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The Decomposition of a House Price index into Land and Structures Components: A Hedonic Regression Approach

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The Decomposition of a House Price index into Land and Structures Components: A Hedonic Regression Approach

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Abstract

The paper uses hedonic regression techniques in order to decompose the price of a house into land and structure components using readily available real estate sales data for a Dutch city. In order to get sensible results, it proved necessary to use a nonlinear regression model using data that covered multiple time periods. It also proved to be necessary to impose some monotonicity restrictions on the price of land and structures.

Key Words

Property price indexes, hedonic regressions, repeat sales method, rolling year indexes, Fisher ideal indexes.

Journal of Economic Literature Classification Numbers

C2, C23, C43, D12, E31, R21.

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1. Introduction

Our goal in this paper is to use readily available multiple listing data on sales of residential properties and to somehow decompose the sales price of each property into a land component and a structures component. We will use the data pertaining to the sales of detached houses in a small Dutch city for 10 quarters, starting in January 1998.

In section 2, we will consider a very simple hedonic regression model where we use information on only three characteristics of the property: the lot size, the size of the structure and the (approximate) age of the structure. We run a separate hedonic regression for each quarter which lead to estimated prices for land and structures for each quarter. These estimated characteristics prices can then be into land and structures prices covering the 10 quarters of data in our sample. We postulate that the value of a residential property is the sum of two components: the value of the land which the structure sits on plus the value of the residential structure. Thus our approach to the valuation of a residential property is essentially a crude cost of production approach. Note that the overall value of the property is assumed to be the *sum* of these two components.

In section 3, we generalize the model explained in section 2 to allow for the observed fact that the per unit area price of a property tends to decline as the size of the lot increases (at least for large lots). We use a simple linear spline model with 2 break points. Again, a separate hedonic regression is run for each period and the results of these separate regressions were linked together to provide separate land and structures price indexes (along with an overall price index that combined these two components).

The models described in sections 2 and 3 were not very successful. The problem is the variability in the data and this volatility leads to a tendency for the regression models to fit the outliers, leading to volatile estimates for the price of land and structures. Thus in section 4, we note that since the median price of the houses sold in each quarter never declined, it is likely that the underlying separate land and structures prices also did not decline over our sample period. Thus we imposed this monotonicity restriction on our nonlinear regression model by using squared coefficients and nonlinear regression techniques in one big regression using all 10 quarters of data. We obtained reasonable estimates for the land and structures components using this technique.

Buoyed by the success of our quarterly model, we implemented the model using monthly data instead of quarterly data in section 5. This is more challenging since we had only 30 to 60 observations for each month. However, the monthly model also worked reasonably well and when we aggregated the monthly results into quarterly results, we obtained quarterly results which were very similar to the results obtained in section 4.

In section 6, we decided to compare our quarterly results with a more traditional hedonic regression model for residential properties. In this more traditional approach, the log of the property price is regressed on either the logs of the main characteristics of the property (the land area and the floor space area) or on the levels of the main characteristics, with dummy variables to represent quarter to quarter price change. We found that the log-log regression fit the data much better than the log-levels regression and the overall index of prices generated by the log-log regression was quite close to our overall index of prices generated by the cost of production model explained in section 4. However, when we used the log-log model to generate separate price index series for land and for structures, the results did not seem to be credible.

Section 7 concludes with an agenda for further research on this topic.

2. Model 1: A Very Simple Model

Hedonic regression models are frequently used to obtain constant quality price indexes for owner occupied housing.[2] Although there are many variants of the technique, the basic model regresses the logarithm of the sale price of the property on the price determining characteristics of the property and a time dummy variable is added for each period in the regression (except the base period). Once the estimation has been completed, these time dummy coefficients can be exponentiated and turned into an index.[3]

Since hedonic regression methods assume that information on the characteristics of the properties sold is available, the data can be stratified and a separate regression can be run for each important class of property. Thus hedonic regression methods can be used to produce a family of constant quality price indexes for various types of property.[4]

A real estate property has two important price determining characteristics:[5]

- The land area of the property and
- The livable floor space area of the structure.

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[2] See for example, Crone, Nakamura and Voith (2000) (2009), Gouriéroux and Laferrère (2009), Hill, Melser and Syed (2009) and Li, Prud'homme and Yu (2006).

[3] An alternative approach to the time dummy hedonic method is to estimate separate hedonic regressions for both of the periods compared; i.e., for the base and current period. See Haan (2008) (2010) and Diewert, Heravi and Silver (2010) for discussions and comparisons between these alternative approaches.

[4] This property of the hedonic regression method also applies to stratification methods. The main difference between the two methods is that continuous variables can appear in hedonic regressions (like the area of the structure and the area of the lot size) whereas stratification methods can only work with discrete ranges for the independent variables in the regression. Typically, hedonic regressions are more parsimonious; i.e., they require fewer parameters to explain the data as opposed to stratification methods.

[5] A third important characteristic is the *location* of the property; i.e., how far is the property from shopping centers, places of employment, hospitals and good schools; does the property have a view; is the property subject to noise or particulate pollution and so on. The presence or lack of these amenities will affect the price of land in the neighbourhood and thus it is important to stratify the sample in order to control for these neighbourhood effects. In our example, the Dutch town of "A" is small enough and homogeneous enough so that these neighbourhood effects can be neglected.

For some purposes, it would be very useful to decompose the overall price of a property into *additive components* that reflected the value of the land that the structure sits on and the value of the structure. The purpose of the present paper is to determine whether a hedonic regression technique could provide such a decomposition.

Diewert (2007) suggested some possible hedonic regression models that might lead to additive decompositions of an overall property price into land and structures components. We will now outline his suggested model (with a few modifications).

If we momentarily think like a property developer who is planning to build a structure on a particular property, the total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S square meters, times the building cost per square meter, β say, plus the cost of the land, which will be equal to the cost per square meter, α say, times the area of the land site, L. Now think of a sample of properties of the same general type, which have prices v_n^t in period t[6] and structure areas S_n^t and land areas L_n^t for n = 1,...,N(t), and these prices are equal to costs of the above type plus error terms η_n^t which we assume have means 0. This leads to the following hedonic regression model for period t where α^t and β^t are the parameters to be estimated in the regression:[7]

$$(1)\ v_n^t = \alpha^t L_n^t + \beta^t S_n^t + \eta_n^t\ ; \qquad n = 1,...,N(t);\ t = 1,...,T.$$

Note that the two characteristics in our simple model are the quantities of land L_n^t and the quantities of structure S_n^t associated with the sale of property n in period t and the two constant quality prices in period t are the price of a square meter of land α^t and the price of a square meter of structure floor space β^t. Finally, note that separate linear regressions can be run of the form (1) for each period t in our sample.

The hedonic regression model defined by (1) is the simplest possible one but it is a bit too simple since it neglects the fact that older structures will be worth less than newer structures due to the depreciation of the structure. Thus suppose in addition to information on the selling price of property n at time period t, v_n^t, the land area of the property L_n^t and the structure area S_n^t, we also have information on the age of the structure at time t, say A_n^t. Then if we assume a straight line depreciation model, a more realistic hedonic regression model than that defined by (1) above is the following model:

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[6] Note that we have labeled these property prices as v_n^0 to emphasize that these are *values* of the property and we need to decompose these values into two price and two quantity components, where the components are land and structures.

[7] In order to obtain homoskedastic errors, it would be preferable to assume multiplicative errors in equation (1) since it is more likely that expensive properties have relatively large absolute errors compared to very inexpensive properties. However, we think that it is preferable to work with the additive specification (1) since we are attempting to decompose the aggregate value of housing (in the sample of properties that sold during the period) into additive structures and land components and the additive error specification will facilitate this decomposition.

(2) $v_n^t = \alpha^t L_n^t + \beta^t(1 - \delta^t A_n^t)S_n^t + \eta_n^t$; $n = 1,...,N(t); t = 1,...,T$

where the parameter δ^t reflects the *depreciation rate* as the structure ages one additional period. Thus if the age of the structure is measured in years, we would expect δ^t to be between 1 and 2%.[8] Note that (2) is now a nonlinear regression model whereas (1) was a simple linear regression model. Both models (1) and (2) can be run period by period; it is not necessary to run one big regression covering all time periods in the data sample. The period t price of land will the estimated coefficient for the parameter α^t and the price of a unit of a newly built structure for period t will be the estimate for β^t. The period t quantity of land for property n is L_n^t and the period t quantity of structure for property n, expressed in equivalent units of a new structure, is $(1 - \delta^t A_n^t)S_n^t$ where S_n^t is the floor space area of property n in period t.

We implemented the above model (2) using real estate sales data on the sales of detached houses for a small city (population is around 60,000) in the Netherlands, City A, for 10 quarters, starting in January 1998 (so our T = 10). The data that we used can be described as follows:

- v_n^t is the selling price of property n in quarter t in units of 10,000 Euros where t = 1,...,10;
- L_n^t is the area of the plot for the sale of property n in quarter t in units of 100 meters squared;[9]
- S_n^t is the living space area of the structure for the sale of property n in quarter t in units of 100 meters squared;
- A_n^t is the (approximate) age (in decades) of the structure on property n in period t.[10]

There were 1404 observations in our 10 quarters of data on sales of detached houses in City A. The sample means for the data were as follows: $\bar{v} = 11.198$, $\bar{L} = 2.5822$, $\bar{S} = 1.2618$ and $\bar{A} = 1.1859$. Thus the sample of houses sold at the average price of 111,980 Euros, the average plot size was 258.2 meters squared, the average living space in the structure was 126.2 meters squared and the average age was approximately 12.6 years. The sample median price was 95,918 Euros.

The results of our 10 nonlinear regressions of the type defined by (2) above are summarized in Table 1 below. The Adjusted Structures Quantities in quarter t, AS^t, is equal to the sum over the properties sold n in that quarter adjusted into new structure units, $\sum_n (1 - \delta^t A_n^t)S_n^t$.

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[8] This estimate of depreciation will be an underestimate of "true" structure depreciation because it will not account for major renovations or additions to the structure.

[9] We chose units of measurement in order to scale the data to be small in magnitude in order to facilitate the nonlinear regression package used, which was Shazam.

[10] The original data were coded as follows: if the structure was built 1960-1970, the observation was assigned the dummy variable BP = 5; 1971-1980, BP=6; 1981-1990, BP=7; 1991-2000, BP=8. Our Age variable A was set equal to 8 – BP. Thus for a recently built structure n in quarter t, $A_n^t = 0$.

