Toward Modelling Poor Cities: A Review of Urban Economic and Planning Models

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As the cities in developing countries and the attendant problems have been growing rapidly, interest in urban analysis has also increased markedly. This paper provides a survey of the main varieties of urban models which have been developed so far, with a view to finding approaches useful for improving our understanding of cities in LDC's. In the process it reflects upon the complexity of urban phenomena which intrinsically cause difficulties in formulating urban economic models. This review should prove of particular interest to several groups of readers, including: (1) researchers experienced in other fields who want to become familiar with the context of urban research; (2) urban researchers more familiar with other aspects of the urban field who want to know more about the state of formal urban economic modelling; and (3) urban policy analysts who wish to evaluate the potential effectiveness of recent urban modelling efforts in assisting policy formulation. A summary of this relatively lengthy paper on a generally unfamiliar subject is provided under cover.

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PREFACE

This paper surveys the main varieties of urban models with a view to finding approaches that can be useful for understanding cities in less developed countries. Some comments are also offered on the complexity of urban phenomena which intrinsically cause difficulties in formulating urban economic models.

Notation is always a problem when comparing a large number of models. Here we have tried to maintain a consistent notation so that the same letter stands for similar variables in different models. One has to say "similar" since each model has its own variation in the definition of essentially the same variable. The notation is explained as it is introduced, and every effort is made to maintain reasonable comparability with the original articles.

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SUMMARY

This paper explores the possibilities of applying current urban modelling techniques to cities in less developed countries. Some problems intrinsic to the modelling of urban areas are discussed first. Uncertainty about the present, unforeseen changes in the future, indivisibilities and economies of scale are posited as the reasons for the existence of cities, and these are the phenomena which are most difficult to codify and to include in models. The peculiarities of cities in LDCs are then described to explain why modelling their activities is an even more complex task than modelling Western cities. These peculiarities stem from unprecedentedly high population growth rates in LDC cities, the co-existence of high and low levels of technology, along with predominantly low incomes and the decline in relative prices of transport and communications as compared with a century ago. Before a review of models is begun the criteria for model evaluation are set out.

Urban models are divided into two classes: analytic models and operational or policy oriented models. The former are mainly based on economic theory and are usually aggregate type models attempting to offer basic insights into urban form. Almost all of these models investigate optimal residential location -- either from the point of view of the household itself or for maximization of social welfare as expressed in a welfare function. Particular attention is paid to the land/transport trade-off; the uniqueness of location; the effects of transport congestion on city form; and the consequences for egalitarian welfare functions. Though these models are at a sufficiently theoretical plane, they should be regarded as conceptual building blocks towards more realistic models.

Operational models characteristically require the use of a computer for their solution. There are two basic strands in such urban modelling: the social physics variety now based on entropy maximization techniques and the behavioral variety mainly drawing on the economic analytic models. The social physics variety of models is introduced with an explanation of the increasingly used entropy maximization techniques. The objective of these models is to distribute activities spatially in a city. A typical output of a model would be allocation of residential and employment location by zones in the city -- often disaggregated by socio-economic types of households, types of residential structures, types of employment, etc. These models reproduce current city structures fairly accurately, after careful calibration. However, without behavioral underpinnings, their methodological basis raises doubts as to their usefulness as predictive or planning tools, given the rapidly changing conditions of LDC cities. The behavioral models reviewed have many pleasingly realistic features; however, they are to unwieldy to operate, are highly data intensive and do not easily permit evaluation of alternative courses of action within a realistic decision-making context. Thus many of the operational models tried so far are found to be of limited practical use in LDC cities.

Some productive approaches to urban modelling in LDC cities are suggested on the basis of three models: each quite distinct in approach. The first is the multi-level approach to modelling of the Master Planning Commission of Stockholm. Their model can be operated at various levels of aggregation and is particularly suited for dialogue with policy-makers. Moreover, it is an evaluative model. The second is the Urban Institute

Housing Model which models the behavior of households, owners, a building industry and government by positing 'model' representatives of each group. Thus the model is small in size and flexible in data requirements, though mathematically complex. It is particularly well suited for modelling the direct and indirect effects of different governmental policies. Finally, Apps' model of housing demand at a disaggregated level is presented as an example of a useful explanatory model to explore behavioral relationships and parameters which can then be used in operational models.

It is the major conclusion of this paper that analytic and operational models for LDC cities have to be developed simultaneously: operational models should be of small, sketch-planning types while analytic models can be more disaggregated.

I. Introduction

1.1 Why Model Cities at All?

> "But as for those who posit the ideas as causes, firstly, in seeking to grasp the causes of the things around us, they introduce others equal in number to these, as if a man who wanted to count things thought he would not be able to do it while they were few, but tried to count them when he had added to their number".
>
> Aristotle (Metaphysics Book 1,Ch.9)

Such was Aristotle's criticism of Plato's Theory of Forms. Plato sought to comprehend reality around him by defining general 'Forms' and interpreting things similar to them as their particular manifestations. The objective was to reduce the size of the problem by having to comprehend only classes of things rather than each by itself. The difficulty with this was, however, a question like, "what is the essence of a table that makes it a table?". Thus ultimately the number of Forms precisely equals the number of 'things around us' and our quest for comprehension of the universe brings us back to our starting point.

Such is also the predicament of the urban model builder. One seeks to build a model of an urban environment with the object of reducing the complexity of the observed world to the coherent and rigorous language of mathematical relationships. (Lowry, 1965). When this language becomes complicated or the size of the 'simplified' model becomes so large that it assumes a complexity of its own, one begins to question the usefulness of such an exercise. Thus when a model of an urban environment is sought it is crucial to keep in mind the objectives of the exercise.

Planners and policy makers in the urban sphere have become prisoners of the idea that in the city everything affects everything else. (Lowry, 1965). If, indeed, everything is interrelated then every

public decision needs to be an informed one if it is to achieve its aims. The urban policy makers must be aware not only of the immediate effects of their decisions but of the indirect effects as well. For example, the provision of a new speedy travel mode has the direct effect of reducing travel time for its users. However, its indirect effects on industry location and consequently on employment and residential location could be far greater in magnitude. The policy-maker, knowing the existence of such interrelations, demands knowledge of their magnitudes. Hence, the demand for models. Models should therefore help the policy maker in understanding the underlying determinants of spatial location within a city; in analyzing the causes of city growth and decay; and in predicting future land uses in specific parts of urban areas. If a model can help to show that commercial development A will take up X hectares of city center land, cause y percent more traffic congestion, and generate z percent less tax income and jobs in ancillary industries than industrial development B, then a policy maker will be interested in that model.

These, however, are a daunting set of demands which social science can scarcely supply even if some of its practitioners pretend that they can. Recent developments in computer technology and in the use of mathematical techniques in the social sciences have raised people's expectations of what can be predicted about the future. Social scientists themselves have been instrumental in raising the level of demands that policy makers make of them. This results in disappointment more often than not.

In the context of urban systems the demand for models is particularly great because of the large number of variables, large number of available

policy parameters, the complex interrelationships among them and the long term consequences of public decisions. The sheer size of the number of variables would not be much of a problem if they were not thought to be correlated. If they were independent, problems could be solved partially or sequentially and there would be no need for models. Planners, therefore, need models for the following purposes:

(i) prediction and projection

(ii) impact analyses of alternative strategies

(iii) plan design

(iv) educating planners

(v) controlling and directing urban change.

A comprehensive model would be one which met all these needs. Such a model has not yet been formulated. Different models meet different needs. The following kinds of models can be distinguished:

(i) theoretical

(ii) policy making

(iii) data manipulating

(iv) educational

(v) measurement devices.

Theoretical models are at a high level of abstraction seeking to gain basic insights into urban structure. Such models can be useful in educating planners as well as in the design of more operational models. Policy making models can be merely predictive, or optimizing or of the impact analysis type: they all seek to help planners and policy makers. Data manipulationg models (e.g. input-output models) are good devices for checking consistency of data while they can also throw light on the structure

of some interrelationshiop. Educational models can be game playing devices with which planners can be educated. While they may not be strictly operational policy makers can use them to test possible effects of policies.

It is evident from this classification that it is not a mutually exclusive nor an exhaustive one. It is presented to illustrate the point that different models do different things which attempt to satisfy the different needs mentioned earlier. Users of models must be as clear about what they cannot do as about what they can. If, for example, a model is merely predictive (in the positive rather than normative sense) it must not be taken to imply what ought to be.

In conclusion, policy makers must be clear about why they want to model cities before they set about financing model builders' dreams. They then have some chance of achieving their objectives.

1.2 Some Reflections on City Phenomena

Having concluded that there is some need for models of the urban environment, it is appropriate to reflect on the intrinsic characteristics of cities which cause problems for model builders. City phenomena are replete which analytic inconveniences such as increasing returns to scale, indivisibilities, interdependencies and minimum size thresholds. Some of these can be captured by models and some cannot. Whatever the case, they are the phenomena that need to be studied and understood.

To comprehend and, perhaps, manipulate the structure of cities we need some understanding of why they exist in the first place. Cities have existed for a long time. Jane Jacobs (1970) has offered some conjectures on the reasons for their existence and their role in economic

development over the ages. She is of the view that cities are the primary economic organs and that most (if not all) economic advances of note have taken place in cities. She even argues that agriculture (as we know it) started in cities and only later spread to the countryside. Furthermore, most agricultural innovations have been city-based and agricultural productivity is therefore a derivative of city productivity. Whatever the merits of her historiography and chain of temporal reasoning some lessons can be drawn from her work. The main one is the idea of cities as centers of innovation. One immediately wonders 'why are cities innovative?' Words such as agglomeration, proximity and minimum threshold size come to mind but more precise explanations do not. The ideas of agglomeration and proximity are at the root of the existence of cities and immediately point to the importance of the spatial structure of cities.

Many cities evolved originally as market or trading centers. This was necessary for product markets to expand and consequently for economic growth. Roland Artle (1972) has suggested that the characteristics of the income elasticity of demand for goods provide us with clues as to why cities invariably seem to go together with economic growth. It is the essentially simple idea that the income elasticity of demand for primary goods like food is low and declining with income while that for urban type goods and services is high and increasing with income. The question then is, 'can these products be manufactured and provided in rural areas?' The answer appears to be negative. Even craft products need markets and capital. Only if a craftsman can expect to sell his product can he afford to invest in capital. Risks are reduced if he has proximity to his customers and more so if the potential number of customers is large. The production process does not need economies of scale for the size of expected market to be a factor. Such a consideration for production is difficult to capture in a

mathematical model.

As markets expand and production increases concern over availability of inputs also becomes important. The likelihood of obtaining labor, more importantly labor of the right type, at a given time is clearly greater in a concentrated population than in one that is dispersed. Other inputs like raw materials and manufactured goods are also easier to obtain. We thus have the notion of interdependencies between products. We must distinguish between two kinds of interdependence. One is the kind represented by an input-output matrix. A dense matrix i.e. one with many non-zero entries represents a high level of interdependence. The process of production is then an intricate mesh: most products require as inputs many other products. This can be termed technological interdependence. Such interdependence does not necessarily produce a city if transport costs are not high. As Moses and Williamson (1967) have attempted to show, cities that grew in the nineteenth century have the structure they do because the cost of moving goods within cities was high relative to (a) moving people within cities and (b) moving goods between cities (by train or water transportation). Industries were thus located very densely and near major transportation modes. The advent of the truck made intra-city transportation of goods cheaper and this is posited as one cause of decentralization. The other kind of interdependence arises from technological interdependence but is more related to 'uncertainty'. Physical proximity of establishments is then of some economic benefit. Vernon (1959) found this to be a major characteristic in relation to the existence of certain types of industries in New York City. Activities which are dependent on changing output demand have to change their own input demands in response. The necessity

to do this in a short time span makes proximity necessary. Risk is spread : a supplier who loses one client can easily switch to another before deciding to change his product line. To summarize, technological interdependence is important for spatial concerns because of transport costs while the other kind of interdependence is important because of information costs and uncertainties.

Moving on to the wider aspects of modern economic activity the interdependence of production with such services as banking, insurance, marketing, etc. has become more important in the agglomeration economies of modern cities. Some of these economies can be dealt with by thinking of indivisibilities (or minimum threshold size) in the size of such activities and can be represented by constraints to a production function. If, however, such activities are regarded as inputs into the production process their representation is not so easy. Agglomeration is necessary for these activities because of the need for face to face contact. One would have expected the telephone and other communication advances to make face to face contact unnecessary, but this does not appear to be so. It is, however, difficult to put economic values on the benefits of such contact and therefore to subsume it in economic analysis.

The existence of economies of scale in certain industries is the next most important reason for the existence of cities. Even if a few industries exhibit economies of scale their effect through backward and forward linkages will be much greater. We thus see whole cities like Detroit, which are based primarily on a single industry which has economies of scale. This can, of course, be represented easily by production functions e.g. by a Cobb-Douglas production function whose sum of exponents is greater than one. The problem created by such functions is that they lose many of the

'nice' properties of constant returns or diminishing returns to scale production functions.

Having dealt with the production side it is also important to recognize the indivisibilities and economies of scale on the consumption side. There are many collective services whose consumption is characteristically joint. Learning and research are activities which are done jointly; moreover, they are tightly correlated and further produce technical change. They thrive on agglomeration. The provision of health services, transportation and recreation also tends to be collective. The more densely an area is settled (within limits) the greater are the potentials for the use of such services. Since such services are collective the private market does not operate very well in handling them. Thus some kind of public authority is necessary to provide them. Consequently, policy makers come into play and ask for guidelines to help in taking decisions.

Finally, there are 'bads' associated with agglomeration. On the production side there are negative externalities like pollution and monopoly resulting from economies of scale. On the consumption side, overcrowding can make the provision of collective services difficult.

In addition to the problems mentioned above, although in part because of them, cities change in rather unpredictable ways. The growth process is of two kinds: one is the multiplication of existing facilities and the other is the initiation of new activities. The stability of a city depends on its ability to cope with change. Most industries have long term cycles: they are created, go through a boom period and then decline. If a city is based on one industry the city itself goes through the same cycle. If, however, the city is based on a variety of industries it is unlikely that all their cycles will be in phase and the

city will then be seen to be more flexible and able to cope with change. At any one time, then, there will be both efficient and inefficient industries. This is, perhaps, what Jane Jacobs means when she says that cities need to have inefficiencies and impracticalities built into them to cope with the uncertainties of chaning technology and the economic environment. This is not unimportant, since urban infrastructure usually lasts about fifty years. Apart from single industry bases, cities built with single transport modes are more vulnerable than others. European cities built when no motorized transport was available have adapted well to the many changes in the modes of transport that have taken place over the past century. In contrast, North American cities built in the last seventy to hundred years are so dependent on automobile transport that they will find the effects of oil price rises more difficult to handle. Models can scarcely capture such characteristics of cities. If a model is an optimizing one and is calibrated on contemporary data its prescriptions may well produce an efficient city for the present but a disastrous one for the future.

Uncertainty about the present, unforeseen changes in the future, indivisibilities, and economies of scale have been posited as the reasons for the existence of cities. These are precisely the phenomena most difficult to codify and to include in models. At best, approximations can be tried. Non-linearities can be built in but they make solution and handling of the model more difficult. Complex interrelationships between variables and the durability of structures also makes the impact of decisions more difficult to analyze. Thus model builders have to be modest about what they are trying to accomplish and policy makers less sanguine about what they can expect from models.

1.3 Cities in Less Developed Countries

Cities in the less developed countries are in many ways even more difficult to handle than those in the Western world. The main reason for these difficulties is that unlike most Western cities the former have grown suddenly and explosively in the present century and have attained sizes comparable to those elsewhere but at lower levels of income. Analysis is more complex because of the coexistence of technology from different centuries. When analyzing transport systems, for example, we have a limited number of modes to worry about in Western cities. In an Asian city, on the other hand, the electric train coexists with the hand-pulled rickshaw, bicycles, scooters, automobiles and, of course, walking.

Even during the period of rapid urbanization in Europe, rates of population growth in cities were on the order of about 0.5 percent per year (IBRD, 1975); whereas the populations of cities in the LDCs are presently growing at the rate of 3 to 7 percent per year. With a growth rate of 0.5 percent European cities had time to adapt and evolve as they grew in size. Diffusion of innovations was slow and economic, social and political institutions emerged to regulate patterns of growth and to govern cities as they grew. This can be seen as an equilibrium process: i.e. one which is roughly at equilibrium at every stage.

The current LDC experience is quite different. A primary factor is that the population growth rate is truly explosive as compared with historical experience. Even with the best of intentions, controlling this growth rate is not easy, especially in the short run, for it has a number of implications. First, the growth rate of cities is high without migration. Second, technical

change in agriculture which raises farm productivity has to provide for the growing rural population in addition to the burgeoning urban population. In times when overall population growth rates were less than 0.5%, a 3% growth in agricultural productivity was tremendous. Now such a growth rate is barely enough to feed the rural population, thereby leaving precious little marketable surplus and hence a sluggishly growing demand for urban products. Cities are therefore not as tightly connected with their hinterlands. Third, a high population growth rate implies that cities can become larger without significant increases in the urbanization levels of a whole country. Fourth, such a pace of increase makes the structural growth of the city even more unpredictable and consequently more difficult to manage.

The second major difference from the European experience is the coexistence of 'high' and 'low' technologies in LDC cities. This makes the demand structure of the rich qualitatively different from that of the poor in a manner more pronounced than that in European cities at similar levels of incomes. Thus the demand pattern of the rich in LDCs corresponds roughly with the current Western rich. They demand, and receive, services and products which use twentieth century technology, while the poor still live in much the same way as they might have a century ago. Thus the 'technological inequality' in consumption between the poor and rich is more pronounced than the income inequality. The latter was probably quite similar in Western cities in their early stages to that existing in LDC cities now.

Unlike the European experience, then, technological growth, income growth and population growth are unbalanced in cities in LDCs today. High population growth and technological growth makes it possible to have cities with a multimillion population at low levels of income. This was just not possible in the last century. The sewage disposal, water supply and transport

problems of a city of 8 million are qualitatively different from those of a city of half a million, which was the size of major western cities at similar income levels.

The third major difference is the decline, in relative prices of transportation and communication costs. This makes for less centralized cities, as seen in some urban densities in Latin America. Workers can live away from their place of work; information and innovations travel faster. Because of other imbalances this diffusion is uneven and urban centered thereby making primate cities more important. The international diffusion of information as well as innovation reinforces this tendency of concentration in primate cities, thereby further exacerbating technology and income inequalities. Elites are internationally mobile: they characteristically have more contact with Western cities than with their own hinterlands. Their demand structure is therefore more in tune with Western factor proportions. The obvious urban effects are those in housing where differences between rich and poor become extreme.

These major differences combine to produce effects which lead us to expect cities which have a different spatial structure from those for which models and plans have been developed. Until now, LDC cities have been lumped together and discussed as if they are all similar, and different as a group from Western cities. It is, however, worthwhile to disaggregate them. Clearly, there are systematic historical and geographical differences among LDCs. The IBRD (1975) classification, based on the urbanization experience, is interesting, but this paper is more concerned with urban form. Our classification below (suggested originally in Mohan, 1975) is a hybrid one and consequently less well ordered logically than the IBRD one:

(a) Pre-Industrial Cities: These are cities which existed in LDCs before colonization, or before the modern era. They grew 'naturally' with highly congested central areas reflecting their existence before the motorized age. Having grown within and along with the traditional economy they have tight links with their immediate hinterland. Their own pattern of production, commercial relationships and transportation patterns can be expected to reflect this. They often have superimposed on them modern influences usually characterized by a modern business center existing along with the old one though spatially separated. Examples of these are some cities in the Middle East and North Africa, e.g., Ibadan in Nigeria, Hyderabad and Lucknow in India, Mombasa in Kenya.

(b) Industrial Cities: These are usually large port cities which were the main colonial importing and exporting centers. They are characterized by a great amount of Western influence. They have links with a wider hinterland, but the links are purely commercial. They have few 'traditional' activities. Their structure is more like Western cities, more geared to motorized transportation. This category should, perhaps, be further disaggregated to reflect varieties of colonial experience. Examples are Singapore, Hong Kong, Calcutta, Bombay, Rio de Janeiro, and Buenos Aires.

(c) Post Industrial Cities: These should really be called administrative cities but are called post-industrial for logical neatness! They are post industrial in the sense that they have really become 'cities' in the last two or three decades. These cities are usually quite sparsely populated and much more planned (initially). They have grown primarily as

capital cities with somewhat questionable 'economic bases'. Some are in the process of attracting more industry and diversified economic activity in addition to governmental activity. Examples are Brasilia and most African capital cities.

The object of giving this brief account of different types of LDC cities is to illustrate the reasons why these cities can be expected to require different kinds of analyses. They also have, large differences in income levels which have to be taken into account.

We now illustrate how the three major differences between LDC cities and Western cities combine to make the life of model builders difficult.

(i) Rapid population growth of these cities at low levels of income but with high and low technology produces segmented markets. Here we focus on the resulting distinctions between the 'formal' and 'informal' labor markets. The model builder is usually accustomed to looking only at the formal markets. The journey to work is then regarded as one of the most important components of urban economic models and as the major transport activity. Whether this is so in LDC cities is not clear. With less organized economic activity employment appears to be more diffuse. Firstly, the large 'informal sector' provides moving employment: hawkers, service oriented people, etc. have no fixed place of work but do have areas of operation. Such employment may well account for about 10% of all urban employment, as in Peru. Their journey to work cannot really be defined and their work location covers a large area. Secondly, small manufacturers, artisans and shopkeepers often live and work in the same location. They are mostly self-employed and could account for another 10% of urban employment. Thirdly, it is often the case that people commute from the neighboring countryside and stay in the city for part of the week and return

to their homes for the other part. Thus it would be surprising if journey-to-work oriented models of urban location are appropriate for LDC cities. (However, precise information on these transportation patterns is scarce so these remarks should be taken as speculative). The informal market is itself heterogeneous and the range of incomes within it is large. Analysis is further complicated by the fact that a majority of informal sector workers are secondary income earners in the household. This makes the derivation of residential location from work location much more complicated . If poorer households characteristically have more than one income earner then this problem is a serious one for models generating residential location distributions. The existence of informal markets also causes difficulties for models which are based on the 'basic' or 'export' oriented employment concept which is then used to generate all other kinds of employment. The relationships between markets in LDC cities needs more investigation to find such multipliers.

(ii) Transportation modes: Analysis and model building is simplified considerably by the assumption of single modes or, at the most two modes. Existence of low incomes along with high technology produces great problems for the model builder in this area. In LDCs, the poorest walk, those slightly better off use a bicycle, then a bus or a train; and the richest own automobiles. There are also other modes like the jeepney, scooter, rickshaws and cycle-rickshaws. Some evidence from Delhi (Sarna, 1975) is worth quoting here. Excluding walking, 42% of commuters used bicycles and about 37% public transport. For low income people at least 35% of work trips were walking trips and about 25% for middle income people. Average work trip length was remarkably short: 12 minutes for fast vehicles, 16 minutes for mass transit and 11 minutes for bicycle users (no data for walkers). This evidence

coupled with the spatial origin of vehicular work trips indicates dispersed employment. Those going to work by fast vehicles live closest to the city center and those not using fast vehicles the farthest. The calculation of urban transport costs and their effect on residential location is then not a simple task but one that requires ingenuity if models are to be developed and are to be of use in LDC cities.

(iii) Housing: High migration rates, low incomes and capital scarcity produce squatter settlements, shanty towns and slum areas. Such housing accounts for about half of total housing in many LDC cities. For model builders this produces two problems. First, data are intrinsically difficult to gather from such neighborhoods since many structures are illegal. Secondly, the variety in housing is as great or greater still than in modes of transport. The production technology for straw huts is clearly different from that for tall buildings. Even if the form of the production function is not different the factor proportions are clearly so. In Western cities this is not the case. At most one can usefully distinguish between low rise and high rise buildings. Production technology is not much different. In LDC cities, different kinds of housing require different kinds of inputs from different markets and the housing market itself is more segmented.

(iv) Factor proportions: The existence of a city implies higher capital-land and capital-labor ratios (although the latter is not obvious). Given that LDCs are capital scarce but have similar sized cities as the West (in population) we should expect their structures to be different. Given that LDC cities do have some skyscrapers it would appear that the mere existence of such structures distorts the allocation of urban resources. Large populations densely packed in, drive up the price of land in central cities, resulting in the use of skyscrapers as a form of capital to substitute for land. From the context of the

whole economy this may not be an optimum allocation of resources. The provision of urban services: sewerage, water supply, electricity, roads, mass transportation etc. is necessary as well as capital intensive. The factor proportions in these activities are probably nearer those in developed countries than those in the country generally. Here the problem is probably of limited technological choice. Nonetheless, it creates allocational and financial problems for LDC cities.

(v) Choice: Low levels of income limit the alternatives available to the poor. Here the model builder is perhaps helped in that some distributions could be over-determined. The limiting case would be one where the poor may be so constrained that their location decision has no element of choice. Usually, though, their choice is expected to be limited, but decisions still have to be made even if they are among a few bad alternatives.

