The Behavior of a Dual Economy under Different "Closing Rules"

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THE BEHAVIOUR OF A DUAL ECONOMY UNDER DIFFERENT ‘CLOSING RULES’

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A subsistence sector which produces output without reproducible capital and is also the only source of labour in the economy opens up the possibility that profit maximisation does not rule everywhere. If, in this context, there is one good and savings determine investment, then equilibrium is unique whether the labour market works classically or neoclassically. If, however, investment determines savings in neo-Keynesian fashion, equilibrium may no longer be unique. These results suggest that the importance of different behavioral specifications cannot be determined independently of how markets get cleared.

1. Introduction

When confronted with the single sector, profit maximisation framework adopted by Taylor-Lysy (1977) and Lluch (1977), it is natural to ask whether their results would carry over to a world in which profit maximisation does not rule everywhere. One world which admits of such a possibility is a dual economy in which there is a ‘capitalist’ sector (a replica of that in Taylor-Lysy and Lluch) and a ‘subsistence’ sector producing output without reproducible capital. Households engaged in subsistence production are the sole source of labour in the economy, so that the labour supply conditions faced by the capitalist sector depend on subsistence production possibilities and the behaviour of the households possessing them.

In keeping with the central question addressed here and the seminal paper by Sen (1963), both the structure and the details of the paper have a strongly taxonomic flavour. Moreover, we shall concern ourselves only with the existence and uniqueness of equilibrium as the basis for comparative statics; for any consideration of adjustment mechanisms and stability would involve major difficulties and take us far afield. It must also be emphasised at the outset that in this paper the price of output is exogenously given whatever the ‘closing rule’. Thus, while there are close parallels between some of the

*I am indebted to Constantino Lluch for long discussions on this subject. Thanks are due also to Frank Lysy, Graham Pyatt, Lance Taylor and an anonymous referee for helpful comments on an earlier draft. All opinions and surviving errors are my responsibility alone.
results for the neo-Keynesian variants considered here and those for the Keynesian variants in T-L, the causal mechanisms are quite different.

The paper is structured as follows: The basic system is set out below. Section 2 is devoted to the classical case in which there are unlimited supplies of labour at a fixed real wage. The next step, in section 3, is to allow the labour market to clear neoclassically: firms maximise profits and households maximise incomes by choosing labour inputs, both taking the real wage as parametrically given. In these two variants, the comparative statics analysis focuses on the effects of capital accumulation on the wage rate, the intersectoral allocation of labour and the share of profits in total income. Sections 4 and 5 deal with three closing rules which are neo-Keynesian in the sense that real investment is fixed exogenously. Two of them preclude maximising behaviour entirely, thus rupturing the usual equality between the marginal product of labour and the wage. The third involves two mutually exclusive logical possibilities. Either capitalists maximise profits perceiving the supply of labour to be perfectly elastic and households maximise income subject to a fixed number of job offers by the capitalist sector, or households face a perfectly elastic supply of job offers and capitalists are confronted by a fixed number of job-seekers. In these two sections, the comparative statics analysis deals mainly with the effects of variations in the level of investment and real wages.

The basic system

The capitalist sector produces output under constant returns to scale using labour, 'machines' (made of the same stuff as domestic output) and an importable. The subsistence sector produces the same commodity, also under constant returns to scale, by combining labour with a resource which can be neither augmented nor traded. Thus, the average and the marginal products of labour increase as labour is withdrawn from this sector. 'Capitalist' households own only 'machines' installed in the capitalist sector; the rest ('worker' households) have equal endowments of labour and the fixed resource.

Finally, there is the matter of factor employment levels. The current stock of 'machines' is given in the short run by investment decisions made in the past; but whatever its size, all machines are fully employed. The members of worker households pool their earnings, whether these are derived from wage employment in the capitalist sector or employment in commodity production in the household itself. There is no disutility of effort, and each individual's labour supply is completely inelastic. We will suppose also that 'campaing out' at the factory gates has no effect on future income, so that no one will

1The assumption that reproducible capital is not employed in the subsistence sector has respectable precedents in the literature; see, for example, Lewis (1954) and Jorgenson (1969).
engage in it. Thus, there will be no open unemployment of labour. At the beginning of each period, those who sought, but failed to get, jobs in the capitalist sector return home to augment the household’s income, for their marginal product is (generally) positive. This opens up the distinct possibility of disguised unemployment in the subsistence sector in the sense that the marginal product of labour therein may be less than the real wage rate ruling in the capitalist sector. We shall return to this matter in some detail in the sections which follow. Suffice it to say for the moment that the total supply of labour to the system is given in the short run, and that anyone failing to find a job in one sector gets employed in the other without further ado.

Using an obvious extension of the notation employed in T–L, the formal counterpart of the above description is as follows:

\[ X_1 + X_2 = C + I + E, \]
\[ V_1 = aX_1, \]
\[ V_2 = X_2, \]
\[ P_X = aP_{V_1} + mP_M, \]
\[ V_1 = F(K_1, L_1), \]
\[ V_2 = G(H, L_2), \]
\[ L_1/V_1 = \Phi(W, rP_X, P_{V_1}), \]
\[ K_1/V_1 = \Psi(W, rP_X, P_{V_1}), \]
\[ P_X C = \gamma_L(WL_1 + P_X X_2) + \gamma_K rP_X K_1, \]
\[ K_1 = \bar{K}, \]
\[ L_1 + L_2 = \bar{L}, \]
\[ mP_M X_1 - P_X E = B. \]

Eq. (1a) is the material balance for the economy. The relationship between value added and gross output in the two sectors is shown in (1b) and (1c); for simplicity, we follow T–L in assuming a fixed input of importables per unit of gross output in sector 1. In eq. (1d) the price of gross output follows Leontief’s rule, the two cost elements being value added (in sector 1) and importables. (Note that the price of value added in sector 2 is identically
equal to the price of gross output.) The production functions for value added are \( F(\cdot) \) and \( G(\cdot) \), respectively, both being homogeneous of degree one. Also, profit maximisation in the capitalist sector yields (1g) and (1h), which are the derived demand functions for labour and capital services, whose arguments are the wage rate \( (W) \), the user cost of capital \( (rP_V) \) and the price of value added \( (P_V) \) — all in nominal terms. Eq. (1i) is the aggregate consumption function, which specifies that a fixed proportion of each kind of income is saved. In the present context, it seems most natural to assume that worker households do not save, that is, \( \gamma_L = 1 \). This is the case we will consider most often. Moreover, if these households do save, it must not be forgotten that they will own part of the stock of ‘machines’, and hence get a share of profits. Eqs. (1j) and (1k) state that both factors are fully employed. Lastly, (1l) shows the level of net foreign borrowing (in domestic units of account).

Now system (1) has twelve equations and sixteen unknowns. It is homogeneous of degree zero in \( (P_X, P_V, P_M, W) \). Also, without loss of generality, we may set both \( H \) and \( L \) equal to unity. We must shed one degree of freedom by choosing a numeraire. In this case, \( P_X \). These steps yield the following set of normalised variables: \( P = P_V, P_X, \pi = P_M/P_X, x_1 = X_1/L, \nu = V_1/L, \theta = K/L, k_1 = K/L_1, l = L_2/L, c = C/L, i = I/L, e = E/L, \) and \( b = B/LP_X, \) \( L \) being normalised to unity, of course.