Table 1: Estimated Land Prices α^t, Structure Prices β^t, Decade Depreciation Rates δ^t, Land Quantities L^t and Adjusted Structures Quantities AS^t

Quarter	α^t	β^t	δ^t	L^t	AS^t
1	1.52015	5.13045	0.10761	380.1	177.5
2	1.40470	6.33087	0.15918	426.9	166.4
3	1.83006	5.13292	0.13410	248.6	111.2
4	1.71757	5.56902	0.14427	285.2	122.0
5	0.70942	8.23225	0.12613	390.2	158.4
6	0.26174	9.94447	0.09959	419.4	168.7
7	2.12605	6.27949	0.13258	368.9	136.5
8	1.71496	7.29677	0.13092	347.3	136.2
9	1.47354	7.86387	0.10507	356.7	156.4
10	2.68556	6.21736	0.18591	402.1	161.6

It can be seen that the decade depreciation rates δ^t are in the 10 to 18% range which is not unreasonable but the volatility in these rates is not a good sign. However, it can be seen that our estimated land and structures prices are not at all reasonable: the price of land sinks to a very low level in quarter 6 while the price of structures peaks in this quarter.

It is of some interest to compare the above land and structures prices with the mean and median prices for houses in the sample for each quarter. These prices were normalized to equal 1 in quarter 1 and are listed as P_{Mean} and P_{Median} in Table 2 below. The land and structures prices in Table 1, α^t and β^t, were also normalized to equal 1 in quarter 1 and are listed as P_L and P_S in Table 2. Finally, we used the price data in Table 1, α^t and β^t, along with the corresponding quantity data, L^t and AS^t, in Table 1 in order to calculate a "constant quality" chained Fisher house price index, which is listed as P_F in Table 2.

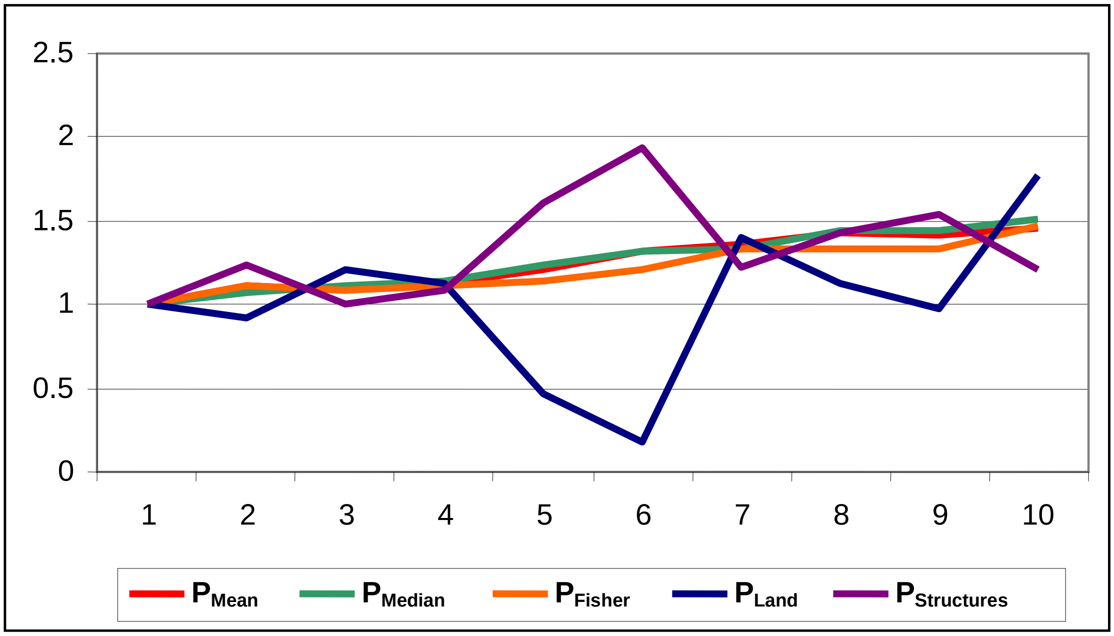
Table 2: Quarterly Mean, Median and Predicted Fisher Housing Prices and the Price of Land and Structures

Quarter	P_{Mean}	P_{Median}	P_F	P_L	P_S
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.11935	1.07727	1.10689	0.92406	1.23398
3	1.07982	1.11666	1.08649	1.20387	1.00048
4	1.13171	1.13636	1.10735	1.12987	1.08548
5	1.20659	1.24242	1.13521	0.46668	1.60459
6	1.31463	1.32424	1.20389	0.17218	1.93832
7	1.36667	1.33333	1.33644	1.39858	1.22397
8	1.43257	1.43939	1.32944	1.12816	1.42225
9	1.41027	1.44242	1.32764	0.96934	1.53278
10	1.45493	1.51515	1.47253	1.76665	1.21185

Note that the median price increases in each quarter while the mean price drops (slightly) in quarters 3 and 9. It can be seen that the overall Fisher housing price index P_F is roughly equal to the mean and median price indexes but again, the separate price series for housing land P_L and for housing structures P_S are not realistic.

The series in Table 2 are graphed in Chart 1 below.

Chart 1: Quarterly Mean, Median and Predicted Fisher Housing Prices and the Price of Land and Structures Using Model 1



It can be seen that while the overall predicted Fisher house price index is not too far removed from the median and mean house price indexes, the separate land and structures components of the overall index are not at all sensible.

One possible problem with our highly simplified house price model is that our model makes no allowance for the fact that larger sized plots tend to sell for an average price that is below the price for medium and smaller sized plots. Thus in the following section, we will generalize the model (2) to take into account this empirical regularity.

3. Model 2: The Use of Linear Splines on Lot Size

We broke up our 1404 observations into 3 groups of property sales:

- Sales involving lot sizes less than or equal to 200 meters squared (Group S);
- Sales involving lot sizes between 200 and 400 meters squared (Group M) and
- Sales involving lot sizes greater than 400 meters squared (Group L).

For an observation n in period t that was associated with a small lot size, our regression model was essentially the same as in (2) above; i.e., the following estimating equation was used:

(3) $v_n^t = \alpha_S^t L_n^t + \beta^t(1 - \delta^t A_n^t)S_n^t + \eta_n^t$; t = 1,...,T; n belongs to Group S

where the unknown parameters to be estimated are α^t, β^t and δ^t. For an observation n in period t that was associated with a medium lot size, the following estimating equation was used:[11]

(4) $v_n^t = \alpha_S^t(2) + \alpha_M^t(L_n^t - 2) + \beta^t(1 - \delta^t A_n^t)S_n^t + \eta_n^t$; t = 1,...,T; n belongs to Group M

where we have now added a fourth parameter to be estimated, α_M^t. Finally, for an observation n in period t that was associated with a large lot size, the following estimating equation was used:

(5) $v_n^t = \alpha_S^t(2) + \alpha_M^t(4 - 2) + \alpha_L^t(L_n^t - 4) + \beta^t(1 - \delta^t A_n^t)S_n^t + \eta_n^t$;
t = 1,...,T; n belongs to Group L

where we have now added a fifth parameter to be estimated, α_L^t. Thus for small lots, the value of an extra marginal addition of land in quarter t is α_S^t, for medium lots, the value of an extra marginal addition of land in quarter t is α_M^t and for large lots, the value of an extra marginal addition of land in quarter t is α_L^t. These pricing schedules are joined together so that the cost of an extra unit of land increases with the size of the lot in a continuous fashion.[12] The above model can readily be put into a nonlinear regression format for each period using dummy variables to indicate whether an observation is in Group S, M or L.

The results of our 10 nonlinear regressions of the type defined by (3)-(5) above are summarized in Table 3 below.

Table 3: Marginal Land Prices for Small, Medium and Large Lots, the Price of Structures β^t and Decade Depreciation Rates δ^t

Quarter	α_S^t	α_M^t	α_L^t	β^t	δ^t
1	0.31648	3.30552	0.87617	6.17826	0.06981
2	0.79113	2.96475	0.78643	6.44827	0.13999
3	1.77147	2.57100	1.27783	4.96547	0.12411
4	0.49927	3.48688	1.02879	6.61768	0.09022
5	0.59573	3.01473	0.44064	7.39286	0.13002
6	0.08365	3.81462	−0.2504	8.38993	0.09269

[11] Recall that we are measuring land in 100's of square meters instead of in squared meters.
[12] Thus if we graphed the total cost C of a lot as a function of the plot size L in period t, the resulting cost curve would be made up of three linear segments whose endpoints are joined. The first line segment starts at the origin and has the slope α_S^t, the second segment starts at L = 2 and runs to L = 4 and has the slope α_M^t and the final segment starts at L = 4 and has the slope α_L^t.

7	1.09346	4.12335	1.26155	6.84204	0.09168
8	2.44028	3.06473	1.29751	5.71713	0.14456
9	2.00417	3.88380	0.88777	6.38234	0.14204
10	3.04236	3.33855	2.30271	5.49038	0.20080

Obviously, the estimated prices are not sensible; in particular, it is not likely that the cost of an extra unit of land for a large plot could be negative in quarter 6!

Looking at the median price of a house over the 10 quarters in our sample, it was noted earlier that the median price never fell over the sample period. This fact suggests that we should impose this condition on all of our prices; i.e., we should set up a nonlinear regression where the marginal prices of land never fall from quarter to quarter and where the price of a square meter of a new structure also never falls. We will do this in the following section and we will also impose a single depreciation rate over our sample period, rather than allowing the depreciation rate to fluctuate from quarter to quarter.

4. Model 3: The Use of Monotonicity Restrictions on the Price of Land and Structures

For the model to be described in this section, the data for all 10 quarters were run in one big nonlinear regression. The equations that describe the model in quarter 1 are the same as equations (3), (4) and (5) in the previous section except that the quarter one depreciation rate parameter, δ^1, is replaced by the parameter δ, which will be used in all subsequent quarters. For the remaining quarters, equations (3), (4) and (5) can still be used except that the parameters $\alpha_S{}^t$, $\alpha_M{}^t$, $\alpha_L{}^t$ and β^t are set equal to their quarter 1 counterparts plus a sum of squared parameters where one squared parameter is added each period; i.e., $\alpha_S{}^t$, $\alpha_M{}^t$, $\alpha_L{}^t$ and β^t are reparameterized as follows:

(6) $\alpha_S{}^t = \alpha_S{}^1 + (\delta_{S2})^2 + ... + (\delta_{St})^2$; $t = 2,3,...,T$;
(7) $\alpha_M{}^t = \alpha_M{}^1 + (\delta_{M2})^2 + ... + (\delta_{Mt})^2$; $t = 2,3,...,T$;
(8) $\alpha_L{}^t = \alpha_L{}^1 + (\delta_{L2})^2 + ... + (\delta_{Lt})^2$; $t = 2,3,...,T$;
(9) $\beta^t = \beta^1 + (\delta_2)^2 + ... + (\delta_t)^2$; $t = 2,3,...,T$;
(10) $\delta^t = \delta$; $t = 2,3,....T$.

