Emmanuel Jimenez

The Economics of Self-Help Housing: Theory and Some Evidence from a Developing Country

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EMMANUEL JIMENEZ

Department of Economics, The University of Western Ontario,
London, Ontario, Canada, and
The World Bank, Washington, D.C. 20433

I. INTRODUCTION

In response to the abject housing conditions in urban areas of the developing world (recent estimates indicate that up to 40% of the population of some of the world's largest cities live in inadequate shelter—see World Bank [21] and Grimes [5]), the multilateral aid agencies, such as the World Bank, concluded that it was

> obvious that considerable emphasis should be placed on "self-help" projects. The personal initiative and work stimulated by such schemes add both to output and savings in a sector where resource limitations are so evidently crucial. (World Bank [21]).

The impact has been large. Since its involvement began in 1972, the World Bank has lent over $1.5 billion to 20 countries in support of these so-called sites and services and slum upgrading projects.

In contrast to the structure-intensive redevelopment projects popular in North America in the 1960's, the projects imply a minimal level of intervention so that costs can be recovered through user charges. The government typically subdivides purchased or expropriated private land and allocates lots to applicant families (chosen according to certain income criteria). Basic infrastructure—water and sewerage connections, either private or communal, electricity, and paved walkways—are then installed. The residents are expected to build the rest of their dwellings at their pace, depending on their available resources. A key aspect of such projects is that participating

households build their dwellings. This approach is thought to be particularly applicable in the low-income setting for several reasons. First, the low-income household may be incapable of mustering the large initial outlay needed to purchase adequate housing because of the inadequacy of financial markets in developing countries. Self-help implies a "progressive" step-by-step development which enables a household to phase in its capital and other resource investments in accordance with a variable income stream. Second, the household is presumed to be able to buy its labor at a price below the current market rate for contracted labor because of a high rate of unemployment. This is consistent with the traditional notion that the opportunity cost of time of marginal workers approaches zero (Lewis [12], Fei and Ranis [3]). Thus, the household invests "sweat" equity rather than financial resources which are in short supply.

Given the size of these programs, several questions need to be answered: What has been the experience regarding the use of self-help labor in the production of housing services? What determines how much of a household's labor is made available? What is the technology underlying this autoproduction process, in particular, regarding the substitutability of inputs in the informal sector? What does self-help imply for overall housing consumption and are present policies emphasizing the appropriate technique? This paper uses the analytical framework of household production to examine these issues.[2] There are trade-offs between using one's time (and learning the necessary skills) in construction and time spent earning income in the market, even for low-income households. (See Mazumdar [14] and Piñera and Selowsky [16] for models of the informal urban sector where employment can easily be found.) An analytical basis for the opportunity cost of household time in housebuilding will be derived. It will also be shown that, because of a nonzero shadow price for time, income and price elasticities of demand for housing may not be interpreted in the usual Hicks–Slutsky sense. For households who engage in self-help housing, supply side parameters feed into the formulation of these elasticities. From the empirical point of view, the theory also leads to testable hypotheses regarding price and income elasticities of demand for housing.

It should be noted that most of the analytical tools which will be developed are applicable to urban issues in the US and other developed countries. First, the problem is directly analogous to the household choice of doing maintenance on one's house with own labor (although the investment motive may be stressed a bit more). This context was recently developed by Mendelsohn [15] with US data. Second, the method is applicable to study-

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[2] The supply and demand parameters affecting home production have been addressed in the literature. Applications include fertility and children's services (Schultz [19]), health (Grossman [6]) and self-sufficiency farming (Hymer and Resnick [9]).

ing the growing homesteading movement in which urban households are able to obtain title to a lot and structure for a nominal sum in a blighted area of the city provided that they develop it within a reasonable time.

The paper first outlines the theoretical considerations of the problem. An empirical section follows based on data from a self-help housing project in El Salvador.

II. THEORETICAL CONSIDERATIONS

The theoretical framework is utility maximization in which households choose to consume housing, leisure, and other goods subject to time and production constraints as well as an income constraint. Several simplifying assumptions are made. Households are assumed to consist of (or to behave as) one person and they allocate all their time among market work, leisure and housebuilding activities. The relevant time horizon will be a two-period world in which houses are constructed in one period. Housing, which is a homogeneous good produced with constant returns to scale technology, is produced only at home;[3] there is no depreciation. All other consumption goods must be bought in the market. The assumptions will focus the model on the determinants of household choice regarding equilibrium housing consumption, and not on the adjustment path to that level.

The representative household maximizes a twice differentiable utility function $u = u(z_1, z_2, h_1, h_2, l_1, l_2)$, where h_i denotes the quantity of housing services consumed in the ith period, l_i is the amount of leisure time taken in the ith period, and z_i represents consumption of all other goods, also in period i. To facilitate the solution of the system, we assume that the utility function is separable and additive across the two time periods.

$$u = u(z_1, h_1, l_1) + \gamma u(z_2, h_2, l_2), \tag{1}$$

where γ is a constant which discounts consumption to the present. All consumption decisions are assumed to be made jointly by the members of the household (or alternatively, decreed by one) in the initial period of the two-period world and the future is faced with perfect foresight. This maximization takes place subject to the following constraints: time, budget, and production.

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[3] This is consistent with the project components of the case study. Note that the model focuses on the consumption rather than the investment side of the consumer choice problem. This seems more appropriate in dealing with a world where most households are owner–occupiers (at least of the structure); where housing services are minimal; where underdeveloped financial systems preclude a mortgage lending market; and where we are dealing with housing projects whose primary objective is to improve households' welfare through an increase in consumption opportunities. Further, many of the projects tend to discourage an investment motive. In El Salvador, the source of our data, to prevent speculation, households were allowed to neither rent nor sell their plots for a five-year period after occupancy.

The total time available to the household in the first period is equal to the duration of the first period ($\bar{T}_1$) times the household size, measured in adult equivalents (N), where N is equal to the number employed (N_e) plus the number unemployed (N_u). This time is taken up by the time devoted to work at home in house building by the household members ($t_1 = \sum_{i=1}^{N-N_u} t_{1i} + \sum_{j=1}^{N_u} t_{1j}$); leisure time ($l_1 = \sum_{i=1}^{N-N_u} l_{1i} + \sum_{j=1}^{N_u} l_{1j}$); and time devoted to market work ($m_1 = \sum_{i=1}^{N-N_u} m_{1i}$):

$$N\bar{T}_1 = t_1 + l_1 + m_1. \tag{2a}$$

It should be noted that the analysis takes N, N_u and thus N_e as exogenous to the system and will focus on the determination of time allocation among families rather than within them. In the second period, the time constraint is a bit different. Because it is assumed that housing is completed in the initial period, in this two-period world, the household will not choose to engage in any housebuilding in the second period. Instead, all of its time will be spent in leisure activities and in the market:

$$N\bar{T}_2 = l_2 + m_2, \tag{2b}$$

where l_2 and m_2 are defined analogously to l_1 and m_1.

