\alpha}-1)^3}{48\bar{\alpha}(2c-1)}, & \text{if } \bar{\alpha} \geq \frac{1}{2(2c-1)}. \end{cases} \quad (69)$$

Finally, we have that, $W_6 = W_6^G + W_6^{S^*} + W_6^R$.

6.3 Forward-looking agents: The COPA ($z = \widehat{3}$)

We denote this case by $\widehat{3}$ (since the sequence is the same as for SRG , and the “ $\widehat{}$ ” denotes the forward-looking scenario. In this case the government chooses the optimal subsidy, given the optimal allocation of reforms and public investments at the later stage. If the optimal subsidy \widehat{s}^* is such that $(c-1-\widehat{s}^* > 1-\bar{\alpha})$, where $1-\bar{\alpha} = x_i : \underline{\gamma}_{RG} = 1$, and thus in the interval $[1-\bar{\alpha}, c-1-\widehat{s}]$ reforms are always preferred to public investments, and the optimal subsidy is the one that maximizes

$$\begin{aligned}
W_{\frac{3}{3}}^{\bar{\alpha}} &= \int_0^{(c-1)(1-\bar{\alpha})} \int_{c-1}^1 (1_i + \gamma_j - c) d\gamma_j dx_i + \\
&+ \int_{(c-1)(1-\bar{\alpha})}^{1-\bar{\alpha}} \int_{\underline{\gamma}_{RG}}^1 (1_i + \gamma_j - c) - c) d\gamma_j dx_i + \int_{(c-1)(1-\bar{\alpha})}^{1-\bar{\alpha}} \int_{\underline{\gamma}_R}^{\underline{\gamma}_{RG}} (1 + x_i + \alpha\gamma_j - c) d\gamma_j dx_i + \quad (70) \\
&+ \int_{(1-\bar{\alpha})}^{c-1-s} \int_{\underline{\gamma}_R}^1 (1 + x_i + \bar{\alpha}\gamma_j - c) d\gamma_j dx_i + \int_{c-1-s}^{\underline{x}_P} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i.
\end{aligned}$$

Notice that a necessary condition for $c - 1 - \hat{s}^* > 1 - \bar{\alpha}$ to hold is that $\bar{\alpha} > 2 - c$. Differentiating (70) with respect to s , we have that

$$\frac{\partial W_{\frac{3}{3}}^a}{\partial s} = 0 \iff \hat{s}_a = -2\bar{\alpha}(c-1) + \sqrt{\bar{\alpha}(1 + \bar{\alpha}(3 + 4(c-2)c))}. \quad (71)$$

In addition, we can show that $c - 1 - \hat{s}_a > 1 - \bar{\alpha} \iff c > \frac{2+(5-2\bar{\alpha})\bar{\alpha}+\sqrt{(1-\bar{\alpha})\bar{\alpha}(1+4(2-\bar{\alpha})\bar{\alpha})}}{4\bar{\alpha}+1} \equiv c_b$. Substituting \hat{s}_a in (70), we obtain

$$\begin{aligned}
W_{\frac{3}{3}}^a &= \frac{1}{6}(((2-c)^2(2c-1) + \bar{\alpha}^2(2c-1)(7-2c(5-2c)) - 2\bar{\alpha}^{3/2}(3-2(2-c)c)\sqrt{\Omega} + \\
&+ 2\sqrt{\bar{\alpha}}\sqrt{\Omega}(1+3S\sqrt{\Omega}-3c\sqrt{\Omega}) + \bar{\alpha}(12\sqrt{\Omega}-4-c(24\sqrt{\Omega}-c(3-c+12\sqrt{\Omega}))))),
\end{aligned}$$

with $\Omega \equiv 1 + \bar{\alpha}(3 - 4(2 - c)c)$.

Assume now that $c < c_b$, so that $(c-1)(1-\bar{\alpha}) < c-1-\hat{s}^* < 1-\bar{\alpha}$. In this case, reforms are preferred to public investments if $\gamma < \underline{\gamma}_{RG}$. Total welfare is now given by

$$\begin{aligned}
W_{\frac{3}{3}}^b &= \int_0^{(c-1)(1-\bar{\alpha})} \int_{c-1}^1 (1_i + \gamma_j - c) d\gamma_j dx_i + \\
&+ \int_{(c-1)(1-\bar{\alpha})}^{c-1-s} \int_{\underline{\gamma}_{RG}}^1 (1_i + \gamma_j - c) - c) d\gamma_j dx_i + \int_{(c-1)(1-\bar{\alpha})}^{c-1-s} \int_{\underline{\gamma}_R}^{\underline{\gamma}_{RG}} (1 + x_i + \alpha\gamma_j - c) d\gamma_j dx_i + \quad (72) \\
&+ \int_{c-1-s}^{\underline{x}_P} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i.
\end{aligned}$$

Differentiating (72) with respect to s , we have that

$$\frac{\partial W_{\frac{3}{3}}^b}{\partial s} = 0 \iff \bar{s}_b = \bar{\alpha}^2(2c-1) - \bar{\alpha}c + \sqrt{(1-\bar{\alpha})\bar{\alpha}(c(4+2\bar{\alpha}-c) - 3 - \bar{\alpha} - \bar{\alpha}^2(2c-1)^2)}. \quad (73)$$

In addition, we can show that $c - 1 - \hat{s}_b > (c-1)(1-\bar{\alpha}) \iff c > \frac{3+2\bar{\alpha}}{1+4\bar{\alpha}}$.