Thus our new parameters $\delta_{S2},...,\delta_{St}$; $\delta_{M2},...,\delta_{Mt}$; $\delta_{L2},...,\delta_{Lt}$ and $\delta_2,...,\delta_t$ and their squares enter equations (6)-(9). It can be seen that this reparameterization will prevent the marginal price of each type of land from falling and it will also impose monotonicity on the price of structures.

The results of the above reparameterized model were as follows: the quarter 1 estimated parameters were $\alpha_S{}^1 = 0.91595$ (0.36782), $\alpha_M{}^1 = 3.3527$ (0.15247), $\alpha_L{}^1 = 0.30862$ (0.049515), $\beta^1 = 5.7653$ (0.57508) and $\delta = 0.14763$ (0.009926) (standard errors in brackets) with an R^2 of .8383. Thus the overall decade depreciation rate was a very reasonable 14.76% and the other parameters seemed to be reasonable in magnitude as well. The only mild surprise was the fact that, at the beginning of the sample period, the marginal valuation of land for small plots was 0.91595 while the marginal valuation for medium plots was 3.3527 which was over 3 times as big. Thus small plots of land suffered a discount in price per meter squared as compared to medium plots of land.[13] Of the 36 squared parameters that pertain to quarters 2 to 10, 22 were set equal to 0 by the nonlinear regression and only 14 were nonzero with only 7 of these nonzero parameters having t statistics greater than 2. The quarter by quarter values of the parameters α_S^t α_M^t α_L^t and β^t defined by (6)-(9) are reported in Table 4 below.

Table 4: Marginal Prices of Land for Small, Medium and Large Plots and New Construction Prices by Quarter

Quarter	α_S^t	α_M^t	α_L^t	β^t
1	0.91595	3.35274	0.30862	5.76531
2	0.95659	3.35274	0.30862	5.83293
3	1.08763	3.35274	0.30862	5.83293
4	1.08763	3.38135	0.30862	6.19944
5	1.17047	3.38135	0.30862	6.19944
6	1.59381	3.38135	0.30862	6.19944
7	1.86173	3.58656	1.06497	6.19944
8	1.93392	3.58656	1.06497	6.19944
9	2.09467	3.58656	1.06497	6.19944
10	2.16906	3.58656	1.68323	6.19944

The above results look sensible (for a change!). The imputed price of new construction, β^t, was approximately equal to a constant 6 over the sample period (this translates into a price of 600 Euros per meter squared of structure floor space).[14] The imputed value of land for a small lot grew from 91.6 Euros per meter squared in the first quarter of 1998 to 216.9 Euros per meter squared in the second quarter of 2000. The imputed marginal value of land[15] for a lot size in the range of 200 to 400 meters squared grew very slowly from 335.3 Euros per meter squared to 358.7 Euros per meter squared over the same period. Finally, the imputed marginal value of land[16] for a lot size greater than 400 meters squared grew very rapidly from 30.8 Euros per meter squared to 168.3 Euros per meter squared over the sample period.

It is possible to work out the total imputed value of structures transacted in each quarter, V_S^t, and divide this quarterly value by the total quantity of structures (converted into equivalent new structure units), Q_S^t, in order to obtain an average price of structures, P_S^t.

[13] This may not be a “genuine” effect; it is likely that the quality of construction is lower on small plots as compared to the quality of medium and larger plots and since we are not taking this possibility into account in our model, the lower average quality of structures on small plots may show up as a lower price of land for small plots.

[14] Thus the imputed value of a new house with a floor space area of 125 meters squared would be approximately 75,000 Euros.

[15] This is our estimate of the value of an extra square meter of land above the threshold of 200 meters squared (and below the threshold of 400 meters squared).

[16] This is our estimate of the value of an extra square meter of land above the threshold of 400 meters squared.

Similarly, we can add up all of the imputed values for small, medium and large plot sizes for each quarter t, say V_{LS}^t, V_{LM}^t and V_{LL}^t, and then add up the total quantity of land transacted in each of the three classes of property, say Q_{LS}^t, Q_{LM}^t and Q_{LL}^t. Finally, we can form quarterly unit value prices for each of the three classes of property, P_{LS}^t, P_{LM}^t and P_{LL}^t, by dividing each value series by the corresponding quantity series. The resulting price and quantity series are listed in Table 5 below.

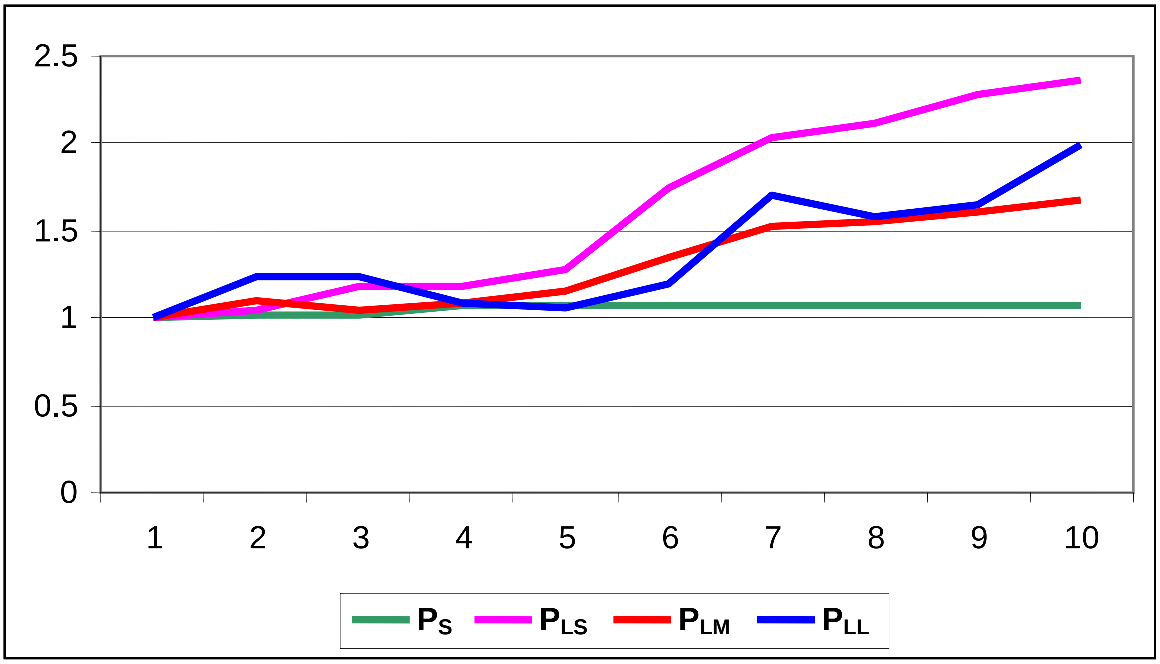
Table 5: Average Prices for New Structures, Small, Medium and Large Plots and Total Quantities Transacted per Quarter of Structures and the Three Types of Plot Size

Quarter	P_S^t	P_{LS}^t	P_{LM}^t	P_{LL}^t	Q_S^t	Q_{LS}^t	Q_{LM}^t	Q_{LL}^t
1	5.76531	0.91595	1.54388	1.21882	174.9	157.0	150.9	72.2
2	5.83293	0.95659	1.69649	1.51438	178.7	141.7	150.5	134.7
3	5.83293	1.08763	1.61619	1.49880	114.5	86.5	104.4	57.8
4	6.19944	1.08763	1.67685	1.32253	124.6	98.4	118.4	68.4
5	6.19944	1.17047	1.78600	1.28946	160.0	111.5	166.3	112.3
6	6.19944	1.59381	2.08535	1.45049	164.0	99.3	190.3	129.8
7	6.19944	1.86173	2.35472	2.07883	138.0	103.6	134.4	130.9
8	6.19944	1.93392	2.39501	1.92618	137.8	89.6	155.3	102.4
9	6.19944	2.09467	2.48640	2.00769	153.2	114.4	151.9	90.4
10	6.19944	2.16906	2.57689	2.42353	180.1	123.4	207.8	71.0

Note that the price of structures series, P_S^t, and the price of land for small plots, P_{LS}^t, in Table 5 coincides with the series of values for β^t and α_S^t listed in Table 4. However, the average prices for land in medium size plots, P_{LM}^t, and for large size plots, P_{LL}^t, listed in Table 5 no longer coincide with the corresponding marginal prices α_M^t and α_L^t listed in Table 4. This is understandable since we have used splines to model how the price of a meter squared of land varies as the lot size varies. Note that P_{LM}^t shows a much greater rate of price increase over the sample period than the corresponding marginal price series α_M^t, which hardly changed over the sample period. This is due to the fact that our model prices the first 200 meters squared of a medium sized lot at the average price of a small lot and the price of small lots increased quite rapidly over the sample period. Another striking feature of Table 5 is the tendency for the prices of land for small, medium and large lots to equalize over time; i.e., at the beginning of the sample period, the price per meter squared of a small lot was 91.6 Euros, for a medium lot, 154.4 Euros and for a large lot, 121.9 Euros but by the end of the sample period, the prices were 216.9 Euros, 257.7 Euros and 242.3 Euros, which was a considerable relative compression in the dispersion of these prices. A final feature of Table 5 that should be mentioned is the tremendous volatility in the quantities transacted in each quarter.

The four price series, P_S^t, P_{LS}^t, P_{LM}^t and P_{LL}^t, were all normalized to equal unity in quarter 1 and they are plotted in Chart 2 below.