Finally, we come to one aspect of LDC cities that is difficult to account for in economic models (except exogenously) but is in some sense the reason for building such models. This is the role of public authorities, in the spatial decisions in the city. Households can be regarded as utility maximizing and firms profit maximizing. Modelling the behavior of public authorities can follow no such rules. Although centralized political authority has long been one of the reasons for the existence of some cities, it has played an especially dominating role since World War II. Governments have become more active in general since then and particularly so in LDCs. The problem is further complicated by the fact that though the effects of governmental decisions are all pervasive in the formal sector they are less visible in the informal sector. We have very little information on these

effects. Theories adequately capturing the interaction between governmental decisions and economic activity have yet to be developed, although governmental reaction functions and the like are now being posited. Some role has obviously to be assigned to governmental activity in urban models of LDC cities. This has to be exogenous as well as endogenous for the effects of exogenous public decisions are easily dealt with by sensitivity analysis.

To summarize, the major differences between LDC cities and Western cities stem from:

a. The unprecedentedly high population growth rates of most LDC cities;

b. the coexistence of 'high' and 'low' technology along with predominantly low incomes; and

c. the decline in relative prices of transport and communication compared with a century ago.

These factors give rise to segmented markets, the coexistence of many transportation modes, squatter settlements, shanty towns and slum areas; while the high intensity of capital in urban areas is not commensurate with factor proportions existing elsewhere in the country. These effects make formal models of LDC cities difficult to develop. Because information on LDC urban phenomena is seriously lacking, these notions are somewhat speculative.

The next section suggests criteria by which urban models may be judged.

1.4 Criteria for Model Evaluation

In reviewing urban models the first question to be asked is the objectives of the models. This has been addressed in the first part of this section. In listing criteria for judging models we present one set for explanatory or analytic models and another for policy-oriented ones because their objectives are different.

A. Explanatory Models

(i) Model Structure: Model structure should be easily comprehensible by those reasonably literate in the field. Since explanatory or analytic models operate at high levels of abstraction making many compromises with reality, their structure must not be obtuse. If a complex reality is being modelled we cannot always ask for the model structure to be simple. But a complex model structure can also be presented so that interrelationships within the model are clearly articulated. This is important because larger policy-oriented models are often built on the basis of these analytic models.

(ii) Correspondence to Reality: As mentioned earlier, it is recognized that analytic models have to make certain compromises with reality to remain explanatory or analytic models within a reasonable size. However, they are explanatory only to the extent that they explain some urban phenomena however aggregated. Thus a good model remains close to reality in its structure and output.

(iii) Output: The results of a good analytic model should give insight into some aspect of urban phenomena. For example, a model can focus on the location decision and illuminate its behavioral determinants. An explanatory model is of little value if it does not accomplish at least as much. It is at this level of modelling that the rationale behind urban structure is seen.

(iv) Normative and Positive Results: Models should not create confusion between what is and what ought to be. Assumptions behind a good model are made explicit. This is important because the results can then be judged on the basis of these assumptions. Moreover the role of the assumptions

in the results can then be clearly seen. If a model is predicting something, a good model should make clear if it is a projection or if it is a prescription for a plan of action. If it is a prescription then the criteria for such a prescription should be spelled out. This is a problem when mere projections of current trends are regarded as targets.

Before proceeding to the criteria for policy oriented models it is useful to see some connections between the two kinds of models. Explanatory models are not necessarily small. They essentially seek to further understanding of the urban process. They cannot usually be directly used in policy planning. Policy-oriented models have to simulate reality reasonably closely to aid in decision making. They have to take account of all interrelationships that have a bearing on the problem under consideration. This usually necessitates sub-models within a larger model. The relationships in these sub-models usually operationalize the behavioral relationships found in explanatory models. An analogy from macro-economics is to use all the research on the consumption function to form the consumption equations in large econometric models. This set of equations could be disaggregated for different product markets and could comprise a set of 30-40 equations. The form of all these equations would have come from the basic consumption function studies. Similarly, the insight of declining land rents as one moves outward from a city center when built into a policy oriented model will probably be disaggregated into segmented markets for different kinds of demands - office space, residence, manufacturing, etc.

Even highly theoretical models are then not totally useless as long as they exhibit the qualities specified above. They ultimately become useful for policy design purposes through their utilization in policy models.

B. Policy Oriented Models

These models are usually more difficult to evaluate since they are usually larger and more complex. They are difficult to understand because of the many bits of inputs that have to be fed into them, the many equations which constitute their structure and the many outputs they provide. They can be judged according to the following criteria:

(i) Objectives: Complex as they are, policy-oriented models must be absolutely clear about their objectives. Clarity of objectives makes it easier for policy makers to use them. They can then judge if the model is accomplishing the objectives it was designed for. A predictive model is quite different in structure from a prescriptive one. A spatially disaggregated one is again different from an aggregated one, and so on.

(ii) Data requirements: In evaluating a model for LDC application its data requirements have to be looked at closely. As described later, half the budgets for policy-oriented models are characteristically devoted to data collection. Since data collection in LDCs is even more of a problem, data requirements of models have to be scrutinized even more carefully. Several issues are involved:

- measurability
- accessibility
- cost

It often happens that some kinds of data requirements are just not measurable. They might fit into neat conceptual slots but are not possible to measure in practice. Other kinds of data are measurable but often not easily accessible. The length of time involved in their collection is a consideration that is particularly important given that the rate of growth of LDC cities is so high. If, for example, a set of data requires two years to collect, it may well become obsolete sooner than it is collected. Resources for such

data collection are scarce, as are the facilities for processing such data. Thus, the more a model uses readily available data, the cheaper they are to obtain and the more quickly the model can be put into operation.

(iii) Inputs: In the description of a model there is often a great amount of confusion between which variables are endogenous and which are exogenous - in other words which variables have to be externally provided. Among the exogenous variables, the distinction between control variables and other variables should be clear. Control variables are the ones that policy makers can play with and should therefore be made explicit. Their operational meanings should also be clear. The clear specification of inputs is also important because it makes it easier to understand what a model is --and is not-- simulating.

(iv) Model Structure: This is the core of the model on which its usefulness really depends. One believes a model to the extent that one has faith in its structure. We discuss various aspects of model structure:

-- Simplicity: The structure of a good model is such that it can be comprehended. It need not be simple in the sense that it does not have complex relationships embedded within it. It should be simple from the viewpoint of explanation. An opaque structure is such that it seems like a black box which cannot be easily explained. Such explanation is facilitated if the model has clear connections with theory and is consistent with it. If it is not consistent it is useful to explain why it is not . Thus the theoretical bases of a good model are made explicit. This is particularly important if a model is to be used as a policy device. Policy makers would be more inclined to believe results if they could comprehend how these results come about.

-- Logical Structure: This is a variation of the issue discussed above. A logical structure aids in comprehension.

-- Flexibility: Models are usually made in relation to one city and then attempts are made to apply them to other cities. On the one hand, the more flexible a model the better it is. On the other hand, flexibility implies that the model is not that closely related to the original city and therefore breeds some suspicion. If a model is claimed to be flexible in this respect justifications should be given about its flexibility. Such justifications depend on the kind of relationships embedded in the model. Is the flexibility due to easily changed parameters which differ from city to city or because the relationships themselves can easily be changed. If a model is lifted from one country to another or from one culture to another, explanations should be given as to the feasibility of such a jump.

Another kind of flexibility is in the changeability of the parts of a model. Where a model is achieving a number of objectives and has a number of sub-models it is useful to know how tightly interrelated these parts are. Are they modules that can be lifted or whose sequence can be changed around to fit different circumstances and objectives? In this sense, the more flexible a model is the better it is as long as the flexibility is explicable.

-- Role of Model Structure in Decision Making:

The structure of the model affects the outputs. Where the output is used for decision making it should be possible to decide whether certain results are due to technical quirks of the model or are believed to be inherent in the city as modelled. In other words, clarity about the model's role in decision making is desirable.

(v) Output: The policy maker receives the output from the model. It should, above all, be intelligible. What it represents should be clear. It should also be reasonable in the sense of correspondence to

reality. If a projection is a wild one in that it is unexpected, then it must be explained. Calibration of the model is used to make its outputs reasonable. Since all outputs are really point estimates of some distribution, some indication should be given about their level of accuracy. They should therefore be accompanied with some tolerance measures. Probability estimates are useful when extrapolation is involved.

Policy makers are also interested in the robustness of outputs. They want to know how sensitive they are to policy changes. Thus a good model should be capable of providing sensitivity tests. In addition, predictive outputs should be clearly distinguished from prescriptive ones.

(vi) Cost of Operation: Costs are of different kinds. Firstly, there is the cost of developing the model and putting it into operation. Secondly, there are time costs, i.e. one likes to know how long it would take to put a model into operation. Thirdly, there are costs of skill. Clearly, one would like all these costs to be minimized. Since skills are in particularly short supply in LDCs this is a crucial variable for consideration. Furthermore, since it requires intimate knowledge of a city to build a good model, it is desirable to find models which can be built with available skill levels. Finally, one is interested in the costs of running the model - usually the cost of running the model on a computer.

In the reviews of models that follow comments on the quality of models are based on the above criteria. To the extent that they are not always made explicit is a shortcoming of this evaluation. It would therefore be useful to refer back to this list of criteria while reading the evaluations.

In concluding this section it is instructive to quote from Garry Brewer (1973) when models should not be used at all:

When
- simpler techniques exist
- data are inadequate
- objectives are not clear
- short term deadlines exist
- problems are minor.

If any of these conditions exist the question of evaluation of models does not arise, since they should not be used at all.

2. ANALYTICAL OR EXPLANATORY MODELS

2.1 The Classical Economic Models

Any review of urban economic models must begin with the seminal contributions of Muth (1969), Wingo (1961), Alonso (1964) and Mills (1967). They are of the same family: utility maximizing households constrained by their budgets trying to find optimum residential locations. The city is located in a featureless plain and possesses a single Central Business District (CBD) where all employment is located.

Muth's households have a utility function

$$U = U\ (x,q) \qquad (1)$$

where x represents consumption of goods other than housing and q is consumption of housing.

Their budget constraint is

$$M = x + p(u)q + T(u,M) \qquad (2)$$

$$T_u > 0$$

where M is household income;

u is distance from the CBD;

p(u) is price per unit of housing, a function of distance from CBD;

T is the cost per trip and is a function of location and income. (and subscripts denote partial derivatives i.e. T_u is $\partial T/\partial u$).

Income includes money value of travel and leisure time. Prices of goods other than housing and transportation are the same everywhere in the city. Housing is regarded as a bundle of services yielded both by structures and the land they are built on. These services are a flow not an asset and thus the price is also of the flow, not of the asset. No distinction is made between owners and renters since they are both seen to be consuming a bundle of services. In the basic model, Muth assumes that households

make a fixed number of trips to the CBD (i.e. it is not a decision variable) and these costs are composed of money costs which vary with distance ($T_u \geq 0$) from the CBD and time costs which are assumed to vary with income ($T_M > 0$).

Maximizing (1) constrained by (2) yields the standard first order conditions:

$$\frac{\partial L}{\partial x} = U_x - \lambda = 0 \tag{3a}$$

$$\frac{\partial L}{\partial q} = U_q - \lambda p = 0 \tag{3b}$$

$$\frac{\partial L}{\partial u} = -\lambda(qp_u + T_u) = 0 \tag{3c}$$

$$\frac{\partial L}{\partial \lambda} = M - \{x + p(u)q + T(u,M)\} \tag{3d}$$

The marginal utilities are in the proportion of the prices (equations 3a, 3b)

$$\frac{U_x}{U_q} = \frac{1}{p(u)} \tag{4}$$

and from (3c)

$$-q\,p_u = T_u \quad \text{or} \quad p_u = -\frac{1}{q} \cdot T_u \tag{5}$$

$$(T_u > 0)$$

(5) shows that, in equilibrium, the result of a small move will not result in any savings. T_u is the marginal change in transport costs and qp_u is the change in housing expenditure occasioned by such a move. Thus a move outwards (i.e. $\delta u > 0$) will increase transport costs by T_u which will be exactly balanced by a saving qp_u in housing expenditure. A small move

inwards balances the saving in transport costs by an equivalent increase in housing expenditure $T_u > 0$ hence $p_u < 0$ i.e. housing costs per unit decline with distance.

Muth then investigates the effect of small changes in each variable and obtains the following results by totally differentiating (5) with respect to u

$$-qp_{uu} - p_u \frac{dq}{du} - T_{uu} \leq 0 \qquad (6)$$

i) $p_{uu} > 0$

The price of housing decreases at a numerically decreasing rate.

ii) $\frac{\partial u}{\partial M} \geq 0$

Optimum location is more distant from the CBD the higher the income. This is not an unambigous result but depends on various assumptions concerning the income elasticity of the demand for housing and the elasticity of T_u with respect to income - assumptions regarded as plausible by Muth. Stated more simply, this result states that the benefits of increased housing consumption outweigh the increase in transportation costs.

This is a very important implication for city structure and change in structure over time. If Muth's assumptions are correct, a general increase in incomes leads to an increase in housing consumption on the part of all households and the city spreads out.

Where transport is slow as it is in developing countries the time costs for higher income people may well be high enough to contravene this assumption. The location of high income people nearer CBDs rather than in suburbs could then be regarded as an optimal location within that framework. This is clearly something that can be tested econometrically if one had good data on housing expenditures and trip times for different income groups by location for some developing country cities.

iii) $\frac{\partial u}{\partial p_o} \leq 0$

where p_o is the price of housing services if they were located at the centre. Optimum location will be nearer the CBD if the price of housing services increases (keeping the price-distance gradient unchanged).

iv) $\frac{\partial u}{\partial T_o} \leq 0$ and $\frac{\partial u}{\partial T_u} \leq 0$

an increase in either the fixed or marginal cost of transport decreases the optimum distance from the CBD.

Muth assumes the income elasticity of demand for housing to be greater than 1 in the derivation of all these results. He supports this assumption by his own empirical work.

Muth then modifies his basic model by relaxing some initial assumptions:

a. The number of CBD trips is made a decision variable by introducing it in the utility function.

b. Similarly, preferences for location are introduced in the utility function, and

c. Uniformly distributed local employment is introduced in addition to the CBD employment.

"a" does not affect the results of the model in any substantive sense. "b" makes the derivation of qualitative results almost impossible. "c" makes possible the derivation of a wage gradient with distance from the CBD. This obviously appears because the local workers incur no transport costs and can therefore accept lower wages to remain at the same utility level at the same location.

Muth then has a production side to his model to describe the behaviour of profit maximizing firms producing housing services:

$$\Pi = pq(L,NL) - rL - \rho(NL) \qquad (7)$$

where Π is profits;

L is quantity of land inputs;

NL is quantity of non-land inputs;

r and ρ are their respective prices; and

q (L,NL) is the production function of housing services.

All the firms are identical and have the same production functions. Thus they all have the same profits irrespective of location. The prices of the inputs vary with locations so differing combinations are used according to location. Equilibrium conditions yield

$$dr^* = \frac{1}{S_L} dp^* - \frac{S_{NL}}{S_L} d\rho^* \qquad (8)$$

(where * indicates natural logarithm of the number);

S_L and S_{NL} are the shares of land and non-land in the firms revenue, i.e. $S_L = \frac{rL}{pq}$ and $S_{NL} = \frac{\rho(NL)}{pq}$

(8) shows that land price is high where the price of housing (e.g. due to location) is high and where the price of other inputs is low (e.g. of raw materials).

We can rewrite (8) as

$$\frac{r_u}{r} = \frac{1}{S_L}\left(\frac{p_u}{p}\right) - \frac{S_{NL}}{S_L}\left(\frac{\rho_u}{\rho}\right) \qquad (9)$$

This shows that the land rent gradient $\left(\frac{r_u}{r}\right)$ is a multiple of the housing price gradient since $S_L < 1$, assuming the gradient of other input prices

(e.g. wages) to be negligible. For land share of 5 to 20 percent the land rent gradient can be anything from 5 to 20 times the housing price gradient. By assuming a Cobb-Douglas production function for housing, Muth then derives the housing price, land rental and population density functions as declining exponentially with distance from the CBD.

This model has been presented in some detail since it is an example of simple economic reasoning stretched to its limit. The assumptions underlying the model are highly unrealistic but that is the price of a simple manageable model. Muth does extensive empirical testing of his propositions: indeed his work is an example of the close relationship of good economic theorizing with empirical work. Many qualitative results are not possible to obtain without the assumption of robust parameter estimates -- the income elasticity of the demand for housing being one example.

Wingo's concern is with the cost and programming of transportation in a city although this is embedded in a larger model of land use and transportation. Indeed, he set out to develop a transportation model but soon discovered that land use and transportation were too interrelated to be separated. Wingo was much more guided by policy considerations than Muth and tried to articulate a model which could be of operational use for policy planners. However, it is more of interest as an analytical model. His approach was guided by three considerations:

(a) A model should have explicit differentiation between policy and structural effects.

(b) It should enhance its analytical value by bringing the main elements of the problems within the framework of economic theory.

(c) It should have conditions for treating the problem of intra-urban

distribution of population as a part of the urban economy as a whole.

His model concentrates on how labor services are organized in space given the characteristics of the transportation system, the spatial arrangements of production, the nature of the labor force and the institutions by which the labor force is articulated with the processes of production. The model provides the following:

1. The concept of transportation demand based on characteristics of the labor force and of the journey to work.

2. A systematic general description of the transportation function.

3. A general transportation cost function.

4. A system of location rents which result from the transportation cost function.

5. The manner in which a household demands space and how supply is equated to demand.

Here we will describe the derivation of the transportation cost function in some detail and neglect the rest of the model. This is because that is really the core of the model. Moreover, the derivation of his cost function is of great interest since it is built up from fairly simple notions but takes account of many of the complexities of transportation. Wingo treated the problems of congestion at that early stage: others rediscovered it almost a decade later. The development of this cost function is a lesson on how a particular part of an urban model requires care and thought as well as empirical observation for a proper specification. Wingo assumes that the journey to work is the most important transportation function and then proceeds to analyze its cost.

He observes that since the journey to work is both spatially as well as temporally concentrated these peaks in demand result in saturation in a given capacity. Other demands on the transportation system can therefore be excluded and the journey to work analyzed. The costs can be broken into two components: the time costs incurred and the actual transportation costs. Both involve a prior

characterization of technology of the urban transportation system. A key assumption made here is that the homogeneity of travellers as well as of carriers (in his case: the automobile). This clearly simplifies the analysis: indeed makes it manageable. The calculation of time spent in the journey to work is done by a function

$$T = T(u,v,n,c) \tag{10}$$

where u is distance travelled;

v is velocity;

n is number of workers; and

c is a measure of capacity of the transportation system.

The specification of the function depends on the mode of transport used. Wingo's contribution here is in the inclusion of n and c in this function to demonstrate the interdependence between the users of a transportation system when demand (represented by n) exceeds capacity (c). The time lost because of this excess demand is because of:

a. <u>ingression</u>: the irreducible minimum of time loss because of aggregation of demand -- this is a technological function;(analagous to changes in water pressure in a pipe according to the volume moving through it).

b. <u>congestion:</u> which arises because of reduction of free flow, e.g. because of bad driving. This is not necessary but arises because of human errors which are proportional to pressure of traffic.

If we define t_n as the time loss to the nth unit entering the transportation system at peak time ,

$$t_n = \frac{n-1}{c} \tag{11}$$

and

$$t = \frac{n(n-1)}{2c} \tag{12}$$

is time lost to all units in queue. This is proportional to the square of demand. Capacity of a system is also determined technologically.

$$C = C\ (v, v^{\beta}, \xi)\ \underline{1/} \qquad (13)$$

where ξ is length of the carrier. The exact specification of the function clearly depends on mode of transport used, condition of roads, etc.

Having derived the time spent in journey to work, its value is derived from the marginal value of leisure function: the supply function of labor. Although a worker is paid according to time spent at work Wingo argues that the wage rate subsumes the time costs of the journey to work. In Figure 1, OP is the time spent at work, PQ is the journey to work. The worker needs to be compensated for OQ hours at hourly wage QJ. However, he gets paid for OP hours only, hence his wage rate may be PH, i.e. W_2W_1 is the "pure" wage rate. $W_1KH\ W_2$ gives a measure of the value of the time spent in the journey to work.

The actual transportation costs, i.e. money costs, have two components:

a. Those that vary according to distance travelled; and

b. Those that vary according to number of trips made.

The sum of these along with the time costs finally gives us the total costs of transportation for an individual.

We can now describe the rest of Wingo's model for the sake of completeness. Since all workers are paid the same wages, the differences in transport costs incurred by each account for the differences in land rental. In other words, land near the center of the city commands a higher rent, the difference being equivalent to the excess transportation costs incurred in living farther out. A household's demand for land depends on the rental value of land and the elasticity of demand is constant. The supply of land is proportional to the distance from the city center and is

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1/ Wingo's expression is $C = \dfrac{v}{\rho(\alpha' V^{\beta} + r'v) + \xi}$

where ρ is a risk coefficient and β is approximately 2-2.5 for automobiles.

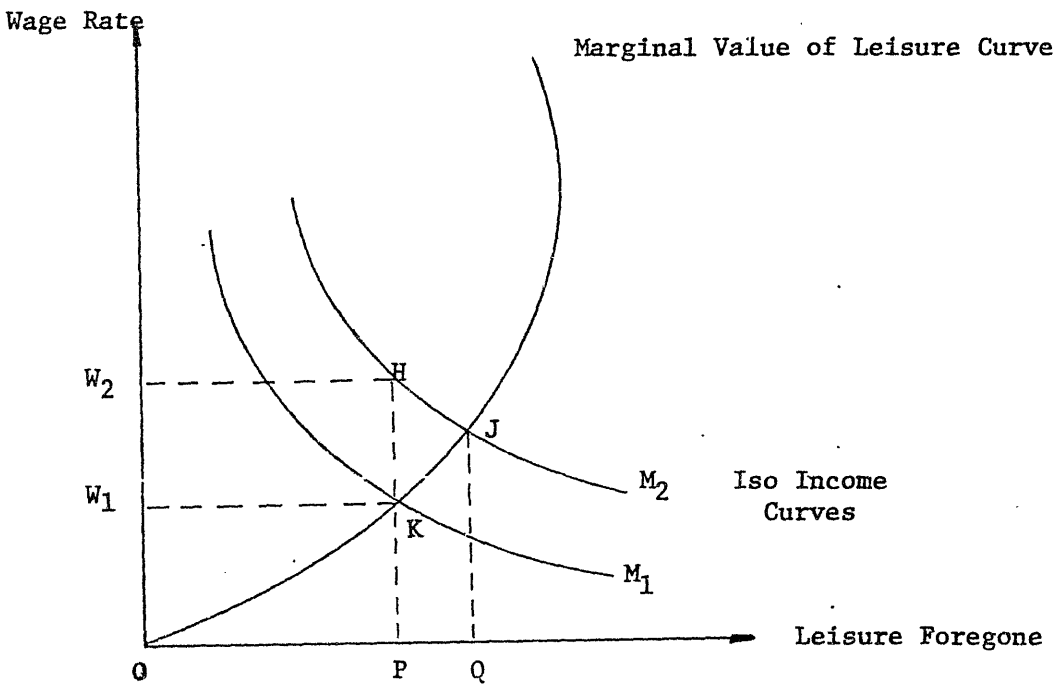

Figure 1: VALUING TIME SPENT IN JOURNEY TO WORK

given exogenously. The model is closed by balancing the supply and demand of residential land. Wingo's condition for locational equilibrium is that the saving in transport cost is equal to the increase in outlay on residential land. This is not strictly correct: the condition should really be that no one can increase his utility by moving. The two are equivalent if the time costs of commuting include the disutility of commuting -- which Wingo does appear to account for.

The importance of Wingo's work lies in the demonstration of the complexity involved in specifying just one component of an urban economic model. The valuation of transport costs involves knowledge of the technological relations of the particular transportation system and of the workings of the labor market. While the assumptions about homogeneity of carriers and travellers are justified at this level of abstraction for developed countries, they are not for less developed countries. The modes of transport are much more mixed -- from walking to bicycles to electric trains -- as are the workings of the informal labor market. Indeed, Wingo himself makes the point that valuing time with money implies fungibility and this is only valid for well operating markets. If time and money are not exchangeable a person may behave as though the scarcity of his time or his money were governing his behaviour -- in either case the other is ineffectual in allocation. Constraints are probably distributed among the population in accordance with income levels. Money is more likely to be the binding constraint for low income groups, time for upper income groups. Such considerations can go a long way in the explanation of location patterns in developing country cities. Modelling this can be done with a Wingo-type approach modified as suggested above.

Alonso (1964) provides a rather complete and general model of urban location and urban land markets. He starts with utility maximizing households constrained by their budgets:

$$U = U(x,q,u) \tag{14}$$

This is similar to Muth's formulation except that q is quantity of land rather than housing and u -- the distance from CBD -- is introduced explicitly in the utility function with $U_u < 0$. Alonso derives a bid price function for each household from the equilibrium conditions. Each household has a bid price curve for a given level of utility. The result of adding u to the utility function is that one of the equilibrium conditions becomes:

$$-\frac{\partial r}{\partial u} = \frac{1}{q}\left(\frac{\partial T}{\partial u} - \frac{1}{\lambda}\frac{\partial U}{\partial u}\right) \tag{15}$$

where λ is the Lagrange multiplier denoting the marginal value of money. This condition says that residential rent compensates for different travel times. Since $T_u > 0$ and $U_u < 0$ the R.H.S. of (15) is positive and the rent (of land) distance function is negatively sloping. The reason Alonso does not go the route of demand curves to analyze the land market is because each location has a different demand curve. The uniqueness of locations makes the derivation of an aggregate demand curve invalid. This problem is circumvented by the use of bid-price curves. A bid-price curve represents the prices a household is willing to pay for land in each location in order to maintain a constant level of utility. A bid-price curve is therefore derived by fixing utility and then varying distance to obtain a function.

$$b = b(u) \tag{16}$$

People with the steeper curves locate nearer the center. The market also yields a price-structure curve showing the market price (rent) of land at each location, r(u). Tangency between these determines a household's location, i.e,

$$p(u^*) = b(u^*) \quad \text{and} \quad p'(u^*) = b'(u^*) \tag{17}$$

for equilibrium.