Money income in the first period is equal to the market wage rate (w) times the amount of labor offered in a given period (m). It is assumed that market wages are constant across time periods. In addition, there is some amount of (exogenously determined) nonwage income (A), which can be assumed to be the same for both periods without loss of generality. Income must be used in expenditures on consumption goods [price (p_z) times quantity (z)]; on materials inputs for housing construction [price (p_x) times quantity (x)]; and on hired (skilled) labor inputs [price (p_L) times quantity (L)]. In the first period, this is equivalent to:

$$A + wm_1 = p_z z_1 + p_x x_1 + p_L L_1. \tag{3a}$$

It should be noted that, throughout this paper, the wage rate (w) refers to the returns per earner in a household from working in the marketplace. The returns to hired labor from working on that family's house will be denoted as p_L. In the second period, again because there is no housebuilding, disposable income will be spent only on consumption goods. It is assumed that, in this model, which concentrates solely on the consumption motive for housing, income cannot be transferred from one period into the next in the form of savings. Again, this seems to be more applicable to the low income setting where there is a lack of investment opportunities in the formal sense

(see Footnote 3):

$$A + wm_2 = p_z z_2. \tag{3b}$$

However, since investment in housing can be transferred to the next period where its consumption stream is enjoyed, housing can be interpreted as the vent for savings. Total income in these two periods must be totally exhausted by consumption.

Because the household has to build its housing needs, it also faces a production constraint. The flow of the quantity of housing services available for consumption in the first period is assumed to be some fixed amount of the initial housing stock, $\overline{H}$, which is assumed to be given, where c is a scaling factor which changes stocks into flows:

$$h_1 = c\overline{H}. \tag{4a}$$

In the second period, consumption is constrained by the amount available for consumption in the first period (assuming no depreciation) plus the additions to the housing stock resulting from construction in the first period:[4]

$$h_2 = h_1 + c\tilde{H}_1, \tag{4b}$$

where the addition to the housing stock is $\tilde{H}_1 \leq f(x_1, at_1, L_1)$ and where $f(\cdot)$ is a twice differentiable production function in materials (x_1) and hired labor (L_1) and own labor used in housebuilding (t_1) adjusted by a productivity coefficient (a). This coefficient transforms labor units into efficiency terms and is necessary to allow for control of any differences in housebuilding skills among the households of the cross-sectional data base. Let $t_1' = at_1$. It is assumed that at least some of one type of labor and materials are necessary inputs into housing production. Finally, in order to simplify the algebra, it is assumed that $f(\cdot)$ exhibits constant returns to scale for any changes in nonzero inputs and is itself nonzero.[5]

The problem then is to maximize Eq. (1) subject to constraints (2) to (4):

$$
\begin{aligned}
u^* = {} & u(z_1, h_1, l_1) + \gamma u(z_2, h_2, l_2) \\
& + \lambda_1(wm_1 + A - p_z z_1 - p_x x_1 - p_L L_1) + \lambda_2(wm_2 + A - p_z z_2) \\
& - \pi_{h_1}(h_1 - c\overline{H}) - \pi_{h2}[h_2 - c\overline{H} - cf(x_1, t_1', L_1)] \\
& + \rho_1(\overline{T}_1 N - t_1 - l_1 - m_1) + \rho_2(\overline{T}_2 N - l_2 - m_2).
\end{aligned}
$$

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[4]Throughout this paper, small letters denote flows. Capital letters denote stocks. Capital letters with tildes denote changes in the stock. It should also be noted that the assumption of a fixed initial housing stock is consistent with the data set.

[5]It is assumed that $\tilde{H}_1 > 0$, which is confirmed by the data.

We set $c = p_z = 1$. If we substitute for h_1 in the utility function, we note that there would now be a constant in the utility function, which we can disregard. Let us also drop the subscripts on the variables which only appear in one period ($x_1 = x$, $t_1 = t$, $t'_1 = t'$, $L_1 = L$, $h_2 = h$, and $\pi_{h_2} = \pi_h$) and substitute for m_i to obtain a simplified format:

$$u^* = u(z_1, l_1) + \gamma u(z_2, h, l_2) + \lambda_1 \big[w(N\overline{T}_1 - t - l_1) + A - z_1 - p_x x$$
$$- p_L L \big] + \lambda_2 \big[w(N\overline{T}_2 - l_2) + A - z_2 \big] - \pi_h \big[h - \overline{H} - f(x, at, L) \big],$$
$$(5)$$

where λ_1, λ_2, and π_h are, respectively, the shadow prices of income in the two periods and housing.

The household chooses optimal consumption levels of housing and other goods. However, it must also choose how to produce housing services—through its own labor and/or materials. The first-order conditions for solving (5) are, then[6]

$$-w\lambda_1 + \pi_h f_{t'} a \leq 0 \qquad \text{or} \qquad t = 0, \tag{6a}$$

$$-p_L \lambda_1 + \pi_h f_L \leq 0 \qquad \text{or} \qquad L = 0, \tag{6b}$$

$$-p_x \lambda_1 + \pi_h f_x = 0, \tag{7}$$

$$u_{z_1} - \lambda_1 = 0, \tag{8}$$

$$u_{l_1} - \lambda_1 w = 0, \tag{9}$$

$$u_{z_2} - \lambda_2 = 0, \tag{10}$$

$$\gamma u_h - \pi_h = 0, \tag{11}$$

$$\gamma u_{l_2} - \lambda_2 w = 0, \tag{12}$$

$$-h + \overline{H} + f(x, at, L) = 0, \tag{13}$$

$$w(N\overline{T}_1 - l_1 - t) - z_1 - p_L L - p_x x = 0, \tag{14}$$

$$w(N\overline{T}_2 - l_2) - z_2 = 0. \tag{15}$$

The interpretation of the first-order conditions is straightforward once we have assumed interior solutions for all the variables. From Eqs. (6a), (6b), and (7), we obtain the familiar expression that inputs x, L, and t will be used until the ratios of their marginal products are equivalent to the ratio of

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[6]In a Kuhn–Tucker formulation, we would first have inequalities for (6a) to (15). Then assume interior solutions for all except (6a) and (6b).

their prices. Also, through (11), the marginal cost of each input in housing production must equal the marginal utility of housing in consumption: $\gamma u_h = \pi_h$. Finally, z_i and h are consumed until the marginal rate of substitution in consumption is equal to the ratio of their shadow prices.

The above system assumes interior solutions hold except for (6a) and (6b); that is, households need to consume some market goods ($z_1, z_2 > 0$), shelter ($h > 0$), leisure ($l_1, l_2 > 0$); and to buy materials inputs ($x > 0$) to construct that shelter. We further assume that at least one equality in either (6a) or (6b) holds, so that some labor in whatever form is needed for construction. Thus, we can distinguish three cases, depending on which of the equalities hold: (a) Case I ($L > 0$, $t = 0$) will hold when the inequality of (6a) and the equality of (6b) hold, so that the value of the marginal product of labor is less than the household's opportunity cost, the market wage: $(\pi_h/\lambda_1)f_{l'}a < w$. Households will thus choose to do no work on housebuilding and instead use only hired labor.[7] (b) Case II ($L = 0$, $t > 0$) will hold when the (imputed) value of the marginal product of hired labor is less than its price: $(\pi_h/\lambda)f_L < p_L$. This is the "pure self-help" case in which the household will find it cost-efficient to use only its own labor in production. (c) Case III ($L > 0$, $t > 0$) will be obtained when both equalities of Eqs. (6a) and (6b) hold. In this case, the values of own labor and hired labor will be equal. Both types of labor will be used.