Chart 2: Prices For Structures P_S^t and for Three Sizes of Plot P_{LS}^t, P_{LM}^t and P_{LL}^t



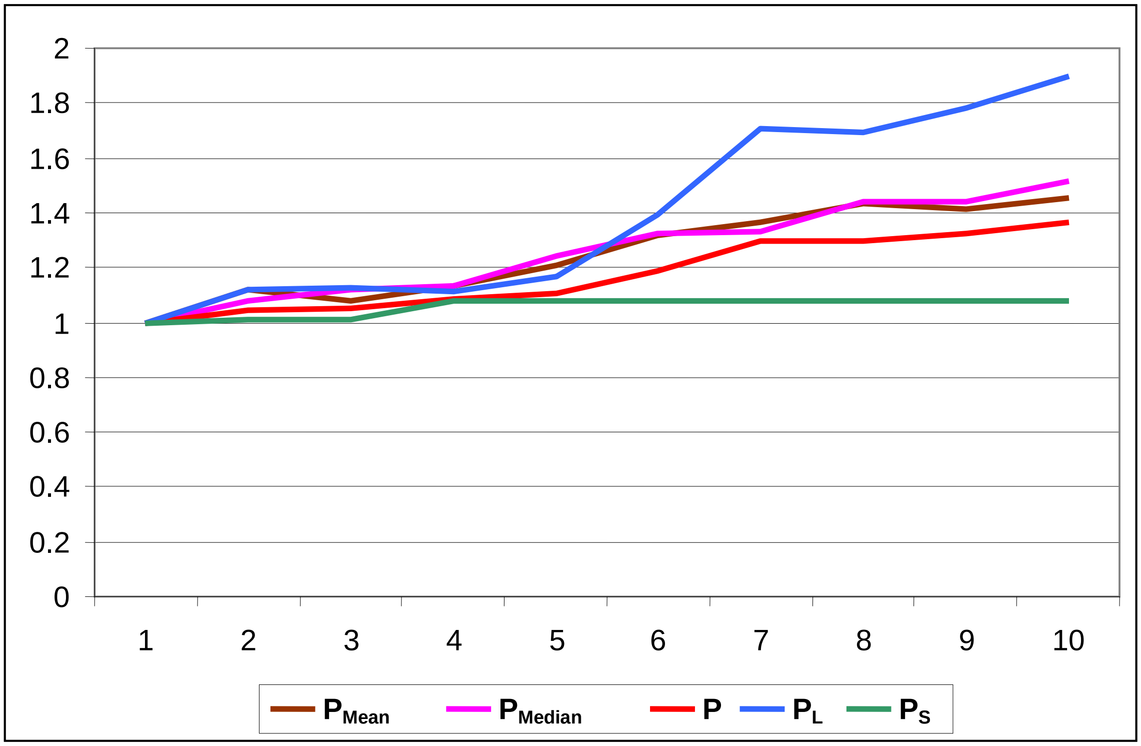
The data listed in Table 5 were further aggregated. We constructed a chained Fisher aggregate for the three land series and the resulting aggregate land price and quantity series, P_L^t and Q_L^t, are listed in Table 6 below along with the structures price and quantity series (normalized so that the price equals 1 in quarter 1), P_S^t and Q_S^t. Finally, a chained Fisher aggregate for structures and the three land series was constructed and the resulting aggregate price and quantity series, P^t and Q^t, are also listed in Table 6.

Table 6: Aggregate Quarterly Price and Quantity Series for Housing

Quarter	P^t	P_L^t	P_S^t	Q^t	Q_L^t	Q_S^t
1	1.00000	1.00000	1.00000	1473.4	464.7	1008.6
2	1.04712	1.11966	1.01173	1563.6	531.3	1030.3
3	1.04959	1.12710	1.01173	969.1	310.0	660.0
4	1.08823	1.11403	1.07530	1074.0	355.5	718.6
5	1.10535	1.16452	1.07530	1415.1	491.5	922.3
6	1.18642	1.39356	1.07530	1483.6	533.4	945.6
7	1.29832	1.70330	1.07530	1260.8	458.9	795.5
8	1.29433	1.69226	1.07530	1233.4	438.7	794.2
9	1.32431	1.77854	1.07530	1320.2	449.1	883.2
10	1.36569	1.90086	1.07530	1531.6	513.0	1038.4

Finally, Chart 3 below plots the aggregate house price series P^t, the land price series P_L^t and the structures price series P_S^t from Table 6 above along with the quarterly mean price series P_{Mean}^t and median series P_{Median}^t.

Chart 3: Quarterly Mean Price $P_{Mean}{}^t$, Median Price $P_{Median}{}^t$, Constant Quality Housing Price P^t, Land Price $P_L{}^t$ and New Structures Price $P_S{}^t$



From Chart 3, it is evident that our estimated constant quality price of housing for City A grew more slowly than the corresponding mean and median series. The major explanatory factor for this difference is probably due to the fact that the average age of the structure in the quarterly sample tended to fall as time marched on.[17]

We have used only 3 characteristics of the property sales: the age of the structure, the area of the land and the floor space area of the house. Real estate data bases generally have information on many other characteristics of the house and these characteristics could be integrated into the above hedonic framework.

In the following section, we will attempt to implement the model explained in this section using monthly data in place of quarterly data.

5. A Monthly Model Using Monotonicity Restrictions

[17] The time series of average age by quarter in our sample was as follows: 1.38, 1.30, 1.24, 1.06, 1.19, 1.21, 1.16, 1.10, 0.957 and 1.18. The average amount of land tended to increase a bit over time; the quarterly averages were as follows: 2.30, 2.60, 2.35, 2.48, 2.69, 2.80, 2.75, 2.78, 2.57 and 2.50. The average structure size transacted by quarter was fairly steady: 1.26, 1.28, 1.26, 1.24, 1.28, 1.27, 1.20, 1.26, 1.24 and 1.29.

Before we repeat the Tables that were listed in the previous section using monthly data instead of quarterly data, it is useful to list the descriptive statistics that describe the monthly data. Thus in Table 7 below, we list various averages for the 30 months of data in our sample as well as N, the number of observations in each month, which range from a low of 26 in month 9 to a high of 63 in month 3.

Table 7: Descriptive Statistics for the Monthly Data

Month	N	Mean	Median	L	S	A	f_S	f_M	f_L
1	55	8.81447	7.4874	2.24109	1.27873	1.45455	0.63636	0.30909	0.05455
2	47	8.59045	7.4420	2.31872	1.21979	1.34043	0.61702	0.34043	0.04255
3	63	9.32068	7.7143	2.34635	1.28254	1.33333	0.57143	0.36508	0.06349
4	46	9.55868	7.7188	2.31326	1.30609	1.26087	0.56522	0.34783	0.08696
5	57	9.60040	7.9412	2.37298	1.24860	1.19298	0.63158	0.26316	0.10526
6	61	10.73692	9.4386	3.03672	1.29590	1.44262	0.45902	0.34426	0.19672
7	42	10.60333	8.4290	2.65738	1.28452	1.11905	0.47619	0.40476	0.11905
8	38	8.74363	8.1680	2.10316	1.24711	1.36842	0.52632	0.42105	0.05263
9	26	9.46656	8.4516	2.19615	1.25500	1.26923	0.65385	0.26923	0.07692
10	37	8.94806	8.1680	2.30027	1.18405	1.24324	0.54054	0.43243	0.02703
11	41	10.96991	8.6218	2.79439	1.27293	1.00000	0.53659	0.34146	0.12195
12	37	10.35631	8.8487	2.31162	1.26081	0.94595	0.54054	0.37838	0.08108
13	51	10.44940	9.3025	2.98471	1.23941	1.43137	0.47059	0.47059	0.05882
14	40	10.12645	8.1680	2.35350	1.28575	1.07500	0.62500	0.25000	0.12500
15	54	11.60774	10.4256	2.66296	1.30981	1.03704	0.40741	0.48148	0.11111
16	40	11.18432	11.1176	2.61050	1.25925	1.35000	0.45000	0.50000	0.05000
17	53	11.49708	9.5385	3.04830	1.27453	1.05660	0.45283	0.37736	0.16981
18	57	12.40321	10.7319	2.69088	1.28018	1.26316	0.38596	0.50877	0.10526
19	46	12.09197	10.3258	2.69978	1.23217	1.26087	0.47826	0.36957	0.15217
20	37	12.28354	9.9378	2.74324	1.18595	1.05405	0.56757	0.29730	0.13514
21	51	12.29845	10.0966	2.80902	1.18529	1.15686	0.45098	0.39216	0.15686
22	36	11.45179	10.4483	2.53528	1.19500	1.41667	0.47222	0.47222	0.05556
23	43	12.75577	10.1647	2.88791	1.27116	1.09302	0.51163	0.34884	0.13953
24	46	13.93129	11.9968	2.86565	1.31087	0.84783	0.36957	0.52174	0.10870
25	36	12.96740	10.8226	2.76778	1.28361	0.80556	0.52778	0.33333	0.13889
26	50	12.95475	10.8453	2.68940	1.24160	1.04000	0.48000	0.44000	0.08000
27	53	12.05086	10.5504	2.31226	1.21755	0.98113	0.52830	0.41509	0.05660
28	61	12.17228	10.7773	2.32656	1.26246	1.21311	0.54098	0.40984	0.04918
29	50	13.33456	10.6638	2.69700	1.31660	1.22000	0.42000	0.48000	0.10000
30	50	13.71641	12.7625	2.50760	1.30640	1.10000	0.44000	0.50000	0.06000

It can be seen that the monthly means and medians no longer steadily trend upwards; there are now many ups and downs in these series. The L and S series are the monthly average amounts of land and structures (in 100s of square meters) sold in each month. There are large fluctuations in some of these averages: L ranges from a low of 2.10 to a high of 3.05 while S ranges from 1.18 to 1.32. The average age in decades, A, ranges from a low of 0.81 to 1.45. The fraction of small lots transacted in a given month, f_S, ranges from a low of 0.370 to a high of 0.654; the fraction of medium sized lots transacted in a given month, f_M, ranges from a low of 0.250 to a high of 0.522 and the fraction of large lots transacted in a given month, f_L, ranges from a low of 0.027 to a high of 0.197. Given the magnitude of these fluctuations, it can be seen that it is unreasonable to expect the mean and median series to give a good approximation to pure price change because the underlying monthly characteristics are changing so dramatically from month to month (and so the mean and median series embody quantity effects as well as price effects).