Only one of the three remaining degrees of freedom is needed for the purposes of this paper, and there are natural ways of discarding two of them. First, we invoke the ‘small country’ assumption to fix the foreign barter terms of trade \( (\pi = \bar{\pi}) \). Secondly, the comparative statics setting suggests zero net foreign borrowing \( (b = 0) \). The former fixes \( \rho \) by (1d). The latter enables us to eliminate \( e \), and hence \( x_1 \) and \( x_2 \), to get the following compact system:

\[
\begin{align*}
(1 - l)p_{r_1} + lr_2 &= c + i, \\
1 &= ap + m\pi, \\
r_1 &= f(k_1), \\
r_2 &= g(h), \quad h = 1 - l, \\
r_1 &= \phi(w, r; p), \\
k_1 &= \psi(w, r; p), \\
c &= \gamma_L[(1 - l)w + lr_2] + \gamma_k r k, \\
(1 - l)k_1 &= k.
\end{align*}
\]

in which there are eight equations and nine unknowns \( (r_1, r_2, c, i, k_1, l, w, r, p) \). In the sections which follow, we make use of this degree of freedom to ‘close’ the system in different ways.
2. A 'Lewis' model: Perfectly elastic labour supply to the capitalist sector at a constant real wage

Let us begin by fixing the real wage in terms of the domestic good: \( w = \bar{w} \). Furthermore, suppose that capitalists can hire as much labour as they please at \( \bar{w} \). This implies that worker households make no active choices concerning the intersectoral allocation of their labour; they simply comply with capitalists' hiring decisions. Thus, the effects of parametric variations in \( \bar{w} \) on employment in the capitalist sector are exactly those set out in section 3 of T-L, which deals with what they call, rather confusingly, a 'neoclassical closure', despite a perfectly elastic supply of labour to the economy as a whole.

What concerns us here, however, is the effects of parametric variations in the capital stock, \( k \). As capitalists are profit maximisers and there are constant returns to scale, \( w = \rho[f(k_1) - k_1 f'(k_1)] \). It follows at once that with \( w \) and \( \rho \) fixed, then \( k_1 \) is likewise -- and hence \( v_1 \) and \( r \) are fixed also. Thus the picture is a very simple one: as the capital stock rises, the capitalist sector expands with the capital-labour ratio, output per man and the rate of profit staying constant. This expansion is fueled by labour supplies from the household sector until this source is exhausted, at which point the growth of the capitalist sector at a constant technique of production can proceed no further.

The total income of worker households is

\[
Y_L = (1 - l)\bar{w} + \rho L r k_1,
\]

where \( \rho L \) is the fraction of the capital stock which they own. If they do not save, then \( \rho L = 0 \), which is the only case considered in both this and the next section. Hence, noting that \( k_1 \) is constant and using (2h), we have

\[
dY_L/dk = \{(1 - l)[\bar{w} - (g - g', h)]\} k.
\]

Thus, \( Y_L \) increases with \( k \) so long as the real wage in the capitalist sector exceeds the marginal product of labour in the subsistence sector.\(^2\) Beyond that point, some of the labour supply to the capitalist sector would be involuntary, and that is the end of the Lewis story.

Let us look at this 'turning point' in a little detail.\(^3\) First, given \( \bar{w} \) and the non-traded endowment, it depends solely on the technical conditions of production in the subsistence sector. Secondly, if \( g(h) \) is well behaved, the

\(^2\) Write \( h = -l \), so that the marginal product of labour in the subsistence sector is \( g(h); hg'(h) = g - g', l \).

\(^3\) This particular turning point is discussed at length in Ranis and Fei (1964). Dixit (1973, p. 341) refers to it as 'the point at which [...] the labour market in agriculture [is] commercialised...'}
marginal product of labour, \((g - g'/l)\), has two properties: it is monotone decreasing in \(l\); and it becomes arbitrarily large as \(l\) becomes arbitrarily small. Hence, if a turning point exists in the interior, i.e. \(\ell \in (0, 1)\), it is unique. Specialising \(g(\cdot)\) to the Cobb–Douglas form, with \(g(1)\) set equal to unity, the turning point is given by

\[ l^* = (\beta / \bar{w})^{1 - \beta}, \]

where \(\beta\) is the output elasticity of labour in the subsistence sector. As one would expect, higher values of \(\bar{w}\) are associated with 'later' turning points, where 'later' is used in the sense that the proportion of the labour force employed in the capitalist sector is relatively large at the turning point. The reverse is true for \(\beta\), high values of which entail 'early' turning points (high \(l^*\)'s). Table 1 provides a summary picture. If \(g(\cdot)\) is CES, then \(\beta\) is not constant: it rises (falls) as more labour is drawn into the capitalist sector if the elasticity of substitution in the subsistence sector, \(\tau\), is less (greater) than unity. Thus, given \(\bar{w}\) and measuring \(\beta\) at \(l=1\), the turning point is hastened (retarded) relative to table 1 if \(\tau\) is less (greater) than unity.

<table>
<thead>
<tr>
<th>(w)</th>
<th>(\beta)</th>
<th>1.3</th>
<th>1.2</th>
<th>2.3</th>
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<tr>
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<td>0.544</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>2/3</td>
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<td>0.563</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.192</td>
<td>0.250</td>
<td>0.296</td>
<td></td>
</tr>
</tbody>
</table>

*This is a corner solution.

We round off this section with a look at the changes in income distribution attending changes in \(k\). Profits per worker employed in the capitalist sector are \(\xi_1 = pf(k_1) - \bar{w}\), which is constant, so that total profits are simply \(Y_k = (1 - l)\xi_1\). If worker households do not save, there is a simple income distributional measure: the ratio of worker household incomes to profits.

\[ \Omega = (\bar{w} + \lambda r_2)\xi_1, \quad (4) \]

where \(\lambda = l(1 - l)\) and \(\xi_1\) is constant. Plainly, \(\lim_{l \to 1} \Omega = \tau\), and at \(l = 0\), \(\Omega = \bar{w}\xi_1\). Also, \(\lambda r_2\) is monotone increasing in \(l \in [0, 1]\) as \(g(\cdot)\) is concave, so that \(\Omega\) is monotone decreasing in \(k\). Thus, income distribution worsens as \(k\) rises, and it worsens rapidly at both ends of the range of \(l\). The rapid

4 Differentiating \(l g(1 - l)\) with respect to \(l\), we have \([tg' (g(1 - l)) + g(1 - l)]\), which is always positive for \(l \in [0, 1]\). It always grows without limit as \(l\) approaches unity, and also as \(l\) approaches zero if \(g(\cdot)\) is well-behaved.
deterioration when $k$ is small ($l$ close to unity) is a ‘structural’ effect: the rapid expansion of capitalist sector output and profits from a small base. At the other extreme, as $k$ approaches the profit-maximising value of capital per man at $w = \bar{w}$ and $l = 0$, the sharp fall in $Q$ is due to the ‘involuntary’ loss of labour from the subsistence sector when the ‘turning point’ has passed; it is not, properly speaking, part of the Lewis story.

3. Neoclassical full employment equilibrium

In the previous section, capitalists’ labour hiring decisions rode roughshod over the labour supply decisions of worker households, whatever the level of the (exogenous) real wage. A less one-sided way of arriving at the inter-sectoral labour allocation is to allow the labour market to clear ‘neoclassically’: the real wage adjusts until worker households’ labour supply decisions are consistent with capitalists’ hiring decisions. Worker households allocate their labour endowment so as to

$$\max_{l} Y_{w} = (1 - l)w + lg(1, l),$$

which yields the familiar condition that the marginal product of labour be equal to the real wage:

$$g(1, l) - g'(1) = w.$$  \hfill (5)

or $l = l(w)$. The supply schedule of labour to the capitalist sector is then simply $1 - l(w)$, and system (2) is closed when we add (5).