On the production side, firms are profit-maximizing. Profit is defined by revenue minus the sum of land and non-land production costs. Alonso uses curious revenue and costs functions:

$$\begin{aligned} \text{Revenue (Volume of Business)} &= R(u,q) \\ \text{Operating (Non-land) costs} &= NL\ (R,u,q) \\ \text{Land costs} &= p(u)q \end{aligned} \tag{18}$$

The firm's bid price function is derived for each level of profits; i.e. the rent a firm is willing to pay for each location in order to make the same profits. Their location is then determined by the equilibrium tangency condition as mentioned above for households. At each location, of course, profit is maximized.

Market equilibrium is achieved when each user's land bid-price is tangent to the price structure. The price structure is the envelope of all bid-price functions. Finally supply of and demand for land should be equal. Alonso analyzes the case where each user's bid-price function is a family of parallel straight lines. He concludes that users will be ranked from the city center according to the ranking of the slope of their bid price lines - steeper ones being nearer.

Alonso's approach is largely diagrammatic though he does give some mathematical analysis. A rigorous mathematical formulation of his model would be quite complex since he allows for different tastes among households; indeed that is what produces different bid-price functions. Mills (1972) has shown that Alonso's assumptions are not adequate to produce a solution to his model. The specification of an equilibrium utility level is necessary for a solution. Particular specification of the form of the utility functions is also necessary to derive stronger implications from the model.

Alonso's work is essentially an extension of Von Thunen's (1826) theory of the values of agricultural land. He has adapted it to an urban area but runs into difficulties precisely because urban land has no intrinsic productivity differences as agricultural land does because of fertility. He thus loses one determining variable and ends up with an n-person, n-firm game. A solution to such a game needs assumptions concerning strategy, permitted coalitions, etc.

Although Alonso's model is not entirely satisfactory it is useful because it once again illustrates the difficulty of modelling urban areas even at a highly simplified level. The difficulties arise from the intrinsic nature of urban areas -- the uniqueness of each location which is created by a complex set of interdependencies.

Drawing on the work of Wingo, Alonso and Muth, Mills attempts to build simple general equilibrium models for urban structure. He has a family of models, all with similar bases but each with a different wrinkle. Here we will review the most complex and earliest of his models (Mills 1967) and then comment on the others.

He begins by speculating about the primary reason for the existence of cities and posits that non-homogeneity of land and non-constant returns to scale in production functions are sufficient to justify the existence of cities. If land is heterogeneous and some land is more productive than other land it will pay to concentrate production on the better land, thus producing a city. This can be represented in models in two ways . One is to introduce variables such as natural resources, topography and climate into the production function and the other is to have just one land input but to associate different efficiency parameters with different sites. Mills chooses the latter for the purposes of this model. Agglomeration economies of different kinds are all broadly interpreted to be scale economies and represented as such in an aggregative model.

The city is assumed to be a homogeneous plain and has 3 activities. The first is the production of goods. The goods production function has non-constant returns to scale. All goods production takes place in the CBD. This represents site advantages of the CBD.

$$X_{1s} = A_1 L_1^{\alpha_1} \cdot N_1^{\beta_1} K_1^{\gamma_1} \quad , \alpha_1 + \beta_1 + \gamma_1 = H_1 \gtrless 1 \qquad (19)$$

where X_{1s} is total output of goods produced; and subscript s denotes supply.

L_1, N_1, K_1 are the land, labor and capital inputs and

$H_1 \lessgtr 1$ represents non-constant returns.

$$X_1 = \int_{CBD} X_1(u)du \qquad (20)$$

where X_1 (u) refers to the amount of goods produced in a ring of width du, u miles from the center.

The other two activities are transportation and the production of

housing. Transportation has only one input - land - with a fixed coefficient.

$$L_2(u) = b\, X_{2s}(u) \tag{21}$$

where X_{2s} is transportation produced.

$L_2(u)$ is land used at distance u.

'Housing' subsumes all goods other than those produced in the CBD. The assumption is that all goods with non-constant returns to scale will be produced in the CBD while the others will be forced by competition to locate adjacent to customers in order to avoid transportation costs. The production function for 'housing' defined thus is

$$X_{3s}(u) = A_3 L_3(u)^{\alpha_3} N_3(u)^{\beta_3} K_3(u)^{\gamma_3}, \quad \alpha_3 + \beta_3 + \gamma_3 = 1 \tag{22}$$

On the demand side X_1 is thought of as an export good with an exogenously given price elasticity λ_1 i.e.

$$X_{1D} = a p_1^{-\lambda_1}$$

A fixed proportion δ of the workers resident at each u work adjacent to their residences in the suburbs -- presumably in housing and transportation. The demand for transportation is then

$$X_{2D}(u) = (1-\delta) \int_{k_o}^{k_1} N(u')du', \quad k_o \leq u \leq k_1 \tag{24}$$

where k_1 is the radius of the city;

k_o is the radius of the CBD;

$N(u')du'$ is the number of people living in a ring of width du' and radius u'.

Within the CBD,

$$X_{2D}(u) = \int_0^{k_o} N_1(u')du' \qquad (25)$$

$$0 \leq u \leq k_o$$

where $N_1(u')du'$ is the number of workers working in a ring of width du' and radius u'.

The demand for housing is constant per worker

$$X_{3D}(u) = N(u)x_3 \qquad (26)$$

Market Conditions

All factor markets are competitive and w the wage rate and ρ the rental rate for capital are given exogenously. Rental rate for land r(u) is exogenous.

In industry 1 we have

$$w = \frac{\partial(P_1X_1)}{\partial N_1} \quad , \quad \rho = \frac{\partial(P_1X_1)}{\partial K_1} \text{ and } r(k_o) = \frac{\partial(P_1X_1)}{\partial L_1} \qquad (27)$$

according to normal marginal productivity conditions. The competition for land is between the CBD industry and suburban uses; thus land use is determined by the rent at the edge of the CBD k_o from the center.

Land being the only transportation input,

$$p_2(u) = a_1 r(u) \qquad (28)$$

i.e. the cost per passenger mile depends only on the rent r(u) at that u, a_1 being a constant.

Housing is produced with competitive input as well as output markets. We have

$$w = \beta_3 \frac{p_3(u)X_3(u)}{N_3(n)} \quad , \quad \rho = \gamma_3 \frac{p_3(u)X_3(u)}{K_3(u)} \quad , \quad r(u) = \alpha_3 \frac{p_3(u)X_3(u)}{L_3(u)} \qquad (29)$$

and

$$p_3(u) = \bar{A}_3 \, r(u)^{\alpha_3} \qquad (30)$$

where

$$\bar{A}_3 = \{A_3 \alpha_3^{\alpha_3} \beta_3^{\beta_3} \gamma_3^{\gamma_3}\}^{-1} \, w^{\beta_3} \rho^{\gamma_3}$$

Other Conditions

The main equilibrium condition is that a worker at u cannot decrease his location costs by moving toward the city center. The decrease in transportation costs would be exactly balanced by an increase in housing costs:

$$p_2(u) + p_3'(u)X_3 = 0 \qquad (31)$$

where

$$p_3'(u) = \frac{dp_3(u)}{du}$$

This is a crucial condition and should really be derived from some maximization conditions. It has embedded in it notions concerning the disutility of transportation and relative prices of housing and transportation.

The rent at the edge of the city is given exogenously and we can assume it to be agricultural rent, i.e.,

$$r(k_1) = r_A \tag{32}$$

In equilibrium all land must be used up:

$$L_1(u) + L_2(u) = 2\Pi u \qquad 0 \leq u \leq k_o \tag{33}$$

in the CBD; and

$$L_2(u) + L_3(u) = 2\Pi u \qquad k_o \leq u \leq k_1 \tag{34}$$

in the suburbs.

Finally,

$$N_1 \equiv \int_o^{k_o} N_1(u)du = (1 - \delta) \int_{k_o}^{k_1} N(u)du \tag{35}$$

which merely makes sure that all workers live somewhere.

Solution

Even though this model is based on highly simplified notions of the structure of the city it does not have a straightforward solution. The solution should provide us with:

a. All output quantitites and prices.

b. All the input quantities and prices.

c. Parameters k_o and k_1 for the size of the city.

It is useful to note that we have a large amount of information given exogenously:

a. All the parameters in production functions.

b. Demand function parameters for goods.

c. Rental rates of labor and capital.

d. Fraction of labor force employed in the suburbs.

e. Demand per worker for housing.

f. Value of agricultural land.

Inspection of the model shows that the rent-distance function $r(u)$ for land is the critical function to be determined; from which many of the other functions can then be derived. Mills provides some interesting insights from the model but does not solve the whole model.

Within the CBD,

$$L_1(u) = \frac{2\Pi}{\lambda r(k_o)} \left(1 - e^{-\lambda r(k_o)u}\right) \tag{36}$$

i.e. the amount of land used in production increases at a decreasing rate as one goes out from the city center even though the amount of total land grows with u.

As a consequence,

$$L_2(u) = 2\Pi u - L_1(u) \tag{37}$$

hence the land required for transportation increases at an increasing rate up to the edge of the CBD. Mills notes that for a very large city $r(k_o)$ and k_o are both large and in the limit

$$L_1(u) \to 0$$

$$\text{and} \quad L_2(u) \to 2\Pi u$$

i.e. all land at the edge of the C B D is required for transportation. We can visualize this result as the requirement for a ring road around the CBD.

The implications of the growth of the city can be found by varying k_o and analyzing the results. Optimal reallocation of land will be provided -- given the assumptions of the model.

For the suburbs, i.e. the city outside the CBD Mills derives

$$r(u) = (A_o + B_o u)^{-\frac{1}{1-\alpha_3}} \qquad (38)$$

where

$$A_o = A_o(r_A, \alpha_3, X_3, k_1)$$

$$B_o = B_o(\alpha_3, X_3)$$

which shows that rent declines as one moves towards the edge of the city, but not exponentially. An exponential decline results only when $\alpha 3 = 1$, i.e. that land is the only input in housing. The implication is that factor substitution makes the land rent profile flatter. We can observe here that this is more likely in earlier stages of development when land <u>is</u> the major input in housing construction.

Finally, Mills derives the population density function:

$$\frac{N(u)}{L_3(u)} = (C + Du)^{-1} \qquad (39)$$

where C and D are functions of r_A, α_3 and k_1 i.e. rent of agricultural land, size of the city and techniques of house construction. The density is thus declining with distance but <u>not</u> exponentially as is often argued, e.g. Clark (1951).

It is of interest to compare this model with some of Mills' own later work. The later modifications are simpler in some ways but more refined in others. This model is curious in a number of ways.

a. The transportation production function has only land as an input.

b. The city is artificially divided into the CBD and suburbs. Workers in the suburbs are imagined to work on housing and transport even though transport has only land in the production function.

c. Housing has a constant per worker demand. This robs the model of part of the flexibility given by the possibility of utility optimization between consumption of space and transportation.
In a later model (Mills, 1969) he simplifies it into 2 sectors -- goods and transportation -- where goods are interpreted to include housing. The production function is made a constant returns to scale function. All the employment is not in the CBD and the center is only seen as a major transportation node through which all exports pass-- this is seen as the justification for the existence of the city. Transportation is now produced by a Cobb-Douglas production function with constant returns to scale. The demand for transportation is now linked to the production of $X_1(u)$ with the assumption that each unit of X_1 generates a fixed demand for transportation. As a result the rent of land, wages and the rental rate of capital are all linked with the cost of transportation -- which itself is a function of land rental. w and ρ are exogenously given. This model is more internally consistent than the other one and the economy is more integrated. The model is mainly used to derive the rent-distance function. It is of a form similar to the earlier model. Land rents decline exponentially if the two production functions (for goods and transportation) have equal exponents. i.e. use equal shares of land. We then have

$$r(u) = r_o \, e^{-Au} \qquad (40)$$

where A is a function of all the other parameters of the two production functions. Everything else, e.g. land use intensity (capital/acre), land used for transportation and output/acre can be derived as a function of land rent. If, then, land rent is a negative exponential function so are all the land use functions.

Finally, Mills in Studies of the Structure of the Urban Economy (1972d) suggests a complex model including congestion. Here he ignores the CBD and concentrates on the suburbs. All employment is now in the CBD; consequently, both the housing and transportation functions do not use any labor. Transportation is again produced with land only. Housing is produced with a Cobb-Douglas production function using land and capital. The demand side is richer: housing demand per worker is made price and income elastic. This way one can investigate the effects of general income changes on the structure of a city. The cost of transportation is now affected by congestion. The congestion function is taken by Mills from earlier work of others (e.g. Walters, 1961) and is really not very different from that of Wingo. It is

$$P_2(u) = \overline{P}_2 + C \left(\frac{X_{2D}(u)}{X_{2S}(u)}\right)^D \qquad (41)$$

where $P_2(u)$ is cost of transportation at u per mile (as before)

$\overline{P}_2$ is some constant (free of congestion) cost; and

C and D are parameters determined technologically by the transportation system.

Much of Wingo's work was interpreted to be the specification of these parameters. D for automobiles is believed to be about 2. The point to note is that congestion cost is seen to be a power function of excess demand. The equilibrium conditions of this model are essentially the same as in the earlier model (Mills 1967).

Mills finds that the introduction of congestion and of elastic housing demand makes the model impossible to solve analytically. The rest

of the book is devoted to a numerical solution and a demonstration of how sensitivity analysis can be performed on such a model. Some of the interesting results are:

a. Technical progress in transportation is seen to induce workers to use more transportation by moving farther out and expanding the city. Land rents fall in the city center.

b. An increase in income elasticity of housing demand increases size of total area and reduces population density -- not surprisingly. However, the magnitude of the effect is surprising: a 9 to 10 percent increase in elasticity causes a 90 percent increase in city area. The disutility of travel is perhaps not taken into full account as suggested earlier for the 1967 model. An increase in income has somewhat similar effects.

c. An increase in D in equation 41 (the elasticity of congestion cost with respect to amount of congestion) has a somewhat paradoxical result. Travel cost increases near the CBD but decreases farther out. This is because this increase amounts to "decongestion" farther out and thus people move farther away. CBD rents rise while they decrease farther out, i.e. the rent distance function increases in curvature.

Much of this model is geared to the explanation of decentralization of U.S. urban areas over time as observed empirically. Increases in incomes and population and technical change in transportation are seen as the main causes.

In summary, Mills' work is interesting because it attempts to see the urban area as a whole in general equilibrium models. It is a

further demonstration that even extremely simple models tend to be mathematically cumbersome. Each addition of complexity results in simplification somewhere else in the model. The more realistic the assumption, the more unlikely it is that an analytical solution is possible. However, an equally important demonstration is that even simple models provide us with insight into prevailing urban structures. He has also incorporated into his general equilibrium model some of the notions of the housing market from Muth, the land market from Alonso and transportation characteristics from Wingo.

These are the 'classical' urban economic models. They are 'classical' because they are pioneering attempts at modelling cities from the economist's vantage point. They are still the most influential in urban economic model building. Even many of the policy-oriented models derive much of their methodological base from these models, as will be evident in later sections. These models bear the same relationship to large economically oriented policy models as do the basic Keynesian macro-economic models to the large macro-econometric models.

The more recent and more theoretical models which have not been quite so influential are reviewed in the following section.

2.2 The "New" Urban Economics

This section reviews a spate of urban modelling work which has appeared in recent years and one which has already been dubbed as "the new urban economics" (Mills and Mackinnon, 1973). It is distinguished from earlier work by being more rigorously theoretical, with a higher disregard for reality. The attempt is to explore the extent of possible conclusions from simple formulations. While the work of Wingo, Muth, Alonso and Mills is also theoretical, it is rooted in extensive empirical work carried out by them. While little operational relevance can be derived from these new urban economic models it is of interest to review them because:

a. Some of them are the work of distinguished economists shifting from other fields.

b. They do offer some counter-intuitive results, e.g. that it is optimal to have an unequal distribution of utility even where all households have the same tastes and income.

c. They demonstrate again the intrinsic complexity of urban areas. Even highly simplified assumptions often lead to models that do not have analytical solutions.

d. The analytical innovation is in the use of control theory or the calculus of variations which is in many ways similar to the growth theory literature. Here, space is the crucial variable over which optimization of one kind or another is taking place -- like time in growth theory.

Almost all the models are monocentric. The city is in a flat plain, travel is equally costly in all directions, and all travel is from home to work. These assumptions make it possible to use one dimensional

analysis with distance from the CBD acting as the main spatial variable. The issues explored are the distribution of rent, residential density and space, consequences of travel congestion on city structure and rents, implications of individuals having different incomes usually on their location pattern, the ability of competitive markets to sustain optimum city structure. All these models are static and thus have no implications for urban growth. The reason for this is probably technical. It is difficult enough to optimize over space; adding time would make the exercise impossible.

These models will be categorized and reviewed under three headings:

a. Distribution of Land Rent, Population Density and Income

Beckmann's (1969) article "On the Distribution of Urban Rent and Residential Density" can probably be regarded as the first in the "New Urban Economics." Beckmann attempted to derive rent, population density and distribution of income groups as functions of distance. Delson (1970) and Montesano (1972) have pointed out various errors in Beckmann's analysis. Here we review the altered results. The model assumes a Pareto income distribution.

$$N(m) = Am^{-a} \tag{42}$$

where m is household income;

N is number of households with incomes greater than m

A,a are positive constants (empirically a has been found to be in the region of 2).

The utility function is

$$U = C_o \log q + C_1 \log u + \sum_{i=2}^{n} C_i \log Z_i \tag{43}$$

where q is amount of land occupied by a household;

u is distance from CBD; and

Z_2 to Z_n are all other goods.

The problem is to maximize (43) subject to the budget constraint

$$r(u)q + tu + \sum_{i=2}^{n} P_i Z_i = m \qquad (44)$$

where t is daily transportation cost per unit distance and P_i is the price of the ith consumption good.

The general problem is similar to that investigated by Alonso but with the crucial difference that everyone does not have the same income now although they do have the same utility function.

In this solution, Beckmann assumes the CBD to have a radius of 1. Delson's correction (1970) asserted that this was an unnecessary assumption. It really does not make much difference if regarded as a normalization procedure making the radius of the CBD the measure of distance. Montesano (1972) provides the correct and complete solution to the problem. Firstly, Beckmann asserts that all households with the same income will locate at the same distance. However, the first order conditions can be used to show only that all households at the same distance have the same income. We have

$$q = -\frac{tu}{\frac{r(u)}{C_o} + u\frac{dr}{du}} \qquad (45)$$

r(u) is only a function of u and so q is also only a function of u. Hence a household located at u pays the same rent, occupies the same space and has the same transportation costs and must therefore have the same income. The converse, however, does not follow.

Secondly, Montesano asserts that we need the explicit assumption $\frac{dm}{du} > 0$, i.e. that income increases monotonically with distance, to solve the model. Montesano calls this an assumption and then proceeds to show at the end of the paper that second order conditions for maxmization of utility require that this be so. Our interpretation here is that this is indeed a result of the model not an assumption. The second order conditions depend on the form of the utility function; the result therefore probably depends on the particular form used.

Montesano shows that the assumption of t=0 i.e. money costs of transport equal to zero leads to multiple solutions. We can see this intuitively as Montesano does at the end of his article after having derived multiple solutions. If t=0 we can derive an expression for the utility function which is only dependent on income; households are then indifferent to location and it is then not surprising that this assumption leads to multiple solutions. The lesson here is that it is sometimes useful to look at second order conditions before tediously solving a model. To the extent that the model tells us anything about urban patterns we can observe that in LDCs where the money costs of transport (i.e. walking to work) are indeed zero for the poor we may have greater difficulty in "optimum" urban design.

With the assumption of t > 0 Montesano does obtain a unique solution which shows:

a. That r(u) is convex but decreasing less than proportionally.

b. That q(u) is increasing, i.e. residential density declines with distance.

c. That y(u) is increasing.

All these are expressed in rather complicated functions with no simple interpretations. None of these results are, of course, surprising; but what is surprising is the complexity of the analysis given a straight-forward utility function, budget constraint and description of the income distribution. This does not augur well for the inclusion of income distribution in models of LDC cities.

b. Congestion and Transportation

The next group of models are concerned with the optimal allocation of the urban area to transportation; the costs of congestion are usually given particular attention. Solow (1972,1973) uses the standard model i.e. maximizes utility with consumption and housing space as arguments in a logarithmically additive utility function subject to a budget constraint. He derives the declining rent-distance function and concludes that "the rent profile must fall fast enough that those living farther from the center and spending on space a fixed fraction of income after transportation cost occupy more space than those living closer to the center." (Solow, 1972) Solow then introduces congestion in the following way. The aggregate width of the road network at distance u is

$$2\Pi u(1-b(u))$$

where b(u) is space devoted to housing. Then annual cost of round-trip travel per person-mile at distance u is

$$C\left(\frac{N(u)}{2\Pi u(1-b(u))}\right)^{D}$$

where N(u) is the number of people living beyond u. Total cost per person is then

$$t(u) = C\ (2\Pi)^{-m} \int_{1}^{u} \left(\frac{N(u)}{u(1-b(u))}\right)^{D} du \qquad (46)$$

where radius of the CBD is normalized as 1. This merely states that congestion costs are proportional to traffic density. This formulation is no different from Mills(1972d) (Equation 41) which itself was taken from Walters (1961). The point to be made here is that this shows explicitly some of the interdependence characteristics of urban areas. N(u) - the number of people living beyond u are dependent on t(u) which itself depends on N(u). Solow solves for the unknown functions t(u), N(u) but has to make the following assumptions:

a. The same fraction of land area is devoted to housing at every distance; i.e.,b(u) = b.

b. The typical person spends half of his total income on housing.

Both are assumptions that deprive the model of its interesting components -- in particular assumption a. Solow himself addressed this problem in a more simplified context of the long narrow city (Solow and Vickrey, 1971). The conclusion reached was that a higher proportion of land would be needed for transporation near the center of the city in the case where city size was limited.

Solow solves his model numerically and finds one interesting result. The introduction of congestion makes the rent profile more convex: the rent falls more sharply as one leaves the CBD and then less sharply near the city limits. It may be recalled that Mills' model, discussed on page 48, found a similar result. Solow investigates this result more thoroughly in the later article (1973). Assumptions (a) and (b) are both relaxed and numerical solutions obtained for different parameter values. The main result is that adding roads near the CBD flattens the rent gradient most strikingly. Since congestion is greatest near the CBD adding roads there reduces congestion costs, hence transport costs and therefore the flattening of the rent gradient, the rent

differences being transport cost differences. Solow finally does some cost-benefit calculations on the allocation of land to roads. In the absence of congestion tolls, market land values reflect differences in private transport costs, not total social costs. Land values do not fall as fast as they should and the market rent function lies everywhere below the "correct" rent function. Land is therefore undervalued and if these values were used in benefit-cost calculations too much land would end up being used for roads.

While this second article does provide some interesting numerical results these models do not provide any new insights into urban structure. Their use, perhaps, lies in Solow's pedagogical style which is a good example of a rather gradual, step-by-step approach to model building. Solow himself suggests that the model could be improved by explicit inclusion of housing in addition to land as a residential cost; the addition of time costs in transportation costs; and the existence of two or more income classes. To this we can add the inclusion of production functions for transportation and housing, i.e. a richer specification of the supply side.

More recent analyses of the congestion cost problem have emanated from the Berkeley group which is heavily influenced by the control engineering approach. Optimizing is now done from the social point of view: total costs of some kind are minimized. Since these are in the form of a city-wide integral, the problem is fairly straightforward one in the

calculus of variations. The solutions are not straightforward and economic implications are often difficult.

Mills and de Ferranti (1971) can really be said to have posed this problem first. Their concern was to find the optimum allocation of land to transportation in the suburbs in the presence of congestion. We use the more general formulation of Livesey (1973) to illustrate this class of models. The usual circular city assumptions are made with N people being given as working in the CBD. The model itself is rather simple:

$$L_1(u) + L_2(u) = \theta u \tag{47}$$

where $L_2(u)$ denotes land used for transportation at radius u;

$L_1(u)$ is land used for residence in the suburbs and for business in the CBD; and

θu is land available.

The density of workers in business is constant at a_c and residential density in the suburbs, a_s is also constant. Thus the number of people working at radius u

$$N_c(u) = a_c L_1(u) = a_c(\theta u - L_2(u)) \tag{48}$$

and the number residing at u are

$$N_s(u) = a_s L_1(u) = a_s(\theta u - L_2(u)) \tag{49}$$

congestion costs are

$$t(u) = \bar{t} + c\left(\frac{T(u)}{L_2(u)}\right)^D \tag{50}$$

where $\bar{t}$ is some constant cost here assumed zero and T(u) is number of travellers at u. This is now a familiar formulation.