The model described in the previous section was rerun using the monthly data so that we now have 30 monthly time periods in place of the old 10 quarterly time periods. The number of parameters to be estimated has skyrocketed to 121 from the old 41 parameters. The results for the monthly model were as follows: the month 1 estimated parameters were α_S^1 = 0.99767 (0.17751), α_M^1 = 3.2434 (0.11801), α_L^1 = 0.29878 (0.043824), β^1 = 5.5632 (0.32939) and γ = 0.14860 (0.00994) (standard errors in brackets) with an R^2 of .8454. Recall that the corresponding quarterly model parameters for quarter 1 were: α_S^1 = 0.91595 (0.36782), α_M^1 = 3.3527 (0.15247), α_L^1 = 0.30862 (0.049515), β^1 = 5.7653 (0.57508) and δ = 0.14763 (0.009926) with an R^2 of .8383. Thus the monthly model has generated parameter estimates that are quite similar to the quarterly model (at least at the beginning of the sample period) but the standard errors for the monthly parameters are all lower (much lower in most cases) than the corresponding standard errors for the quarterly model (and the R^2 has also increased modestly). This is very encouraging! Of the 116 squared parameters that pertain to months 2 to 30, 99 were set equal to 0 by the nonlinear regression and only 17 were nonzero with 8 of these nonzero parameters having t statistics greater than 2. The month by month values of the parameters α_S^t α_M^t α_L^t and β^t defined by (6)-(9) are reported in Table 8 below.

Table 8: Marginal Prices of Land for Small, Medium and Large Plots and New Construction Prices by Month

Month	α_S^t	α_M^t	α_L^t	β^t
1	0.99767	3.24338	0.29878	5.56322
2	0.99767	3.24338	0.29878	5.70008
3	0.99767	3.24338	0.29878	5.70008
4	0.99767	3.24338	0.29878	5.70008
5	1.13331	3.24338	0.29878	5.70008
6	1.13331	3.24338	0.29878	5.70008
7	1.19700	3.24338	0.29878	5.70008
8	1.19700	3.24338	0.29878	5.70008
9	1.19700	3.24338	0.29878	5.70008
10	1.19700	3.24338	0.29878	5.70008
11	1.19700	3.24338	0.29878	6.16317
12	1.19700	3.24338	0.29878	6.16317
13	1.19700	3.24338	0.29878	6.16317
14	1.19700	3.24338	0.29878	6.16317
15	1.39826	3.24338	0.29878	6.16317

16	1.39826	3.24338	0.29878	6.16317
17	1.39826	3.24338	0.29878	6.16317
18	1.81248	3.59946	0.98436	6.16317
19	1.81248	3.59946	0.98436	6.16317
20	1.94070	3.59946	0.98436	6.16317
21	1.94070	3.59946	0.98436	6.16317
22	1.94070	3.59946	0.98436	6.16317
23	1.94070	3.59946	0.98436	6.16317
24	2.04119	3.59946	0.98436	6.16317
25	2.04119	3.59946	0.98436	6.16317
26	2.07040	3.59946	1.51651	6.16317
27	2.11420	3.59946	1.51651	6.16317
28	2.11420	3.59946	1.65081	6.16317
29	2.11420	3.59946	1.65081	6.16317
30	2.25443	3.81381	3.71600	6.16317

Comparing the entries in Table 8 with the corresponding quarterly entries in Table 4, it can be seen that the monthly results agree fairly well with the quarterly results with the exception of the sudden surge in the marginal price for large lots in month 30 of Table 8 from 1.65 in month 29 to 3.72 in month 30 (the corresponding surge from quarter 9 to 10 of the marginal price of large lots recorded in Table 4 was 1.06 to 1.83). This discrepancy could be due to the fact that the fraction of large lots sold is rather small and so the estimate of the marginal price of large lots is particularly uncertain. Another possible explanation for the large surge in the marginal price for large lots in both the quarterly and monthly models is the fact that nonparametric time series models tend to be unreliable at the endpoints of the sample period because there is a tendency for the model to fit the errors at the endpoints. Our model is very close to being a nonparametric time series model since it has many free parameters for each time period and thus, it may be subject to this type of bias.[18]

As in the previous section, it is possible to work out the total imputed value of structures transacted in each month, V_S^t, and divide this monthly value by the total quantity of structures (converted into equivalent new structure units), Q_S^t, in order to obtain an average price of structures, P_S^t. Similarly, we can add up all of the imputed values for small, medium and large plot sizes for each month t, say V_{LS}^t, V_{LM}^t and V_{LL}^t, and then add up the total quantity of land transacted in each of the three classes of property, say Q_{LS}^t, Q_{LM}^t and Q_{LL}^t. Finally, we can form monthly unit value prices for each of the three classes of property, P_{LS}^t, P_{LM}^t and P_{LL}^t, by dividing each value series by the corresponding quantity series. The resulting (average) price and quantity series are listed in Table 9 below.

Table 9: Average Prices for New Structures, Small, Medium and Large Plots and Total Quantities Transacted per Month of Structures and the Three Types of Plot Size

[18] This hypothesis could be checked by adding some additional months of data to the original sample.

Month	P_S^t	P_{LS}^t	P_{LM}^t	P_{LL}^t	Q_S^t	Q_{LS}^t	Q_{LM}^t	Q_{LL}^t
1	5.56322	0.99767	1.60698	1.30112	58.4	54.8	46.7	21.8
2	5.70008	0.99767	1.47684	0.91554	48.0	44.7	40.7	23.6
3	5.70008	0.99767	1.61759	1.38761	68.3	57.5	63.5	26.8
4	5.70008	0.99767	1.64109	1.64760	51.5	40.0	44.8	21.6
5	5.70008	1.13331	1.79016	1.57229	61.1	56.1	43.6	35.6
6	5.70008	1.13331	1.81513	1.46894	66.0	45.7	62.0	77.5
7	5.70008	1.19700	1.70577	1.38617	47.0	31.0	45.3	35.3
8	5.70008	1.19700	1.60423	1.81919	39.7	29.9	40.0	10.1
9	5.70008	1.19700	1.74733	1.54141	27.7	25.6	19.1	12.4
10	5.70008	1.19700	1.70836	1.01640	37.0	31.7	42.7	10.7
11	6.16317	1.19700	1.77644	1.23196	46.1	34.3	39.1	41.2
12	6.16317	1.19700	1.68211	1.69702	45.9	35.8	40.7	18.3
13	6.16317	1.19700	1.7993	0.79731	52.4	38.0	68.0	46.3
14	6.16317	1.19700	1.73924	1.69010	45.0	39.3	27.2	27.6
15	6.16317	1.39826	1.89336	1.56091	62.4	34.3	71.1	38.5
16	6.16317	1.39826	1.97569	1.21322	42.3	28.5	58.2	17.7
17	6.16317	1.39826	1.88742	1.33043	59.2	36.6	54.4	70.6
18	6.16317	1.81248	2.26538	1.97975	62.3	34.2	77.7	41.5
19	6.16317	1.81248	2.33078	2.13814	48.1	34.5	47.9	41.8
20	6.16317	1.94070	2.44863	1.98785	38.1	34.2	31.7	35.6
21	6.16317	1.94070	2.38913	2.05146	51.7	34.9	54.8	53.6
22	6.16317	1.94070	2.33266	1.72763	35.4	27.5	44.5	19.2
23	6.16317	1.94070	2.41604	1.88852	47.7	34.7	42.1	47.4
24	6.16317	2.04119	2.51182	2.01176	54.5	27.3	68.8	35.7
25	6.16317	2.04119	2.40728	1.94534	41.9	30.1	31.4	38.2
26	6.16317	2.07040	2.51694	2.12495	54.4	37.6	62.2	34.7
27	6.16317	2.11420	2.47966	2.43401	56.8	46.7	58.4	17.5
28	6.16317	2.11420	2.49731	2.43394	66.0	56.1	67.4	18.5
29	6.16317	2.11420	2.51566	2.29851	56.8	31.8	65.8	37.2
30	6.16317	2.25443	2.76893	3.18049	57.1	35.5	74.6	15.3

Comparing the monthly prices in Table 9 with their quarterly counterparts in Table 5, it can be seen that the prices of structures and the (average) prices of small lots are very similar in the two tables. However, there are some substantial differences between the quarterly and monthly average prices of medium and large lots. Moreover, in both tables, it can be seen that there are some fluctuations in the average prices of medium and large lots, with the fluctuations being quite substantial in the case of monthly prices. These fluctuations are due to the smaller sample sizes in the monthly model compared to the quarterly model and to the nature of our spline model for the cost of land. The marginal price of land for an extra unit of land for a medium lot is greater than the marginal price for an extra unit of land for a small lot. Thus if the average size of a medium lot increases going from one period to the next, then the average price for medium lots will increase. Similarly, the marginal price of land for large lots is less than the marginal price for medium lots. Thus if the average size of a large lot increases going from one period to the next, then the average price for large lots will decrease. Since monthly sample sizes can be small for medium and large lots, substantial fluctuations in the average size of lots sold in each month within these two categories of lot size will lead to substantial fluctuations in the average prices for these two types of lot.[19] This type of fluctuation can be controlled by making the lot size ranges smaller so that divergences between marginal and average prices within each lot size category would be reduced.[20] Another method for controlling these spline model induced fluctuations would be to drop the spline model for the price of land and simply have one price of land for all lot sizes. However, we are reluctant to do this since our results for the Dutch city "A" indicate that the price levels and trends for the different sized lots differed substantially.

A final method for controlling spline model induced fluctuations in the price of land would be to value the entire *stock* of detached houses in the city using our model. Since the stock of houses changes very little from month to month, this would eliminate large fluctuations in the average amount of land for medium and large lots.[21]

We note that our model could serve many purposes. As indicated in the above paragraph, the model could be used to provide up to date valuations for the entire stock of detached houses in the city, provided that we had information on the age, land area and floor space area for each house in the city. The model could also be used to value new additions to the city's housing stock provided that information on the age, land area and floor space area for each newly constructed house in the city was available.[22]

The data listed in Table 9 were further aggregated. We constructed a chained Fisher aggregate for the three land series and the resulting aggregate land price and quantity series, P_L^t and Q_L^t, are listed in Table 10 below along with the structures price and quantity series (normalized so that the price equals 1 in quarter 1), P_S^t and Q_S^t. Finally, a chained Fisher aggregate for structures and the three land series were constructed and the resulting aggregate price and quantity series, P^t and Q^t, are also listed in Table 10.

Table 10: Aggregate Monthly Price and Quantity Series for Housing

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[19] Analogous fluctuations for small lots (and for structures) cannot occur because for these commodities, average and marginal prices coincide.

[20] A possible problem with this strategy is that the sample sizes within each category of lot would decline and become zero in some cases. However, this is not necessarily a problem since our spline model does not really require that the sample size within each lot size category be nonzero; i.e., our spline model shifts the entire schedule of lot size costs up (or down if we entered the squared terms in equations (6)-(8) into the model with negative signs instead of positive signs) and we do not require actual transactions in a given period for all possible lot sizes. Thus the main cost of increasing the number of spline segments appears to be the fact that a large number of additional parameters would have to be estimated.