Substituting \widehat{s}_b in (72), we obtain

$$W_{\frac{3}{3}}^b = 1/6(-2\bar{\alpha}^4(2c-1)^3 + 2\bar{\alpha}^3(1-3c+4c^3) + (c-1)(3(c-2)^2 + 6\sqrt{\Phi} - 2c\sqrt{\Phi}) + \quad (74)$$

$$- 2\bar{\alpha}^2(2c-1)(4 - \sqrt{\Phi} - c(4-c-2\sqrt{\Phi})) - \bar{\alpha}(2\sqrt{\Phi} - c(9 + 3(c-4)c + 4\sqrt{\Phi}))),$$

with $\Phi \equiv (1 - \bar{\alpha})\bar{\alpha}(c(4 + 2\bar{\alpha} - c) - 3 - \bar{\alpha} - \bar{\alpha}^2(2c - 1)^2)$. Let now consider the case in which $c < \frac{3+2\bar{\alpha}}{1+4\bar{\alpha}}$, so that public investments are always preferred to reforms because of (20). Total welfare is given by

$$W_{\frac{3}{3}}^c = \int_0^{c-1-s} \int_{c-1}^1 (1_i + \gamma_j - c)d\gamma_j dx_i + \int_{c-1-s}^{\underline{x}_P} \int_0^1 1 + x_i + \gamma_j - s(c-1) - c)d\gamma_j dx_i. \quad (75)$$

Differentiating (75) with respect to s , we have that

$$\frac{\partial W}{\partial s} = 0 \iff s = \frac{(3-c)(c-1)}{4c-2} \equiv \widehat{s}_c. \quad (76)$$

It is also easy to verify that $c-1-\widehat{s}_c > 0$. Substituting \widehat{s}_c in (76) we obtain

$$W_{\frac{3}{3}}^c = \frac{(c-1)(c(63 + c(-43 + 9c)) - 25)}{8(2c-1)}. \quad (77)$$

Summarizing our findings, we have that:

$$W_{\frac{3}{3}}^c = \left\{ \begin{array}{ll} W_{\frac{3}{3}}^c, & \text{if } c < \frac{3+2\bar{\alpha}}{1+4\bar{\alpha}}, \\ W_{\frac{3}{3}}^b, & \text{if } c \geq c \frac{3+2\bar{\alpha}}{1+4\bar{\alpha}}, \\ W_{\frac{3}{3}}^\alpha, & \end{array} \right\} \quad \left. \begin{array}{l} \text{if } c < c_b; \\ \text{if } c \geq c_b. \end{array} \right.$$

6.4 Proof of Proposition 1

(i) For low enough values of $\bar{\alpha}$, no reforms are undertaken under *GSR*, *SRG*, *SGR*, while they always are under *RSG*, *RGS*, and *GRS*. We also have that, at $x = \underline{x}_P$, the welfare changes associated with having reforms, instead of subsidies, is given by

$$DW_{SR} = \int_{\underline{x}_P-s}^{\underline{x}_P} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c)d\gamma_j dx_i - \int_{\underline{x}_P-s}^{\underline{x}_P} \int_{\underline{\gamma}_R}^1 (1 + x_i + \bar{\alpha}\gamma_j - c)d\gamma_j dx_i. \quad (78)$$

Differentiating (78) with respect to x_i , we have that

$$\frac{\partial DW_{SR}}{\partial x_i} = \frac{s(2(c-1-x_i) + s)}{2\bar{\alpha}} > 0. \quad (79)$$

Thus, a necessary condition for $DW_{SR} > 0$ is that $DW_{SR}|_{x_i=c-1} > 0$. In addition, we have that

$$\frac{\partial DW_{SR}}{\partial s} \Big|_{x=c-1} = \frac{1}{2} \left(1 - \bar{\alpha} - 4(c-1)s - \frac{s^2}{\bar{\alpha}} \right), \quad (80)$$

and

$$\lim_{s \rightarrow 0} \frac{\partial DW_{SR}}{\partial s} \Big|_{x=c-1} = \frac{1 - \bar{\alpha}}{2} > 0 \quad (81)$$

so that a small subsidy strictly improves welfare, notwithstanding the fact that it crowds out reforms. Hence, for small values of $\bar{\alpha}$, we necessarily have that $SGR \succ RSG$ and $SRG \succ RGS$. Also, since $SGR = SRG$, we necessarily have that $SGR = SRG \succ Max\{RSG, RGS\}$.