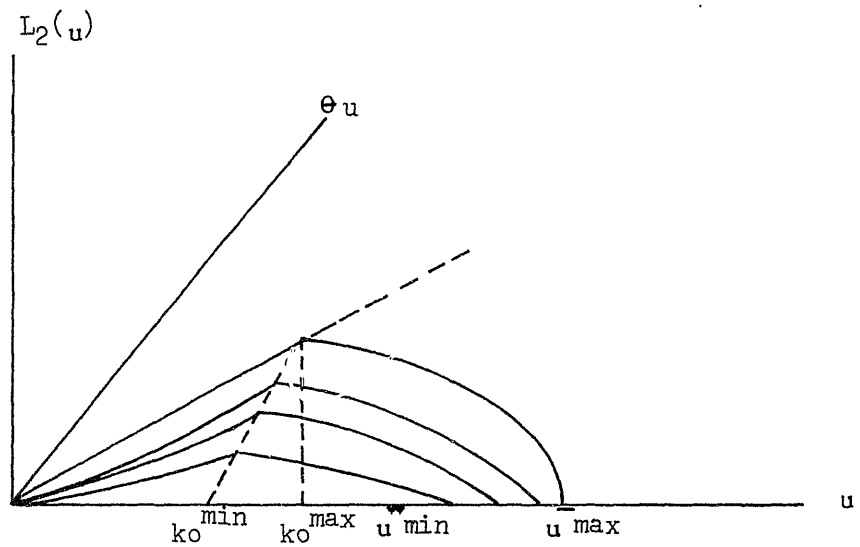

Figure 2: OPTIMAL ALLOCATION OF LAND FOR TRANSPORTATION IN BOTH THE CBD AND THE SUBURBS FOR A FIXED WORKING POPULATION (Livesey Model)

(ko - radius of CBD)

Land value is taken as a constant R_a which is opportunity cost of agriculture use regarded as the relevant alternative use.

Hence total social cost is

$$\int\{C\ T(u)\ C(u)^D + R_a\theta u\}\ du \qquad (51)$$

when congestion cost $C(u) = \frac{T(u)}{L_2(u)}$

This is the integral to be minimized subject to the given constraints. The problem is first tackled separately for the CBD and suburbs and then in a unified way for both parts of the city. The form of the solution is best seen in a diagram (Figure 2) for the analytical expressions are quite cumbersome and uninformative. Figure 2 shows that the optimal allocation of land for transportation as we move out from the center to the edge of the CBD is a monotonically increasing concave function and then a monotonically decreasing convex function until the boundary of the city is reached. The maximum is at the edge of the CBD.

This is really quite an uninteresting model for it has very little economic and behavioural content. The opportunity cost of land being taken as constant deprives the model of any pretense as a serious (though abstract) model of an urban area. It would be consistent if all business activity were equally spread over the city and all employment were local. But there would then be no transportation either. The assumptions of constant business and residential densities are equally restrictive. The model would clearly be made too complex, perhaps unmanageable if more reasonable assumptions were used. As it is it should only be called a problem in the calculus of variations and not suggested as shedding any light on the structure of a city.

A subsequent model from the Berkeley group by Legey, Ripper and Varaiya (1973) extends the analysis by allowing for substitution between land and capital. Housing and transportation are produced by Cobb-Douglas production functions using land and capital. Demand for housing and transportation is perfectly inelastic, i.e., everyone demands the same amount of each commodity. The total social cost now includes capital costs. The interest rate on capital is taken as given. Land value is again taken to be constant reflecting the alternative agricultural value. The sum of transport costs (which have the usual formulation (Equation 50)), capital costs and land costs is to be minimized for optimality. The solution gives the magnitudes of land and capital devoted to housing and transportation and the optimum size of city given the population. Two solutions are obtained: the optimal solution corresponding to what a central planning board would do and a market solution. The central result is that a market city would be more spread out than an optimal city. If, however, congestion tolls are charged, the market city could be the same as the optimal one.

This model is of somewhat greater interest even though the demand side is primitive. The criticism of using constant agricultural value holds again.

Another model inspired by Solow-Vickrey's "Long Narrow City" is Marvin Kraus' "Circular City" (1973). It is concerned with questions of optimal land use in an urban environment with particular emphasis on transportation rights of way and the pricing procedures necessary to utilize them efficiently. This model has no residential sector and

business valuation of sites reflects only travel on the city's roads. Demand for trips between any two units of business area is inelastic. The only costs of roads are the value of the land they cover and tolls can be levied on all roads in a costless way. All intersections are signalized and these are costless too. The object is to minimize total transport costs in the city incurred per unit of time. The total area to be allocated to business is given but the city's radius is to be determined as is the distribution of business area with the circunference. Each trip uses a route which minimizes the price to the trip taken which is taken to include money as well as time costs. Optimization of the signalization and toll system minimizes the total cost since trip demands are inelastic.

Radial as well as circumferential traffic is allowed in this model. Each unit area generates a demand of g trips per hour which are uniformly distributed over all units of business area. All land within a central disc, a circle of radius u_o concentric with the city's circumference (radius u_1) is allocated to a circumferential inner road.

Let $y(u)$ be area of land devoted to radial roads and $s(u)$ be the area devoted to business within the ring bounded by circles of radius u_o and u.

Now

$$f_y(u) = \frac{y'(u)}{2\Pi u} \quad , \; y'(u) \geq 0 \tag{52}$$

and

$$f_s(u) = \frac{s'(u)}{2\Pi u} \quad , \; s'(u) \geq 0 \tag{53}$$

These functions characterize the intensity with which land is allocated to alternative uses. We also have

$$y\ (u) + z'(u) \leq 0 \tag{54}$$

Traffic: Let $V_1(u)$ be volume of radial traffic through an arc of radius r and length dr; and

$C_1(u)du$ be the capacity of such an arc.

Similarly $V_2(u)$ is the volume of circumferential traffic through a radial segment of infinitesimal length dr at radius r; and

$C_2(u)du$ the capacity of such a segment.

The cost per trip mile in direction i, u miles from the center

$$AC_i(u) = f\left(\frac{V_i(u)}{C_i(u)}\right) \qquad i = 1,2 \qquad (55)$$

which is the familiar volume divided by capacity type function. Here

$$f'(\cdot) > 0$$

$$f''(\cdot) > 0 \qquad (56)$$

$$\text{and} \quad f(0) > 0$$

i.e. $f(\cdot)$ is an increasing strictly convex function and is non-zero at zero density; V_o is the hourly flow of circumferential traffing crossing any radial line segment of the inner ring road and ring road hourly travel costs are given by a function

$$2\Pi G(V_o, U_o)$$

Thus total travel costs for the city per unit time are

$$2\Pi \int_{u_o}^{u_1} \sum_{i=1}^{2} uv_i(u)\, AC_i(u)dr + 2\Pi b(V_o, U_o) \qquad (57)$$

This is the expression to be minimized.

Further specification of road capacity relates it to land and green time (when signals are green) allocation patterns. Further

$$P_i(u) = AC_i(u) + T_i(u) \qquad i = 1,2 \qquad (58)$$

where $P_i(u)$ is the price of a trip mile and $T_i(u)$ is the toll per trip mile.

Before the problem can be solved rules for trip patterns are provided.

In the solution, analytical expressions are first obtained for

$$V_1(u),\ V_2(u) \text{ and } V_o$$

i.e. the traffic follows in each of the directions.

We can summarize the final results for optimality:

a. In the absence of an inner ring road, <u>any</u> configuration of trip prices inducing travel through the city center leads to explosive travel costs.

b. Not surprisingly, toll charges should be the difference between average and marginal costs.

c. On every circle, the marginal rates of substitution of land for variable trip costs be equal in the production of radial and circumferential travel. This ensures the optimal allocation of land between radial and circumferential roads.

d. Lastly, a similar condition holds for the allocation of green time to radial and circumferential roads. The marginal rates of substitution of the value of green time for variable trip costs should be equal for travel in both directions.

This model is not noted for its realism either. It is however, a noteworthy attempt to relax the general assumption of all travel being

radial. The treatment of the two directions in travel can be extended to the modelling of different modes of travel. This would be particularly important for LDC cities where modes are, indeed, heterogeneous.

c. 'Optimal Towns'

The last group of models considered in this section are those concerned with deriving the conditions and consequences of "optimal towns." They are somewhat difficult to compare since optimality clearly depends on the welfare function used and the type and extent of constraints in the model. The welfare function reflects the moral or other preferences of the modeller while the constraints constitute his conception of the city.

Mirrlees (1972) is the orginator of this group of models and his approach is somewhat different from the others. He poses the problem in almost the simplest form possible. The welfare function is the sum of all individual utilities. Individual utilities depend on consumption of goods, space of residence and location:

$$U = U(x,q,u) \tag{59}$$

where x is consumption of goods other than housing;

q is amount of space used in housing; and

u the distance from the center is the location variable.

We may recall that this is the same formulation as Alonso (1964). Further,

$$\begin{aligned} q &= q(u) \\ \text{and} \quad x &= x(u) \end{aligned} \tag{60}$$

are assumed but are also required for optimality. The main conclusion of the paper is that optimal allocation requires that utility is a function of distance. We have the seemingly surprising result that with identical individuals

welfare maximization requires unequal treatment except in some special cases. In fact, this turns out not to be so surprising, because u is included explicitly in the utility function. As Mirrlees shows, when u is not explicitly included, it is possible to achieve equal utility. The problem arises because identical individuals have to be placed in different locations. They can all have the same utility if some trade-offs are possible. If, for example, more space is traded off against higher transport costs equal utility becomes possible; but explicit preferences for location have to be dismissed. A richer specification of locational preferences which can be traded would also allow equal utilities. When location is dependent on u alone only a special case where the rent gradient and the utility function are such that the changing consumption of goods and housing are always peculiarly balanced against distance changes allows equal utilities.

This is an interesting theoretical exercise even though the nature of an urban area is in a very rudimentary form: distance from the center is the all important variable. Otherwise the problem is straightforward utility maximization (summed to form the social welfare function) with budget constraints. It is of interest because it shows forcefully that:

a. Cities imply inequality even if all individuals are identical and if the city is uniform.

b. With identical individuals this is possible to mitigate only if locational preferences are more complicated than distance from the city center. This implies that the city would then be less homogeneous.

c. As a corollary, we can also have equality of sorts if people have different utility functions; equality is then difficult to define operationally except by income.

Riley (1973) has a similar model but includes the number of leisure hours explicitly in the individual's utility function. Since these vary with transportation time, distance is now a "pure" location preference variable in the utility function. It does not include the disutility of commuting. Riley makes the social welfare function rather more egalitarian by making it logarithmically additive, i.e.

$$W = \prod_{i=1}^{N} U^i \tag{61}$$

where W is total social welfare;

U^i is the utility of individual i; and

N is the population of the city.

Riley finds that utility increases exponentially with distance i.e. optimality requires unequal treatment of identical individuals even with an egalitarian social welfare function. The reason here is not quite the same as in Mirrlees' model since distance is now a "pure" location variable in the utility function, depending on the parameters

$$\frac{\partial U}{\partial u} > 0$$

or

$$\frac{\partial U}{\partial u} < 0$$

in this problem. Riley's explanation of his result is that the fact that an individual can live at only one location (and not at two) causes a non-convexity which is not present in the usual case of utility and

welfare maximization. Here everyone does have the same marginal utility of income but since everyone cannot have an identical consumption bundle (nor can he have an identical total utility level) the degree of inequality is primarily governed by the elasticity of utility with respect to distance. Riley uses

$$U = x^{\alpha} q^{\beta} h_3^{\gamma} u^{\delta} \qquad \alpha, \beta, \gamma > 0 \qquad (62)$$

where h_3 is number of leisure hours and the other symbols are the usual ones. Inequality is an increasing function of δ. If this is positive, the degree of inequality decreases with β -- the elasticity of utility with respect to residence area. If δ is negative the degree of utility increases with β. Congestion costs are not considered and transport costs are linearly proportional to u in this model.

Dixit (1973) and Oron, et.al. (1973) have the same concern as Mirrlees and Riley but have more developed models of the urban area. Both use substantively similar models, Dixit following Oron, et. al.

Dixit's main theme is that optimum city size is determined by the balance between economies of scale in production and diseconomies in transport, congestion being an important part of the latter. We can summarize the two models, as follows:

Goods are produced with increasing returns to scale

$$X = AN^{\alpha} L^{\beta} \qquad (63)$$

where $\alpha + \beta > 1$, $0 \leq \alpha, \beta \leq 1$ and N is number of man-hours worked. (Oron, et. al. have only labor as input since they fix CBD size).

Housing and transportation are produced with land only: $L_1(u)$ and $L_2(u)$ respectively at distance u.

Dixit's major additions are:

a. He includes congestion in the conventional way, i.e. a power of traffic density plus some constant as in Equation (41) (Mills). Oron, et. al. assumed that these costs were directly proportional to traffic density.

b. The individual's utility function is

$$U(u) = x^{\sigma/1+\sigma} q^{1/1+\sigma} \tag{64}$$

where $x=x(u)$ and $q=q(u)$ follow our usual symbols. Oron, et. al. assume $\sigma=1$.

This form of the utility function means that expenditure on goods and housing occurs in the proportion $\sigma:1$ with a given income and prices. Dixit's point, which is well taken, is that σ is at least 3.

Workers supply a fixed number of hours devoted to work and commuting. It would be more realistic to assume that work hours are fixed .

Dixit's social welfare function is

$$\int_{k_o}^{k_1} - U(u)^{-m} \{-n'(u)\} \tag{65}$$

where $m > 0$,

$n(u)du$ is the number of people between u and $u + du$;

k_o is the radius of the CBD, and

k_1 is the radius of the city.

m is a parameter which controls the level of inequality in the optimum allocation. A higher m means less inequality and in the limit, $m \to \infty$ means full equality. Oron, et. al. constrain their model to this case.

Dixit derives analytical functions for traffic density, residential density and utility level, all in terms of consumption x(u). x(u) is then expressed in terms of u for which analytical expressions are obtained for the case of pure congestion costs in travel.

Residential density: $\text{const } (C_1 + C_2 u)^{-\{1 + \frac{a}{D(a-1)}\}}$ (66)

where C_1, C_2 are constants;

D is the exponent in the congestion function and

$$a = \frac{D\ (1+m)\ (1+\sigma)}{(1+D)\ m}$$

This (66) is a negative exponential form if a=1 but this case has no straightforward interpretation. Solow regards (66) as the more general form. Dixit's innovation over Mills (1967) is in making the land proportion used in housing and transportation endogeneous and in using a social welfare function. According to Dixit, a more developed housing production function yields similar results.

The rent function

$$r(u) = \text{const } (C_1 + C_2 u)^{-\frac{(1+D)a}{D(a-1)}} \tag{67}$$

and

$$\text{traffic density} = \text{const } (C_1 + C_2 u)^{-\frac{a}{D(a-1)}} \tag{68}$$

are again negative exponential if a=1.

Finally,

$$U(u) = \text{const } (C_1 + C_2 u)^{-\frac{1}{m(a-1)}} \tag{69}$$

which says unambigously that

$$\frac{dU}{du} > 0$$

i.e households located farther away have higher utility. The degree of inequality, of course, depends on m. Only $m \to \infty$ implies equality; hence more usual values like m = 1 imply considerable inequality.

Dixit's model is important in several respects:

a. It is the most developed model of its kind combining the approaches of Mills and Mirrlees, i.e., general equilibrium and optimality.

b. More variables are made endogenous than most models.

c. Analytical results are obtained even with congestion.

d. The allocation of income between housing and other goods is realistic.

It would be useful to bring capital into a similar model to get a "more" complete general equilibrium analysis. The curious assumption of fixed leisure hours should be dropped for fixed work hours. It is difficult to see how the assumptions affect the results.

Dixit does some numerical calculations on his model. He shows how transport costs are crucial in determining optimum city size. Lower transport costs are instrumental in making possible larger cities which allow greater advantage from economies of scale.

2.3 Summary

This section has reviewed the main strands of the development of urban economic models over the past decade and a half. There was almost no work of this kind before this period.

It is worth noting that there is a surprising unity of concerns among the different models that have been reviewed. Almost all these models investigate optimal residential location - either from the point of view of the household itself or for maximization of social welfare as expressed in a welfare

function. Particular attention is paid to the operation of the land market and the effects of congestion in transportation. Here, the salient features of these models are brought together to give a better idea of what has been gained from them.

Muth illustrates how relatively simple economics can be utilized to understand the structure of urban areas - in particular the housing market. He maximizes utility which has only housing and other things as arguments: location is not an explicit argument in the function. He finds that a consequence of market equilibrium is declining rent gradient with distance from city center. He also shows how capital/land substitution operates to make the housing price gradient much less (by an order of magnitude) steep than the land price gradient. Wingo concentrates on transportation and illustrates the complexities of urban modelling by deriving a plausible specification of a transportation cost function. He also obtains a decline in rent gradient as a consequence of transportation costs. Alonso turns the problem around and emphasizes the uniqueness of each urban location. This argument makes it invalid to derive aggregated demand functions for urban land. The consequence of this argument is that a 'pure' location variable must be included in the utility function. The implications of doing this are brought out in later work by Mirrlees, Dixit and others who show that this characteristic of urban land makes inequality inevitable if everyone has the same utility function. A corollary is that different utility functions would make equality possible.

Mills' contribution was to bring together many strands of work in a general equilibrium model of a city. His conclusion also is that the land rent profile is crucial to the allocation of activities within a city, that rent declines with distance from the city center and that factor substitution makes the land rent

profile flatter. He also finds that congestion makes the land-rent distance function more convex - a finding corroborated in later work by Solow. Mills also demonstrates that even a highly simplified urban general equilibrium model becomes mathematically cumbersome very quickly. Beckmann extends earlier work by positing an income distribution function for households and concludes that income increases with distance as Muth had by comparative static analysis along with additional assumptions. Beckmann's work also illustrates how a relatively simple income distribution function makes the mathematics cumbersome. Solow rediscovers the rent-distance relation but his contribution is in a lucid exposition of how a step by step approach to urban model building can be followed. In a benefit cost framework he finds that if congestion is neglected land would be undervalued. This conclusion is taken further by Legey, Ripper and Varaiya, who conclude that a market city is more spread out, if congestion costs are not somehow internalized by the actors in the urban market. Kraus' circular city makes a significant attempt to allow other than radial travel and the resulting model illustrates the costs of adding such simple attributes of reality into a model.

Though these models are at a sufficiently theoretical plane they should be regarded as conceptual building blocks towards more operational models. Each of the concerns exhibited in these models: the land/transport trade-off; the uniqueness of location; the effect of transport congestion on city form; the consequences of egalitarian welfare functions, etc. is a real problem which has first to be dealt with at a general level before operationalization into policy models.

In concluding this discussion of theoretical urban economic models we can remark that higher levels of generality and general equilibrium type models do yield some insights into urban form, as distinguished from particularistic and/or partial equilibrium models. This is what we would expect from highly complex and inter-related phenomena.

3. OPERATIONAL OR POLICY-ORIENTED MODELS

This section reviews, cursorily, some of the work done on policy-oriented models over the past fifteen years. These models are rather more difficult to review than the explanatory models because of their sheer size. In a paper such as this it is not practicable, nor of benefit, to present all the technical details. Indeed, it is difficult to present them technically at all because of the complexity of their notation. Here our objective is to appreciate the essence of the methodology used in each of these models rather than achieving a detailed understanding of each. Thus technical details are provided wherever it is considered necessary for this objective and symbolic notation is used only when it facilitates exposition.

The models reviewed in this section are mostly large in the sense that the only practical way to operate them is on a computer. They are spatially disaggregated to a greater or lesser extent and allocate activities to geographic zones. They pertain to metropolitan areas and their concerns are intra-metropolitan. Regional models are not considered here.

Such models have had a checkered history. They came to the fore in the early 1960s in the U.S. as concern over the declining central cities mounted and various planning solutions were sought. It was thought that these models would help planners in their professional roles as advisors to public decision-makers, with emphasis on objective plan evaluation. It was also expected that they would have an educational role in developing better theory of urban spatial structure as well as in giving planners as well as decision-makers more systematic ideas of urban areas. These models have

failed with respect to the first objective and been only partially successful with respect to the second. Great disillusionment had set in in the U.S. by about 1968, but the challenge was enthusiastically taken up across the Atlantic in Britain at about the same time. Meanwhile, the new urban modelling in the U.S. has been done more by economists than others.

Most urban models are focused on land utilization, the types of structures erected on the land, the prices of the land and structures, the types of households which occupy these structures, and the impact of changes in the transportation network on this system. Part of production and employment often called 'basic' or 'export-based' is regarded as exogenous. The number and types of households in the city and their living place locations are derived from the number and types of workers and their workplace locations. This is usually the most important part of large policy-oriented urban models.

While by no means exhaustive this section reviews some of the major strands of policy-oriented modelling. The Lowry model is presented first since it was the first of its kind and is still regarded as the high point of modelling experience in 15 years. Many of the distributions generated by it were based on the gravity concept of interaction. Later developments in Britain have used the entropy maximization technique to give better theoretical basis to these techniques. The concept of entropy maximization is therefore introduced in an elementary way to facilitate understanding of the basis of the models that follow. These are models developed by Marcial Echenique and his associates.

These models probably represent the highest development of the Lowry framework and are representative of many such efforts in Britain as well as the U.S. Moreover, they are of special relevance to us because they are among the very few comprehensive modelling efforts so far attempted

in the LDCs. The NBER model reviewed next is the most ambitious effort based on the use of behavioural relationships--mostly economics-- that has yet been developed. This model derives much of its rationale from the 'classical' economic models discussed earlier. Finally Edwin Mills' policy-oriented planning model is reviewed. This is an optimizing model using a mixed integer programming framework.

3.1 The Lowry Model

The model was developed as part of a study of the Pittsburgh region with the purpose of aiding the regional planning effort. The objectives of this model are best stated in Lowry's words:

> "The object of this research has been the development of an analytical model capable of assigning urban activities to sub-areas of a bounded region in accordance with those principles of locational interdependence that could be reduced to quantitative form. The model is not designed to project regional aggregates such as total employment or population, but rather to allocate such aggregates to locations within the region. Properly adapted, it should be useful for the projection of future patterns of land development and for the testing of public policies in the fields of transportation planning, land use controls, taxation, and urban renewal."
>
> (Lowry, 1964, p.2)

The model has 3 sectors: a basic sector, a retail sector and a household sector. The basic sector is the export sector whose employment and location is not affected by local events. These are activities whose location and employment levels are assumed to be given. The retail sector has local clients whose employment levels and location are closely tied in with access to local residents. The location of households is powerfully influenced by the residents' place of work. In

addition, the location and number of households also depend on the location of retail establishments and vice versa, i.e., they are interdependent. The structure of the model is therefore quite simple. It follows the methods of social physicists more than those of economic theorists. In other words, it seeks to replicate the urban environment by observing statistical regularities rather than explaining them. The main principle used in location of retail enterprises and distribution of households is an analogue to Newton's law of gravity.

The level of interaction is directly proportional to the mass of interacting bodies and inversely proportional to the distance between them -- usually the square of distance.

The city was divided into a grid composed of one mile squares and these were the smallest areas that the model handled. The model distinguished four types of land use:

$$A_j = A_j^U + A_j^B + A_j^R + A_j^H \qquad (70)$$

where A_j is area of tract j;

U refers to unusable land;

B to area used by the basic sector;

R to retail sector; and

H to household sector, i.e., residential

A_j and A_j^B are given as is the employment provided by A_j^B i.e. E_j^B.

The retail sector is divided into types of establishment each of which has an employment function of its own:

$$E^k = a^k N \tag{71}$$

i.e. population N of the city generates employment E^k for type k retail establishments. For each tract j

$$E_j^k = b^k \{ \sum_{i=1}^{n} \left(\frac{c^k N_i}{T^k_{ij}}\right) + d^k E_j \} \tag{72}$$

This is in many ways the central part of the model. The size of establishments of type k in tract j is determined by accessibility of households over the whole city but only local employment in the tract E_j. T^k_{ij} is a measure of distance between tracts i and j and N_i is population of tract i. a, b, c, and d are constants. This says that the likelihood of household shopping trips declines with distance and market potential for a tract is a weighted index of the number of households in surrounding tracts. Locally employed individuals make only short range retail trips.

$$E^k = \sum_{j=1}^{n} E_j^k \tag{73}$$

$$E_j = E_j^B + \sum_{k=1}^{m} E_j^k \tag{74}$$

i.e. total employment in tract j is a sum of basic and total retail employment.

The land A_j^R occupied by the retail sector in each tract is then determined through an exogenously specified employment density coefficient (e^k) for each type of establishment. Thus

$$A_j^R = \sum_{k=1}^{m} e^k E_j^k \tag{75}$$

Total population is simply a function of total employment in the city.

$$N = f \sum_{j=1}^{n} E_j \tag{76}$$

The number of households in each tract is a function of that tract's accessibility to employment opportunities.

$$N_j = g \sum_{i=1}^{n} \frac{E_i}{T_{ij}} \tag{77}$$

Total population, is, of course, the sum of tract populations

$$N = \sum_{j=1}^{n} N_j \tag{78}$$

The model then has some constraints to control establishment size and densities.

$$E_j^k \geq Z^k \qquad \text{or} \qquad E_j^k = 0 \tag{79}$$

The size of type k establishment must be greater than some number Z^k.

$$N_j \leq Z_j^H A_j^H \tag{80}$$

places a constraint on maximum residential density for each tract (which may vary from tract to tract).

$$A_j^R \leq A_j - A_j^U - A_j^B \tag{81}$$

restrains the amount of land used by retail establishments to that available.