First, we establish sufficient conditions for the existence and uniqueness of equilibrium, given $k$ and $p$. If $g(\cdot)$ is ‘well-behaved’: (i) $l$ is continuous and monotone decreasing in $w$; (ii) $\lim_{w \to \infty} l(w) = 0$; and (iii) at $l = 1$, $w = g(1) - g'(1) > 0$. Now consider the capitalists’ demand for labour, $\lambda$, which is given by $w = p[f'(k_1) - k_1 f''(k_1)]$ or $\lambda = \lambda(w; k, p)$. If $f(\cdot)$ is ‘well-behaved’: (i) $\lambda$ is continuous and monotone decreasing in $w$; (ii) $\lim_{w \to \infty} \lambda(w) = 0$, and (iii) at $\lambda = 1$, $w = p[f'(k_1) - k_1 f''(k_1)] > 0$. Thus, under the conditions stated, there exists some positive $w$ such that $1 - l(w) = \lambda(w; k, p)$ with $l \in (0, 1)$. Moreover, the equilibrium $(w, l(w), \lambda(w; k, p))$ is unique.

A word of warning before the detailed analysis which follows: Nothing in the above argument depends on $F(\cdot)$ and/or $G(\cdot)$ being CES. Indeed, if the usual definition of ‘well-behaved’ is adopted, only the Cobb Douglas case is admissible.\(^5\) In particular, we know that if $F(\cdot)$ is CES and $\sigma < 1$, then

\(^5\) Let the production function be $F(X)$, where $X$ is a vector of inputs. To be ‘well-behaved’, $F(\cdot)$ must satisfy, \textit{inter alia}, the conditions

$$\lim_{x_i \to 0} F_i = x_i$$

and

$$\lim_{x_i \to x} F_i = 0.$$
lim_{L \to 0} F_2(K, L) = \alpha_L^{\sigma(/ \sigma - 1)}. An immediate conclusion is that if \( \sigma < 1 \), the capitalist sector may never get going, the critical condition being

\[ p\alpha_L^{\sigma(/ \sigma - 1)} > g(1) - g'(1). \]

This condition is less likely to hold: the lower is \( \sigma \); the greater is the elasticity of substitution in the subsistence sector; the more generous is the non-traded endowment relative to population; and the more adverse are the barter terms of trade (the lower is \( p \)). Fig. 1 illustrates an equilibrium without a capitalist sector. (*Mutatis mutandis*, the conditions for an equilibrium without a household sector are self-evident.) Note that no infusion of extra capital -- by act of God or the U.S. Congress -- will bring the capitalist sector to life. For \( p\alpha_L^{\sigma(/ \sigma - 1)} \) is independent of the capital stock, and the effect of a rise in \( k \) is to cause \( \lambda(\cdot) \) to twist clockwise about the pivot \((0, p\alpha_L^{\sigma(/ \sigma - 1)})\) in the \((\lambda, w)\) plane, yet retaining its downward slope. However, population growth will do the trick eventually, for the resulting fall in the per caput non-traded endowment entails a steady downward shift in \( l(w) \). Naturally, expropriation of the subsistence sector’s endowment will do just as well for this purpose. But the kindliest midwife would be a sufficiently favourable movement in the barter terms of trade.

Recalling the ‘classical’ case, \( w = \bar{w} \) in section 2, it is clear that \( w \) must not exceed \( p\alpha_L^{\sigma(/ \sigma - 1)} \), a restriction which was noted neither there nor in T L. Of course, if worker households are without subsistence production possibilities, however these are specified, the whole of the labour force is offered inelastically to the capitalist sector and the wage is given by the marginal produce of labour at full employment: \( w = p[f(k) - k f'(k)] \). Once the first machine has made its appearance, the capitalist sector is launched.

![Diagram](image)

**Fig. 1.** The capitalist sector cannot yet come to life.

We turn now to the relationship between the overall capital labour ratio, the intersectoral allocation of labour and the real wage. By differentiating (5) totally, we obtain:

\[ E(l, w) = wl^2 / g'' < 0, \]

\( E(a, b) \) denotes the elasticity of \( a \) with respect to \( b \). Note that \( L(b, a) = 1 \cdot E(a, b) \).
that is, a rise in the wage is associated with a fall in employment in the subsistence sector— as intuition suggests. From (2h), we have

\[ E(k, w) = E(k_1, w) - lE(l, w)/(1 - l), \]

where \( E(k_1, w) = \sigma/\nu, \nu \) being the share of profits in value added in the capitalist sector.\(^7\) It follows at once that \( E(w, \bar{k}) > 0, \) i.e., a rise in \( \bar{k} \) is associated with a rise in the real wage.

Now we know that \( g'/g'' = -\tau/(1 - u) \), where \((1 - u) \) and \( \tau \) are, respectively, the imputed share of labour and the elasticity of substitution in the subsistence sector. Substituting into \( E(l, w) \) above, we obtain \( E(l, w) = -\tau/u. \) Hence,

\[ E(w, \bar{k}) = \nu v(1 - l)/(\sigma u(1 - l) + \nu v l), \]

and is always positive. As expected, the real wage is less responsive to changes in the capital stock than in the single sector framework because the labour supply to the capitalist sector is not completely inelastic: a rise in \( k \) produces a quantity adjustment (in \( l \)) as well as a change in the real wage.

Let us examine the end points: \( l = 0, l = 1 \) in turn. In the former case, \( E(w, \bar{k}) = \nu/\sigma: \) if there is no subsistence sector, the single sector result applies. At the other extreme, the nascent capitalist sector faces a real wage rate of \( g(1) - g'(1), \) the marginal product of labour in the subsistence sector when all labour is employed therein. The rate of profit is certainly positive—otherwise, the capitalist sector would be dormant rather than nascent—so that \( v > 0. \) Hence, \( \lim_{l \to 1} E(w, \bar{k}) = 0. \) Note that \( E(w, \bar{k}) > 0 \) for all \( l \) in the interior, unless the imputed share of labour in subsistence sector output is unity, in which case the capitalist sector always faces a perfectly elastic supply of labour.

The key question here is: under what conditions is Lewis' assumption a good approximation, given that the wage is that which clears the market? This is equivalent to ascertaining whether or not \( E(w, \bar{k}) \) is small in some sizeable domain of \( l. \) We have seen already that when virtually all workers are employed in the subsistence sector, \( E(w, \bar{k}) \) approaches zero. Elsewhere, eq. (6) provides the answer. Although it contains too many parameters for a compact tabular presentation of \( E(w, \bar{k}) \), we can rewrite it as follows:

\[ E(w, \bar{k}) = (v \cdot \sigma) v[l + \tau v l, \sigma u(1 - l)], \]

which is the basis for the layout of table 2. The choices of \( v/\sigma \) reflect the fact that \( v \) will usually be about 0.5 and \( \sigma \) will generally be between 0.5 and 1.

If \( F(\cdot) \) is CES, eq. (2e) becomes \( w' = (x, p) v r_1. \) Hence, \( \sigma dw = dr_1, r_1 = p' \cdot dk_1/e_1. \) Rearranging, \( E(k_1, w) = \sigma r_1 (p k_1 / e_1) = \sigma r_1. \)
Those of $\tau/u$ reflect the feeling that $\tau$ may reach 1.5 or more, while $u$ will be 0.5 or a little less. Table 2 suggests that Lewis' assumption will be 'acceptable' when: $\tau$ is large relative to $\sigma$; $\sigma$ itself is 'large' (close to unity); and at least half the work force is employed in the subsistence sector. We see that the associated adjustments in the allocation of labour between sectors keeps the responsiveness of real wages to rises in the capital stock well below that in the single sector world, even when two-thirds of the labour force are employed in the capitalist sector. It is also clear that if population growth is admitted into the picture, then $E(w, k)$ will be less than the value given by eq. (6) unless population growth is fully offset by land-augmenting technical progress in the subsistence sector.

Table 2
$E(u, k)$ for different labour allocations.