[21] This is our preferred method for controlling price fluctuations due to the changing composition of the houses sold from period to period. However, this method requires information on the total stock of housing for the neighbourhood under consideration. Alternatively, one could simply use the characteristics of a "representative" dwelling unit for the neighbourhood.

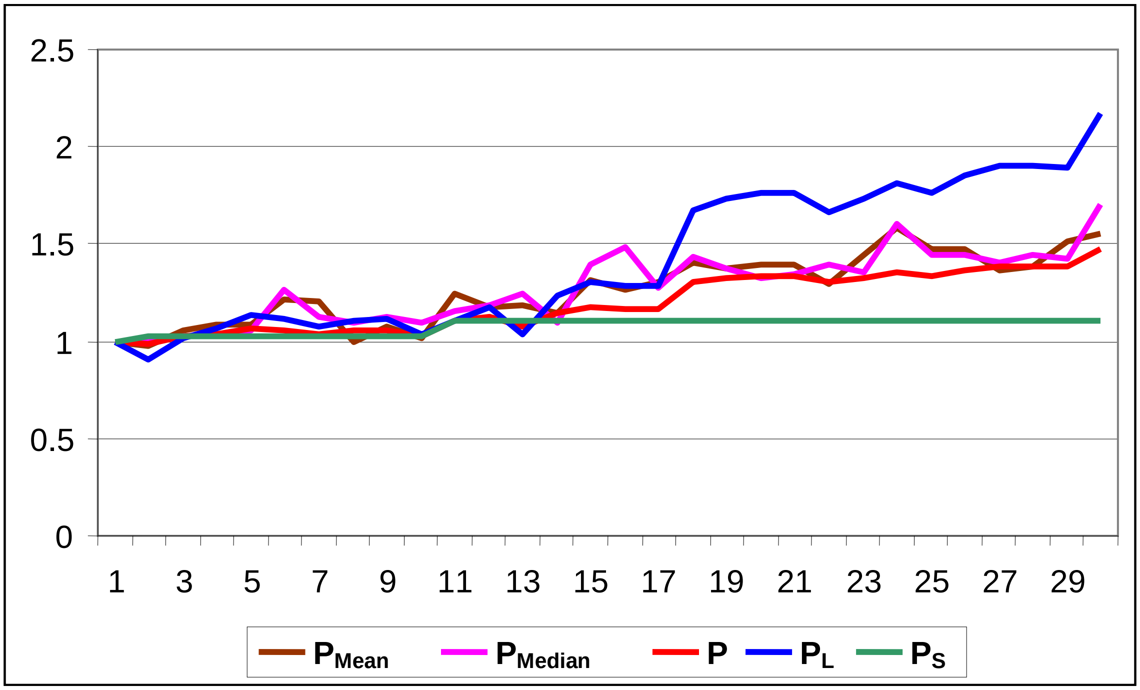
[22] If the country uses the acquisitions approach to the treatment of housing in a Consumer Price Index where only the price of the new structure is to enter the index, then it can be seen that our suggested model could be very useful in this context. For a review of alternative ways of treating housing in a CPI, see Diewert (2002; 611-121) (2007).

Month	P^t	P_L^t	P_S^t	Q^t	Q_L^t	Q_S^t
1	1.00000	1.00000	1.00000	483.1	158.0	325.1
2	0.98367	0.90296	1.02460	406.3	139.8	266.9
3	1.02322	1.01875	1.02460	573.4	193.7	380.1
4	1.03842	1.06444	1.02460	426.0	140.0	286.3
5	1.06182	1.13450	1.02460	513.8	174.1	339.7
6	1.05410	1.11416	1.02460	620.7	249.7	367.0
7	1.03870	1.07414	1.02460	415.0	152.0	261.3
8	1.05201	1.11031	1.02460	327.3	106.5	220.7
9	1.05492	1.11921	1.02460	228.6	74.3	154.2
10	1.02777	1.03874	1.02460	323.5	117.2	205.7
11	1.10324	1.10087	1.10784	403.6	146.4	256.4
12	1.12844	1.17182	1.10784	377.0	121.4	255.5
13	1.07897	1.04048	1.10784	488.8	196.7	291.3
14	1.14840	1.23650	1.10784	364.5	114.1	250.6
15	1.17216	1.30861	1.10784	535.0	185.3	347.2
16	1.16331	1.28324	1.10784	375.9	137.4	235.6
17	1.16251	1.28103	1.10784	527.1	193.4	329.5
18	1.30446	1.67238	1.10784	539.6	191.4	346.4
19	1.32683	1.73494	1.10784	421.9	151.9	267.4
20	1.33544	1.75877	1.10784	336.6	122.1	211.9
21	1.33615	1.76073	1.10784	469.3	175.2	287.5
22	1.30038	1.66369	1.10784	314.5	114.5	197.2
23	1.32457	1.73082	1.10784	417.0	149.4	265.2
24	1.35520	1.81700	1.10784	469.5	165.3	303.2
25	1.33672	1.76372	1.10784	351.2	119.8	233.1
26	1.36767	1.85284	1.10784	470.2	166.3	302.4
27	1.38407	1.90045	1.10784	459.6	150.5	316.0
28	1.38631	1.90728	1.10784	532.8	173.9	367.2
29	1.38116	1.89227	1.10784	483.8	168.2	315.8
30	1.47653	2.17090	1.10784	465.6	154.4	317.9

Comparing the monthly price series in Table 10 with the corresponding quarterly price series in Table 6, it can be seen that they are reasonably close with the exception of the price of housing in the last three months (1.38631, 1.38116 and 1.47653) and the price of land in the last three months of our sample period (1.90728, 1.89227 and 2.17090). The corresponding prices of housing and land for the last quarter in the quarterly model were 1.36569 and 1.90086, which are well below the unweighted average of the corresponding 3 monthly prices. However, as we will see in Table 11 below, when we take appropriately weighted averages of the monthly data, these discrepancies are greatly reduced.

Chart 4 below plots the monthly aggregate house price series P^t, the land price series P_L^t and the structures price series P_S^t from Table 10 above along with the monthly mean price series P_{Mean}^t and median series P_{Median}^t.

Chart 4: Monthly Mean Price P_{Mean}^{t}, Median Price P_{Median}^{t}, Constant Quality Housing Price P^{t}, Land Price P_{L}^{t} and New Structures Price P_{S}^{t}



From Chart 4, it is evident that our estimated constant quality price of housing for City A grew more slowly than the corresponding mean and median series. As was the case with the quarterly Chart 3, the major explanatory factor for this difference is due to the fact that the average age of the structure in the sample tended to fall as time marched on.

It is of interest to take the *monthly* data from Table 9 and aggregate these data into *quarterly* unit value prices and the corresponding *quarterly* quantities. This was done, generating three aggregated quarterly land price and quantity series and the aggregated quarterly structures price series, AP_{S}^{t}. These three aggregated land price series were then aggregated into an overall aggregated quarterly price series AP_{L}^{t} using chained Fisher aggregation. Finally, the three aggregated land price series and the aggregated constant quality structures series AP_{S}^{t} were aggregated into an overall aggregated quarterly housing price index, AP^{t}, which is listed in Table 11 below along with AP_{L}^{t} and AP_{S}^{t}. For comparison purposes, the corresponding quarterly price series for housing, land and structures, P^{t}, P_{L}^{t} and P_{S}^{t}, from Table 6 in section 4 (i.e., the estimates from the original quarterly regression model) are also listed in Table 11.

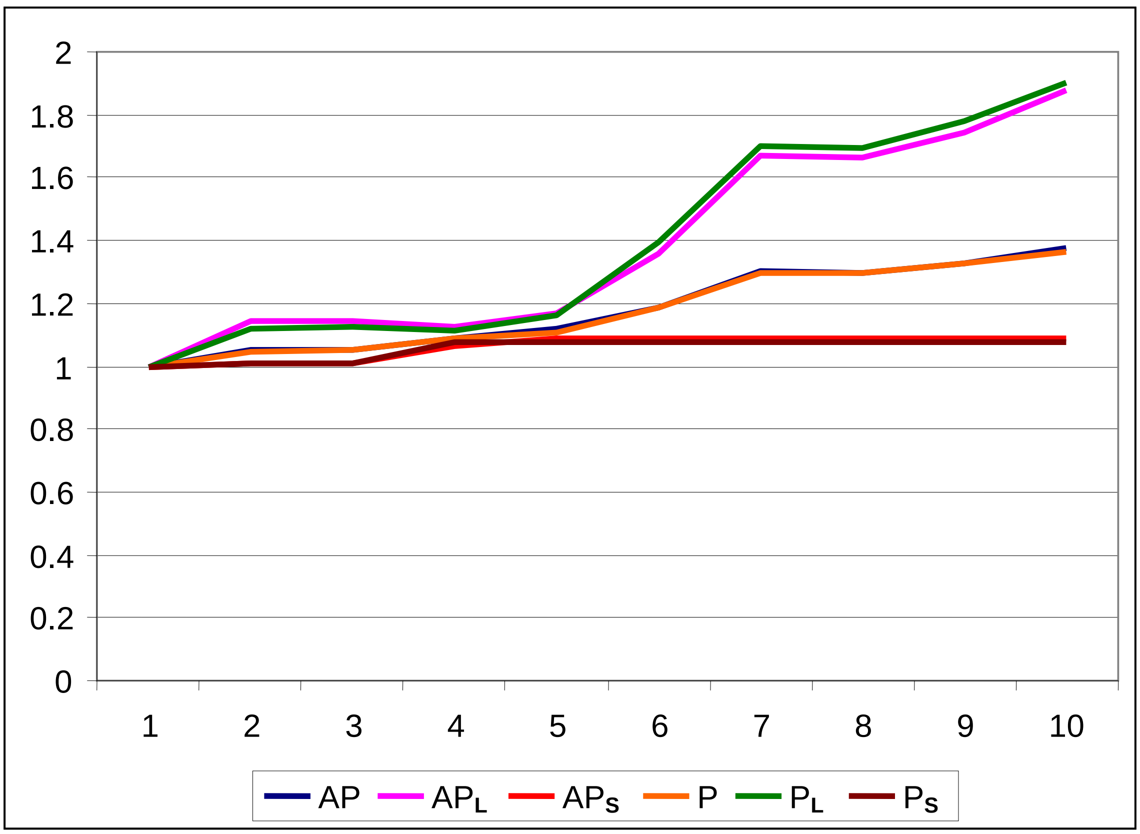
Table 11: Quarterly Price Series for Housing P^{t}, Land P_{L}^{t} and for Structures P_{S}^{t} and Aggregated Quarterly Price Series for Housing AP^{t}, Land AP_{L}^{t} and for Structures AP_{S}^{t} from the Monthly Model

Quarter	AP^{t}	AP_{L}^{t}	AP_{S}^{t}	P^{t}	P_{L}^{t}	P_{S}^{t}
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

2	1.05456	1.14511	1.00810	1.04712	1.11966	1.01173
3	1.05324	1.14122	1.00810	1.04958	1.12710	1.01173
4	1.08638	1.12440	1.06652	1.08823	1.11402	1.07530
5	1.11630	1.16636	1.09000	1.10535	1.16452	1.07530
6	1.18687	1.36005	1.09000	1.18642	1.39355	1.07530
7	1.30256	1.67089	1.09000	1.29833	1.70333	1.07530
8	1.29930	1.66214	1.09000	1.29433	1.69226	1.07530
9	1.32929	1.74573	1.09000	1.32432	1.77854	1.07530
10	1.37454	1.87564	1.09000	1.36570	1.90086	1.07530

The above series are graphed in Chart 5 below.