The model is shown to satisfy the necessary conditions for solution i.e., the number of unknowns is equal to the number of equations, and a solution method is suggested using the constraint inequalities.

We note that even such a simple model with few behavioural relationships is quite demanding in terms of data and computer capacity. The city was divided into 456 tracts, it had 1.5 million people divided into 448,000 households and 550,000 jobs. It distinguished between 5 land uses: basic, residential, retail, unusable and agricultural or vacant. It was found that retail trade had to be clustered into only 3 types: neighborhood facilities like food stores and gasoline services; local facilities like eating and drinking places, medical and health services, etc.; and metropolitan facilities with larger versions of local facilities like department stores, financial services, etc. Almost all manufacturing was regarded as basic. A great amount of data were needed to generate trip distribution functions. Space use standards had to be derived to generate estimates of area demanded by retail employment. It is clear that such data are difficult to find in developing countries. Furthermore, gross coefficients (e.g. space standards) would be difficult to observe because of a far greater heterogeneity in types of retail establishments.

The model was successful in generating plausible co-distributions of employment and residential population given its very simple structure and methodological underpinnings. Lowry himself is very cautious in claiming usefulness of the model and really regards it as a first generation effort leading to better work. Its map of the city is filled partly by hand and only partly by its own structure. It is not easy to transplant from one environment to another since the structure of the model is very sensitive to the data base on which it is built.

While the Lowry model was seen to have great promise it has not been possible to use it operationally in many places. Goldner (1971) reviews the aftermath of the model in an appreciative vein but really ends up hoping that future models could be more useful. The descendants of Lowry's model observed the basic/retail dichotomy; the causal chain from basic employment, to residential population to retail employment; and the multiplier relationships of all other employment to basic employment. Despite a great amount of modelling effort in the U.S. it is remarkable that only one reached anywhere near operational use. Innovations suggested have ranged from higher disaggregation of tracts and model parameters to finer specifications of household and employment types.

Most of the operational models have emerged in Britain where it is worth noting that the number of tracts used in most models is in the region of 100 as compared with Lowry's 456. Wilson (1974) and Batty (1972) provide a good review of the theoretical and practical developments in this field of modelling in Britain: though there was an explosion of model building based on the Lowry framework in the U.S. too it was largely unsuccessful and few models, if any, reached the operational stage. Useful reviews of these developments are found in Kendrick (1972) and Brewer (1973) in addition to Goldner (1971) mentioned above. It would appear that American disillusionment, in large measure, resulted from unrealistic expectations about what could be quickly learned from urban simulation models, serious underestimates of the difficulties of constructing truly useful models and the lack of an appropriate and long term financial commitment to their development. Being tied to policy and planning requirements of particular communities and studies, virtually all efforts to date have had to deal with unrealistic deadlines and other limitations. Judging from the British experience one could also say the opposite:

there has been too much money available for the development of these models in the U.S. The result has been unwieldy models which never succeeded in being operational for policy use. There were therefore many disasters and the model 'movement' died in 1968. Many obituaries and post mortems have been written[1/] but D.B. Lee's (1973) is, perhaps, the most insightful.

In Britain, on the other hand, descendents of the Lowry model only started to appear around 1968 and were developed by Michael Batty and A.G. Wilson under the auspices of the Centre for Environmental Studies in London. Practical work on the models has been proceeding in concert under the direction of Lionel March and Marcial Echenique and their associates at the Centre for Land Use and Built Form Studies in Cambridge. They have been made operational by the Cambridge group in 5 towns in England - all of which have been of less than 500,000 population. In Britain it is conjectured that constraints of time, money and computer use had led to models which are smaller and therefore easier to put into effect. Even so it is not clear from any of the documentation if they were actually used for policy purposes.

Batty (1975) has estimated that total resources expended in urban modelling in the last 8 years in Britain have not amounted to more than $600,000. About 20-30 models have been developed during this time. If this estimate is correct, the achievements have been truly remarkable. A review of British modelling is found in Batty (1972).

The Echenique group has gone on to develop models in Latin America, starting with Santiago and Caracas, and is now developing one in Sao Paulo. These models are extensions of their work in the 5 towns in Britain and are essentially Lowry derivatives with Wilson's entropy maximization formalism. We review the Santiago and Caracas models in some detail but present a simple introduction to entropy maximization techniques before proceeding further.

1/ See Brewer (1973), Department of Transportation (1973) among others.

3.2 Entropy Maximization

A.G. Wilson introduced the concept of entropy maximization to urban modelling in the context of Lowry type models. He wanted to find a better theoretical basis for the use of gravity models in distribution and allocation. He explains and develops this in his 1970 publication. The gravity model is based on a Newtonian analogy. A characteristic function of urban models is to find levels of interaction between spatially separated zones, e.g. the pattern of movements of people or goods between zones. The gravity model posits the interaction between zones i and j as proportional to each of a mass at i, a mass at j , and inversely proportional to some function of the distance (or travel cost) between them -- such as equation (72) -- in the Lowry model. The gravity analogy deals in aggregates and the formulation is deterministic. Entropy maximization deals directly with the components of the system of interest and obtains interactions as statistical averages. If we are interested in the journey-to-work, the gravity model takes a residential population in zone i and jobs in zone j as its masses; the entropy maximizing method deals with individuals, assesses their probability of making a journey and then obtains statistical averages. This formulation is therefore probabilistic.

The aim of this section is to give an intuitive understanding of the entropy maximizing method and of its role in urban modelling. Before this can be done, entropy itself needs to be defined and purged of its thermodynamic connotations.

Entropy is a precise mathematical concept measuring the "amount of uncertainty" represented by a probability distribution. Nothing more nor less can be read into the concept. Given a probability distribution:

$$p_1, \; p_2, \; \ldots\ldots\ldots\ldots p_n$$

associated with a random variable

$$X_1, \; X_2, \; \ldots\ldots\ldots\ldots X_n \qquad (82)$$

$$S = - \sum_{i=1}^{n} p_i \ln p_i$$

is the entropy of the system. It is a unique measure of the amount of uncertainty in the given distribution. It measures how uniform a distribution is. Our intuition or common sense tells us that a uniform distribution has a large amount of uncertainty. Furthermore, the higher the number of possible states, i.e. larger the n, the more uncertainty there is. The probability of any state occurring is $\frac{1}{n}$ i.e. $p_i = \frac{1}{n}$ (83)

Entropy should therefore be a monotonically increasing function of n. Equation (82) obeys this rule as well as some other conditions which make it a unique, unambiguous measure. Any change in the direction of equalizing the different probabilities will increase the entropy.

Maximizing entropy amounts to saying that we want to make the distribution as uniform as possible subject to whatever constraints exist. The most familiar use of entropy is in thermodynamics. The characteristic problem there is that some average level of energy, say $\overline{E}_i$ in a system is known; there are many different quantum levels E_i and we want to assign probabilities to each of these quantum levels. This problem is solved by maximizing entropy subject to the condition that the expected value of E_i is $\overline{E}_i$. The solution to this constrained maximization problem assigns these probabilities to each quantum level. What the method achieves is that, given the mean value, it provides the most uniform distribution possible.

Another example presents a more intuitive understanding of the entropy maximization technique. Suppose there are 1000 cars parked bumper to bumper and they occupy the full length of, say, 3 miles. We also know the total weight of these 1000 cars. We can also find out the length and weight of each make of car that may be in this cluster of cars. The problem is: given only this information, can we make any inferences about the number of cars of each make that are in this cluster. We can convert this into an entropy maximization problem and the solution will give us the most likely distribution of makes. If n_i is the number of cars of make i, N the total number of cars, then the distribution of

$$\frac{n_i}{N}, \quad i = 1, \quad \ldots m$$

where there are m makes, is the distribution we are looking for. The total weight and length of all cars and the weight and length of each make is the information we have. This information comprises the constraints to the entropy maximization problem.[1/]

The function of the technique is therefore to provide the most plausible unbiased distribution given rather sketchy information. It makes sure that no other information or bias other than that subsumed in the constraint set occurs in the predicted distribution.

In the context of urban systems, the use of entropy maximization is well illustrated in relation to a system dealing with movements of people, for example the journey to work. A state of the system is defined as an assignment of individual persons to the movement channels in the

1/ This method of presentation is taken from: E.T. Jaynes "Probability Theory in Science and Engineering" Dallas, Texas: Feld Research Laboratory, 1958.

system such that it does not violate any of the constraints on movements. A distribution of the system is a macro-property of the system -- a distribution of total movements regardless of the movements of individual persons. There are three levels of resolution in this analysis. First, the micro-state is the assignment of each individual to particular work trip categories. Second, the meso-state is the number of individuals going from each origin i to each destination j. Third is the macro-state which describes only how many people work in each destination j and how many live in each origin i. Many combinations of micro-states can give rise to the same meso-state and many combinations of meso-states can give rise to the same macro-state. This problem is rather similar to the car problem described above. Given a macro-state we can use entropy maximization to find the most likely distribution of meso-states and given a meso-state we can predict the most likely micro-states. Since many states of the system can form one distribution, then on the assumption that all the states are equiprobable, the model is based upon the most probable distribution of person movements subject to any constraints.

Generating a journey-to-work distribution through the use of entropy maximization amounts to saying: if we repeatedly ask the population of a city to choose work and residence locations aimlessly (or randomly), though subject to some constraints, the distribution that we get most often is the distribution that entropy maximization provides. The constraint set can, of course, contain information which divides people according to income classes, race, employment type etc.: analogous to the information on length and weight of each make in the car example above. The journey-to-work distribution that is obtained is in this sense not totally random. However, beyond the constraint set, the 'randomness' prevails and it is this implication

of the entropy maximization technique that is seen here as the most problematical in the context of urban modelling. Even within classifications of employment, income, race, etc., people do not locate themselves aimlessly but through the operation of some preference functions (implicit though they may be), and subject to the external market forces. If all this information can be included in the constraint set of the entropy maximizing procedure the same result is obtained. That this is unlikely will now be demonstrated mathematically. To this it should be added that intuitively this would appear to be an extremely difficult task, if indeed it is possible at all. In any case in practice thus far, the constraint set has seldom included any preference functions and even the market information included has been rudimentary. Thus, characteristically, entropy maximization models which utilize mean values to generate various location distributions can scarcely be regarded as having any predictive value.

Mathematically, entropy maximization does the following:

Given a random variable

$$x_1, x_2 \ldots\ldots x_n$$

with probability

$$p_1, p_2 \ldots\ldots p_n$$

We maximize entropy.

$$S = -\sum_i p_i, \ln p_i \tag{85}$$

$$\text{s.t. } \sum p_i f(x_i) = E\left\{f(x)\right\} \tag{86}$$

$$\text{and } \sum p_i = 1 \tag{87}$$

This gives

$$p_i = \exp\left\{-\lambda - \mu f(x_i)\right\} \tag{88}$$

$$\text{and} \quad e^{\lambda} = \sum \exp\left\{-\mu f(x_i)\right\} \tag{89}$$

$$(\text{since } p_i = 1)$$

where λ and μ are the Lagrange multipliers.

This shows that if we know averages we can generate trip distributions. This is the basic entropy maximization procedure: clearly the number of constraints can be increased to include more information.

The following now demonstrates the conditions in which a utility maximizing technique gives similar results. This treatment follows Wilson (1970).

In the standard utility maximizing problem, we maximize:

$$U = U\ (X_1, X_2, \ldots, X_n, M) \tag{90}$$

s.t.

$$x_i p_i = M \tag{91}$$

where U is the utility of the consumer

x_i are amounts of goods consumed

p_i are prices

and M is the income of the consumer

To obtain a solution we maximize , get first order conditions, solve the resulting equation system and then find the optimal quantities.

If we now define:

$$y_i = \frac{x_i\ p_i}{M} \tag{92}$$

y_i can be a probability distribution and we have

$$S = - \Sigma y_i \text{ In } y_i \tag{93}$$

We can also write and maximize

$$U = U\left(\frac{y_1 M}{P_1}, \frac{y_2 M}{P_2}, \ldots, \frac{y_n M}{P_n}, M \right) \tag{94}$$

$$\text{S.t.} \quad \sum_i y_i = 1. \tag{95}$$

and find a solution

$$y_i = y_i \quad (p_1, p_2 \quad \ldots, p_n, M) \tag{96}$$

which defines the same system as the standard utility maximization system.

Now let there be constraints

$$f_k \quad (y_1, y_2, \ldots \quad y_n) = g_k \qquad k = 1, \ldots m \tag{97}$$

and maximize

$$S = -\sum y_i \ln y_i \tag{98}$$

s.t.

$$y_i = 1 \tag{99}$$

and constraint set above (97)

Wilson then suggests that maximizing S is equivalent to maximizing

$$U = S + \sum \mu_k (g_k - f_k) \tag{100}$$

$$\text{s.t.} \quad \sum y_i = 1 \tag{101}$$

Here under certain conditions μ_k are the Lagrange multipliers of the earlier maximizing S problem.

If entropy S plays no role in the utility function this will exhibit itself in two ways:

(i) The parameters μ_k will be large compared with S, thereby reducing S to insignificance;

or (ii) There will be as many constraints (components of the utility function) as there are variables y_i, in which case the set of constraint equations can be solved directly for the y_i s without reference to entropy.

What is really being said here is that if we put all the information that we have in a utility function into constraints in the entropy maximizing problem the solution will be the same. To understand this better: if a utility function orders behavior in a rather deterministic fashion and this can be transformed into a constraint set there will be little allowance for uncertainty in the system. Hence maximizing a measure of uncertainty (entropy) would make no difference and the same answer would be obtained. The argument is therefore that if the utility function does not have this kind of information then uncertainty should be taken account of: entropy maximization does this while utility maximization does not. Therefore the former is a more general procedure.

The problem is that it may not always be possible to transform the information in a utility function into a usable constraint set for entropy maximization. If this in fact can be done there would be no difference in the solution and there would be no meangingful choice. However, maximizing entropy in a system which is completely determined by the constraint set serves no purpose. Thus the correspondence between utility maximization and entropy maximization is illusory: it happens only when entropy maximization loses its meaning.

It is hoped that the above gives an intuitive as well as partial mathematical understanding of the entropy maximization approach. It gives some idea of the role of mean values and of the various parameters that are used in distributions in the Echenique et. al models that are reviewed next. Some basis is also provided for the methodological criticism that is offered for these models.

3.2 The Echenique et al Models

Some of the main attempts to apply modelling techniques to L.D.C. cities have been made by Marcial Echenique 1/ and his associates in cities in Latin America. They have built and calibrated models for Santiago and Caracas and are now doing so for Sao Paulo. These models are essentially built around a Lowry framework embellished by entropy maximization distribution techniques while each model has its special characteristics and objectives. Here we describe in some detail the Caracas and Santiago models though technical detail is kept at a minimum. The Sao Paulo model is the latest one being developed and no documentation is available as yet .

These models are extensions of the urban stock and activity model developed at Cambridge (Echenique et al., 1969) which had been applied to new towns in Britain. The Santiago model expands the original framework of the Cambridge model by modelling the inputs to the urban model ; that is, the location of basic employment, the transport cost and parameters such as the labor participation rate and service employment ratio. This is done by coupling the urban model with a regional model and a detailed transport model. The original model gives at every time interval the changes in employment by economic sectors and the population change. The transport model interacts with the stock and activity model determining the level of accessibility in each zone by a detailed assignment of vehicles to the networks.

The Caracas model attempts to disaggregate the urban model in order to explore policies related to income distribution and squatter housing. In

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1/ Of the Martin Centre for Urban and Architectural Studies, Cambridge University, England.

order to do this an economic framework is used to explain the urban processes: given the location of basic employment, the socio-economic groups and income groups of these employees are calculated. The income of workers determines the housing type and transport that they can afford. The model attempts to simulate the operation of a simplified land market, establishing rents and land values by competition of different land uses.

The Santiago Model

The structure of the model is illustrated in Fig.3.

The Regional Model predicts changes in population given birth and death rates, migration data and demand for labor in each region. Employment is regarded as proportional to the amount of investment which is considered exogenous. In this case only public sector investment is taken into account since it comprises 77% of total investment and is, perhaps, more predictable. The output from the regional model is fed into the intra-urban model at each time period. The results of different investment policies e.g. different allocation mixes between regions can be tested. The model gives employment in each region by different sectors: agriculture, mining, manufacturing and administration.

The Intra-urban model is divided into various parts:

(i) Basic Employment Model: This distributes the total basic employment given by the regional model through zones and employment sectors. Agricultural employment is a function of

- agricultural land available
- fertility
- accessibility to consuming zones

and is subject to maximum density constraints.

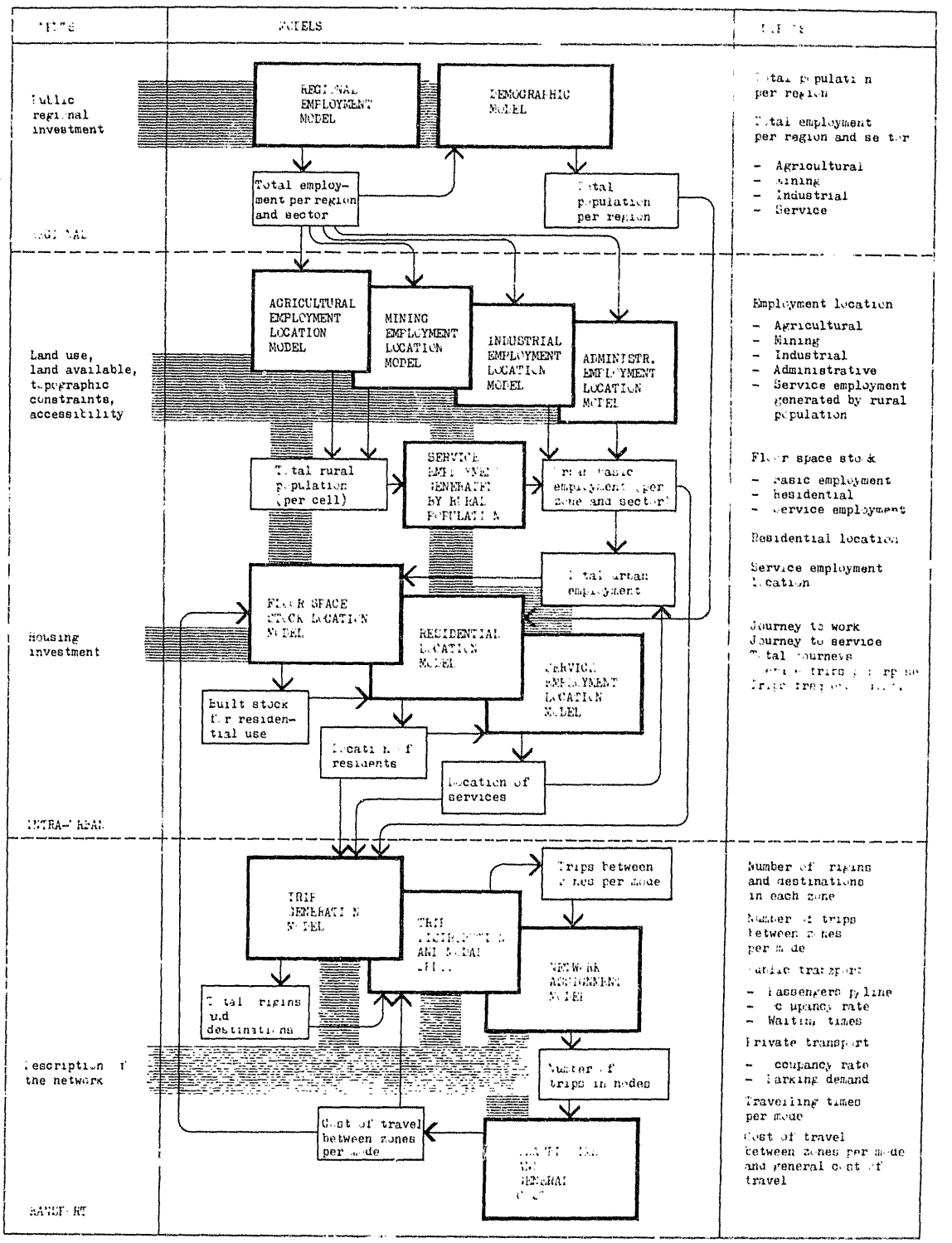


Figure 3: THE STRUCTURE OF THE SANTIAGO MODEL

(ii) Industrial employment is a function of

- industrial land available
- regional accessibility of zones

and an additional parameter is introduced to simulate the existing cluster behavior of industry.

Administration and finance employment is merely a function of urban accessibility. Finally, some service employment which is generated by the rural population (agricultural and mining) is also regarded as basic employment.

(iii) The urban stock and activity model: this takes account of the spatial characteristics of the city to distribute the population to residential areas and to generate service employment. It takes as inputs

- amount of land available for development in each zone
- transportation network
- total amount of floor space per employee
- basic employment space standards

The model first distributes the existing stock of floor space, then the location of residential activity and finally the location of service employment generated by the residential activity. This process has to be iterated until the service employment reaches the total given by the regional model. These distributions are done by the entropy maximization techniques.

(iv) The transportation model:

Given the distribution of activities and the transportation network the transportation model can simulate the travel activities of the city. Since the distribution of activities depends on the transportation network and vice versa the transportation model can be linked to the stock and activity location model and iterated to simulate the changes resulting from changes in either. The model has four stages:

(a) Trip generation: this gives the total number of trips emanating from and coming to each zone.

(b) Distribution and Modal Split: this provides the distribution T^k_{ij} i.e. the number of trips from zone i to zone j by mode k. This requires a cost matrix to operate. The modes considered were bus, motor car and pedestrian.

(c) Network assignment: this assigns the trips through the network defined as a set of modes and links . Each mode has a different network: buses follow pre-established routes, cars avoid congestion while pedestrians follow the shortest routes since no congestion is allowed for them.

(d) Generalized cost: this assigns costs to each of the modes and routes. These costs are then fed back into step b. and iterated until convergence.

The output of the transportation model is essentially the inter-zonal accessibility matrix which is fed back into the activity stock and location model and the whole model is then iterated.

Simulation for Santiago : Since the governmental changes in Chile this model has been abandoned. It would appear that not much work was done on it after calibration . Work on the model stated in 1970 and it was finally operational in 1973.

The simulation was done over a sixty zone irregular pattern. Data were obtained from various sources.

- population from the 1970 national survey
- employment from 1969 and 1972 origin destination surveys and made compatible with the 1970 census
- floor space from the taxation office
- roads from the Ministry of Public Works and Transport.
- maps.

All information was based on 1970. The regional model was run to simulate the period 1965-70. It was run under 3 different investment assumptions for prediction up to the year 2000

- continuation of part investment pattern
- no investment in Santiago
- no investment in service activity in Santiago

All the simulations were found to be quite accurate as compared with actual data for the relevant years. There is no evidence nor information on the efficiency of the model as a predictive device. Since the model is calibrated on 1970 data it is not surprising that it simulates the past well.

The Caracas Model: A Disaggregated Model of a Metropolitan Area

This model is said to combine the macro-scale or social physics approach with the micro-scale or economic approach. It attempts to simulate interaction of supply and demand in the land market. Given the location of 'basic' employment (manufacturing, government and agriculture) the model distributes employees to their residential places. As in the Santiago model and other models with the Lowry structure, the residential population generates demand for services which generates more employment and more services and iterates until equilibrium is reached.

It has 5 sub-models:

(i) Employment: This determines the socio-economic group according to employment type. Each socio-economic group has an income distribution which determines the housing type they can afford as well as the transport they use.

The model has 4 employment types

- service
- industrial
- government

- agricultural

and 5 socio-economic group types:

- managerial
- professional and technical
- clerical
- manual
- agricultural worker

We have

$$E_i^{xy} = E_i^x \; p^{E(xy)}$$

where E_i^{xy} is number of employees working in zone i of employment type x and socio-economic group y. Given E_i^x, the number of employees of type x in zone i, and $p^{E(xy)}$, the proportion of employees of type x who are in socio-economic group y, E_i^{xy} is obtained.

Once this is obtained the income of each group is determined. To do this two parameters have to be known.

mean income of employees in group y,

a parameter of the distribution in group y.

The distribution is done over 5 income ranges. The next step in the procedure is to calculate E_i^{zho} the number of employees working in zone i in income group z living in house type h using transport type o. Here there are two types of housing h

- normal housing
- squatter housing.

Transport types are also two

- car owners
- non car owners.

This calculation can be done with a knowledge of the following parameters:

- value of mean income in income group z
- mean income of those using transport type o.

The distribution parameters generating the distributions from these mean values have to be calibrated.

The output of this sub-model therefore gives (a) the number of employees

$$E_i^{xy}$$

working in zone i of employment type x and socio-economic group y

(b) E_i^{zho}

those working in zone i income group z living in home type h and travelling by transport type o

(c) various combinations and summations of the above.

(ii) Land : This sub-model distributes land for different uses depending on rent paying ability and total supply of land.