<table>
<thead>
<tr>
<th>$\tau/u$</th>
<th>$v/\sigma$</th>
<th>$l$</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
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<td>0.06</td>
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<td>0.5</td>
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</tr>
<tr>
<td>2</td>
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<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>0</td>
<td>0.17</td>
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<td>0.38</td>
<td>0.44</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>0</td>
<td>0.06</td>
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<tr>
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We now go in search of a 'turning point', this time in connection with the income distribution measure, $\Omega$. To make the analysis tractable, $g(\cdot)$ is specialised to Cobb–Douglas with $g(1) = 1$, so that (5) takes the form $l = (\beta/w)^{(1-\beta)}$. Hence, if there is to be a turning point, the ratio of worker household incomes to profits must attain a minimum for $l \in (0, 1)$.

Differentiating (4) logarithmically, some manipulation yields:

$$E(\Omega, w) = [1, 1, (\beta + A)] E(A, w) + (1 - \sigma), v.$$  

From $l = (\beta/w)^{1/(1-\beta)}$ we obtain

$$E(A, w) = - (1 + 1)(1 - \beta) = - 1 \{1 - l(1 - \beta)\}.$$  

Whence,

$$E(\Omega, w) = (1 - \sigma), v - l_1 \{ (\beta + l(1 - \beta))(1 - l)(1 - \beta) \}. \quad (7)$$

Noting that $\xi = pf(k_1) - w$, we have

$$d[\log(w, \xi)] = E(k_1, w) - w \{ p_1 \cdots w \} = [1, E(k_1, w) + w, \xi] w.$$  

As $E(k_1, w) = \sigma, v$, and $w, \xi = (1 - v)/v$, the result in the text follows at once.
Now \( E(\Omega, k) = E(\Omega, w) \cdot E(w, k) \), where \( E(w, k) \) is given by (6) and is always positive. Thus, if we are seeking a ‘turning point’ for \( \Omega \), it suffices to find a turning value for \( E(\Omega, w) \). We see at once from (7) that if \( \sigma \geq 1 \), \( E(\Omega, w) < 0 \), \( \forall l \in (0, 1) \), so \( \sigma < 0 \) is a necessary condition for a ‘turning point’. That is to say, the share of wages in value added in the capitalist sector must rise with accumulation if there is to be such a point.

Next, let us examine \( E(\Omega, w) \) as \( l \) tends to zero and unity, respectively. In the former case, \( E(\Omega, w) \to (1-\sigma)l \); for if the capitalist sector accounts for virtually all employment in the economy, the share of wages falls (rises) with the real wage according as \( \sigma \geq 1 \), as in the single sector story. At the other extreme, we have \( E(\Omega, w) \to -\tau \) as \( l \to 1 \), which is the ‘structural’ effect of the growth of the capitalist sector from a small base. Given the assumptions on the technology in the capitalist sector, \( E(\Omega, w) \) is continuous, so if \( \sigma < 1 \) there will exist at least one real wage rate such that \( E(\Omega, w) = 0 \). Concerning uniqueness, the term in braces in eq. (7) is monotone increasing with \( l \in (0, 1) \).

Also, \( (1-\sigma)l \) is monotone decreasing as \( l \) increases if \( \sigma < 1 \), so that there is just one value of \( l \in (0, 1) \) which yields \( E(\Omega, w) = 0 \).

This is a ‘turning point’ à la Kuznets. Worker households lose ground relatively as the capitalist sector expands from scratch at the expense of the subsistence sector. But if \( \sigma < 1 \), the wage share of value added in the former increases at the same time and this eventually offsets the effects of the changes in economic structure. Once again, then, the interesting question is: when does the turning point arrive, given that \( \sigma < 1 \)?

In table 3, the smaller value of \( v \) is pitched low enough to allow for the fact that it decreases with \( k \) if \( \sigma < 0 \), although it appears that variations in \( v \) have a rather modest effect on the turning point anyway. Changes in the output elasticity of labour in the subsistence sector also have a weak impact on the location of the turning point, although the direction accords with intuition: stronger concavity in labour inputs alone hastens the turning point. Predictably, a rise in \( \sigma \) delays the turning point. The overriding impression, however, is that the turning point arrives late in the game – if it arrives at all. It is a gloomy prospect if the proportion of the workforce engaged in the household sector must shrink to one-third or less before the unequalising effects of structural change are overcome by continuing accumulation.

### Table 3

<table>
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<tr>
<th>( \sigma )</th>
<th>( v )</th>
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4. Investment exogenously determined: Variations on a neo-Keynesian theme

In true Kaldorian style, let us now toss out maximising behaviour (and hence the supply and demand curves which such behaviour generates; in favour of fixing real investment exogenously \((i=\bar{I})\). To keep track of the ensuing degrees of freedom, if any, it is convenient to set out the system under consideration. Let \(s_K = (1-\gamma_K)\) and \(s_L = (1-\gamma_L)\). Then:

\[
(1-l)p v_1 + l v_2 = c + i, \quad \text{(8a)}
\]
\[
1 = ap + m \bar{\pi}, \quad \text{(8b)}
\]
\[
v_1 = f(k_1), \quad \text{(8c)}
\]
\[
v_2 = g(1/l), \quad \text{(8d)}
\]
\[
s = s_K (1-l)p v_1 + s_L l v_2 - (s_K - s_L)(1-l)w, \quad \text{(8e)}
\]
\[
s = i, \quad \text{(8f)}
\]
\[
(1-l)k_1 = \bar{k}. \quad \text{(8g)}
\]

Eq. (8f) is the new twist to the story: savings are brought into equality with the exogenously determined level of investment. We have eight unknowns \((v_1, v_2, k_1, c, s, l, w, p)\) and seven equations. As before, once the foreign barter terms of trade are fixed, \(p\) is likewise. Also, given the capital stock, (8g) enables us to describe resource allocation in terms of \(l\).

**Variant 1.** One way of closing the system is to fix the labour allocation: \(l=\bar{I}\). This may be interpreted either as complete immobility of labour in the short run, or as a regime in which capitalists perceive labour supply as rationed, i.e., there is a quantity signal. Once \(l\) is fixed, \(v_1\) and \(v_2\) are determined, and savings investment equilibrium must be brought about through changes in the real wage rate. Combining (8c) and (8f), we obtain the real wage.

\[
w = \frac{s_K (1-\bar{I})p f(\bar{k}/1-\bar{I}) + s_L \bar{f}(1/\bar{I}) - \bar{i}}{(s_K - s_L)(1-l)}. \quad \text{(9)}
\]

Thus, given \(\bar{k}, \bar{I}, p\) and \(\bar{i}\), there is a unique real wage.

**Cases A, B.** \(s_K > s_L \geq 0\). If \(i\) is not too big, \(w \geq 0\). Also, \(w\) rises with \(p\) and \(\bar{k}\), but falls as \(\bar{i}\) increases.

**Case C.** \(s_K = s_L\) (Solow-Swan). Here, there is just one pair \((\bar{I}, \bar{i})\) which yields
an equilibrium: $\bar{I}$ and $w$ must take their market clearing values ($l^0, w^0$) given $k$ (see section 3), and $\bar{I}$ must be equal to the resulting volume of savings $s = s_k(w^0_1 + v^0_2)$. As nothing else will do, equilibrium is improbable.

This latter case illustrates the key role of shifts in income distribution in a rather paradoxical fashion. For if shifts in income distribution do not affect savings, and output is fixed, then changes in the real wage are immaterial to attaining equilibrium – unless the allocation of labour is 'neoclassically' right.

**Variant 2.** In the light of variant 1, the obvious alternative is to fix the real wage ($w = \bar{w}$) and allow $l$ to float, a formulation akin to that of Taylor and Bacha (1976). It has no direct counterpart in $T-L$; but there is a certain similarity to their 'Keynesian $I$', in which changes in prices bring about a level of total output that is consistent with (real) savings investment balance. Here, all prices are fixed. However, changes in the labour allocation lead to changes in total value added and its distribution.