Chart 5: Quarterly Constant Quality Housing Price P^t, Land Price $P_L{}^t$ and New Structures Price $P_S{}^t$ and the Corresponding Quarterly Aggregates Generated by the Monthly Model, AP^t, $AP_L{}^t$ and $AP_S{}^t$



It can be seen that the original quarterly overall house price index series, P^t, coincides so closely with the corresponding aggregated series from the monthly model, AP^t, that the two series can barely be distinguished from each other in Chart 5. Similarly the original quarterly constant quality structures price index, $P_S{}^t$, can barely be distinguished from its aggregated counterpart from the monthly model, $AP_S{}^t$. Finally, the original quarterly series for land, $P_L{}^t$, lies slightly below its monthly aggregated counterpart, $AP_L{}^t$, for the first half of the sample period and slightly above for the second half. Our conclusion is that the monthly and quarterly hedonic regression models are in fairly close agreement with each other. Both models seem to give sensible results.

It can be seen that the logic behind our functional form assumptions for our hedonic regression model come from the supplier perspective; i.e., we justified our model from the perspective of a builder who buys a lot at a given price per squared meter and then builds a structure on the lot at another price per squared meter of floor space of the structure. But it is important to justify a hedonic regression model from a consumer or purchaser perspective as well[23] and in the following section, we explore such an approach.

6. Hedonic Regressions for Housing from a Consumer Perspective

A very simple way to justify a hedonic regression model from a consumer perspective is to postulate that households have the same (cardinal) utility function, $f(z_1,z_2)$, that aggregates the amounts of two relevant characteristics, $z_1 > 0$ and $z_2 > 0$, into the overall utility of the "model" with characteristics z_1, z_2 into the scalar welfare measure, $f(z_1,z_2)$. Thus households will prefer model 1 with characteristics z_1^1,z_2^1 to model 2 with characteristics z_1^2,z_2^2 if and only if $f(z_1^1,z_2^1) > f(z_1^2,z_2^2)$.[24] Thus having more of every characteristic is always preferred by households. The next assumption that we make is that in period t, there is a positive generic price for all models, ρ^t, such that the household's *willingness to pay*, $W^t(z_1,z_2)$, for a model with characteristics z_1 and z_2 is equal to the generic model price ρ^t times the utility generated by the model, $f(z_1,z_2)$; i.e., we have for each model n with characteristics z_{1n}^t,z_{2n}^t that is purchased in period t, the following *willingness to pay* for model n:[25]

$$(11)\ W^t(z_{1n}^t,z_{2n}^t) = \rho^t f(z_{1n}^t,z_{2n}^t).$$

In order to relate the above model to sales in the Dutch city of "A", identify the first characteristic with the size of the land area of the house n sold in period t, L_n^t, and the second characteristic with the quality adjusted (for the age of the structure) size of the structure, AS_n^t, so that

$$(12)\ AS_n^t \equiv (1 - \delta A)S_n^t$$

where S_n^t is the *unadjusted size of the structure*, δ is the *depreciation rate* for structures and A is the *age of the structure*. Finally, set the willingness to pay for the housing unit,

[23] Purchaser preferences for properties are perhaps more important than producer costs of production since a property will not be purchased unless the utility of the property to the buyer is equal to or greater than its cost.

[24] It is natural to impose some regularity conditions on the characteristics aggregator function f like continuity, monotonicity (if each component of the vector z^1 is strictly greater than the corresponding component of z^2, then $f(z^1) > f(z^2)$ and $f(0,0) = 0$.

[25] For more elaborate justifications for household based hedonic regression models, see Muellbauer (1974) and Diewert (2003).

$W^t(L_n{}^t,(1 - \delta A)S_n{}^t)$, equal to the *selling price of the property*, $v_n{}^t$ and we have the following hedonic regression model:

$$(13)\ v_n{}^t = \rho^t f(L_n{}^t,(1 - \delta A)S_n{}^t).$$

There remain the problems of choosing a stochastic specification for the hedonic regression model (12) and of choosing a functional form for the hedonic utility function f. The simplest choices for f(L,AS) are that (i) f is a *linear function* of L and AS or (ii) f has a *Cobb-Douglas* functional form. These two choices lead to the following hedonic regression models after adding mean zero error terms $\eta_n{}^t$ to each choice:[26]

$$(14)\ v_n{}^t = \rho^t(\alpha L_n{}^t + \beta(1 - \delta A_n{}^t)S_n{}^t) + \eta_n{}^t\ ; \qquad n = 1,...,N(t);\ t = 1,...,T;$$
$$(15)\ \ln v_n{}^t = \ln\rho^t + \alpha + \beta \ln L_n{}^t + \gamma \ln[(1 - \delta A_n{}^t)S_n{}^t] + \eta_n{}^t\ ; \qquad n = 1,...,N(t);\ t = 1,...,T.$$

In order to identify all of the parameters, we require a normalization on the hedonic prices ρ^t. It is natural to set ρ^t equal to one in the first period:

$$(16)\ \rho^1 = 1.$$

It can be seen that the hedonic regression model defined by (14) and (16) is essentially a reparameterization of our first simple regression model explained in section 2 above (with some additional restrictions on the parameters). However, the Cobb-Douglas model defined by (15) and (16) is a new model and we will use our 10 quarters of data in order to estimate the nine time dummy parameters, ρ^2, ρ^3, ... , ρ^{10}, and the 4 remaining parameters, α, β, γ and δ. This model is essentially a standard log-log time dummy hedonic regression model.[27]

The results of the above reparameterized model were as follows: the quarter 1 estimated parameters were $\alpha = 1.7662$ (0.016564), $\beta = 0.49941$ (0.011127), $\gamma = 0.50163$ (0.024201) and $\delta = 0.12609$ (0.0072004) (standard errors in brackets) with an R^2 of 0.8244.[28] The parameters $\ln\rho^2$, $\ln\rho^3$,..., $\ln\rho^{10}$ were 0.0376, 0.0265, –0.0034, 0.0496, 0.0669, 0.0554, 0.0217, 0.0161 and 0.0314 respectively and the standard errors for all of these time dummy variables was very close to 0.02. Thus the estimate for the quarter 4 time dummy, $\ln\rho^4$, turned out to be negative but it was not significantly negative since the t statistic was only –0.15.

[26] Note that the linear f(L,AS) that is defined in (14) is linearly homogeneous in the variables L and AS. The Cobb-Douglas f that is defined in (15) will be linearly homogeneous if $\beta + \gamma = 1$.
[27] The only unusual feature of this model is the nonlinearity that arises from the use of quality adjusted structures as a characteristic rather than the use of unadjusted structures and age as explanatory variables.
[28] Recall that in the quarterly model estimated in section, the R^2 was .8383 and the estimated depreciation rate was 0.14763 which is a bit higher than our present estimated decade depreciation rate of 0.12609

The most interesting feature of our quarterly log-log regression is that our estimated Cobb-Douglas hedonic aggregator function exhibited virtually *constant returns to scale* in the two characteristics; i.e., our parameter estimates for β and γ summed to 1.00104.[29]

Our estimated time dummy variables were exponentiated and are reported as the ρ^t series in Table 12 below.[30] These estimated overall house price indexes can be compared with our earlier estimates listed in Table 6 above; see the series P^t listed there. It can be seen that the correspondence between ρ^t and our earlier price series for housing P^t is fairly close; see Chart 6 below.

We now encounter a problem with the log-log hedonic regression model as compared to the linear hedonic regression model explained in section 4 above: the linear model generated separate estimates for the price of land and for the price of quality adjusted structures whereas the present model does not seem to be able to generate these separate estimates for the price of land and structures. However, it is possible to use the log-log model (or any other hedonic model based on a hedonic utility function $f(z_1,z_2)$) in order to generate *imputed estimates* for the price of land, ρ_L^t, and for quality adjusted structures, ρ_S^t. The basic idea is to take the consumer's period t willingness to pay function, $W^t(z_1,z_2)$, and differentiate it with respect to z_1 and z_2. These two partial derivatives will give us estimates of the consumer's increase in well being in period t, valued at the period t price for the hedonic aggregate, due to a marginal increase in the quantities of z_1 and z_2; i.e., we will have imputed prices for extra units of z_1 and z_2 in period t. Thus we define ρ_L^t and ρ_S^t as follows:

(17) $\rho_L^t \equiv \partial W^t(z_1^{t*},z_2^{t*})/\partial z_1 = \rho^t \partial f(z_1^{t*},z_2^{t*})/\partial z_1$; $\quad t = 1,...,T$;
(18) $\rho_S^t \equiv \partial W^t(z_1^{t*},z_2^{t*})/\partial z_2 = \rho^t \partial f(z_1^{t*},z_2^{t*})/\partial z_2$; $\quad t = 1,...,T$

where z_1^{t*} and z_2^{t*} are the average amounts of land and quality adjusted structures for the properties sold in period t; see the last two columns of Table 12 for a listing of these average quantities for our sample of 10 quarters of data. Note that the average amount of land series is more volatile than the average quantity of quality adjusted structures series. We use (17) and (18) to generate imputed price series for land and quality adjusted structures, using our estimated coefficients for α, β and γ in order to form an estimated $f(z_1,z_2)$ function.[31] The average utility or quantity of housing in each quarter generated by our log-log model, $u^{t*} \equiv f(z_1^{t*},z_2^{t*})$, is also listed in Table 12.