Symbolically, the model calculates L_j^g the quantity of land used by activity g in each zone j. The supply of land is considered as a function of total land available in the zone, L_j, and the demand by each activity. Here the activities are of six types

- service
- industrial
- government
- agricultural
- normal housing
- squatter housing

i.e. the sum of employment and housing categories.

Once again various mean values and standards are required and distribution parameters to be calibrated:

- land standards according to each activity
- level of each activity in each zone
- mean rent paying ability of each activity in each zone
- amortization rate of expenditure in land according to activity.

The total value of land in each zone is calculated as the sum of all expenditures on land in each zone.

(iii) Residential location: This set of equations determines E_{ij}^{zhok} i.e. the likely distribution of employees working in zone i living in zone j in income group z, housing type h, transport type o and using transport mode k. For this to be accomplished, various other calculations have to be made first.

- monthly cost of location which consists of monthly cost of transport to work according to mode, monthly average cost of transport to services according to income groups, value of time spent in travelling according to income group and mode, and the average rent for housing type h in zone j
- mean location cost for each income group

Each of the location cost components are calculated as usual with the help of various mean values and distribution parameters.

Once the employees are distributed to residential zones according to income group, housing type and mode of transport the model transforms them into households and thence the total population in each zone.

(iv) Transport : This sub-model calculates the transport costs needed for the residential sub-model.

The cost of the journey to work is merely an average cost per mile

for each mode multiplied by the distance and frequency of trips between zone i and j.

The cost of transport to service locations is somewhat more complicated since it has to be calculated according to household location and income groups. Furthermore, service trips from workplaces have to be accounted for also.

(v) Service location: This sub-model calculates

$$p_{ji}^{sfhok}$$

i.e. the number of people living in zone j, travelling to zone i for services, belonging to household income group f, housing type h, having transport type o and travelling by mode k.

The calculation involves a knowledge of mean monthly transport costs to services and the calibration of a distribution parameter.

Having calculated the distribution of population making service trips the model generates

$$E_i^s$$

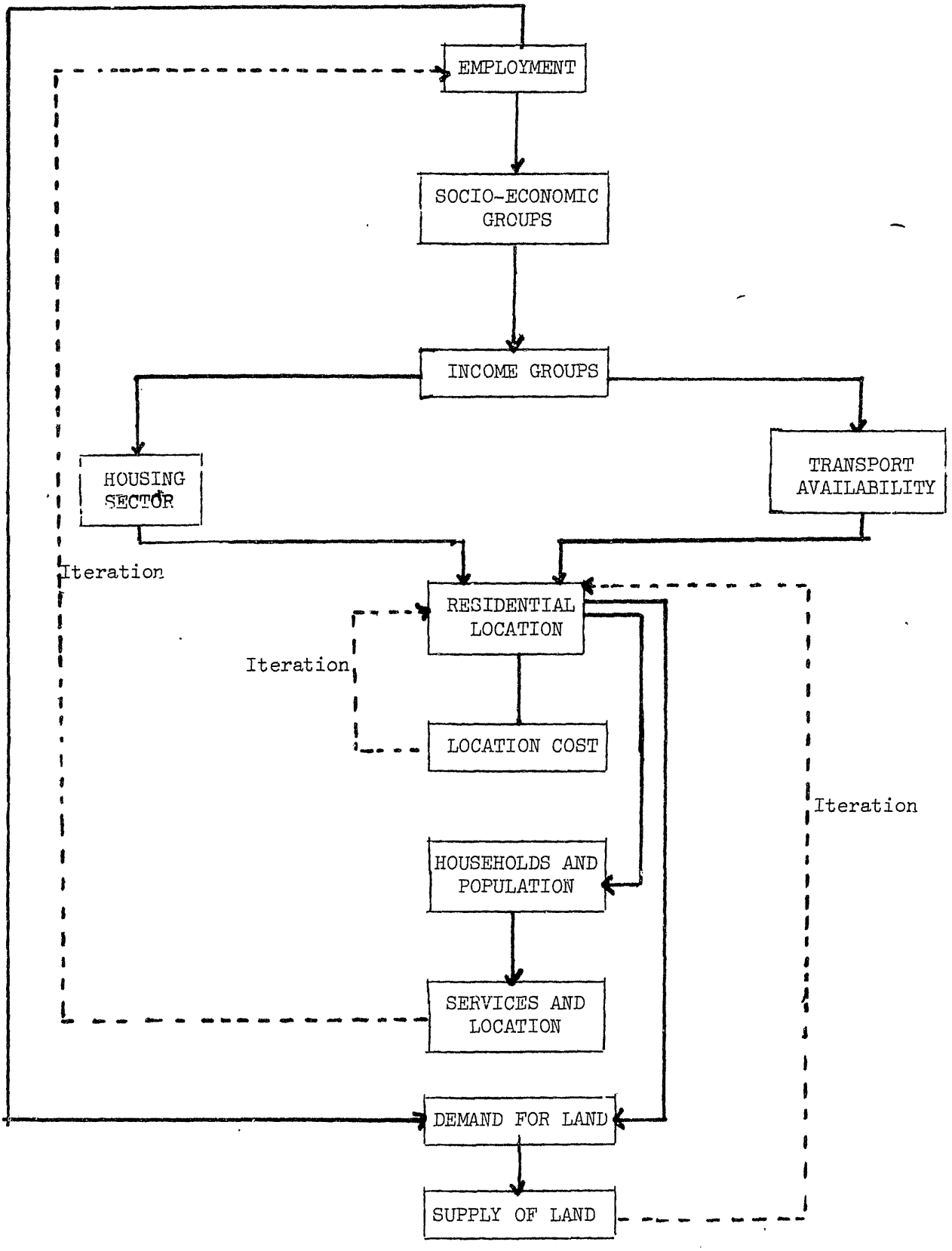
the number of employees required for services in zone i.

Solution of the Model

Figure 4 illustrates the structure of inter-relationships in the model and the iterative process used to solve it. It also well illustrates the central importance of the residential location sub-model. A solution of the model requires the following equilibria.

(a) Employment-Population: Given employment, residential population is generated resulting in a demand for services, hence service employment is added to original employment and the process iterated until equilibrium is reached.

Figure 4: CARACAS MODEL: STRUCTURE OF ITERATION PROCESS



(b) Location cost - residential location: is largely determined by location costs of which one component is rent. The rent is a mean for all residents and therefore depends on who lives in the zone. This process is iterated to reach an equilibrium.

(c) Residential Location - Land: The demand for land is generated by mean rent which determines the amount of land residents are able to buy in competition with other activities. The new land is distributed until there is no further change in the distribution of residents across zones.

The order of solution is as follows:

Given employment

(i) location cost - residential location equilibrium is achieved

(ii) service location and distribution of land is achieved

(iii) new increase in employment causes more residents

and the whole process is repeated again and again until full equilibrium is achieved.

It is not clear from the summary mathematical statement of the model why the model should be expected to converge to an equilibrium.

Application to Caracas

The city was divided into 30 zones. Service and government employment was concentrated in one zone, industrial employment in three and agricultural employment only in peripheral zones.

All land with less than 60% slope was regarded as available but excluding land for such public purposes as parks, military uses, cemeteries, university and for roads. The employment structure was found to be

51% service

32% manufacturing

16% government and services

1% agriculture

Most of the data came from a 1966 5% sample survey covering some 60,000 people. The data required were

(a) Basic inputs:

- employment located in each zone according to employment type
- land available in each zone
- distrance matrix for each pair of zones

(b) Coefficients:

The nature of these has been mentioned in the description of the model.

(c) Output values: essentially the outputs are

E_{ij}^{zhko} - employees working in i living in j, income group z, living in housing type h, having transport availability o and travelling by mode k

H_j^{fho} - households living in j, income group f, living in housing type h and having transport availability o.

$c_{ij}^{loc\ zhko}$ - cost of location of an employee living in j, working in i, income group z, house h, using transport mode k and owning transport type o .

Various summations of the above can also be obtained for different variables.

In addition, transport costs, housing costs, land values, land availability, trip lengths etc. can also be obtained in disaggregated form.

Simulation of the model gives good results in terms of the closeness of output values to the actual 1966 data.

Evaluation

The development of the Caracas model was begun in 1968 and, perhaps, finished in late 1973 or early 1974. It is not clear how much it cost nor how many professional man-years were spent on its development. The total financial as well as professional costs are likely to be substantial, especially if the basic model development costs in Cambridge, England are also included.

The basic criticism of these models is of their model structure and methodology. Despite Wilson's innovative explanation and development of entropy maximization techniques, their basis for urban simulation remains suspect. What the technique essentially does is to take the mean value of some variable and then generate some most likely distribution around it subject to whatever constraints are chosen.

To illustrate, one step in the procedure in the employment sub-model (p.94) is to calculate E_i^{zho} -- the number of employees working in zone i, in income group z, living in house type h, and using transport type o. This calculation is carried out by using:

- value of mean income in income group z
- value of mean income in housing group h
- value of mean income of those in transport group o

and two distribution parameters for distributing housing group h and transport group o. These parameters are calibrated from the base data. Thus, various mean values pertaining to different groups are used along with calibrated distribution parameters to obtain the required E_i^{zho} distribution. A similar procedure is employed in all the other distributions obtained in the model.

As such, the model merely replicates observed data and it is then not surprising that its simulation is remarkably close to actual values. At best, such a model can be viewed as a set of reduced form estimates of behavioral structures that are not specified, at worst it is a collection of spurious, accidental or temporary relations between variables. If the latter is the case, the model has no content and could be dangerously misleading in forecasting. If the former is true, then the model is useful as long as the underlying behavioral structure is unaltered. The problem is that it is impossible to judge which is the case and even if the more optimistic assumption is true, we have no basis for knowing when or for what reasons the unspecified underlying structure changes.

The usefulness of this model as a policy tool is therefore under serious question. The Caracas model was calibrated for 1966 data. It has not been updated for any later year because of lack of data. If this had been done there would have been some basis for an informed evaluation. Model simulation for a later year based on 1966 calibration could have been tested on actual values. On the basis of this test some faith in the invisible underlying behavioral structure might have been generated. As it is there is no basis for even this modest assurance and the lack of post 1966 data merely underscores the difficulty of obtaining data for such a model of an LDC city.

The Santiago model never went into real operation because of the governmental changes in Chile. It was developed under governmental sponsorship so the expectation was that it would have been used for policy purposes. While it is true that the governmental change in question was of a radical nature, this circumstance nevertheless illustrates an important problem in the use of such a large model for policy purposes. Any 'good' large model requires time to develop as well as for data collection. Three years is almost a minimum for the model to become operational. Yet policy makers change over such periods as a matter of routine. LDCs in particular are more prone to rapid change. Models such as the Santiago and Caracas models, which have a rather opaque methodological base, are then all the more difficult to explain and 'sell' to successions of competing policy makers. If their bases were less opaque, it could be easier to convince new policy makers of their continuing validity.

The Echenique group is now developing a model for Sao Paulo. It is expected that this model will take less than a year to develop, utilizing about six professional man years in the process. This model will be a more aggregated version of the Caracas effort. Documentation is still not available, so not much can be said. If it is developed within a year, that would be an encouraging sign. However, the strictures aginst its methodology would remain.

It must be pointed out that the Caracas model was developed as an educational device in a university environment. Strictures against its use as a policy tool may therefore be unfair to the authors, although they proffer good advice to potential users.

The problem of data has already been mentioned . It has two dimensions. First is the sheer lack of and the difficulty of collecting such detailed data. Second is the time-factor. With the rapid pace of change in LDC cities, no sooner is a detailed set of data collected, than it is obsolete. What this implies is not that data should not be collected and modeling given up but that smaller, more manageable sets should be sought for smaller , more manageable models.

The basic use of the Echenique models for policy purposes is as forecasting tools. They can give planners information on the possible consequences of their actions as well as stimulate thought about new directions. They have no normative content. They do not help in evaluating any consequences. To the extent that the planners have faith in their forecasting structure the models are clearly useful, to the extent that they do not, the models are unusable. It is then left to the modellers to find ways of defending their model structure and explaining it to planners. This process is easier for models with more comprehensible structures. Models placing greater stress on behavioral relationships can be just as misleading if these relationships are badly estimated, but they are easier to test since these relationships are easier for the non-modeller to appreciate or reject. In this respect it is encouraging to note that the Caracas model has as an objective simulation of the land market according to micro-economic concerns, though the actual market simulation is somewhat primitive.

Before concluding this section,it is worth mentioning that Nathaniel Lichfield and Partners have developed similar models somewhat

further as described in Christopher Turner (1975) and Lichfield(1975). In addition to a few structural modifications, their major extension in the Urban Growth Simulation Model for North Central Texas is the application of evaluative sub-models to the output of the main model. These sub-models evaluate the consequences of alternative policies on the cost of public utilities, on air pollution, accessibility to urban resources and 'social deprivation'. This is an encouraging development in making these models more directly relevant for policy concerns.

3.4 The N.B.E.R. Simulation Model 1/

This is perhaps the most ambitious of all urban modelling attempted. It follows a somewhat different family of models pertaining to transportation and urban land use. Six of these built in the late fifties and through the sixties for the Puget Sound, Southeastern Wisconsin, Atlanta, Detroit and the San Francisco Bay area (2) are well reviewed in Brown, et. al. (1972). Here we merely describe some of their unifying characteristics as a prelude to the N.B.E.R. simulation model.

The objective of these models was to help policy makers in the planning of transportation. They are characterized by the assumption of an undirectional relationship between land use and transportation. Thus considerable effort is devoted to modelling land use in some detail to derive transportation requirements. Regional population and employment forecasts are taken as exogenous. Input-output methods are used to forecast future employment by industry. Retail employment is derived from these forecasts and households located according to family type. Different models employ various levels of disaggregation for dwelling types and household types and their assignation. These results for projected land

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1/ This section reviews the work described in Ingram et. al. (1971, 1972), Kain and Ingram (1974) and Brown et.al. (1972).

use then provide the basis for future transportation plan design. Some use supply and demand concepts for equilibrating the housing market, taking into account volume of housing stock and pattern of filtering with age of structures. The San Francisco models are somewhat different in that they follow the Lowry model structure.

All these models have heavy data requirements ,but still have few behavioral relationships embedded in them. They are in the genre of mechanistic forecasting models which find it difficult to cope with technical changes and innovations which affect the structure of cities in a crucial way. Interdependencies are usually modelled in a sequential manner: the tension between sequential and simultaneous relationships has been observed earlier. All of these models were expensive:

(i) Atlanta $1.75 m

Data collection	36%
Analysis and models	24%

(ii) South Eastern Wisconsin $ 1.99 m

Data collection	62%
Analysis and models	14%

(iii) Bay Area Transportation Study $5.54m

Data collection	60%
Analysis and models	18%

(iv) Detroit Talus $ 4.70m

Data collection	46%
Analysis and models	19%

(v) Puget Sound Transportation Study $1.7 m

(further breakdown not available)

We note that a major part of the expense was always on data collection thus underscoring the importance of looking critically at data needs of models.

The NBER simulation model was embarked on to improve on earlier work. It was deeply rooted in economic theory with utility maximizing households and profit maximizing firms. Its goals were to:

(a) enrich economic theory;

(b) advance the art of model building;

(c) evaluate problems of urban growth and decay; evaluate specific problems and policies; and consider broad strategies for dealing with U. S. cities.

This effort has been quite successful in realising (a) and (b), but somewhat less so in realizing (c), despite the fact that it has been handsomely supported financially as well as intellectually. The model does not appear to have reached the stage of evaluating public policies.

We now describe the model itself:

The Model

The NBER model incorporates the theoretical approach of the traditional analytic models of residential location and urban spatial structure into a framework with more realistic and less restrictive assumptions. It is more realistic in the following ways:

(i) It drops the monocentric assumption of analytic models and explicitly incorporates multiple work places.

(ii) It abandons the long run equilibrium framework of analytic models which characteristically ignore the effects of durability of capital. The NBER model overcomes this by representing the standing stock of physical capital in the city and models the supply side of the housing market in detail.

(iii) Finally, it takes account of externalities such as neighborhood effects and racial discrimination.

It was designed to simulate major changes in urban spatial structure

that occur over periods of from 10 to 50 years. It simulates effects on spatial structure of long term trends in the level and distribution of employment, changes in transportation, technology and increases in income. It provides a description of spatial structure at a point in time and modifies this over a period of years by simulating location and investment decisions of firms, households and home suppliers.

The model is primarily a model of urban housing markets. It does represent other urban phenomena, such as industry location and changes in the demographic structure of the population, but the behavior of the housing sector is its central concern. This involves modelling the behavior of housing consumers, suppliers and the 'market' in some detail. In doing this it is claimed to improve on previous models (most of the social physics variety) which were elaborate statistical descriptions with little or no theoretical justification.

The model can be described in terms of a demand sector, a supply sector and a market clearing sector. The activities in each of these sectors are carried out in one or more of seven sub-models.

(a) Demand Sector

(i) Employment Location Sub-Model: Given an exogenous change in total employment, it revises the level and composition of employment at each work place and by each of nine industry types. It translates employment changes by industry to changes in employee characteristics.

(ii) Movers Sub-Model: According to the results of (i) above this model generates movers, i.e. households vacating housing units in response to employment changes, and modifies them to produce households seeking housing. This has the effect of preserving some of the city structure and changing it

only incrementally.

(iii) Demand Allocation Sub-Model: This is where the model allocates households to housing types. It takes account of the costs of the journey to work and expected housing prices to form gross housing prices. Thus each housing type is associated with a gross housing price in relation to every work place. Finally, household allocation is performed by the use of demand functions.

(b) The Supply Sector

(i) Vacancy Sub-Model: As a result of the movers (a,ii above) this sub-model generates vacancies in each zone by housing type. This also includes new construction.

(ii) Filtering Sub-Model: In response to expected housing prices formed in (a,iii) above quality classifications of available housing stock are changed. This also takes account of expected maintenance costs and therefore relative profitability of different maintenance strategies.

(iii) Supply Sub Model: This also responds to (a,iii) above and performs stock transformation according to profitability of construction and transformation activities from expected prices and exogenous building costs.

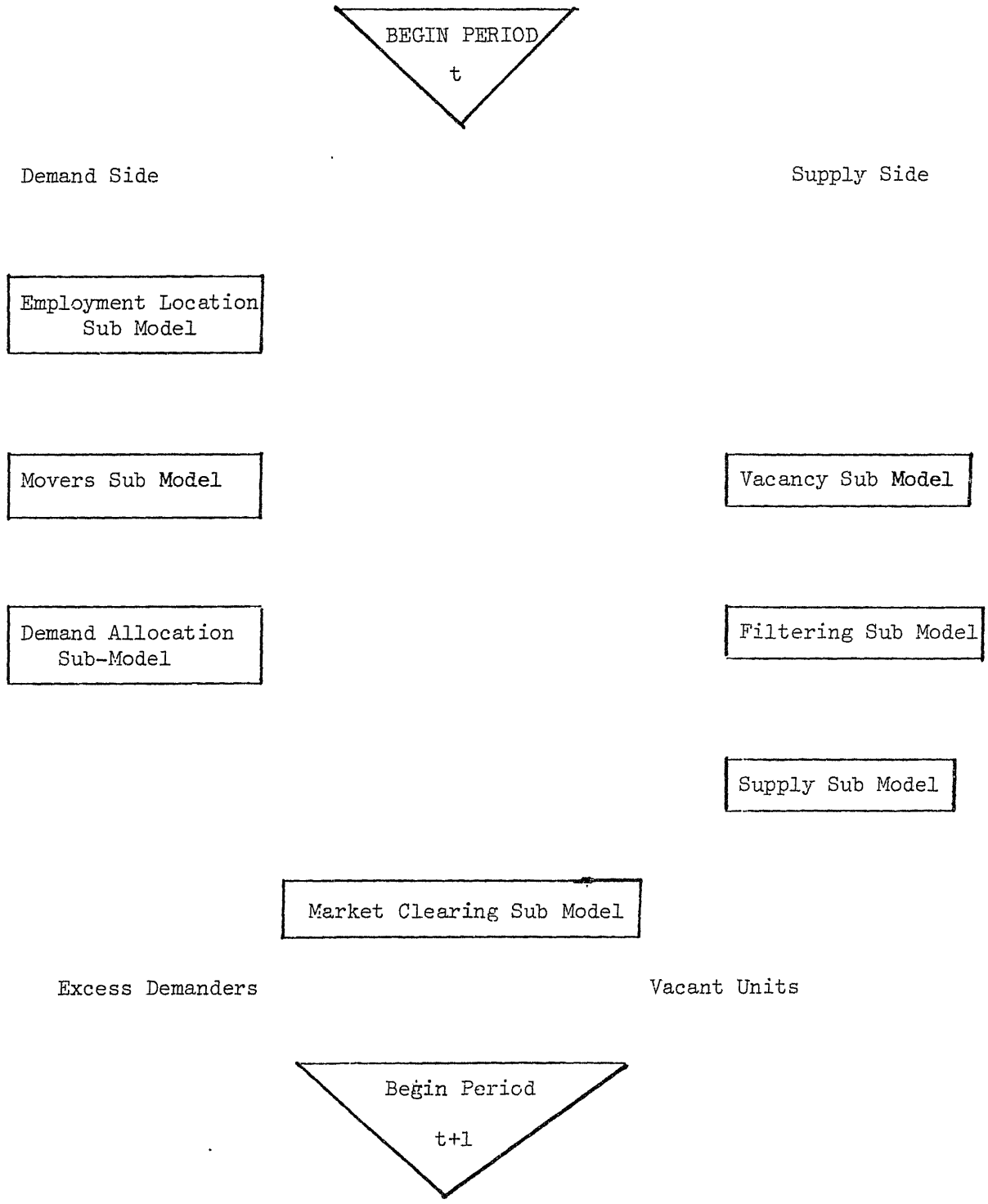
(c) Market Clearing Sector

Market Clearing Sub-Model : Moving households are matched to available units of the types chosen in the demand allocation sub-model. Each house-type is solved for as a separate sub-market. Shadow prices are used to generate prices for the next time period. Work-trip patterns are also updated.

The model structure described above is illustrated in Figure 5.

Embedded in the model structure described above are the following special features not found in most other models:

Figure 5: STRUCTURE OF THE N.B.E.R. MODEL



(i) Most other models locate the entire population at once to produce target year solutions. This model allows adjustments in employment and residential locations, alterations to the housing stock and changes in the distribution of work trips in an incremental fashion only. This has the effect of letting the existing city structure influence the future which is a pleasing simulation of reality.

(ii) The model does not force equilibrium of each housing sub-market in each period. Individual housing sub-markets are allowed to have excess supply of units (vacancies) or excess demand which, in turn, affect prices in the next period.

(iii) The model produces expected housing price by housing type and zone for each period. These prices affect the behavior of households seeking housing as well as firms producing them.

In the 'Detroit Prototype' version of the model the region is divided into 19 workplaces and 44 residence zones, the former being aggregates of 32 inner residence zones. It distinguishes households by family size, family income and education and age of head resulting in 72 household classes. Housing is distinguished by structural type, number of rooms, quality and lot size,resulting in 27 types. There are two modes of travel.

We now describe some of the processes determining the distributions generated by the model.

(a) Behaviour of the Consumer: The consumer is essentially seen as the classical utility maximizing household. In this model the following assumptions are made:

(i) The household has a fixed and predetermined set of demands for travel to known destinations. The journey to work predominates in travel costs and households place monetary value on travel costs.

(ii) Households have preferences for housing 'attributes' which they buy in a finite number of combinations of 'housing' bundles.

(iii) Housing bundle prices vary by location and these price surfaces are known to consumers who act as price takers.

The consumer's problem is then posed as a cost minimization problem. Since travel costs are subsumed in gross housing prices, the optimal location for the household is that location which has a 'housing bundle' whose price is the minimum. These minimim prices, furthermore, are the result of its demand based on income, household characteristics and taste in the traditional demand analysis way.

(b) Determination of Housing Prices and Quantities

As mentioned earlier, the NBER model differs radically from earlier approaches in recognizing the durable nature of urban structures and allowing city structure to change only marginally in each period. This also has the result of allowing disequilibrium in various housing sub-markets. However, it follows traditional economic theory in assuming that housing production is responsive to market demands and prices, that suppliers are profit maximizing and that households are price takers in both output and factor markets. It assumes further that

(i) Housing outputs are heterogeneous and are produced using combinations of existing durable structures, current inputs and neighborhood attributes.

(ii) Most of the supply in each period is from used structures.

(iii) Some of the housing attributes are not produced by competitive firms e.g. they are supplied by local governments. This has the effect of placing constraints on suppliers' actions.

The production function for housing consequently reflects these assumptions in including existing structures and neighborhood characteristics as inputs in the function. This is a radical departure from standard practice which usually includes land, capital and labor in inputs in a production function.

(c) Market Clearing

The household'sselection of a housing bundle is represented in the NBER model by econometrically estimated demand functions. These equations express the probability that a particular household will consume a particular type of housing as a function of the household's socio-economic-demographic characteristics and the minimum gross price of the bundle. Solution of these functions gives the number of persons employed at each workplace who will demand each type of housing bundle. The demand for each type of bundle is summed over all workplaces, and firms then attempt to satisfy this demand in each location on the criterion of profit maximization.