Eqs. (8e) and (8f) yield

$$\bar{I} = s(l) = s_k(l - l)pr_1 + s_Llr_2 - (s_k - s_L)(1 - l)\bar{w}, \quad (10)$$

a non-linear equation in $l$ which may certainly possess more than one root in $(0, 1)$. To pursue this possibility of multiple equilibria, observe that $(1 - l)r_1$ and $lr_2$ are the levels of total output ('quantity' value added, to be precise) in the two sectors. Given the capital stock and the non-traded endowment, the assumptions about the technologies imply that each of the sectoral output levels is strictly concave in $l$. In turn, this implies that the sum of the first two terms of $s(l)$ in (10) is strictly concave in $l$. It follows at once that $s(l)$ is a strictly concave function. Furthermore, $s(0) = s_kp/\bar{k} - (s_k - s_L)\bar{w}$, which may be of either sign: and $s(1) = s_Lg(1)$, which is positive unless $s_L = 0$. As $s(l)$ is strictly concave, it has a unique maximum on $[0, 1]$ and attains its minimum either at $l = 0$, or at $l = 1$.

If $s(0) > 0$, $s(\cdot) \geq 0$ on $[0, 1]$, then there will be at least one equilibrium provided $s_{\min} \leq \bar{I} \leq s_{\max}$. If, further, $s$ attains its maximum at $l = 0$ or $l = 1$, then equilibrium, if it exists, is also unique. If, however, $s$ attains its maximum in the interior, then for each and every $\bar{I} \in (\max[s(0), s(1)], s_{\max})$, there will be two equilibria – that is to say, there will be two labour allocations consistent with that particular value of exogenous investment. In the case where $s(0) < 0$, the fact that investment is necessarily non-negative implies that $s$ must take an interior maximum if $s_L = 0$ and there is to be a set of $\bar{I}$ for which equilibrium is possible. (For each $\bar{I}$, there will be two equilibria, as we have just seen.)

Hence, it is of interest to establish conditions under which $s$ will take an interior maximum. Given the assumptions on the technology, $s'(l)$ is continuous on $[0, 1]$. Hence, a necessary and sufficient condition for $s$ to take an
interior maximum is $s' (0) > 0$ and $s' (1) < 0$, since the maximum, if it exists, will be unique. Straightforward differentiation of (10) gives

$$\frac{ds}{dl} = - s_K p [ f - k_1 f'] + s_L [ g - g' l ] + (s_K - s_L) \bar{w},$$

which yields the required conditions. It is easily seen that a sufficient condition for an interior maximum is that both $F(\cdot)$ and $G(\cdot)$ are well behaved.

For the sake of simplicity, we illustrate the possibilities for the 'classical' savings case: $s_K > s_L = 0$. The analysis for the cases $s_K > s_L > 0$ and $s_K = s_L$ follows similar lines and yields broadly similar results. Figs. 2 and 3 correspond to $s(0) > 0$ and $s(0) < 0$, respectively. In both cases, there will be at least one equilibrium provided $\bar{i}$ is not greater than $s_{\text{max}}$. If $\bar{i} > s(0) > 0$, there will be two equilibria, as depicted in fig. 2; if $s(0) > \bar{i} > 0$, equilibrium is unique. If $s(0) \leq 0$, there will always be two equilibria provided $\bar{i} < s_{\text{max}}$, of course.

Now as $s_L = 0$, we see from (11) that $s_{\text{max}} = s(l)$ is given by $p [ f - k_1 f'] = \bar{w}$, that is, $\bar{i}$ is such that the marginal product of labour in the capitalist sector is equal to the exogenously given wage. As $\bar{k}$ and $p$ are given exogenously, the marginal product of labour exceeds the wage for all $l > \bar{i}$, and conversely for $l < \bar{i}$. This provides an illuminating characterisation of the various equilibrium possibilities, for when there is only one equilibrium, it is clear that $l > \bar{i}$.

An important conclusion of the 'Keynesian solution' in Bruno's single sector framework is that a rise in investment will always increase employment, and hence output (as the capital stock is fully employed). Here, however, there is always full employment, and the effects of changes in investment upon the level of employment in the capitalist sector are simply intersectoral ones. In particular, employment in the capitalist sector does not always increase with investment, a result which parallels that in Taylor and Bacha. A glance at figs. 2 and 3 reveals that if the initial equilibrium is one
in which the marginal product of labour in the capitalist sector is less (greater) than the wage rate, then a rise in investment will lower (increase) employment in that sector. The reason for these results is perfectly straightforward. If profits are the only source of savings, then a rise in investment must be met by an increase in profits. If $k$, $p$ and $\bar{w}$ are given, this can only come about through a fall (increase) in capitalist sector employment when the marginal product of labour in that sector is less (greater) than the wage rate.

Where variations in the wage rate are concerned, we see from (10) that a rise in $\bar{w}$ will cause $s(l)$ to shift downwards while twisting about $(1,0)$ in the $(l,s)$ plane. Hence, a rise in $\bar{w}$ will increase $l_1$ and decrease $l_2$. In this setting, therefore, a rise in the real wage will increase (decrease) capitalist sector employment if the marginal product of labour in that sector is greater (less) than the real wage.

To complete the list of parametric variations, a rise in $k$ (or $p$) will cause $s(l)$ to shift upwards while twisting about $(1,0)$ in the $(l,s)$ plane. This is shown by the broken curve in fig. 3. If investment stays unchanged, the movements in the equilibrium labour allocations are in opposite directions: $l^1$ decreases, but $l^2$ increases. Thus, if the adjustment process leads to the establishment of an equilibrium in the neighborhood of that existing before the shift in $k$, a rise in $k$ will be associated with a rise (fall) in employment in the capitalist sector if the wage is greater (less) than the marginal product of labour. Another possibility is that capitalists’ ‘animal spirits’ wax with rising $k$. In this event, the shifts in $l^1$ and $l^2$ are not immediately clear, being the outcomes of two offsetting effects; for a rise in $i$ (given $k$) will increase $l^1$ and decrease $l^2$.

In all this, we must not lose sight of the fact that total output in the economy will certainly change if the allocation of labour between sectors changes, so we ask: does a rise in investment always increase value added? In variant 1, $l$ is fixed and hence total output with it. Turning to variant 2, let $q = (1 - l)pr_1 + r_2$, so that

$$dq = -p[f - k_1 f'] + [g - g' l],$$

i.e., $q$ rises with employment in the capitalist sector if the marginal product of labour in the capitalist sector exceeds that in the subsistence sector. Noting that $i = s(l)$ and (11), we obtain

$$\frac{dq}{dl} = \frac{p[f - k_1 f'] - [g - g' l]}{s_k p[f - k_1 f'] - s_l[g - g' l] - (s_k - s_l)w}. \quad (12)$$

If $s_k = s_l$, savings always increase with total value added and hence $q$ always rises with $i$, unless $q$ is already at its maximum, when a rise in $i$ is
These results parallel those in Bruno's single sector framework. If $s_L = 0$, however, then $q$ will rise with $\bar{T}$ provided the differences between: (a) the two marginal products of labour, and (b) the marginal product of labour in the capitalist sector and the exogenous wage, have the same sign at the equilibrium value(s) of $(\bar{\omega}, \bar{T}, \bar{k})$. Otherwise, $q$ will fall as $\bar{T}$ rises, which is a possibility that cannot be ruled out. In particular, there are equilibria in which the wage is greater than the marginal product of labour in the capitalist sector; but the latter may still exceed its counterpart in the subsistence sector.