The household choice calculus involves 10 equations (the equality of (6b) and (7) to (10) for Case I; the equality of (6a) and (7) to (10) for Case II) in 10 unknowns for Cases I and II. There are 11 equations and unknowns for Case III. We can now derive reduced form relationships for each of the dependent variables in terms of the independent variables according to the Cases. Of particular interest to the analysis are the equations for the derived demand of inputs into the production process and the equation for housing consumption (the latter being, in Grossman's [6] terminology, a "hybrid equation" which embodies demand and supply parameters). In general form, they are:

$$L_J = L_J\left(w, a, N, N_u, \mu; p_L, p_x, \overline{H}, \gamma\right), \tag{16}$$

$$t_J = t_J\left(w, a, N, N_u, \mu; p_L, p_x, \overline{H}, \gamma\right), \tag{17}$$

$$x_J = z_J\left(w, a, N, N_u, \mu; p_L, p_x, \overline{H}, \gamma\right), \tag{18}$$

$$h_J = h_J\left(w, a, N, N_u, \mu; p_L, p_x, \overline{H}, \gamma\right), \tag{19}$$

where $J =$ I, II, III for the cases and variables listed to the right of the

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[7]It must be noted that this case must be strictly defined because households always have to put in some amount of supervisory time. The data do not allow for differentiation between supervisory time and actual construction time for own labor.

semicolon are not expected to vary across observations because of the nature of the data base. Of course, $L_{II} = t_1 = 0$. The term μ represents characteristics which also condition preferences and are unmeasured by the researcher but known to the household.

Comparative Statics

Equations (16) to (18) are the estimating equations. The data base will not allow the direct estimation of the housing consumption equation, (19), but it is shown below that its parameters can be derived from the input demand equations.

The most interesting results to be derived have to do with the effect of wage rates on input demand and housing consumption. This effect will differ, depending on the type of household. Cramer's Rule can be applied for each of the cases and *a priori* notions of the sign and magnitude of the wage coefficient in (16) to (19) can be obtained. The derivation of the results is outlined in the Appendix.

The wage rate elasticities of demand for hired labor and materials when own labor is not used (Case I) are:[8]

$$\hat{\epsilon}^{I}_{Lw} = \bar{\eta}_{hw} + \eta_{hI}S^{I}_{w}, \tag{20}$$

$$\hat{\epsilon}^{I}_{xw} = \bar{\eta}_{hw} + \eta_{hI}S^{I}_{w}, \tag{21}$$

where $\bar{\eta}_{hw}$ is the utility compensated cross price elasticity of demand for housing with respect to changes in the price of leisure; η_{hI} is the Hicks–Slutsky income effect; and S^{I}_{w} is the share of wage income in total income. The intuitive explanation of (20) is relatively straightforward. A higher wage rate will imply a higher level of earned income. This has a positive income effect on housing consumption which will cause labor inputs to rise in some proportion (depending upon the exact form of the production function) since households are committed to acquiring houses only through self-help methods. This is captured by the second term on the right-hand side of (20). In addition, a rise in the wage rate will cause leisure to become more expensive relative to the other arguments of the utility function, (1). Assuming some substitutability among the various consumption goods in any one period, this will reinforce housing demand and will further boost demand for hired labor. In cross-section terms, households with higher wage

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[8]Throughout this paper, the symbol "ϵ_{ij}" will denote a full equilibrium (equating demand and supply) elasticity—the percentage change in the use (in the case of inputs) or consumption (in the case of consumption goods) of the "ith" variable with respect to a unit change in the "jth" variable. The symbol "η_{ij}" will denote only demand elasticities for consumption goods in the partial equilibrium sense. A bar over such an elasticity will mean that the measure is utility compensated. The "$\hat{\ }$" signifies that the elasticity can be estimated directly from the input demand equations.

rates will tend to demand more hired labor, all other things given. The explanation for (21) is similar.

The effect on housing consumption of a unit change in the wage rate can be similarly derived. In this world in which the household makes supply and demand decisions simultaneously, demand and supply elasticities will be related:

$$\epsilon_{hw}^{\mathrm{I}} = \hat{\epsilon}_{Lw}^{\mathrm{I}}. \tag{22}$$

Intuitively, this holds because an increase in the wage rate will have positive income and leisure-substitution effects. This would be analogous to solutions derived in most housing demand models (e.g., [2]), except for the presence of the leisure substitution term. The response is equivalent to that of input demand since housing is only available through building. Assuming constant returns to scale, the equality must hold.

For Case II the effects of a change in the wage rate on own labor supply and housing consumption yield different results. One of the primary justifications for self-help housing projects is that they make use of a relatively plentiful input (household labor), while minimizing the demands on households' financial resources. Because of supposedly high unemployment rates, this can presumably be done at little cost to the earning potential of the household. This is equivalent to an assumption that the opportunity cost of labor is zero. However, experience has shown that project participants tend to use contract labor for many of the tasks which were felt to be manageable by self-help methods (see [7]). If the wage rate is a measure of the shadow price of self-help labor, the solution of the system yields results which incorporate both demand and supply side parameters because of the presence of the production function as a constraint of consumption. In particular, the behavior of variables, such as the amount of labor the household sets aside for housebuilding and the consumption c·' housing services, will depend, not only on demand elasticities, but also on parameters like the elasticity of substitution of inputs in the production of housing.

The magnitude of the elasticities of own labor and materials used with respect to the wage rate for Case II are respectively:

$$\hat{\epsilon}_{tw}^{\mathrm{II}} = \bar{\eta}_{hw} + \eta_{hI}S_w^{\mathrm{II}} + \bar{\eta}_{hp_h}K_t^{\mathrm{II}} - \sigma_{tx}K_x^{\mathrm{II}}, \tag{23}$$

$$\hat{\epsilon}_{xw}^{\mathrm{II}} = \bar{\eta}_{hw} + \eta_{hI}S_w^{\mathrm{II}} + \bar{\eta}_{hp_h}K_t^{\mathrm{II}} + \sigma_{tx}K_t^{\mathrm{II}}, \tag{24}$$

where $\bar{\eta}_{hp_h}$ is the utility compensated price elasticity of housing (the pure substitution effect); σ_{tx} is the elasticity of substitution of own labor for other inputs; and K_i^{II} are factor shares in total output, $i = t, x$. The effect of the wage rate on time spent at home (i.e., Eq. (23)) can be decomposed into income and substitution effects. The income effect is, of course, positive. A

rise in wage income would lead to a rise in housing demand, thus enhancing the need for own labor, t, in producing it. As before there is also a positive cross price elasticity of leisure effect. We have called these two effects the "leisure–income" effect. The substitution effect has two components. First of all, an increase in the wage rate will increase the price of household time and will lead to a substitution of other inputs for the use of own time. This is captured by the elasticity of substitution, σ, and has a negative impact on t. Secondly an increase in the price of time will lead to an increase in the implicit price of housing (and the opportunity cost of staying away from the market). Housing consumption will decline in favor of other consumption. This will obviate the need for t. Thus, both substitution effects are reinforcing in the negative direction.