Spatial competition among households competing in the same housing sub-market is represented in this model by a linear programming algorithm which minimizes aggregate travel expenditure for households competing in the same sub-market. Thus the procedure also yields shadow prices for each bundle type in each residence zone in the solution.

The output of the model essentially gives the workplace and location of each household and the bundle of housing it consumes. Further it produces expected prices for each type of housing in each residence zone during each period.

Included among the inputs to the model are employment; costs of performing various supply activities (i.e. an array of supply costs with a given technology and fixed factor costs); zoning constraints like limits

in residential density; amount of available land in each zone; and forecasts of demand over current period.

Evaluation

The NBER model is being developed still further and has now reached a third version,'Pittsburgh II',but little documentation is available after the Detroit Prototype. The major modification is an expansion of the number of neighborhoods in the model which (because of the way housing bundles are defined) reduces the number of housing bundles that have to be coped with in each zone. The model has really developed into a research process rather than a product.

It must be recognized in conclusion that the NBER model is an admirably ambitious attempt to incorporate urban realities within a basic framework of urban economic theory to produce a model of city structure. It is not, however, clear yet how successful or how expensive this effort has been. Its development was begun in 1968 and has cost, at a minimum, one million dollars. There has not yet been a policy simulation with the full model. Its data requirements are clearly high: detailed demographic data are needed to generate household types and their characteristics; housing price data and construction costs data are needed in detailed fashion for the supply sector. The model, as a whole, is quite unwieldy and does not yield many fruitful results in comparison with the effort involved in building it.

It is difficult to enumerate reasons why an enterprise such as the NBER model effort has not become an "operational" model. It was started with clear-cut objectives, had ample and long term financial support and has had some of the best urban practitioners involved in its conception. One reason is merely that it was over-ambitious. Another reason is, perhaps,

that its objectives have changed over the course of its development. While it was originally envisaged that it would be a useful policy-making or policy-helping device, it has slowly become an almost wholly research device. The specification of each sub-model has been a project in itself and has illustrated the use of economic theory in modelling. In this respect it has been very instructive. It has spawned a host of side studies which have enriched the state of the art of urban studies.

It is probably fair to conclude that the NBER model has been more successful as an analytical or explanatory model rather than as a policy-oriented model. This is as should have been expected since its size is too large and data requirements too intensive and detailed for the time and resource constraints of policy making. Its usefulness for LDC cities lies in some of the ways it has adapted existing theory to reality. In particular, its enumeration of housing-bundle types may be a good approach to the extremely varied bundles found in LDC cities. Partial research efforts in these cities could be modelled after parts of the NBER model - applying it fully would be foolhardy.

3.5 Mills' Optmizing Programming Model (Mills, 1975).

Finally, in this section we present yet another kind of model: Mills' recent optimizing programming model. Although one would particularly expect policy oriented models to have some normative content, such models have been more the exception than the rule.

The paradigm for normative models in economics is that first some market resource allocation is derived, then tested against some welfare criterion and then government policy prescribed if this allocation turns out to be sub-optimal. The assumption is always that the government operates in the public interest while everyone else furthers his own interests. In this model Mills explores the issue of the desirable extent of governmental regulation and interference in the urban system. It introduces assumptions that indicate why it is desirable for government to undertake certain activities. It then specifies the effect of government activity on private resource allocation, and the effect of private activity on the use of the public service. It permits calculation of an optimum allocation of both public and private resources. Finally it demonstrates that competitive markets sustain an optimum allocation of resources if the public sector provides its service in optimum fashion.

The model concentrates on the provision and pricing of transportation. It is formulated in a non-linear programming framework and is a development of earlier models by Mills (Mills, 1972b, 1974).

The Model

Unlike other models presented in this section this one makes a larger number of simplifying unrealistic assumptions but it still is rather complex. The city is seen as a homogeneous plain stretching in all directions from a central export point. The model determines the amount and

production technique of each of an arbitrary number of goods and services to be produced at each location in the urban area. Each good that is produced must be exported or transported to the point where it is consumed. This model unlike others therefore also takes account of traffic in goods.

Space in the area is represented by a square grid centered on the central export point. All squares at a given distance from the center have identical patterns of production, consumption and transportation. Transportation is only between squares and only in the north-south and east-west directions.

$\bar{r}$ goods are produced in the city. Of these, two are high income housing and low income housing. All other goods are exported and the export amounts are given exogenously. Apart from the center goods can also be exported from suburban nodes $\hat{u}$ miles from the center. The first set of equations in the model are identities to ensure that

(a) total exports of each good are at least as much as exogenously prescribed and

(b) shipments into each square plus production in the square equal outward shipments plus use as input or final consumption in the square. It is important to note that this implies a set of $\hat{u} \times r$ equations in each square.

The next set of inequalities builds the transportation system. They determine $t_{rk}(u)$ which is the number of unit miles of good r shipped at congestion level k per square at u. The congestion level helps in determining the width of roadway required - higher congestion means a narrower roadway. Once again, this is a set of $r \times \hat{u}$ inequalities. In addition there

is one to ensure that land demanded by transportation is less than total land for transportation. A further set of inequalities and equations ensures that there is only one level of congestion in each square. Finally, there is an inequality to ensure that the land used for all purposes does not exceed the total available.

A feature of the model is that it permits an arbitrarily large number of production techniques used to manufacture goods. This is done by concentrating on the capital-land ratio which is represented by the height of buildings. A large number of storeys means high capital-land ratio. Labor input-output coefficients are independent of building heights.

Such a view of the production process implies that a city's labor force and total output of all goods are determined by the export requirements via the input-output matrix. Transportation costs, land and capital requirements, however, are endogenously determined. The objective function is then to minimize the sum of land, capital, transportation and exports costs needed to produce the required export goods. It is assumed that R_A rent for land, at the city periphery is uniform, and the city can therefore be expanded at this uniform unit cost. Similarly, capital can be acquired without limit at a fixed rental rate R.

The model is solved using mixed integer programming techniques. The only non-linearity in the model is the integers used to represent congestion levels.

Solution of the Model. The model is solved for a hypothetical U.S. city of about 1 million population.

It has 5 sectors i.e. $\bar{r} = 5$

- office activities
- retail firms
- manufacturing firms
- low income housing
- high income housing

$\bar{u} = 11$ implying an urban area of about 250 sq. miles.

$\bar{s} = 20$ implying that the highest building has 20 floors.

Income groups are the lower half of the population and the richer half

K_A = \$4000/acre and

R = \$1m rental rate per \$10 m worth of capital value.

An entire input-output matrix was constructed for the 5 sectors. Approximate values of automobile travel and goods movement were used. Time costs varied with income.

With such a simplified model there are 788 variables excluding slack variables. Of these, 30 are integer valued. There are 219 constraints in the example. The model was solved on an IBM 360/91 computer and took 6 minutes (\$70) of computer time and used about 400 K of the core of the computer. (The capacity of the core is about 1100K).

The solution results in a city rather typical of a U. S. city as intended. The central square is devoted to office activities in 17 story buildings and transportation is at the highest congestion level. The next square has retail firms, some low income housing, in 5 story buildings . Square two has high and low income housing as well as some office activities. The city center is ringed by low income housing as is typical of U.S. cities. All other squares have some of all activities except that the suburban export

nodes($\hat{u}$ = 7) has no manufacturing firms.

The congestion pattern is unsatisfactory: high in square zero, low in square one and than high again after square five. Land rent falls rapidly near the center and then slowly as is typical of most cities.

Evaluation

In many ways this model can interestingly be compared with the Echenique et al approach. Conceptually, they do rather similar things. Both specify some basic sector exogenously around which a city is generated. Echenique et al specify this sector as one that is unrelated to local or residential location while Mills specifies exports. Both models then distribute activities and transportation by maximizing some function subject to some constraints. Both involve the guessing of a large number of mean values and parameters specifically. This model must be fed with

- total export of each good
- input-output matrix coefficients i.e. input of good q per unit output of good r using production technique/s.
- land standard for transportation at each congestion level.
- capital-land ratio for transportation
- unit cost of exporting each good from squares at each distance
- total cost of shipping goods per unit at each congestion level
- rental rate of peripheral lands
- rental rate of capital.

Mills' model is more simplified by regarding the city as symmetric, thereby reducing the spatial problem to a unidimensional one. His model permits generalizing by allowing the identification of each square uniquely, but computation will then become cumbersome.

The key conceptual difference between the two models is in their objective functions. Entropy maximization merely does some 'most probable' distribution within given constraints, while here costs are minimized. The latter is more appealing because it is easier to understand clearly what is being minimized. One therefore appreciates the reason behind whatever distribution of activities, transportation, etc. is generated. Furthermore, in the policy making context the policy maker understands such a procedure readily. With entropy maximization, at best, the modeller himself has some idea of the generating force in his model. It is easier for a modeller to interact with a policy maker when he also understands what is going on. Parameters in this model also have easy meanings while they are merely some 'distribution parameters' that are calibrated in the entropy maximization approach. As a result, interaction with a policy maker could yield changes in parameters and in the objective function that could easily be incorporated into Mills' model while it would be difficult to do so in the Echenique et al efforts.

A caveat here is necessary. Not intuitively understanding the procedure in a model does not necessarily mean that its results are less reliable than one that is comprehensible. It does, however, mean that it is more difficult to evaluate and that its results demand a greater degree of faith.

The main drawbacks in Mills' model are its monocentric assumption and its unimodal transportation. As has been argued earlier in this paper, such assumptions may have some validity in the U.S. context but almost none in a L.D.C. situation. Further, its production functions are of the fixed coefficient type which do great violence to urban reality. Mills recognizes these defects and suggests the following by way of extensions of the model:

(i) Transportation: introducing a second mode which has a

different right of way like a subway is easy since we merely have another set of I.O. coefficients etc. which can be used to represent the second mode. Buses, rickshaws, pedestrians etc. would be more complex since the same roadway would be used.

(ii) Scale Economies: Mills suggests that these are possible to include by the use of more integer variables and constraints specifying threshold levels of production. However, he conjectures that such modifications would require identification of each square and the computations required could well exhaust even a large computer.

In its present form the model can be used easily to test policies such as land pricing, transportation pricing, land taxation and land use controls with manageable modifications. In each case, the primary output of the model is the amount and production technique of each good to be produced in each square, the origin and destination of each shipment and the congestion level in each square. This suffices as the profile of a city.

Before concluding it is worth noting that the number of variables and constraints in this model is large despite its conceptual simplicity. It needs a late generation large computer like the IBM 360/91 which is often still not available in LDCs.

In conclusion, we regard such a model as an interesting departure from other standard approaches with some advantages over them. Its drawbacks, however, do not make it a fruitful approach to LDC cities at present.

3.6 Summary

This section has reviewed policy-oriented models which are regarded as representing the main strands of urban modelling. Each of the models reviewed is quite different in approach from the others and none are regarded as successful policy models - particularly for possible uses in LDC cities.

The objective of each of the models is to distribute activities spatially in a city. A typical output of a model would be the allocation of residential and employment location by zones in the city. This would be disaggregated by socio-economic types of households, types of residential structures, types of employment etc. The main features of each of the models are now summarized.

The Lowry model is the parent of many models that have followed it since its first publication. Its conceptual contribution was to posit some basic sector of activity which is exogenously given and is the driving force behind a city. This activity requires households to supply labor which, in turn, require retail services. The provision of retail services requires more households and the process is iterated until equilibrium is reached. The principle behind the generation of location distributions is the gravity model of interaction. The original Lowry model was partly filled in by hand to produce plausible employment-residential co-distributions. Lowry himself was not too sanguine about its potential usefulness but his contribution must be regarded as the single most important and influential one in non-economic urban modelling. The Lowry model descendants in the U.S. have largely been failures and have been subject to heavy criticism from within the profession, consequently leading to disillusionment.

More innovative developments have been carried out in Britain where A.G. Wilson has given a more respectable theoretical base to the gravity model through the method of entropy maximization. Echenique and his associates have developed models based on Lowry and entropy maximization and attempted to apply them to South American cities. While these models have reproduced city structures fairly faithfully after careful calibration their value as planning or predictive tools remains suspect. They have scant behavioral underpinnings which makes one sceptical about their use as predictive devices. Moreover,

they are heavily data intensive which makes their applications in LDCs limited.

The NBER model can be regarded as the economists' answer to urban planners. This model has a behavioral structure based on micro-economic theory and borrows much from the analytical models of Muth and Alonso discussed in the last section. It makes them more realistic by dropping the monocentric assumption; by taking account of the existence of disequilibrium in markets, the effects of the durability of capital structures and of externalities like neighborhood effects. Despite these pleasing features it can scarcely be called an operational model. It is unwieldy to operate, it is not evaluative and there are some indications that it has developed into a research process rather than a product. Its data requirements are even more intensive than the Echenique et al. models. However, much can be learned from the formulation of its sub-models.

The last model discussed - Mills' programming model - differed from the others in that it is an optimizing model and has much more simplifying assumptions. It regards the city as mono-centric and allows a unimodal form of transport. Despite its simple and unrealistic assumptions it becomes mathematically complex and large. It has 788 non-slack variables of which 30 are integer valued and has 219 constraints. Its solution requires non-linear programming techniques and a late generation computer.

Large models of urban areas are expensive in terms of data as well as technical expertise, particularly for LDCs, while their use is of dubious value. The essential conclusion of this section is a somewhat paradoxical one. While large scale urban models may be of some use as research methods

exploring urban form and structure and their underlying rationale they are of little use as practical policy devices. This is more so in the context of LDCs where change is so fast that models are soon rendered obsolete for policy purposes. The next section suggests some fruitful approaches which are more modest in scope though not without their own limitations.

4. SOME FRUITFUL APPROACHES TO URBAN MODELLING

Finally, in this section we review some work whose approaches, though quite different in each case seem to give some hope for useful urban modelling. It should be regarded as an illustrative section merely suggesting some fruitful approaches.

It must be emphasized here that these models are not considered directly transferable to LDC cities. They are too rooted in their particular institutional settings for such transferability, as indeed they should be. We start with a description of some of the work of the Transportation and Location Analysis Group in the Master Planning Commission of Stockholm as reported in Lundquist (1973a and 1973b) and Andersson (n.d.). Their model is an evaluative one and is particularly suited to the Swedish environment where the government has considerable control over urban form. The second model discussed is the Urban Institute Housing model as described in various publications of that Institute. This model reflects the U.S. institutional system focusing on the urban housing market. It was originally designed to help the Department of Housing and Urban Development in testing policies such as the housing subsidy scheme. It also is therefore highly policy oriented and is quite different in structure from other models discussed .

Lastly, we discuss an explanatory model which seeks to find behaviorial relationships which explain the residential pattern in a city in the British context. It is presented here to illustrate how the development of policy-oriented and explanatory models should go hand in hand to increase our understanding of cities. The relationships and parameters found in an Apps type model can then be fed into policy-oriented models as inputs. Such a model is also an example of how a data gathering effort could usefully be organized systematically.

All these models are described in semi-technical detail for ease of understanding.

4.1 The Andersson-Lundquist Stockholm Model

Their approach is a realistic one concerned with evaluating policy alternatives for the city of Stockholm. The main policy variables are the provision of transportation and residential housing. The urban problem is therefore viewed as one that can be analyzed in terms of dynamic interdependent investment planning. Scarce resources are to be allocated to physical investments in order to provide maximum social and economic welfare. The high degree of complexity in urban structure is essentially caused by strong interdependencies over space and time. Thus the problem can be made manageable only if it is decomposed hierarchically. In other words, we should first divide a city into a fairly small number of zones, study the interactions between them, derive approximate plans at this level of aggregation and only then disaggregate further. The reason for this approach lies in technical realism. Interdependencies that are believed to exist in the urban system can only be modelled with non-linear mathematics and these are manageable only if the problem size is kept small. We can thus have a mathematically complex model at an aggregate level and then have a sequential transition to disaggregated linear models which can be of larger size.

The authors of this model see the crucial concerns of urban modelling to be:

a. Planning under uncertainty.

b. Normative welfare criteria.

c. Individual behaviour.

d. Spatial and sector disaggregation.

e. Explicit treatment of interdependencies.

Uncertainty about the future is a characteristic feature in the planning of LDC cities. This is currently caused by their explosive growth rates as well as political instability in many places. The latter is particularly important in the formulation of normative welfare criteria that are to be used in an optimizing model. If one is interested in the implementation of planning ideas, the welfare criteria must reflect those of the political policy makers. At the same time, since these can be expected to be myopic (e.g. because of election cycles), if the modellers' interest is in longer term welfare, the results obtained from the use of different welfare criteria should be robust. In this way, uncertainty about the future is also better taken care of.

In specifying the model the city is seen to be composed of:

a. Building stock;

b. Transportation systems;

c. Recreation land.

The interplay between transportation network structure and urban location pattern is seen as fundamental. Activities are to be allocated within a slowly changing building stock. The modelling is done at 3 levels:

a. First, interdependencies between building stock capacity and communication network design are explored. This is Model 1.

b. Second, interdependencies between activity location and use of transportation network are specified. This is model 2.

c. Finally, the results of a and b are further disaggregated.

In model 1, the welfare function is defined in terms of indices of:

a. Accessibility: good opportunity for interaction that is promoted by a compact urban structure.

b. Environment: space standards measured by level of pollution, degree of segregation, population density, etc.

c. Costs of investment resources, operating activities, etc.

The Stockholm model has N disjoint zones and if we have

B_i - amount of building stock in Zone i

t_k - (0,1) variable denoting absence or existence of transportation network link we can formulate costs of interaction as a measure of accessibility:

$$I = \sum_{i,j}^{N} (B_i \, d_{ij} \, (\bar{t}) \, B_j)/\bar{B}^2 \qquad (102)$$

where d_{ij} is distance between zone i and j and $\bar{B} = \sum_i B_i$, total building stock being given.

Similarly, we can formulate costs of congestion:

$$C = \sum_{i}^{N} \{(\frac{B_i}{A_i}) \cdot B_1\}/\bar{B} \qquad (103)$$

where A_i is the area of Zone i; and $\frac{B_i}{A_i}$ is thus a measure of congestion.

The problem is then to minimize some weighted sum of these two costs. We have a trade-off between the ease of interaction and the unease of congestion.

We minimize:

$$W = \alpha I + \beta C \qquad (104)$$

subject to

$$\alpha + \beta = 1 \qquad (104.1)$$

$$\Sigma B_i = \bar{B} \qquad (104.2)$$

Capital required for $\overline{B} \leq \overline{K}$ (104.3)

Labor required for $\overline{B} \leq \overline{L}$ (104.4)

$$B_i = B_i^o + \Delta B_i \quad (104.5)$$

and $\Delta B \geq 0$ (104.6)

Here α and β are the weights assigned to each of the costs. These weights can be varied to test the effects of different degrees of importance attached to environmental concerns and accessibility concerns. The first constraint merely ensures that the weights sum to unity. The second constraint (104.2) ensures that the amount of building stock in all the zones sums to the given total building stock.

Constraints (3) and (4) (104.3 and 104.4) are capital and labor availability constraints to reflect the local capital and labor markets. The left hand sides really specify production functions or parameters for labor and capital derived from production functions. They specify the technology being used. In this model Cobb-Douglas production functions are used to derive capital (K) and labor (L) coefficients.(104.5) shows that new building stock in each zone is existing building stock B_i^o plus the additional building ΔB_i planned for the current period. (104.6) ensures that <u>some</u> building does take place.

The aggregate building stock demand levels,capital, labor availability magnitudes, etc. are assumed to be obtained from national or regional projections or planning models.

The objective function in this problem is a highly non-linear combinatorial one which cannot be dealt with by conventional optimization techniques. Thus a heuristic tree searching procedure is suggested for finding a solution. The interdependencies between building stock location and transportation network design are considered by a nested procedure where the layout of building stock is guided by its implications for

network performance. In simpler terms, a systematic trial-and-error procedure can be used -- specially since the transportation network is described by a simple (0,1) system and thus has a finite number of possibilities.

Stockholm was divided into 12 zones for the purposes of this model and 25 transportation projects were considered. Model solutions give level of building stock in each zone and the existence or non-existence of transportation links between zones. The model is flexible in the following ways:

a. Model structure: We can vary the specification of the objective function as well as the constraints to test the robustness of results. In particular, values of α and β can be systematically varied to observe the effects of differing priorities.

b. Exogenous data: Capital, labor coefficients can be varied to represent different technologies. Given desired stocks can be varied as well as growth rate projections.

c. Heuristic procedures: The solution procedure can be varied to test for sensitivity of results.

These are precisely the kinds of decision variables that one is interested in exploring in the context of LDC cities. The objective function can easily be modified to reflect concerns of such cities: investment and maintenance costs, for example, would probably be more important than certain kinds of environmental problems. The most appealing part of this (general) model is its low requirement of data. We need to divide a city into only a few zones. As is evident from (104) the kind of parameters and data needed are at such aggregated levels that it should be easy to estimate or guess for LDC cities. Furthermore, we can test the robustness of results by varying the data.

Lest this sound too optimistic we have to recall that the model is highly complex mathematically and that the solution is not straightforward. High

levels of skill are therefore needed which are not often available in developed countries, let alone the less developed ones. In addition, the input requirements, and the objective function formulation is such that the modeller be intimately acquainted with the city being modelled since many judgments are involved.

We describe model 2 with even less rigour. The information of interest is posited as a vector:

$$X^T = (L^T, R^T, P^T) \tag{105}$$

where L^T is zonal supply of land;

R^T is residential activity; and

P^T denotes production activities.

The objective function is of the form

$$\alpha X^T D X + (1-\alpha)\ X^T Y X$$

where D is an interaction matrix;

Y is a congestion matrix; and

α is a trade-off parameter.

The constraints include:

a. Exogenously determined growth rates of production levels.

b. Limited availability of labor and capital as before.

c. Balancing of supply and demand of land.

d. Depreciation rates.

e. Activity relocation.

f. Balanced growth of residential and productive activity. This is achieved by a set of equations relating the labor market to the housing market.

The model has a quadratic programming structure with a quadratic objective function and linear constraints. Global optimum cannot be guaranteed because of the existence of non-convexities and various initial solutions have to be tried.

The model was tried for the Stockholm region using 7 zones. All economic activity was grouped into 3 production activities: manufacturing industry, retail service, public services. The data and parameters used were obtained in various ad hoc ways; e.g. from

a. Regional projections: production levels, total labor force, spatial requirements.

b. National data: technological coefficients

c. Fictive values: friction of distance, interaction intensities and regional variation of technology.

The model tells us the location of manufacturing and residential activity given the overall constraints and objective function.

Once again, as with model 1, we observe that the model appears to be of manageable size without a high degree of information requirements. We do not describe its structure as a model to be followed but as a realistic approach worth learning from. The delineation of activities and the specification of the objective function can easily be modified to reflect LDC cities.

Model 3 does not seem to have been developed and tested yet. Since it is at a highly disaggregated level it will probably suffer from the normal information and other difficulties discussed in Section 3 in the context of other policy-oriented models.

In a planning context, models such as model 1 and 2 would appear to be particularly useful since they evaluate possible courses of action in a way that can easily be discussed with a policy maker. Moreover, the policy-maker's objective function can be probed by varying the coefficients in the objective function. It could also be useful in informing policy makers what their objective functions are when these are unarticulated but implicit from appreciation of particular sets of results. These models are also at a level of aggregation at which policy decisions are often made e.g. should a bridge be constructed or shouldn't it. Once such courses of action in terms of transportation links and levels of desired building stock by zones have been suggested, further disaggregation can either be left to normal market processes or to a next disaggregated level model. The latter requires a deeper knowledge of people's preferences with regard to housing and location: one method of investigating these is given in the Apps model in Part 3 of this section.

4.2 The Urban Institute Housing Model 1/

The Urban Institute Housing Model is presented here to illustrate yet another approach to modelling and one which augurs well for the future and for applicability to LDC cities. The model represents the population of an urban area by a relatively small number (30 to 50) of 'model' households, the housing stock by a relatively small number of 'model' dwellings, and market behavior by the interaction of these model households and dwellings over a ten year interval. Households choose among all available dwellings,

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1/ This section reviews the work described in De Leeuw(1972), De Leeuw and Struyk (1975) and De Leeuw et al (1974).

taking into account their income and family type, dwelling quality and price, and neighborhood characteristics. Owners of existing dwellings decide how much to upgrade or depreciate their dwelling so as to maximize expected profits. Builders are prepared to construct new dwellings demanded at a price which covers costs and a normal profit rate. Governments can affect market outcomes in a wide variety of ways ranging from complex subsidy formulae to rent controls to outright prohibitions of certain dwellings in certain locations.

The model has been applied to six different metropolitan areas to obtain key parameters of household and landlord behavior. It is recognized that to convey a credible description of urban housing markets the model must account for a host of details and characteristics of these markets which cannot be captured in a simple way. For example, it must be recognized that there exist distinctions between new stock and existing stock as do the distinctions among housing sub-markets of different qualities and locations. Particular attention is devoted to two characteristics of housing which make it different from other goods: durability and neighborhood effects.

The 'model' dwellings resemble the actual housing stock at the start of the decade by location and level of housing services. 'Model' households resemble the actual population of a metropolitan area in income distribution and in demographic and racial composition. The model,in effect, is a description of the process by which households and dwellings get matched within an urban area. The other agents in the model are the building industry which supplies the new dwellings, and the govenment which can regulate the housing market in a variety of ways. Thus there are four economic agents participating in the model:

- households deciding which dwellings to select
- the owners of existing dwellings deciding what quantities to supply at what cost
- the builders constructing new dwellings
- government constraining or facilitating outcomes in various ways.