Where variations in the real wage are concerned, we have

$$d_q \, d\bar{\omega} = [d_q \, ds(l)] \cdot [ds(l) \, d\bar{\omega}],$$

where, from (10), $ds \, d\bar{\omega} = -(s_K - s_L)(1 - l)$. In the light of (12), it is clear that if $s_K > s_L$, then the effect of a rise in the wage upon value added will be opposite in sign to that induced by a rise in investment. If $s_K = s_L$, $q$ will be invariant with respect to $\bar{\omega}$, since savings are independent of the distribution of income. Hence, if investment is fixed, total output is too.

In closing this section, it should be recalled that the second variant considered here shares an important feature with the 'Keynesian' variants in T-L, namely, that savings investment equilibrium is brought about by adjustments in output. It cannot be too strongly emphasised, however, that the character of this neo-Keynesian system is also fundamentally different from their 'Keynesian' variants in that here the wage rate equals the marginal product of labour in the capitalist sector only by the merest chance, whereas in T-L it always does so by virtue of the flexibility of the real wage. It is this consideration which prompts a third neo-Keynesian variant, to which we now turn.

### 5. A third neo-Keynesian variant with maximising behaviour

Throwing out the supply and demand schedules for the only factor whose allocation can be varied may be too much for some readers to stomach. But they cannot keep both if investment is fixed, because the system will then be overdetermined. However, the discussion in sections 1 and 3 provides a clue as to how maximising behaviour can be reinstated.

If the ruling real wage is above that which would clear the labour market ($\bar{\omega}^0$, say), some workers from the subsistence sector will be disappointed on arriving at the factory and will then return home, where they will be 'underemployed'. Capitalists are on their demand curve for labour, for the marginal product of labour in the capitalist sector is equal to the wage, which capitalists take as parametrically given while (correctly) perceiving labour to be in perfectly elastic supply. Households are certainly not on their
labour supply curve, for the returning workers push the marginal product of labour in the subsistence sector below the prevailing wage. Conversely, if the real wage is below \( w^0 \), the factory gates will stay open all day because worker households, by keeping enough workers at home to bring the marginal product of labour into equality with the wage, do not supply enough workers to the capitalist sector for capitalists to achieve equality between the marginal product of labour and the wage. In this case, capitalists are not on their demand curve for labour; but worker households are on their labour supply curve.

From eq. (3) we see that if worker households save, their aggregate income from profits is affected by how much labour they supply to the capitalist sector, since both \( r \) and \( k \) depend on \( l \). But as there are many such households, it is reasonable to suppose that each of them ignores this connection when making decisions about how much labour to supply to the capitalist sector. That being so, we have the following rule for closing system (8):

\[
\begin{align*}
  w &= g(1, l) - g'/l, & w < w^0. \\
  &= p[f(k_1) - k_1 f'(k_1)], & w > w^0.
\end{align*}
\]

At \( w = w^0 \), both conditions hold simultaneously, and we are back in section 3; but there is equilibrium only if \( \bar{I} = s(w^0; k, p) \). If \( w = w^0 \), either worker households are maximising income by unrestricted choice of \( l \) given \( w \), or capitalists are maximising profits by unrestricted choice of \( \lambda \) given \( w \); but not both at the same time. Thus, the equilibrium possibilities \( (l, w) \) of the economy must be derived from the relevant sections of \( l(w) \) and \( \lambda(w; k, p) \), as depicted in fig. 4.

In contrast to variant 2 of section 4, in which equilibrium is achieved through changes in labour allocation at a fixed wage, the natural way of searching for an equilibrium in this system is to ask: can we find a real wage which will be associated with a volume of savings equal to the exogenously determined level of investment demand? Once the wage rate is found, the
labour allocation follows at once from (13). Moreover, as the wage rate affects both output and income distribution, the chances of getting a unique answer look slim.

It is convenient to work with the total output of each sector, so write \( pV_1(1-l; k) = (1-l)pr_1 \) and \( V_2(l) = lV_2 \). If \( w < w^0 \), we have

\[
s(w; E, p) = s_K pV_1[1-l(w)] + s_L V_2[l(w)] - (s_K - s_L)[1-l(w)]w. \tag{14}
\]

Differentiating with respect to \( w \) and noting that \( V'_2 = w \), we get

\[
ds/dw = -s_K(pV'_1 - w)(dl/dw) - (s_K - s_L)(1-l).
\]

Now, as \( f(\cdot) \) and \( g(\cdot) \) are strictly concave, \( l'(w) < 0 \) and \( l''(w) > 0 \); \( pV'_1 - w \) \( \geq 0 \) and decreases with \( w \). Hence, \( s'(w) < 0 \) if \( s_K \geq s_L \). Further, \( V'_1[1-l(w^0)] - w^0 = 0 \). Also \( \hat{w} \) is the wage rate which is just high enough to induce an infinitesimal labour supply to the capitalist sector when the whole of the population is engaged in the subsistence sector, i.e., \( \hat{w} = g(1) - g'(1) \).

Now suppose that capitalists are on their demand curve for labour \( (w \geq w^0) \), so that \( pV'_1 = w \). Then,

\[
s = s_K pV_1[\hat{\lambda}(w)] + s_L V_2[1 - \hat{\lambda}(w)] - (s_K - s_L)\hat{\lambda}(w)\hat{w}. \tag{15}
\]

Whence,

\[
ds/dw = s_L(w - V'_2)(d\hat{\lambda}/dw) - (s_K - s_L)\hat{\lambda}.
\]

Here, \( (w - V'_2) \geq 0 \) and increases with \( w \); also, as before, \( \hat{\lambda}'(w) < 0 \), \( \hat{\lambda}''(w) > 0 \). Thus \( s'(w) < 0, \forall w \geq w^0 \), provided \( s_K > s_L \). [If \( s_K = s_L \), we have \( s'(w) \leq 0 \), with equality at \( w^0 \).]

From the above and the analysis of the appendix, it is easy to sketch savings as a function of the real wage. Figs. 5, 6 and 7 show \( s(w) \) for the classical savings case with \( \sigma > 1 \), \( \sigma < 1 \) and \( F(\cdot) \) well-behaved, respectively. Qualitatively, the other cases are no different except that \( s(w) \) has a positive lower bound for large \( w \) whatever the value of \( \sigma \). If the exogenous level of real investment is ‘too high’ or ‘too low’ (for \( \sigma > 1 \), at least), there is no equilibrium. On the other hand, should equilibrium exist, it will not be unique unless \( \bar{I} \) happens to equal \( s_{\text{max}} \), or \( \bar{I} \) is sufficiently low and \( \sigma \leq 1 \). Otherwise, there will be two values of the real wage capable of supporting \( \bar{I} \). If \( \bar{I} \) is high enough, both of them will be less than \( w^0 \); at lower values of \( \bar{I} \), one will be below, and the other above, \( w^0 \).

Of course, there is no way of telling which real wage rate will prevail, because nothing has been said concerning the disequilibrium behaviour of the system. Nevertheless, it is still desirable to compare the two equilibria.
which may rule for given $\bar{k}$ and $p$. In the appendix, it is shown that profits increase with the real wage if the latter is sufficiently small, reaching a unique maximum for some real wage $w^*$ less than $w^0$, but falling thereafter. Furthermore, the incomes of worker households increase with the real wage until some $w^{**} \in [w^0, \infty)$, but decrease thereafter. As for the income distribution measure $\Omega$, this decreases as the wage rises if $w$ is sufficiently small, but begins to rise before $w^*$ and is still rising at $w^{**}$. Given $\bar{\tau}$, if the 'high wage' equilibrium has a real wage rate less than $w^{**}$, worker households would certainly prefer it to the alternative 'low wage' equilibrium; otherwise, their preference is unclear, although the examples which follow suggest that the income of worker households will be higher in the high wage equilibrium.