Notice that the components of the expression for the wage rate elasticity of demand for materials, (24), are similar to those of (23). The difference lies in the sign and the weight of the input substitution term. An increase, say, in the wage rate will cause the household to substitute materials for own labor.

These findings regarding labor supply have profound effects on the effect of a wage rate change on housing consumption. Income elasticities of demand for housing have occupied a large portion of the theoretical and empirical literature (see [2] and [17] for thorough surveys of the empirical state of the art). Almost all of the studies postulate that housing is a normal good, i.e., that an increase in permanent (or current) income would lead to an increase in housing demand (recent US evidence indicates that this is a less than proportional increase). The theoretical work underlying these estimates is based on classical consumption theory. However, in a housing project where the population builds at least a portion (if not all) of its own housing needs, this framework may be inadequate because it concentrates solely on the demand side. The theory outlined in the preceding section, in which part of the production choice is left up to the consumer, admits the possibility that housing (the self-help type) may be an inferior good, at least with respect to *wage* income, since the effect of a change in the wage rate upon housing consumption is, in elasticity terms:

$$\epsilon_{hw}^{II} = \bar{\eta}_{hw} + \eta_{hI} S_w^{II} + \bar{\eta}_{hp_h} K_t^{II}, \tag{25}$$

where ϵ_{hw}^{II} is the wage elasticity of housing consumption, $\bar{\eta}_{hp_h}$ is the utility compensated own-price elasticity of housing, and K_t^{II} is the share of labor in total output. The sign of the expression is ambiguous and can be divided into income and substitution effects. The first term on the right-hand side of (25) is the previously explained "leisure–income" effect, which is positive, assuming that housing is a normal good in the Slutsky definition (with income exogenously determined), and some substitutability in consumption

between leisure and housing. However, an increase in the wage rate will influence the implicit price of housing. Its tendency to do so will depend on the share of labor in total output. This substitution effect would have a negative impact upon housing consumption. Thus, because of the fact that an increase in wage income also implies that the opportunity cost of time has increased, the effect on the consumption of housing that is built with self-help labor is ambiguous. As a result, self-built housing can appear to be "inferior" with respect to changes in wage income.

The elasticities of the use of hired labor, own labor and materials inputs with respect to the market wage for Case III can be shown to be, respectively:

$$\hat{\epsilon}_{Lw}^{III} = \bar{\eta}_{hw} + \eta_{hI}S_w^{III} + \bar{\eta}_{hp_h}K_t^{III} + \sigma_{tL}K_t^{III}, \tag{26}$$

$$\hat{\epsilon}_{tw}^{III} = \bar{\eta}_{hw} + \eta_{hI}S_w^{III} + \bar{\eta}_{hp_h}K_t^{III} - \sigma_{tL}K_L^{III} - \sigma_{tx}K_x^{III}, \tag{27}$$

$$\hat{\epsilon}_{xw}^{III} = \bar{\eta}_{hw} + \eta_{hI}S_w^{III} + \bar{\eta}_{hp_h}K_t^{III} + \sigma_{tx}K_t^{III}. \tag{28}$$

All the symbols have been defined earlier, except for σ_{tx} and σ_{tL} which are, respectively, the Allen partial elasticities of substitution of own labor with materials inputs and with hired labor inputs in housing production. These influence the wage rate elasticity of own labor use negatively since the household will react to a change in the price of own labor by substituting for the relatively cheaper input. The extent to which it can do so is limited by technology, as represented by these elasticities. The "leisure–income" and compensated price affects are present in all three elasticities as they affect input demand through output (housing) demand. The wage elasticity of housing consumption for Case III is:

$$\epsilon_{hw}^{III} = \bar{\eta}_{hw} + \eta_{hI}S_w^{III} + \bar{\eta}_{hp_h}K_t^{III}. \tag{29}$$

The last (negative) substitution term is present, as in Case II, because own labor is used.

The results described above are summarized in Table 1. In addition, it can be shown that the expression for the elasticity of housing consumption and inputs with respect to nonwage income will contain the same expression for all cases:

$$\epsilon_{hA}^{j} = \eta_{hI}S_A^j = \hat{\epsilon}_{iA}^j, \tag{30}$$

where $j = I, II, III$, S_A^j is the share of nonwage income in total income, and $i = x, t, L$.

TABLE 1

Summary of Main Theoretical Results[a]

	Wage rate elasticity of input demand	Wage rate elasticity of housing consumption
Case I ($t = 0, L > 0$)	(20) $\hat{\epsilon}_{Lw}^{I} = \bar{\eta}_{hw} + \eta_{hI} S_w^{I}$	
	(21) $\hat{\epsilon}_{xw}^{I} = \bar{\eta}_{hw} + \eta_{hI} S_w^{I}$	(22) $\epsilon_{hw}^{I} = \bar{\eta}_{hw} + \eta_{hI} S_w^{I}$
Case II ($t > 0, L = 0$)	(23) $\hat{\epsilon}_{tw}^{II} = \bar{\eta}_{hw} + \eta_{hI} S_w^{II} + \bar{\eta}_{hp_h} K_t^{II}$	
	(24) $\hat{\epsilon}_{xw}^{II} = \bar{\eta}_{hw} + \eta_{hI} S_w^{II} + \bar{\eta}_{hp_h} K_t^{II} + \sigma_{tx} K_t^{II}$	(25) $\epsilon_{hw}^{II} = \bar{\eta}_{hw} + \eta_{hI} S_w^{II} + \bar{\eta}_{hp_h} K_t^{II}$
Case III ($t > 0, L > 0$)	(26) $\hat{\epsilon}_{Lw}^{III} = \bar{\eta}_{hw} + \eta_{hI} S_w^{III} + \bar{\eta}_{hp_h} K_t^{III} + \sigma_{tL} K_t^{III}$	(29) $\epsilon_{hw}^{III} = \bar{\eta}_{hw} + \eta_{hI} S_w^{III} + \bar{\eta}_{hp_h} K_t^{III}$
	(27) $\hat{\epsilon}_{tw}^{III} = \bar{\eta}_{hw} + \eta_{hI} S_w^{III} + \bar{\eta}_{hp_h} K_t^{III} - \sigma_{tL} K_L^{III} - \sigma_{tx} K_x^{III}$	
	(28) $\hat{\epsilon}_{xw}^{III} = \bar{\eta}_{hw} + \eta_{hI} S_w^{III} + \bar{\eta}_{hp_h} K_t^{III} + \sigma_{tx} K_t^{III}$	

[a]Symbols explained in the text.