The time-frame of the model is a decade i.e. the model predicts the situation of end-of-decade households and dwellings given the housing stock at the start of the decade and given travel times, construction costs and certain other information during the decade.

A solution to the model is a set of locations, quantities and prices in which no one has any incentive to change.

The model is firmly based on past theoretical work which it extends by bringing various strands together into a comprehensive housing model. The authors of the model identify three strands of past work as of particular relevance:

(a) Alonso(1964) and Muth(1968) and their followers who stress the interrelation between transportation costs and housing demand and the implications for rent gradients and location patterns.

(b) 'Filtering': They have drawn on a second strand of the literature which deals with quality differences among existing dwellings, their changes, and the occupancy patterns accompanying them.

(c) Neighborhood Effects: The development of the literature on the effects of neighborhood racial composition and wealth on household choices and household prices has been influential in the inclusion of zonal wealth and racial composition in household utility functions in this model.

Thus the construction of this model admirably shows the link between explanatory models of the simple type and policy-oriented models which

typically tend to be rather more complex. It is distinguished from other such models by the following characteristics:

(a) The scope of the ideas incorporated into the model, ranging from accessibility-housing cost trade-offs to constraints on market outcomes imposed by the existing stock to the influence of neighborhood, race , and wealth on household choice;

(b) The relatively small size of the entire model compared to other simulation models;

(c) The empirical application of exactly the same theoretical model to six areas; and

(d) The range of housing, income, and land use policies which can easily be analyzed within the framework of the model.

The Model

We will now present a semi-technical description of the model structure. Space does not permit a full description while a non-technical description would not be useful here. We pay particular attention to the meanings of particular parameters used.

The model is driven by two kinds of behavior: utility maximization by households and profit maximization by owners of existing dwellings:

(i) Household Utility Functions: Each household maximizes utility:

$$U_{ij} = H_{ij} X_{ij} Z_1 Z_2 Z_3 \quad \ldots \quad \ldots \quad \ldots \qquad (106)$$

where

U_{ij} is the utility of the jth dwelling to the ith household

H refers to housing services

X to other goods and services

Z_1 , Z_2 , Z_3 are neighborhood or zone characteristics.

Now

$$H_{ij} = \left[Q_j - \gamma_1 \alpha_i \quad M_i / P_n\right]^{\alpha_i} \tag{107}$$

$$X_{ij} = \left[(M_i - Q_j P_j) - \gamma_1 (1 - \alpha_i) M_i\right]^{1 - \alpha_i} \tag{108}$$

where

Q_j is quantity of housing services offered by dwelling j

γ_1 is a parameter affecting the degree to which households will alter their housing choice in response to a price discount,

α_i is a parameter expressing the strength of housing preferences versus preference for other goods for households of type i,

M_i is household i's 'model' income (after adjustments for taxes and transfers)

P_n is the price per unit of service of newly constructed dwellings and P_j is the price per unit of dwelling j.

'Model' income

$$M_i = (M_i^a)^{.6} (\overline{M})^{.4} \tag{109}$$

where M_i^a is the actual annual income of household i and $\overline{M}$ is the median income of all households of type i .

Note that if $\gamma_1 = 0$ the utility function reduces to a Cobb-Douglas type utility function. Here other goods are merely represented as income minus housing with the implication that the price of other goods is normalized at 1. $\gamma_1 > 0$ has the effect of posing a minimum level of housing services only an excess over which gives positive utility. These minimum levels vary with income unlike a constant term which would be found in a Stone-Geary utility function. The value of γ_1 is the critical determinant of the degree of substitutability in demand among housing

sub-markets. A high γ_1 raises the minimum level of housing and other goods that is acceptable and therefore narrows the options available to a household.

Neighborhood Characteristics

$$Z_1 = (200 - T_j)^{.5 + \alpha_i - \alpha_1} \qquad (110)$$

where Z_1 represents accessibility

T_j average travel time in a month

α_i is once again the strength of housing preferences for household type i

and α_1 is the value of α_i for households of type 1 i.e. white non elderly families

$200 - T_j$ is an approximation of the monthly leisure time available to a worker in Zone j. The exponent .5 is based on studies suggesting that people value leisure time at about half their wage rate.

Z_2 represents the average net rent (gross rent less operating costs) of a zone relative to the average net rent in an SMSA (Standard Metropolitan Statistical Area). More precisely

$$Z_2 = \left[\frac{\left\{ (P_j - P_o)\, Q_j \right\}_{\text{zone}}}{\left\{ (P_j - P_o)\, Q_j \right\}_{\text{SMSA}}} \right]^{.01\gamma_2} \qquad (111)$$

where P_j is the price per unit of housing services

P_o is minimum operating costs per unit

Q_j is quantity of housing services

The numerator is the zonal average and the denominator is the SMSA average.

γ_2 is a parameter expressing willingness to pay in exchange for living in a wealthy zone.

The higher γ_2 the higher the utility people get by living in a wealthy neighborhood and consequently more they are willing to pay for this privilege.

Finally,

Z_3 is a zonal characteristic capturing the effect of racial composition of neighborhoods on utility.

$$Z_3 = R_{ij} + \left[1000/(100\,\gamma_3 + 1)\right] \qquad (112)$$

R_{ij} is the proportion of households in the zone of dwelling j belonging to the same racial group as household i

γ_3 is a parameter expressing the strength of preferences for racial homogeneity.

The larger γ_3 the more sensitive utility is to changes in R_{ij}. For example if $\gamma_3 = 0$ Z_3 can vary from only 1000 to 1001 while if $\gamma_3 = 1$ Z_3 can vary from 100 to 101 - a much larger percentage variation.

Thus the utility function takes account of utility gained by households due to

- quantity of housing services
- quantity of other goods
- accessibility of neighborhood they live in
- average wealth or income of neighborhood and
- racial composition of neighborhood

each expressed in appropriate functional form.

(ii) <u>Existing Dwelling Supply Functions</u>

The supply curve for existing dwelling j is specified as follows:

$$Q_j = \left[\beta_1 + \frac{2}{3} \beta_2 \left(\frac{P_j - P_o}{P_c} \right) \right] Q_o \qquad (112)$$

where

Q_j is the level of housing services currently orovided by dwelling j

Q_o is the level provided ten years ago

P_j is the price per unit of housing services offered by dwelling j

P_o is operating costs per unit

and

P_c is capital costs per unit of service for a new dwelling.

All prices are in flow terms i.e. on a monthly basis.

β_1 and β_2 are parameters to be determined empirically.

This expression is derived from profit maximization subject to a production function. The profit maximization procedure is actually maximization of expected profits. This embodies within it the behavior of the landlord over a long time horizon.

β_1 can be interpreted as some measure of depreciation over a decade.

β_2 is a parameter in the production function and determines the slope of the supply function.

(iii) Model solution

In essence, once we are given plausible parameter values

γ_1, γ_2, γ_3;

β_1 and β_2

and the α_i s

the model can be used to simulate the housing market.

The key exogenous variables in the model are

P_n the price of new housing. This includes P_c and P_o the capital cost and operating cost components.

P_o the operating cost component is very important as it affects strongly the number of removals from the housing stock.

Q_m the minimum quantity standards for new housing.

Y_i the household incomes.

Q_o the quantity of initial housing services.

T_j the travel time associated with each zone.

The endogenous output of the model comprises:

prices and quantities for each existing and new dwellings,

numbers of new dwellings and removals,

assignments of households to dwellings, zonal averages of prices, quantities,incomes and racial proportions.

Zonal averages of net rents and of racial proportions are fed back into the model and affect household behavior as explained earlier. Ultimately all the endogenous output is determined by exogenous variables, by government regulations and by the behavioral parameters for households and owners.

While the structure of the model is relatively simple and comprehensible its solution is a complex operation and will not be described here.

(iv) Data Requirements and Inputs:

One of the features of this model is its great flexibility. It can be run with a range of data quality. On the one hand the estimation of model parameters could be done after the availability of extensive and detailed data. The estimation of each one of the parameters: $\alpha_i S$, γ_1, γ_2, γ_3, β_1, β_2 needs detailed econometric work and each can comprise a research

project by itself. Similarly, the constructinn of 'model' households, household incomes, 'model' dwellings, travel times, etc. all need data that is readily available in the U.S. census and Federal Housing Administration (FHA) surveys. Such data are not, in general, so readily available in L.D.C. cities. On the other hand, all the parameters can be guesstimates as well as all the exogenous inputs and the model could still be used for policy purposes. The guesstimates could then be successively improved. For example, rough notions of income distribution could be utilized to construct 'model' households; similarly 'model' dwellings can be constructed from impressions of the variety of existing dwellings. This can be done primarily because the model is small in size: it is difficult to 'construct' plausible data when the model needs thousands of households but it can be done by hand when it only needs 30 - 50. Indeed, the entropy maximization models use similar averages and then generate the distributions in an attempt to be more realistic. Here no attempt is made to squeeze more information from the data than exists and averages remain averages.

(v) Outputs

The model is quite clear in identifying which are the parameters, the exogenous variables and the endogenous variables. The parameters have ideally to be estimated and reflect the behavior of households. The exogenous variables are the ones that we play with to simulate government policies. For example, a housing allowance program would be simulated by changing the Y_is, while a construction subsidy would be simulated by changing the P_n - the price of new households. Quantity constraints are subsumed by Q_m. The model then gives us the effect of different variables by forecasting changes in endogenous variables.

(vi) Evaluation

The Urban Institute housing model has been calibrated for six metropolitan areas of the U.S. - Durham (N.C.), Washington D.C., Chicago, Portland (Oregan), Pittsburgh and Austin. Except for some instances parameter values for different cities are not very different giving some faith in the belief that they do reflect behavioral relationships. The forecasting abilities of the model have been tested by using parameter values of the other 4 cities on Washington D.C. and Chicago with encouraging results: about 18% error in prediction. The composite results have then been used to simulate urban areas for policy testing. The simulations allow for variations in policies, types of cities and in behavioral parameters.

The suggestion here is not that the Urban Institute Housing Model is ripe for application to an LDC city. Indeed, it is too based on the character of the U.S. housing market for such a transplantation. Its institutional structure is heavily that of the U.S. - particularly that of a relatively free housing market albeit a segmented one. It is, however, an excellent example of an approach which is at once sophisticated and relatively simple as well as fruitful for policy concerns. This model does not give any welfare conclusions though its outputs could easily be fed into evaluative sub-models . Lest any wrong impression has been created this is not a simple model to solve. Indeed, a typical run of the model takes about $120 to run with about forty 'model' households. Increasing the number of these households to about 75 would probably quadruple costs. Thus expanding the model is expensive. It has taken about 10 professional man-years over a period of about 3 years to make the model operational at a total cost exceeding $600,000. These man-years have, moreover, been those of some of the best people in the field. Each city application costs about $20 - 40,000 and a period of about 4 months to put into effect. This presumes the existence of

a U.S. type data base which can readily be adapted for the model.

The recommendation here is that the approach embodied in this model - the logical structure and its policy applicability - is a good one to follow. It was built up from a base of clear objectives and explicit recognition was made of the theoretical bases used. Such clarity is rarely found in urban models. Its decadal span is also a sensible one in view of the durable aspects of city structure. One must, however, beware of the temptation to transfer the model as it is to an L.D.C. city since it is firmly rooted in the U.S. experience and institutional system. Its methodology is what should be transplanted.

4.3 A Model of Housing Demand at a Disaggregated Level

Paradoxically enough, fruitful policy-oriented models are likely to be at higher degrees of aggregation while explanatory models may have to be disaggregated. Policy-making is time-bound and policies are characteristically made at an aggregated level. People's behavior varies in all kinds of ways with different socio-economic characteristics, family cycle concerns, etc. It is then not really surprising that models which study this behavior are likely to be more detailed than those which prescribe or test policies. Moreover, the results of detailed explanatory models make it easier to feed policy-oriented models with more realistic data, parameters and coefficients which are often otherwise mere guesstimates.

Patricia Apps developed a theoretical framework for a model of housing demand at a disaggregated level and tested it for the city of Reading, England. The structure of the model is based on the economic theory of market demand which states that the individual's consumption patterns are the result of his preferences, income constraints and housing prices. It is also based on the assumption that differences in preferences are caused by social status, household size and stage in a family cycle of the household. The model thus predicts changes in housing demand resulting from changes in factors such as:

a. Number of households.

b. Income.

c. Social status distribution.

d. Size of household or composition.

The analysis is conducted at two levels. At the first level, housing is seen as a single commodity and a set of income -- consumption on Engel curves are derived for each household type. As the next level there is a two dimensional set of Engel Functions; one dimension for different household types and the other for different housing commodities. Working through conventional consumer theory of utility maximizing households with budget constraints the Engel curve is obtained

$$C_h = \Pi_h X_h = C_1 \quad (M, \Pi_1, \ldots \Pi_n) \tag{113}$$

where C_h, the allocation of expenditure on housing (commodity X_h) with price Π_h, is specified for household i of type k.

M is household income; and

Π_i, i=1,..., n are the relative prices of other commodities.

The form of the function for household type k for different incomes reveals preferences for that household type.

At the next level, utility is a function of housing characteristics considered as a set of commodities. We then have, for household j of type k

$$\begin{aligned} U = U_1(X_{11}, X_{12} \ldots X_{1m}) + U_2(X_{21}, X_{22}, \ldots X_{2m}) \\ + \ldots + U_h\ (X_{h1}, X_{h2}, \ldots X_{hm}) + \ldots \\ U_n(X_{n1}, X_{n2}, \ldots X_{hm}) \end{aligned} \tag{114}$$

where U_i, i=1 to n is the utility provided by the ith group of commodities;

U_h is the utility provided by housing with $X_{h1} \ldots X_{hm}$ being the m housing characteristics.

We then obtain the second level of housing consumption income Engel curves

$$C_h = \sum_{f=1}^{m} \Pi_{hf} X_{hf} = \sum_f C_{hf}$$

$$C_{hf} = C_{hf} \; (C_h, \Pi_{h1}, \Pi_{h2}, \ldots, \Pi_{hm}) \qquad (115)$$

where Π_{hf} is the price of housing commodity f;

X_{hf} is the amount of housing commodity f consumed by household j of type k; and

C_{hf} is the expenditure on housing commidity f now expressed as a function of total housing expenditure and the relative prices of different housing commodities.

Given data on:

a. Total housing expenditures

b. Characteristics of housing

e.g. i) location variables including employment accessibility; shopping accessibility; quality of neighborhood schools;

ii) dwelling characteristics such as age of structure, type of structure and condition of structure; and

iii) space of dwelling.

Equation (115) can be estimated by multiple regression methods to obtain Π_{hi} s i.e. the implicit prices of different housing services.

Finally, having obtained these prices, demand functions for each of these characteristics can be estimated using household types as the independent variables.

This procedure therefore reveals household consumption patterns for different housing characteristics subject to income and price constraints. A by-product of this model is that the system is easier to handle since the number of housing characteristics is much smaller compared to housing types. This is particularly true of LDCs where the variety of housing types is even greater -- from straw shacks to tall apartment buildings.

Apps was quite successful in obtaining the housing characteristic prices from careful regression analysis but was not able to obtain good demand functions in the next stage. We have the normal problem of linear assumptions; housing characteristics are assumed to be non-interacting. Apps speculates that deficiencies in the data may be causing her problems.

As should be evident from the description of this model, it is extremely data intensive. Apps used a good 10 percent sample survey of Reading and a host of other data sources to build all the indices of housing characteristics. The appeal of this procedure is that it reveals the preferences of consumers. It is this kind of knowledge that is necessary for the disaggregation of results obtained from Lundquist-Andersson type models. In particular, some appreciation of the differences in behaviour of different income groups is obtained. This information is necessary for robust planning so that plans do not get subverted by disgruntled groups whose preferences have been grossly violated -- as has happened often in relocation of low income groups in various cities. However, the question of data availability remains, though consumer household surveys of a detailed nature are being made increasingly now.

Such work is particularly important for LDC cities since, as is suggested earlier, preference of different income groups, ethnic groups, etc., are likely to be more disparate in LDC cities than, say in the U.S. The results of such models are particularly important since planners and policy makers do not generally have intuitive notions of preferences of people outside their class and income experience. With huge slum and squatter populations, such organized information would be invaluable in LDC cities. Good estimates of such magnitudes as income and price elasticities of demand for housing and transportation are the kind of inputs that both Urban Institute and Stockholm models need for useful operation.

One sector of urban activity that has been ignored here and throughout this paper is the location pattern of industry in cities. This is not accidental. It is because there is little work on the subject even in Western economics. We mention in passing here the work of Bergsman, Greenston and Healy (1972, 1975) which could usefully be applied to LDC cities. They have attempted to redefine the Standard Industrial Classification (S.I.C.) system according to location clusters of different industries. They have used data from the Census of Manufacturing and used techniques of factor analysis to obtain their clusters. Their classification turns out to be quite different from the S.I.C. which is used internationally. It is this kind of work that is needed in LDC cities to test the speculations offered in Section 1. One obtains insights into the nature of scale economies and agglomeration from such work and consequently a better understanding of the structure of cities. Their work does not shed any light on intra-urban industrial location but does offer insights into industrial cluster patterns.

5. MODELLING POOR CITIES: WHAT SHOULD BE DONE

How LDC cities should be modelled is very much dependent on what the objectives of the exercise are. The lessons from the last fifteen years experience of urban modelling in developed countries is more explicit in telling us what not to do than in what should be done. One of the main lessons is that at this stage we should be modest in our expectations of the capabilities of modelling. Consequently, limited objective models are more likely to be operationally useful than comprehensive models. Based on the following perception of the Bank's institutional role, a fruitful strategy towards the modelling of poor cities is now suggested.

The World Bank's main activity consists of lending funds for projects and programs at concessional as well as market interest rates. Thus models of urban structure should help in identification as well as evaluation of urban projects. Identification of projects requires models which predict and/or prescribe the future structure of a city. Evaluation requires cost-benefit analyses and urban models can help in tracing some inter-relations as well as in optimization. To feed such models explanatory or analytical models also are needed to promote basic understanding of the urban process. Behavioral relationships and parameters can then be used in the policy oriented models utilimately needed.

It is the major conclusion of this paper that policy-oriented models should be of the 'sketch-planning' type i.e. small and manageable while the analytical or explanatory models can be larger and disaggregated. The reasons for this have been documented throughout the paper. Some concerns are, however, worth emphasizing. A useful policy model needs to have fast turn-around time to make it more responsive to policy needs and issues. It is

only then that modellers and decision-makers can interact and test for various alternatives. Policy makers have had no involvement in models developed in the U.S. It is then not surprising that these models have not been used for policy purposes. Thus if we are now serious about developing policy-oriented models this major shortcoming must be remedied .

The Stockholm model and the Urban Institute Housing Model are two examples of approaches that meet some of these requirements. They are both small and manageable though mathematically complex. The Stockholm model attempts to have a structure conducive to planner/modeller interaction. It is therefore a step in the right direction. It was also developed with fairly specific objectives. Its policy applications were in helping gross decisions like building or not building a bridge. That is a realistic approach in the sense that policy decisions are usually about some large investments rather than about detailed residential locations. Policy makers are interested in gross impacts rather than in disaggregated allocation. This view of the policy making process is more relevant in countries not having command economies as is the case for almost all LDCs.

The Urban Institute Housing Model is more based on behavioral relationships but it utilizes the idea of telescoping a large system into a small, more manageable and comprehensible one. Its data requirements are flexible in that it can be operated on guesstimates as well as on accurate data. Thus, in LDCs, where speed in decision-making needs to be encouraged and where data is scarce, this type of a model can be usefully utilized in an incremental fashion. Systematic policy planning can be begun with rough data and the model successively refined as better and better data is available. One other aspect of the Urban Institute model deserves comment. It seeks to be a general behavioral model so that the same structure

can be calibrated for different cities. It is not self-evident that such an approach can work across continents and cultures. This needs more research on the explanatory model side and should be conducted concurrently. One's presumption here is that within the U.S. where the institutional structure is essentially common across cities this is not an unjustified approach. If a model is to be useful it is important that attention be given to the particular institutional structure of the country concerned. Thus adequate account must be taken of the larger public sectors in LDCs and of the consequent constraints on the private market. Similarly, the higher segmentation of LDC markets has to be recognized. Local nationals can be expected to be more conversant with such institutional differences than expatriates. Thus it is suggested that local participation in modelling is of the utmost importance. This should aid in interaction with policy makers as well.

Urban models should be seen as a process rather than as products. The use, for example, of an Urban Institute type model which can involve successive improvements in data as well as model is clearly a process. This process is educational for modellers as well as policy-makers. If seen in this way modelling will suffer less from short time horizons. This makes it even more important that local nationals be involved in modelling efforts rather than quick kibbitzing expatriates.

It is mentioned above that policy-models require inputs from explanatory or analytic models. These inputs are both data oriented as well as of behavioral relations. Continuing research aimed at achieving greater understanding of existing patterns as well as of future changes is necessary for the development of good policy models. Such research might help in appreciating what cities are for and why they exist. The Apps

model which is very disaggregated has been suggested as one fruitful approach to understand the behavioral relations implicit in housing. In such efforts it is almost inevitable that extremely disaggregated data is needed since one is interested in the whys and wherefores of behavior. We can afford to do this in this context because the usefulness of such modelling is not time-bound. Specifically, the kind of information severely lacking in LDCs is of the following nature.

A. Transportation

Some ideas on transportation in LDC cities were discussed in the introduction. Continuing that discussion one would like detailed information on the following:

(i) Analysis of Movements

The hustle-bustle characteristic of an LDC city has to be systematized. Information is needed on the origin destination pattern, traffic variability at different times and, as a result, the importance of the journey to work.

(ii) Costs of Different Modes:

We need detailed information on operating (and fixed) costs of different modes - walking, bicycles, automobiles, buses, etc. In addition, the income classes which use these modes have to be determined in order to calculate time costs. Using some notion of efficiency, is the prevailing pattern efficient?

(iii) Congestion:

Movement in central areas of LDC cities is painfully slow. To what extent is this relevant as a transport cost?

Such information is crucial to the understanding of location patterns in LDC cities. Various conjectures concerning the movement of people were offered in the introduction: they can be tested with the help of such information. Mean values, cost parameters and even preference patterns can be derived to be then used as inputs in operational models.

B. Housing

(i) Production:

Considerable information is already available on this score. Further quantitative information is needed on self-help housing to be able to construct production functions for the housing sector.

(ii) Preference Structures:

This is a rather muddy area of research which needs further theoretical as well as empirical clarification. One has to disentangle the effects of incomes from those of tastes in peoples choice of housing. The Apps kind of work is one approach to doing this. For a more intuitive grasp of housing preferences and patterns as they change with income and time, Homer Hoyt's studies (1959, 1966) of residential neighbourhoods in the U.S. provide good examples.

(iii) Housing-Transport Trade-Offs:

Studies of preferences and income variation with regard to housing will also provide information on the effect of transport costs on location choices. It is clear from existing models that we get different urban location patterns depending on the elasticity of demand for space and the imputed cost of time. More measurement in this area would provide valuable insights in the design of urban economic models.

Housing for the poor has become a much discussed problem. Most LDC cities have a majority of the people living in slums, squatter settlements and shanty towns. Such housing is often regarded by planners as deficient and as a situation to be remedied. Since such housing comprises a significant magnitude and hard information on it is not yet available, such information is crucial to the good design of the housing sector in urban models.

C. Industry and Employment

(i) The Informal Sector:

While more and more information is slowly being made available on the content of the informal sector not much progress is being made on its spatial characteristics. One would, for example, like to know the kind of movements of goods and of people it generates. What degree of flexibility does it have to the urban structure? To what extent is it innovative in overcoming traditional spatial problems of transport costs, etc.?

(ii) Small-Scale Industry:

Bergsman et. al. (1972, 1975) have performed a valuable service in the U. S. by attempting to find clusters of activity which always seem to go together. A similar analysis of patterns in LDCs would provide a great amount of interesting information on the nature of economic activity in these cities. The interconnection between activities within these clusters would provide clues on the reasons for agglomeration.

(iii) Links with the Hinterland:

It has often been suggested that the degree of primacy of large cities in LDCs is higher than elsewhere. This coupled with the vast dual economy literature would give some impression of self-sufficiency in these

cities. Since the Lowry type models in particular and others as well often take some basic "export" sector as given and as the real driving force in the model structure, it is important to identify the nature and extent of linkages that LDC cities have with their immediate hinterland as well as more far off ones.

The informal sector has been mentioned in the introduction. Its existence, it has been suggested, makes many market activities in LDCs different in nature from those in Western cities. Its employment patterns have a large effect on location patterns - both residential as well as employment. Thus information on its spatial characteristics is essential to the design of useful urban models.

Each of these is a substantive research project in itself. Moreover, as outlined in the introduction account has to be taken of the differences between cities within the less developed world. Information gathering must be done within some kind of framework to be most useful. That is merely another word for modelling.

The 'modelling agenda' suggested above is eminently practical. It involves concurrent investment in the accretion of knowledge as well as in policy aids. Some of this research is messy, but necessary if we are to expand our understanding of poor cities in order to articulate sensible policies.

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