[29] We also estimated the log-linear variant of a hedonic regression; i.e., our estimating equation for this model was $\ln v_n^t = \rho^t(\alpha L_n^t + \beta(1 - \delta A_n^t)S_n^t) + \eta_n^t$. Our Dutch data did not support this model at all; the final log likelihood for this model was only 226.3350 as compared to the final log likelihood for the log-log model of 587.2707. The R^2 for the log-linear model was only 0.7064 as compared to the R^2 of 0.8244 for the log-log model. Both the log-linear and the log-log model have the same dependent variables so their log likelihoods and R^2 can be compared.
[30] We set $\rho^1 = 1$.
[31] We also use our estimated δ coefficient in order to form quality adjusted structures for each observation in our sample.

Table 12: Quarterly Price Series for Housing ρ^t, Land ρ_L^t and for Structures ρ_S^t and Average Quantity of Housing u^{t*}, Land z_1^{t*} and Adjusted Structures z_2^{t*}

Quarter	ρ^t	ρ_L^t	ρ_S^t	u^{t*}	z_1^{t*}	z_2^{t*}
1	1.00000	0.59342	1.31578	2.73702	2.30339	1.04346
2	1.03838	0.58707	1.43397	2.94696	2.60311	1.07046
3	1.06632	0.63251	1.40368	2.78595	2.34557	1.06163
4	1.06271	0.61942	1.42368	2.89453	2.48009	1.08384
5	1.11679	0.62730	1.55238	3.02638	2.69076	1.09214
6	1.19406	0.65592	1.69707	3.07518	2.79573	1.08537
7	1.26213	0.67971	1.82945	2.96912	2.75336	1.02753
8	1.28984	0.71421	1.81872	3.08031	2.77816	1.09584
9	1.31074	0.75710	1.77193	2.96772	2.56590	1.10122
10	1.35251	0.79026	1.80755	2.92238	2.49783	1.09691

The land and quality adjusted structures prices generated by the log-log model were normalized to equal unity in quarter one and these normalized series are listed as ρ_L^t and ρ_S^t in Table 13 below. These series can be compared to the price of land P_L^t and quality adjusted structures P_S^t that were generated by our quarterly linear model listed in Table 6 of section 4 above, which are also listed in Table 13.

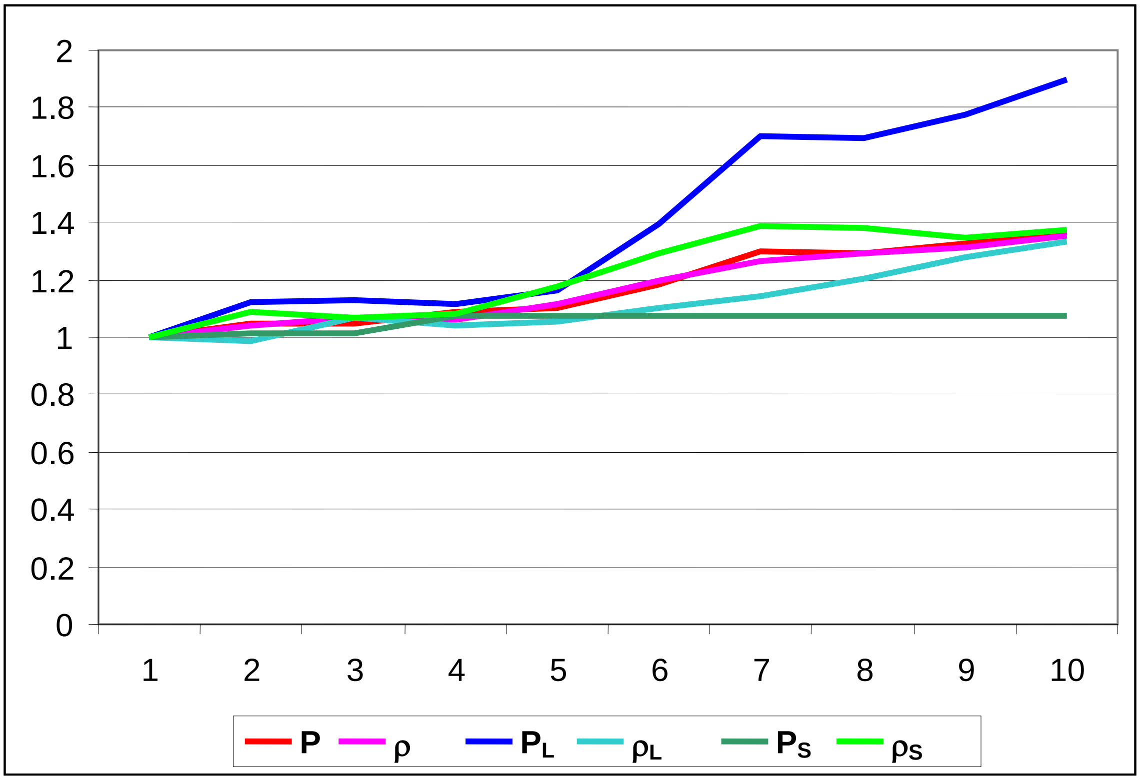
Table 13: Quarterly Price Series for Housing P^t and Imputed Prices for Land P_L^t and for Constant Quality Structures P_S^t from the Linear Hedonic Model and their Counterparts, ρ^t, ρ_L^t and ρ_S^t from the Log-Log Model

Quarter	P^t	ρ^t	P_L^t	ρ_L^t	P_S^t	ρ_S^t
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.04712	1.03838	1.11966	0.98930	1.01173	1.08982
3	1.04959	1.06632	1.12710	1.06587	1.01173	1.06681
4	1.08823	1.06271	1.11403	1.04380	1.07530	1.08201
5	1.10535	1.11679	1.16452	1.05708	1.07530	1.17982
6	1.18642	1.19406	1.39356	1.10532	1.07530	1.28978
7	1.29832	1.26213	1.70330	1.14541	1.07530	1.39039
8	1.29433	1.28984	1.69226	1.20355	1.07530	1.38223
9	1.32431	1.31074	1.77854	1.27582	1.07530	1.34668
10	1.36569	1.35251	1.90086	1.33170	1.07530	1.37375

The series listed in Table 13 are graphed in Chart 6.

Looking at Table 13 and Chart 6, it can be seen that our overall estimates of house price inflation from the linear hedonic model, P^t, and from the log-log hedonic model, ρ^t, are very close to each other. However, the two hedonic models produce very different estimates of land and structures inflation: the estimates of land price inflation from the linear model, P_L^t, are well above the corresponding log-log estimates, ρ_L^t, whereas the estimates of structures price inflation from the linear model, P_S^t, are well below the corresponding log-log estimates, $\rho_S{}^t$. The question naturally arises: which set of estimates is closer to the "truth"?

Chart 6: Quarterly Price Series for Housing P^t and Imputed Prices for Land $P_L{}^t$ and for Constant Quality Structures $P_S{}^t$ and their Counterparts, ρ^t, $\rho_L{}^t$ and $\rho_S{}^t$



We believe that the estimates from the linear model are more credible. Evidently, there was a bit of a house price "bubble" in the Netherlands during these 10 quarters. The log-log model attributes more than half of the bubble to increases in the price of structures whereas the linear model attributes most of the bubble to increases in the price of land. A look at construction prices in the Netherlands shows that construction prices did not increase dramatically during these 10 quarters starting at the first quarter of 1998.[32] Thus the linear model is more consistent with the actual pattern of construction prices and the price of raw land during this period. One could argue that this is irrelevant: what counts are household, or more generally, purchasers valuations of the characteristics and these valuations do not have to coincide with market prices for units of the characteristics purchased separately. However, a situation where a purchaser's valuation of an extra unit of land is well below the market price of land and where the valuation of an extra unit of structure is well above the market price of building that extra unit should not persist

[32] The Statistics Netherlands (national) Construction Price Index for new dwellings for the same period took on the following values (with Q1 in 1998 normalized to equal unity): 100, 100.4, 100.4, 100.8, 101.0, 101.7, 102.7, 103.1, 104.5,105.1.

indefinitely: there will be a tendency for purchasers to buy houses with more floor space and less land in order to move their marginal willingness to pay for land and structures closer to the corresponding market prices for land and structures.

7. Conclusion

Our tentative conclusion at this point is that hedonic regression techniques can be used in order to decompose the selling prices of properties into their land and structure components but it is not a completely straightforward exercise. In particular, monotonicity restrictions on the parameters will generally have to be imposed on the model in order to obtain sensible results[33] We found that our model worked fairly well on monthly data as well as on quarterly data. Our results also indicate that stable coefficients cannot be obtained using just data for one quarter. An open question is: how many quarters (or months) of data do we need to run in the one big nonlinear regression in order to obtain stable imputed prices for land and structures?

Here is a list of topics where further research is required:

- Can we adapt our method into a rolling year method; i.e., we use only the data for a full year plus one additional time period and use the results to update our previous series?[34]
- We did not eliminate any outliers in our preliminary research. Do we get similar results if outliers are eliminated?[35]
- Is it worthwhile to consider more characteristics?
- How does our suggested method compare to the repeat sales method[36] (using the same data set)?
- Can our method be generalized to deal with the sales of condominiums and duplexes?[37]

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[33] In our data set, it was reasonable to assume that prices never declined. However, at times, real estate prices do decline and thus when it is suspected that a decline is taking place at a certain time period, the algebra associated with equations (6)-(9) must be suitably modified.
[34] This rolling year and updating methodology has been investigated in the context of scanner data and it seems likely that it would work in the present context as well; see Ivancic, Diewert and Fox (2009) and de Haan and van der Grient (2009).
[35] About 15-20% of our observations could be classified as outliers; i.e., the predicted sale price differs from the actual sale price by more than 20,000 Euros. We did run a quarterly regression that eliminated outliers and obtained similar results to our results in section 4.
[36] The repeat sales method is due to Bailey, Muth and Nourse (1963), who saw their procedure as a generalization of the matched model methodology that was used by the early pioneers in the construction of real estate price indexes like Wyngarden (1927) and Wenzlick (1952). Case and Shiller (1989) further modified the repeat sales method.
[37] In the case of condominium sales, there are some subtle problems associated with the allocation of the common land area of the structure to the individual units in the apartment block.

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