The last step is to show that $SGR \succ GSR$ or, since reforms are never implemented for small enough values of $\bar{\alpha}$, that $SG \succ GS$. From (54), (55), (64), (67) after some algebra, we obtain that:

$$W(SG) - W(GS) = \begin{cases} \frac{(c-1)(c(43-24c)-15)}{8(2c-2)}, & \text{if } c < \frac{3+\sqrt{5}}{4}; \\ \frac{(2-c)(3-c-c^2)}{8(2c-2)}, & \text{if } c \geq \frac{3+\sqrt{5}}{4}; \end{cases} \quad (82)$$

expression that is positive for $c \in (1, 2)$.

(ii) Let us now consider the case $\bar{\alpha} \rightarrow 1$. Now, there are no costs associated with reforms, and they can finance the entire infrastructure gap. Consequently, we have that $W(RGS) = W|_{\bar{\alpha} \rightarrow 1}(RSG)$ is necessarily the best choice.

(iii) For intermediate values of $\bar{\alpha}$, let us take $\bar{\alpha} = \frac{1}{2}$. In this case

$$\begin{aligned} W|_{\bar{\alpha} \rightarrow 1/2}(GSR) &= \frac{((c-1)(-15+c(46+c(-35+8c))))}{8(2-c)}; \\ W|_{\bar{\alpha} \rightarrow 1/2}(GRS) &= \frac{1}{24(-1+c)(49+c(-50+13c))}; \\ W|_{\bar{\alpha} \rightarrow 1/2}(RSG) &= \begin{cases} \frac{(c-1)(14+c(-16+5c))}{12}, & c < \frac{3}{2}; \\ \frac{(-55+2c(48+c(-27+5c)))}{24}, & c \geq \frac{3}{2}; \end{cases} \\ W|_{\bar{\alpha} \rightarrow 1/2}(RGS) &= \begin{cases} \frac{(c-1)(14+c(-16+5c))}{12}, & c < \frac{3}{2}; \\ \frac{(-55+2c(48+c(-27+5c)))}{24}, & c \geq \frac{3}{2}; \end{cases} \quad (83) \\ W|_{\bar{\alpha} \rightarrow 1/2}(SRG) &= \begin{cases} \frac{(c-1)c(2c-3)}{2}, & c < \frac{3+\sqrt{5}}{4}; \\ \frac{11+2c(-51+2c(84+c(-135+2c(57+4(-6+c)c))))}{12(2c-1)^3}, & \frac{3}{2} > c \geq \frac{3+\sqrt{5}}{4}; \\ \frac{(-55+3/(1-2c)^2-1/(-1+2c)^3+2c(48+c(-27+5c)))}{24}, & c \geq \frac{3}{2}; \end{cases} \\ W|_{\bar{\alpha} \rightarrow 1/2}(SGR) &= \begin{cases} \frac{(c-1)c(2c-3)}{2}, & c < \frac{3+\sqrt{5}}{4}; \\ \frac{9+2c(-28+c(41-22c+4c^2))}{8(2-c)}, & \frac{3}{2} > c \geq \frac{3+\sqrt{5}}{4}; \\ \frac{27+c(-276+c(1026+c(-1815+2c(795-330c+52c^2))))}{24(2c-1)^3}, & c \geq \frac{3}{2}. \end{cases} \end{aligned}$$

By plotting the different expressions, it is immediate to verify that the optimal myopic sequencing is *SRG* or *SGR* (which are identical), when the cost of resources c is low ($c < \frac{3+\sqrt{5}}{4}$), *SGR* for intermediate values of c , and *SRG* when c is large.

6.5 Proof of Proposition 2

The proof that subsidies should be implemented before reforms is along the same lines as the proof of part (i) of Proposition 1. If a subsidy crowds out public investments, the change in welfare is given by

$$DW_{SG} = \int_{x_i-s}^{x_i} \int_0^1 (1 + x_i + \gamma_j - s(c-1) - c) d\gamma_j dx_i - \int_{x_i-s}^{x_i} \int_{c-1}^1 (1 + \gamma_j - c) d\gamma_j dx_i. \quad (84)$$

Differentiating (84) with respect to x_i , we have that

$$\frac{\partial DW_{SG}}{\partial x_i} = s > 0. \quad (85)$$

Thus, a necessary condition for subsidy to dominate public investments is that $DW_{SG}|_{x_i=c-1} > 0$. In addition, we have that

$$\frac{\partial DW_{SG}}{\partial s} \Big|_{x=c-1} = \frac{2s - 3 + c(4(1-s) - c)}{2}, \quad (86)$$

and

$$\lim_{s \rightarrow 0} \frac{\partial DW_{SG}}{\partial s} \Big|_{x=c-1} = \frac{(3-c)(c-1)}{2}, \quad (87)$$

so that a small subsidy strictly improves welfare, notwithstanding the fact that it crowds out public investments. This implies that subsidies should be implemented before public investments.

Finally, the only situation in which the cascade and the *COPA* coincide is when reforms are not part of the *COPA*. When this is the case, we necessarily have that no reforms are undertaken in the cascade. The reason is simple. Assume that reforms were undertaken, then a positive reform will improve welfare given the optimally chosen subsidies and investments. But, if this were the case, the *COPA* would be strictly dominated by another allocation. A contradiction.

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