III. EMPIRICAL FINDINGS

This section has two main goals. The first concerns the role of the market wage in determining the amount of unpaid labor which households utilize in housebuilding. One of the crucial properties of the model is that it disputes the assumption that the shadow price of labor in poor countries is negligible. In particular, the market wage rate is considered to be a primary determinant of the amount of work that households will put into housebuilding (i.e., the opportunity costs of time are nonzero) and the amount of labor they will hire. Further, the wage rate elasticities of labor demand and supply will differ depending upon the household's mix of labor inputs. The response of own-labor supply to changes in the wage rate will be less than the response of demand for hired labor. This is so because of the existence of substitution effects resulting from the increased implicit price of own time. Because consumers produce at least a portion of what they consume, demand and supply elasticities are linked. This hypothesis will be examined through regression analysis of Eqs. (16) to (18). The independent variables which are expected to vary across observations are the wage rate and weekly nonwage income, as well as household size and proxy variable for ability in housebuilding.[9]

[9]Various forms were attempted. The linear, semi-log and double-log formulations all yielded the same results. In the interests of space and because the coefficients are readily interpreted as elasticities, only the double-log form results are reported.

The other goal of this section, is to derive a set of elasticities which will yield some insights into certain parameters of demand and supply of low-cost housing. The results outlined in the preceding paragraph have important repercussions on the wage income elasticity of housing demand. This parameter is affected by opportunity costs. This implies that the elasticity is lower for those who participate in self-help housing than for those who do not, and may even be negative, if the income effects are outweighed by substitution effects which are occasioned by these positive opportunity costs. The data base does not allow the direct estimation of a housing consumption equation which would yield such an elasticity. The reason is that, because the houses in the project have not been appraised or sold, housing value cannot be used as a dependent variable in the equation. However, the theoretical framework allows us to infer the values of both housing demand and supply parameters (such as the elasticity of substitution between materials and labor) from estimated coefficients of input demand.

The Sample

The sample which is used for this study is of 353 families in a project in two medium-sized cities, Santa Ana and Sonsonate, of El Salvador.[10] This project is a sites and services project in which households commence habitation from uniform plots, on which is a stone slab to serve as the floor of a one-room house, provision for running water and sewerage and four walls. Project participants then develop the house through self-help means. The survey is part of a panel study which is meant to follow the pattern of housing consolidation of the families in the project. The data base is taken from a survey applied some $1\frac{1}{2}$ years *after* the households took possession and started construction.[11]

As explained in the earlier section of the paper, the households of the sample divide into three types: those who do only self-help (Case I), those

[10] The data are based on socioeconomic surveys which were carried out by research teams in El Salvador sponsored jointly by the World Bank and the International Development Research Centre (IDRC) of Canada. It is part of a larger research effort (four other countries are involved) to attempt to measure the impact of these urban development projects on the low-income population they serve. The main goal of these surveys is to measure, via an experimental design, the effects of the projects on health, demographic, and other socioeconomic variables. The case of El Salvador was chosen over the other countries (the Philippines, Senegal, and Zambia) because of availability and reliability. Of the original 363 participant families interviewed, ten were dropped from the sample because of inconsistent replies on some key questions.

[11] This time period is probably sufficient to circumvent a possible conceptual problem—a problem which states that what we have measured are "incomplete" housing units, i.e., households might be intending to put more labor into housebuilding. It is assumed that one year is enough to finish a habitable house—to a level which is perceived by the household as such when it makes its initial choice.

TABLE 2

Average Values of the Variables[a]

	Case I	Case II	Case III	All	
No. in Sample	178	98	77	353	
PAIDL	9.26	—	6.67	6.12	No. of man-weeks of paid
	(8.92)		(6.54)	(8.54)	labor in households.
UNPAIDL	—	8.93	9.10	4.47	No. of man-weeks of own
		(9.83)	(11.3)	(8.65)	labor in house bldg.
AVWKWAGE	82.48	72.13	84.31	80.01	Average weekly wage rate
	(40.03)	(33.39)	(45.26)	(39.74)	of household (in colones)
WNONWAGE	9.84	8.63	5.80	8.62	Weekly nonwage inc. of
	(22.55)	(24.28)	(19.57)	(22.43)	household (in colones)
NUNEMPLY	0.30	0.42	0.39	0.35	No. of people in the HH
	(0.72)	(0.76)	(0.76)	(0.72)	declared to be unemployed
AGEH	41.2	40.5	41.8	41.1	Age of household
	(10.7)	(11.0)	(10.6)	(10.7)	head.
SEXH	0.49	0.61	0.66	0.56	Prop. of household head
	(0.55)	(0.49)	(0.47)	(0.49)	who are male.
HHSIZE	5.1	6.2	5.9	5.6	Household size
	(1.7)	(2.1)	(2.1)	(2.0)	
NCONS	0.03	0.42	0.12	0.16	No. of persons in HH who
	(0.21)	(0.61)	(0.32)	(0.42)	have been employed in
					const. sector.
NPRIMES	2.3	2.6	2.7	2.5	No. of persons in HH
	(1.0)	(1.1)	(1.2)	(1.1)	aged 17–60.
SEXRATIO	0.52	0.49	0.50	0.51	No. of females to
	(0.22)	(0.18)	(0.18)	(0.20)	No. of males.
MATS	2811.47	1868.16	2956.68	2581.26	Amount of materials
	(2515.90)	(1884.98)	(1900.97)	(2267.47)	used in construction
					(in colones)

[a]During the sample period U.S. $1 = 2.5¢ (colones). Standard deviations are in parentheses.

who build their entire dwellings using only hired labor (Case II) and those who used both (Case III). As can be seen from Table 2, most households are of the Case I variety. Those who use both represent only 22% of the sample.

In comparing Cases I, II, and III, the pure self-help households have household heads who are slightly younger and who are predominantly male. Further, household size tends to be larger and, within households, there is a substantially greater average number of members who have been employed in the technical or construction field. The availability of workers at home also appears to be a consideration as the average number who are unemployed is greater for both Cases II and III households. Finally, the self-help households have a slightly smaller proportion of females. While these characteristics point to a distinct tilt of housebuilding ability towards Case II households, some (such as the age and sex variables) are also indicators of

ρ higher earning capacity. Despite this, average wages for Case II households are still lower, which lends some initial support to the hypothesis that the opportunity cost of time is indeed important for these households. It should also be noted that Case I households, who use *only* hired labor in construction, do not have the highest average wages. This is undoubtedly due to the fact that there are households within Case I who may be using solely hired labor, not only because the opportunity cost of own time is so great, but also because some are relatively inept housebuilders.

Results of the Estimation

The results of the estimation of the derived input demand equations are summarized in Tables 3a and 3b.[12] For Case I households, the wage rate, as expected, has a positive impact on the demand for hired labor. This is to be interpreted, according to the theoretical results, as the "leisure–income" effect. It is the sum of the Hicks–Slutsky income effect and a substitution effect which is positive as a result of the rise in the relative price of leisure. Also, the *a priori* result that the wage rate elasticities of labor and materials demand are similar is confirmed.