Except for $\bar{\tau}=s_{\text{max}}$, one equilibrium will be Pareto-superior to the other. Moreover, in the light of the foregoing analysis of capitalists' and workers' incomes, the Pareto-superior equilibrium may be preferred by both sorts of households. Even so, that reason alone is not enough to establish that the equilibrium in question will actually rule.

To illustrate the foregoing, let us derive $s(w; \bar{k}, p)$ and related variables for a special case. There is classical savings behaviour ($s_k=1$, $s_l=0$), so that savings equal total profits. Also, $\bar{k}=p=1$ a. 4 $\beta_1 = \beta_2 = 0.5$, which implies

![Fig. 5. ($\sigma > 1$).](image1)

![Fig. 6. ($\sigma < 1$).](image2)

![Fig. 7. ($F(\cdot)$ well-behaved).](image3)
$l^0 = 0.5$, $w^0 = 1/\sqrt{2}$. Using eqs. (14) and (15), we get table 4. For any value of $\bar{t}$, the high wage equilibrium is certainly Pareto-superior, and all the gains accrue to worker households, for $\bar{t}$ is given and only capitalists save. If $\bar{t}$ is relatively low, the spread between the two equilibrium wage rates is large. Also, the proportion of the workforce engaged in the capitalist sector is higher in the high wage equilibrium, because if capitalists alone save, higher wages must be accompanied by higher output (and hence profits) in the capitalist sector if the given level of investment is to be matched by total savings. To take a specific example, if $\bar{t} = 0.25$, then $w = 0.525$ and $w = 1.0$ are the two equilibria, the corresponding values of $l$ being 0.907 and 0.75, respectively. Output is 8.7 percent larger in the latter (high wage) equilibrium, which corresponds to an 11.5 percent increase in the incomes of worker households.

### Table 4

$s(w)$ and related variables: An illustration.

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<th>$s$</th>
<th>$l$</th>
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<th>$(pV_1 + V_2)$</th>
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</tbody>
</table>

To illustrate the effects of rises in $k$ (or $p$ for that matter), let us rework the above example with $k = 2$, which entails $l^0 = 0.33$, $w^0 = 0.866$. Omitting the tabular details, if $\bar{t} = 0.25$, the equilibrium values of $w$ are 0.51 and 2.0, and the corresponding values of $l$ are 0.961 and 0.875, respectively. Comparing the two low wage equilibria, total output and its distribution are unaffected by the rise in $k$. Comparing the high wage equilibria, total output rises by only 5.1 percent (due to the fall in employment in the capitalist sector), and $Y_L$ rises by only 6.1 percent, despite the doubling of wages, because employment in the capitalist sector is very small.

In terms of the Solovian parable of ‘islands’, let us look at the relationship between investment and the real wage. Given the same endowments and capital stocks, a higher level of investment is associated with a higher (lower) real wage if the latter is less (greater) than the real wage at which savings attain a maximum ($w^{\text{m}}$, say). Hence, if we compare ‘high wage’ equilibria, a rise in investment is associated with a fall in the real wage and a rise in
employment in the capitalist sector. Plainly, this relationship between the real wage and employment in the capitalist sector parallels that in section 3, although the functioning of the economy in the two cases is quite different. The effects of changes in investment on total output are also easily established, for any movement in \( w \) towards \( w^0 \) entails a rise in total output because \( l \) moves towards \( l^0 \). Hence, if an equilibrium exists in the interval \((w^{00}, w^0)\), a (small) rise in investment will decrease total output. Otherwise, investment and total output will move together. Moreover, employment in the capitalist sector moves with investment exactly as total output does. Hence, with the exception noted above, investment, output and capitalist sector employment are associated in the same way as they are in the single sector Keynesian story.

We must now take note of the fact that if net accumulation is going on, the savings function is changing over time. This takes us into dynamics proper. However, if the adjustment process is rapid and the new equilibrium is established in the neighbourhood of the old one, it is possible to say something about the change if the exogenous investment level is constant. First, note that \( s(w) \) shifts upward for all \( w > \dot{w} \), as \( \beta(w; k, p) \) shifts upwards with the increase in \( k \). In this case, therefore, the real wage falls with time if, initially, \( w < w^{00} \). Of course, if capitalists' animal spirits are dampened as their failure to attract in workers becomes more glaring, the decline in the wage rate will get a new impetus, and equilibrium at \( \dot{w} \) (presumably, with zero savings) may be the sad fate awaiting the economy. By contrast, if, initially, \( w > w^{00} \), the press of disappointed would-be workers still outside the factory gates when capitalists have hired enough to bring the marginal product of labour and the wage into equality may arouse a great urge to accumulate, with a consequent rise in investment. Thus, there are two forces in operation: rising animal spirits, which tend to lower the real wage; and the consequent rise in the capital stock, which (given \( l \)) will raise the real wage. We have pushed our speculations far enough at this point, and they must await a considered analysis of dynamic processes, which lie beyond the scope of this paper.

6. Conclusions

In sections 2 and 3, this dual economy yielded up two 'turning points'. The first marks the onset of neoclassical clearing in the labour market, which is the end of the Lewis story. The second exists only if the elasticity of substitution in the capitalist sector is less than unity: it is the point at which the share of worker households in national income ceases to fall with capital accumulation. Both 'turning points' occur when the proportion of the workforce employed in the subsistence sector is quite small for plausible parameters, at least. These are mildly interesting findings, but saving the
effects of labour supply to the capitalist sector being responsive to the real wage in the neoclassical story, sections 2 and 3 do not contain anything startlingly different from their counterparts in Lluch or T-L.

It might be thought that this failure to discover such differences can be traced to the specification of the subsistence sector. Yet I do not find this a convincing argument. Suppose, for example, that subsistence production took place under sharecropping arrangements with labour inputs being chosen by worker households. As these tenants receive only a fraction of the marginal product of labour in the subsistence sector, their willingness to supply labour to the capitalist sector is greater than in the family-owned farm system featured above. Hence, given the labour force, the nontraded endowment and the capital stock, the equilibrium wage will be lower than in section 3. But none of this alters the fact that there is still a perfectly good labour supply schedule to the capitalist sector, so that the story in section 3 is unchanged in its essentials. Nor does the fact that resource allocation under this form of sharecropping is inefficient have any bearing on the matter either. Of course, following the introduction of landlords (who take a fraction $\xi$, say, of subsistence sector output), there are now three kinds of households in the economy. This entails a minor modification to the consumption function (2g), which now reads

$$c = \gamma_L(w + (1-l)\omega_2) + (\gamma_H - \gamma_L)\xi(1-l)\omega_2 + \gamma_1 r^1k_1.$$  

Again, there would be some changes in detail in sections 4 and 5, but the qualitative conclusions there would be untouched.

A little reflection suggests that if the central issue is how the endogenous variables are related to one another when comparing one equilibrium with another, then neither the institutional structure of the subsistence sector nor the decision rule of worker households is of any great consequence provided the resulting supply curve of labour to the capitalist sector is well-behaved. Equilibrium will be established through a unique supply demand cross in the labour market, and the ensuing comparative static propositions should harbour no surprises.

What, then, is the source of the character of the results obtained in the neo-Keynesian cases? The answer lies in the way in which the goods market clears. As Marglin (1976) has emphasised, a fixed level of investment amounts to a prior claim on the demand side of the market for goods, so that the level and distribution of income must take a configuration which is consistent with that prior claim assuming that the latter is producible. Thus, if either the real wage or the intersectoral allocation of labour is fixed as in section 4 there is no room for the labour market to operate independently. The set of endogenous labour market variables adjusts in a purely passive way, and the marginal product of labour in the capitalist section.
sector will not, in general, equal the wage. The other possibility is that both the real wage and the intersectoral allocation of labour are free to float (section 5), and in this case maximising behavior reappears on a leash: labour market equilibrium is established either on the demand curve or on the supply curve (whatever their institutional-cum-behavioural origins).