For Case II, the wage rate elasticity of the use of own labor is smaller than the elasticities of Case I. This is due to the substitution effects described in the theory, which include the increased use of other inputs as the price of own labor inputs rises with a rise in wage, as well as the overall effect of decreased demand for housing as input prices and costs rise. Both would tend to depress the need to supply labor to build housing. It is interesting to note that the substitution effects do not swamp the income effects. The theory also led us to expect Table 3's result that the wage rate elasticity for own labor in Case II should be less than that for demand of materials inputs.

For Case III, the wage rate coefficients of hired labor and materials inputs exceed that of own labor, as predicted by Table 2 because of the presence of negative substitution terms in production in (27) as opposed to (26) and (28). Aside from the income effects, a rise in the wage rate would make own labor more expensive relative to hired labor, leading to substitution of hired labor for own labor. An interesting observation is that the substitution effects appear to outweigh the income effects for the own labor equation. While the coefficient is not significantly different from zero, it

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[12]An alternative method of specifying the empirical model would be to account for the probability of not using one type of labor or the other. Heckman's [8] two-stage estimation technique considers this problem when there is one criterion to determine the choice of cases. Here we would need a more complex maximum likelihood estimation technique (unavailable to the author) since there are two criteria to determine the three cases. Preliminary results from a simplified model with an additive labor input into production indicate that selection is not a problem for the sample.

TABLE 3a
Labor Input Equations[a]

	CASE I		CASE II		CASE III		CASE III	
	LN(PAIDL)	LN(PAIDL)	LN(UNPAIDL)	LN(UNPAIDL)	LN(PAIDL)	LN(PAIDL)	LN(UNPAIDL)	LN(UNPAIDL)
CONSTANT	−0.2337	−0.6586	0.6806	0.9806	−1.4169	−1.3910	1.7746[+]	3.3902*
	(0.8012)	(0.9706)	(0.6822)	(0.7868)	(1.4253)	(1.8009)	(1.4767)	(1.8094)
LN(AVWKWAGE)	0.4838**	0.4909**	0.2309*	0.2768*	0.6611*	0.5756*	0.0039	0.0928
	(0.1757)	(0.1852)	(0.1405)	(0.1509)	(0.3487)	(0.3097)	(0.3504)	(0.3209)
LN(WNONWAGE)	0.0855*	0.1046*	0.0697	0.0959*	0.1129	0.1466[+]	−0.0484	−0.0180
	(0.0527)	(0.0568)	(0.0850)	(0.0885)	(0.1189)	(0.1186)	(0.1232)	(0.1191)
NUNEMPLY	−0.1687[+]	−0.1796[+]	−0.1654[+]	−0.2262[+]	−0.1745	−0.3139[+]	−0.0569	−0.1156
	(0.1178)	(0.1185)	(0.1531)	(0.1600)	(0.1889)	(0.2033)	(0.1957)	(0.2042)
HHSIZE	−0.0204	−0.0330	0.0135	0.0247	0.0446	0.0114	0.0458	0.119
	(0.0475)	(0.0483)	(0.0532)	(0.0545)	(0.0668)	(0.0737)	(0.0692)	(0.0740)
AGEH		0.0092[+]		0.0046		0.0195[+]		−0.0021
		(0.0078)		(0.0105)		(0.0158)		(0.0159)
SEXH		0.2872*		−0.1641		−0.0324		−0.0822
		(0.1635)		(0.2499)		(0.3292)		(0.3308)
NCONS		0.3110		0.1330		−0.3044		0.3288
		(0.3732)		(0.1915)		(0.4585)		(0.4606)
CITY		−0.0577		−0.4570		−0.5668*		−0.9460**
		(0.1567)		(0.7868)		(0.3027)		(0.3042)
R^2	0.06	0.09	0.04	0.09	0.07	0.14	0.02	0.14
N	178	178	98	98	77	77	77	77

[a]Standard errors are in parentheses.
**indicates 95% significance.
*indicates 90% significance.
[+]indicates that the coefficient exceeds the standard error.

TABLE 3b

Materials Input Equations

	CASE I		CASE II		CASE III	
	LN(MATS)	LN(MATS)	LN(MATS)	LN(MATS)	LN(MATS)	LN(MATS)
CONSTANT	5.5198**	4.6940**	5.5131**	5.9915**	6.8749**	7.5311**
	(0.6954)	(0.8298)	(0.5492)	(0.6289)	(0.7991)	(1.0213)
LN(AVWKWAGE)	0.5054**	0.5070**	0.3148**	0.2909**	0.1671	0.1822$^+$
	(0.1525)	(0.1584)	(0.1163)	(0.1206)	(0.1978)	(0.1746)
LN(WNONWAGE)	0.0960**	0.1000**	0.1094*	0.1514**	0.0421	0.0503
	(0.0461)	(0.0486)	(0.0674)	(0.0707)	(0.0668)	(0.0673)
NUNEMPLY	−0.1954**	−0.2291**	−0.2956*	−0.2549**	−0.0788	−0.0784
	(0.0123)	(0.1013)	(0.1233)	(0.1279)	(0.1055)	(0.1153)
HHSIZE	−0.0257	−0.0239	0.0591$^+$	0.0626$^+$	0.0244	0.0225
	(0.0413)	(0.0413)	(0.0428)	(0.0436)	(0.0386)	(0.0418)
AGEH		0.0089$^+$		−0.0088$^-$		−0.0044
		(0.0067)		(0.0084)		(0.0090)
SEXH		0.3263**		0.1677		−0.0059
		(0.1398)		(0.1998)		(0.1867)
NCONS		−0.2635		0.2910*		−0.0657
		(0.3191)		(0.1531)		(0.2600)
CITY		0.2013$^+$		−0.2110$^+$		−0.2466
		(0.339)		(0.831)		(0.1717)
R^2	0.09	0.14	0.14	0.20	0.04	0.07
N	178	178	98	98	77	77

should be kept in mind that the size of this a negative substitution effect could have substantial implications for the wage rate elasticity of housing consumption. If most of these negative effects are attributable to the utility compensated price elasticity of housing demand, and if this latter price elasticity were large enough, then a negative value for the wage rate elasticity of housing consumption would result. This implies that, for households who perform self-help in housebuilding, housing may be an "inferior" good with respect to wage income.

The effect of the other variables can be briefly reviewed. The effect of the number of household members unemployed, *given* household size, has a negative effect on all inputs. However, the size of the coefficient is not larger for the self-help cases than Case I. Of the productivity variables, age and sex appear to be important in determining the amount of labor hired in Case I. Older household heads who are male tend to hire more. The amount of experience in construction (which is not corelated with wages) does not seem to have a significant impact on input demand. The negative correlation between age of head and wages may be affecting the significance of the former. Sex of head does not appear to be correlated with wages.

Some Derived Elasticities

As stated before, the elasticities derived from the preceding discussion can be used to obtain estimates of other parameters, such as the individual components of the income and substitution effects, the elasticity of substitution between labor and materials in the production of housing, and the wage income elasticity of housing consumption. They are derived from the manipulation of the equations in Table 1 and thus do not require the estimation of specific functional forms of a production function or housing demand equation (Such estimates are handicapped by the difficulties of deriving a suitable dependent variable.) The results are summarized in Table 4. The estimates used are those for the equations with proxies. Because of the relative stability of the coefficients within each case, the results are not significantly altered when other (or no) measures of productivity are used.

The wage rate elasticity of housing consumption can be easily computed for each of the cases from Table 2.[13] As expected, this derived elasticity is greater for Case I (0.4909) than for Cases II and III (0.2890 and 0.1788, respectively) since the pure hired labor case is not affected by the price substitution effects. Moreover, the pure income effect can be derived for Case I by using (30), where $\hat{\epsilon}_{LA}^{I} = \hat{\epsilon}_{xA}^{I} = \bar{\eta}_{hI}S_{A}^{I}$, which implies that the Hicks–Slutsky income elasticity of housing demand ranges from 0.78 to unity. These estimates are not far off the mark of some recent estimates (using direct estimates of demand curves) of permanent income elasticities of housing demand in other developing countries such as Korea [23] and Columbia [13], which range from 0.5 to unity. Another finding then, although one which is very preliminary because of the difficulty in making a strict comparison, is that estimates of income elasticity are similar even in the face of disparate samples and estimating techniques. Finally, it should be noted that the substitution effects do not swamp the income effects in the wage rate elasticity of housing consumption for Cases II and III.

The left-hand column of Table 1 can be used to obtain the Allen partial elasticities of input substitution for Cases II and III.[14] The estimates for

[13] The results are:

$$\epsilon_{hw}^{I} = \hat{\epsilon}_{Lw}^{I} = 0.4909.$$

$$\epsilon_{hw}^{II} = \hat{\epsilon}_{tw}^{II}K_{t}^{II} + \hat{\epsilon}_{xw}^{II}K_{x}^{II} = 0.289.$$

$$\epsilon_{hw}^{III} = \hat{\epsilon}_{Lw}^{III}K_{L}^{III} + \hat{\epsilon}_{tw}^{III}K_{t}^{III} + \hat{\epsilon}_{xw}^{III}K_{t}^{III} = 0.1788.$$

[14] The results are:

For Case II:
$$\sigma_{tx} = \hat{\epsilon}_{xw}^{II} - \hat{\epsilon}_{tw}^{II} = 0.0071.$$

For Case III:
$$\sigma_{tx} = \hat{\epsilon}_{xw}^{III} - \hat{\epsilon}_{tw}^{III} + \frac{K_{L}}{K_{t}}\left(\hat{\epsilon}_{xw}^{III} - \hat{\epsilon}_{Lw}^{III}\right) = 0.0188$$

$$\sigma_{tL} = \hat{\epsilon}_{Lw}^{III} - \hat{\epsilon}_{tw}^{III} + \frac{K_{x}}{K_{t}}\left(\hat{\epsilon}_{Lw}^{III} - \hat{\epsilon}_{xw}^{III}\right) = 2.6558.$$

TABLE 4

Derived Estimates of Elasticities

Cases	Case I	Case II	Case III
Estimated elasticities[a]			
$\hat{\epsilon}_{Lw}$	0.4909**	—	0.5756*
$\hat{\epsilon}_{tw}$	—	0.2768*	−0.0928
$\hat{\epsilon}_{xw}$	0.0570**	0.2909**	0.1822
Estimated factor shares[b]			
K_L	0.1981	—	0.1184
K_t	—	0.2564	0.1816
K_x	0.8019	0.7436	0.7000
Estimated income shares			
S_w	0.89	0.93	0.97
Derived elasticities[c]			
ϵ_{hw}	0.4909	0.289	0.1788
σ_{tx}	—	0.0071	0.0188
σ_{tL}	—	—	2.6558

[a] From Tables 3a and 3b; *indicates that the point estimate is significant at 90% confidence. **indicates that the point estimate is significant at 95% confidence. Elasticities derived from equations which include productivity variables. " — "indicates Not Applicable.

[b] Average Proportion of Estimated Total Cost. It is assumed that skilled labor is paid at the rate of 15¢/day, which is thrice the official minimum daily wage and is consistent with the results of the survey.

[c] The equation for the wage rate elasticity of housing demand and the elasticities of substitution of own labor for other inputs in housing production are derived from Table 1.

both Cases II and III (the last two rows of Table 4) indicate that there is not much scope for substitution between self-help own labor and materials inputs in housing production. This is consistent with aggregate studies of the elasticity of substitution between labor and capital which range from 0.1 to unity (see [20] for a review of these studies). However, there is evidence of a relatively elastic rate of substitution between the two labor inputs (which may be interpreted as the difference between skilled and unskilled labor).

IV. SUMMARY AND CONCLUSIONS

The current research effort attempts to explain, in rational economic terms, the parameters which face households when they decide whether or not to engage in self-help housing production. Further, it derives the

determinants of the household choice to carry out various types of self-help activities, such as the optimal mix of own versus hired labor. The wage rate and its relationship to the price of hired labor and the productivity of the individual household in construction work become the primary factors in the decision-making.

One of the results shown is that, for the sample in question, the wage rate is indeed a determinant of the type of self-help housing which these households prefer to do. Further, this influence leads to strong substitution effects for households who use some of their own labor directly in construction activities. The wage rate acts as a measure of opportunity costs and the amount of time which households set aside for housebuilding is adversely affected by them. For the "self-help" cases (Case II and III) this adverse effect may outweigh income effects to such an extent that it leads to a negative relationship between housing consumption and the wage. Although such a negative relationship was not found in our sample, the wage rate elasticity of housing consumption is estimated to be significantly lower for the self-help samples than other estimates.

This implies that a policy which attempts to stimulate self-help housing through increases in wage income (many projects sponsored by multilateral aid agencies incorporate some sort of income–employment generation schemes) may lead to lower than expected levels of housing consumption. Although our theoretical result is not strong enough to make a long-run statement, at the very least, households may postpone consumption of housing services by not building immediately. Our samples also provide evidence that there is little scope for substitution between labor and materials inputs in building low-income dwelling units in developing countries. Production is characterized by relatively fixed technical coefficients for these two inputs. However, substitution is relatively flexible between the two types of labor inputs. This has implications for the types of accompanying policies which can be used to assist participant families. For example, a policy which provides subsidized loans to buy only materials may not lead to a desired increase in production if the supply of the concomitant labor factors remain constrained due to a fixed supply of unpaid labor and unavailability of funds to hire labor.