Formally, of course, the case of equilibrium on the labour demand curve appears to be identical to the neoclassical case in section 3. But these economies are functioning quite differently. In the neoclassical case, the real wage adjusts so as to establish equilibrium in the labour market: incomes and hence savings follow. There is no problem in the (passive) goods market because savings determine investment. In section 5, the direction of causation is exactly reversed, the level(s) of the real wage being determined by the level of investment. Certainly, the move from a single sector to a dual economy is responsible for the richer possibilities found in the neo-Keynesian analysis. But this paper suggests that the importance of the specification of institutional structure and agents' behaviour cannot be determined independently of how markets get cleared.

Appendix

In the third neo-Keynesian variant, we see at once from fig. 4 that the allocation of labour in the economy is single-valued in the real wage, but the converse is not true (except at \( l^0 \)). Plainly, given the allocation of labour and the capital stock, output in the economy is independent of the real wage; but profits are higher in the case where \( w < w^0 \). Given \( k \), let \( i \) now float freely so that we may look at the effects on incomes and savings of parametric variations in the real wage.

Suppose \( w < w^0 \), so that equilibrium is established on \( l(w) \). Differentiating \( Y_L \) with respect to \( w \) and recalling that \( g - g' l = w \) we get

\[
\frac{dY_L}{dw} = 1 - l(w) \geq 0,
\]

as expected. Profits in the capitalist sector are

\[
Y_k = P L (1 - l(w)) - (1 - l(w))w,
\]

and so,

\[
\frac{dY_k}{dw} = -[P L' - w]dw + 1 - l(w).
\]

In this case, the marginal product of labour in the capitalist sector exceeds the wage \((P L' > w)\) and \( dl/dw \) is negative, as worker households are willing to supply more labour to factories as the wage rises. Thus, profits may rise.
or fall with increases in the real wage. At \( w = \hat{w}, \ l = 1 \); then \( Y'_K(\hat{w}) > 0 \). Also, \( Y'_K(w^0) = -[1 - I_1(w^0)] < 0 \). Under our assumptions, \( (pV'_1 - w)_l, I(w) \) and \( |dl/dw| \) are continuous and decreasing in \( w \), which suffices to establish that total profits in the capitalist sector attain a unique maximum for some \( w^* \epsilon (\hat{w}, w^0) \). Furthermore, \( dY_L \ dw = 0 \) at \( l = 1 \), so that \( \Omega \) falls as \( w \) rises for \( w \) sufficiently small.

The effects of variations in \( w \) on accumulation also depend on the level of \( w \). If both \( Y_L \) and \( Y_K \) increase with \( w \), as will be the case for \( w \) sufficiently small, then savings will rise too. However, once profits start to fall, savings will tend to fall if \( s_K > s_L \) and the gain in \( Y_L \) is not sufficient to offset the decrease in profits. To be exact, as \( s = s_K Y_K + s_L Y_L \),

\[
ds \ dw = s_K(pV'_1 - w)(dl/dw) - (s_K - s_L)[1 - I_1(w)].
\]

If \( s_L = s_K \), \( s \) rises with \( w \) until \( w^0 \) (production is rising as the strength of the distortion weakens and the propensity to save is independent of the distribution of income). If \( s_K > s_L > 0 \), then \( s'(w^0) \) is certainly negative. Further, \( s' \) changes sign for some \( w > w^* \), so that maximising the rate of savings in the economy requires a higher real wage than that needed to maximise current profits.

We now turn to the other half of the story: \( w > w^0 \), and equilibrium is established on \( \lambda(w; k, p) \), the capitalists' derived demand curve for labour. In this case, \( V'_1 = w \), and \( Y'_L(w) = -\lambda(w) < 0 \); i.e., profits decline monotonically as the real wage increases. What of the incomes of worker households, meanwhile? We have

\[
Y_L = \lambda(w) \cdot w + [1 - \lambda(w)] \cdot g[1 - \lambda(w)].
\]

Differentiating with respect to \( w \), we get

\[
dY_L \ dw = \lambda(w) + \{w - g + g' [1 - \lambda(w)]\}(dl/dw).
\]

As \( \lambda'(w) < 0 \) and the wage exceeds the marginal product of labour in the subsistence sector, \( Y'_L(w) \) may take either sign. Now \( Y'_L(w^0) = \lambda(w^0) > 0 \). Also, \( \lambda(w) = 0 \) for some finite real wage if the elasticity of substitution in the capitalist sector tends to a value less than unity in the limit \( \lambda \to 0 \) (see section 3). Our assumptions ensure that \( Y'_L(w) \) is continuous in \( w \). It is also readily seen that \( Y'_L(w) < 0 \) when \( \lambda(w) = 0 \), so that \( Y_L \) takes a maximum for some real wage greater than \( w^0 \). If the elasticity of substitution equals or exceeds unity, then

\[
\lim_{w \to x} Y'_L(w) = \{1 + E(\lambda, w)\} \lambda = [1 - E(k_1, w)] \lambda = [1 - \sigma \ 'v] \lambda.
\]
Thus $Y_L(w)$ approaches zero from below as $w$ becomes very large ($v \to 1$ if $\sigma > 1$). The continuity of $Y_L(w)$, coupled with the fact that $Y_L(w^0) > 0$, implies that $Y_L$ takes a local maximum for some finite $w^* > w^0$.

We now analyse $s(w; k, p)$ in section 5 for three savings functions. For $w < w^0$, we have:

**Case (A).** $s_K > 0, s_L = 0$.

We have $s'(\tilde{w}) > 0$ and $s'(w^0) < 0$; so $s$ takes a unique maximum for some $w \in (\tilde{w}, w^0)$. From (14), $s(w) = \overline{s}_K p\overline{V}_1[0] \geq 0$ according as $\sigma \geq 1$. Hence, $s(w) = \overline{s}_K p\overline{V}_1[0]$, $\forall w \in [0, \tilde{w}]$.

**Case (B).** $s_K > s_L > 0$.

Again, we have $s'(\tilde{w}) > 0$ and $s'(w^0) < 0$, so $s$ takes a unique maximum for some $w \in (\tilde{w}, w^0)$ - but lower than the corresponding $w$ in the previous case. Also, $s(w) = \overline{s}_K p\overline{V}_1[0] + s_L \overline{V}_2[1]$, $\forall w \in [0, \tilde{w}]$.

**Case (C).** $s_K = s_L$ (Solow-Swan).

Here, $s'(\tilde{w}) > 0$ and $s'(w^0) = 0$, so savings take a maximum at the market clearing wage (certainly a case of having one's cake and eating it). Further, $s(w) = \overline{s}_K p\overline{V}_1[0] + \overline{s}_L \overline{V}_2[1]$, $\forall w \in [0, \tilde{w}]$.

For $w > w^0$, we have:

**Case (A).** $s \to s_K p\overline{V}_1[0]$ as $w \to \infty$ if $\sigma > 1$ in a CES technology. If $\sigma < 1$, $s = 0$, $\forall w \geq x^a_1(\sigma^{-1})$ (see above). If $F(\cdot)$ is well-behaved (or $\sigma = 1$), $s \to 0$ as $w \to \infty$.

**Case (B).** $s \to s_K p\overline{V}_1[0] + s_L \overline{V}_2[1]$ as $w \to \infty$ if $\sigma > 1$; $s \to s_L \overline{V}_2[1]$ if $F(\cdot)$ is well-behaved. If $\sigma < 1$, $s = s_L \overline{V}_2[1]$, $\forall w \geq x^a_1(\sigma^{-1})$.

**Case (C).** $s = s_K [p\overline{V}_1 + \overline{V}_2]$. The behavior of $s$ as $w$ increases without limit follows at once from the preceding cases.

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