APPENDIX

Total differentiation of the system described in (6) to (16) will yield:

$$[A][Y] = [G],$$

where

$$[A] \equiv \begin{bmatrix} \pi_h a^2 f_{t't'} & \pi_h a f_{t'L} & \pi_h f_{t'x} & 0 & 0 & 0 & 0 & 0 & a f_{t'} & -w & 0 \\ \pi_h a f_{Lt'} & \pi_h f_{LL} & \pi_h f_{xL} & 0 & 0 & 0 & 0 & 0 & f_L & -p_L & 0 \\ \pi_h a f_{xt'} & \pi_h f_{Lx} & \pi_h f_{xx} & 0 & 0 & 0 & 0 & 0 & f_x & -p_x & 0 \\ 0 & 0 & 0 & u_{z_1 z_1} & u_{z_1 l_1} & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & u_{l_1 z_1} & u_{l_1 l_1} & 0 & 0 & 0 & 0 & -w & 0 \\ 0 & 0 & 0 & 0 & 0 & u_{z_2 z_2} & u_{z_2 h} & u_{z_2 l_2} & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & u_{h z_2} & u_{hh} & u_{h l_2} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & u_{l_2 z_2} & u_{l_2 h} & u_{l_2 l_2} & 0 & 0 & -w \\ a f_{t'} & f_L & f_x & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -w & -p_L & -p_x & -1 & -w & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -w & 0 & 0 & 0 \end{bmatrix},$$

$$[Y] \equiv \begin{bmatrix} dt \\ dL \\ dx \\ dz_1 \\ dl_1 \\ dz_2 \\ dh \\ dl_2 \\ d\pi_h \\ d\lambda_1 \\ \\ d\lambda_2 \end{bmatrix}, \quad [G] \equiv \begin{bmatrix} \lambda_1 \, dw - \pi_h(f_{t't'} at + f_{t'}) \, da \\ \lambda_1 \, dp_L - \pi_h f_{Lt'} \, da \\ \lambda_1 \, dp_x - \pi_h f_{xt'} \, da \\ 0 \\ \lambda_1 \, dw \\ 0 \\ -u_h \, d\gamma \\ \lambda_2 \, dw - u_{l_2} \, d\gamma \\ -f_{t'} t \, da \\ -dA - (\overline{T}_1 N - t_1 - l_1) \, dw - w(N \, d\overline{T}_1 + \overline{T}_1 \, dN) \\ \qquad -L \, dp_L - x \, dp_x \\ -(\overline{T}_2 N - l_2) \, dw - w(N \, d\overline{T}_1 + \overline{T}_1 \, dN) \end{bmatrix}$$

Case I. The appropriate system for Case I can be obtained from (A1) by deleting the first row and column of $[A]$, $[\dot{Y}]$, and $[G]$ and setting $t_1 = 0$ to form

$$[A^I][Y^I] = [G^I]. \tag{A2}$$

The determinant of this system is:

$$A^I = \pi_h f_{Lx} h^2 \Delta / Lx, \tag{A3}$$

where

$$\Delta = (\pi_h^2/\lambda^2)\big(u_{z_1 z_1} u_{l_1 l_1} - u_{z_1 l_1}^2\big)\big(-2u_{z_2 l_2} w + u_{l_2 l_2}^2 + w^2 u_{z_2 z_2}\big)$$

$$+ \big(2w u_{z_1 l_1} - u_{l_1 l_1} - w^2 u_{l_1 l_1}\big)\big(-u_{hh} u_{l_2 l_2} - w^2 u_{z_2 z_2} u_{hh} + u_{h l_2}^2$$

$$+ w^2 u_{z_2 h} - 2w u_{z_2 h} u_{h z_2} + 2w u_{z_2 l_2} u_{hh}\big).$$

Stability conditions require that $A^I > 0$. The effect of a change in the wage rate upon the use of hired labor in housebuilding can be derived by applying Cramer's Rule to the system in (A2):

$$(\partial L/\partial w) = \left[-\lambda_1 A_{41}^I + \lambda_2 A_{71}^I - (\overline{T}_1 - l_1)A_{91}^I + (\overline{T}_2 - l_2)A_{10}^I\right]/A^I,$$

$$(A4)$$

where A_{ij}^I is the minor of $[A^I]$ defined by the element in the ith row and jth column.

We note that, if we assume that opportunity costs reflect market costs, the expressions in (A3) and (A4) can be written in terms of Hicks–Slutsky demand elasticities. This can be shown by solving the standard demand-side two-period utility maximization problem in which income cannot be transferred between periods. Deriving the first order conditions for the problem and then totally differentiating them would yield a system which has a determinant equal to Δ. It is then straightforward to show that (A4) can be written in terms of the Hicks–Slutsky income and substitution effects by comparing the expressions for the minors $[A_{ij}^I]$ the minors of the standard demand-side system (see Jimenez [11] for proofs). Further, the wage rate elasticity of housing consumption can be expressed in the same demand elasticity terms with a similar argument:

$$(\partial h/\partial w) = \left[\lambda_1 A_{76}^I - \lambda_2 A_{76}^I + (\overline{T}_1 - l_1)A_{96}^I - (\overline{T}_2 - l_2)A_{10,6}^I\right]/A^I.$$

$$(A5)$$

Case II. When Case II applies, the system can be obtained from (A1) by deleting the second row and second column of $[A]$, $[Y]$, and $[G]$, and setting $L = 0$ to form:

$$[A^{II}][Y^{II}] = [G^{II}].\qquad (A6)$$

The derivation of the expressions for the wage rate elasticities of own-labor use and housing consumption (as well as demand for materials inputs) follows the argument for Case I. However, on evaluation of these elasticities, one finds that they must be written, not only in terms of the Hicks–Slutsky demand side parameters as in Case I, but also in terms of certain supply side parameters (see [11]). These are defined as:

$$\sigma_{xt} = xtf_{t'}f_x/f_{xt'}h,\qquad (A7)$$

$$K_t = f_{t'}t/h,\qquad (A8)$$

where σ_{xt} can be interpreted as the elasticity of substitution of own labor for

other inputs in housing production, given our assumptions regarding the production function; and K_l is the share of labor in total output.

Case III. The system of differential equations required to perform comparative static operations when Case III applies is precisely described by (A1). The derivation of the expressions for the wage rate elasticities of the use of the two types of labor and materials inputs, as well as of housing consumption follows the arguments for Case I and Case II (i.e., the application of Cramer's Rule to (A1)). It can be shown (see [11]) that these elasticities can be written in terms of Hicks–Slutsky income and substitution effects and the expressions for K_i; and

$$\sigma_{ij} = hF_{ij}/ijF, \tag{A9}$$

where $i, j, = x, L, t'$; F is the bordered Hessian determinant of the three-input production function $h = f(x, t', L)$; and F_{ij} is the cofactor of the element in the ith row and jth column of $[F]$; such that

$$[F] = \begin{bmatrix} 0 & af_{t'} & f_L & f_x \\ af_{t'} & a^2 f_{t't'} & af_{t'L} & af_{t'x} \\ f_L & af_{t'L} & f_{LL} & f_{Lx} \\ f_x & af_{t'x} & f_{Lx} & f_{xx} \end{bmatrix}$$

and $F = \det[F]$. It is then obvious that (A9) can be interpreted as the partial elasticity of substitution of input "i" for input "j" in housing production, as derived by Allen [1].

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