WPS7578

Policy Research Working Paper 7578

Illicit Activity and Money Laundering from an Economic Growth Perspective

A Model and an Application to Colombia

Edgar Villa
Martha A. Misas
Norman V. Loayza

World Bank Group
Development Research Group
Macroeconomics and Growth Team
February 2016

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Abstract

This paper contributes to the economic analysis of illicit activities and money laundering. First, it presents a theoretical model of long-run growth that explicitly considers illicit workers, activities, and income, alongside a licit private sector and a functioning government. Second, it generates estimates of the size of illicit income and provides simulated and econometric estimates of the volume of laundered assets in the Colombian economy. In the model, the licit sector operates in a perfectly competitive environment and produces a licit good through a standard neoclassical production function. The illicit sector operates in an imperfectly competitive environment and is composed of two different activities: The first activity produces an illicit good that nonetheless is valuable in the market (for example illicit drugs); the second does not add value to the economy but only redistributes wealth (for example robbery, kidnapping, and fraud). The paper provides a series of comparative statics exercises to assess the effects of changes in government efficiency, licit sector productivity, and illicit drug prices. From the model, the analysis derives a set of estimable macroeconometric equations to measure the size of laundered assets in the Colombian economy in the period 1985 to 2013. The paper assembles a data set whose key components are estimates of illicit income from drug trafficking and common crime. Illicit incomes increased drastically until 2001, reaching a peak of nearly 12 percent of gross domestic product and then decreasing to less than 2 percent by 2013. The decline overlaps not only in a period of high economic growth, but also after the implementation of Plan Colombia. The data set is used to estimate the volume of laundered assets in the economy by applying the Kalman filter for the estimation of unobserved dynamic variables onto the derived macroeconometric equations from the model. The findings show that the volume of laundered assets increased from about 8 percent of gross domestic product in the mid-1980s to a peak of 14 percent by 2002, and declined to 8 percent in 2013.

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Illicit Activity and Money Laundering from an Economic Growth Perspective: A Model and an Application to Colombia*

Edgar Villa[†], Martha A. Misas[‡] and Norman V. Loayza[§]

JEL code: O10, O54, K14, K42

Keywords: Crime, Economic Growth, Drug Trafficking, Money Laundering, Colombia

*This paper is the result of collaboration between the Government of Colombia, members of the Colombian academic community, and the World Bank. We gratefully recognize the UIAF (Colombia), especially director Mr. Luis Edmundo Suarez and deputy director Mr. Javier Gutiérrez for their support. We thank María Camila Castellanos and, especially, Angela Hurtado for their superb assistance in building the data set used in this project; Wilson Martinez, former Prosecutor General of Colombia, for his practical advice; and Claudia Meza-Cuadra for her careful editorial assistance. We gratefully acknowledge the financial and technical support of the World Bank in undertaking and completing this project. In this respect, we are particularly indebted to Yara Esquivel (Senior Financial Specialist), Jean Pesme (Financial Markets Integrity Practice Manager), Luis Serven (Senior Advisor Development Research Group), Emile van der Does de Willebois (Senior Specialist), Eva Gutierrez (Finance and Markets Program Leader), Daniel Lederman (Lead Economist), Barbara Cunha (Senior Specialist), Marjin Verhoeven (Senior Specialist), Quy-Toan Do (Senior Economist), Emily Adeleke (Finance and Markets Specialist), Alfonso García Mora (Practice Manager) and Roberto Aleu Biel (Consultant). We are grateful for the helpful comments of Alvaro Montenegro (Universidad Javeriana, Bogotá), Leonardo Duarte (Universidad Nacional de Colombia), Gonzalo Hernández (Universidad Javeriana, Bogotá), Munir Jalil (Universidad Nacional de Colombia), and Franz Hamman (Banco de la República de Colombia). We also wish to thank Daniel Flechas, Stephany Santacruz, Alejandro Gaviria, and Sebastián Zárate for superb research assistance for this project.

†Associate Professor of the International School of Economics and Business Administration. Universidad de La Sabana, Campus Puente del Común. Chía, Colombia. Email: edgar.villa@unisabana.edu.co.

‡Associate Professor of the Economics Department. Pontificia Universidad Javeriana. Calle 40 No. 6-23, Edificio Gabriel Giraldo. Email: mmisas@javeriana.edu.co.

§Lead Economist of World Bank, Development Economics. Email: nloayza@worldbank.org.

1 Introduction

Money laundering has increased substantially in the last 20 years among developed and developing economies.[1] The economics literature on the long-run effects of money laundering is limited as only a few papers consider the measurement and effect of illicit activities–and associated laundering transactions—on capital accumulation and economic growth. Moreover, there is no consensus on whether there is a negative or positive relation between the funneling of "dirty money" into the financial system and the growth prospects of the economy.[2] Some experts consider that illicit activities and money laundering have a negative economic effect because they i) generate economic distortions that decrease the productivity of licit factors in the economy; ii) erode the financial and real sectors that are infiltrated by asset laundering, generating bankruptcy risks and ultimately financial crises; and iii) undermine government institutions through corruption and capture. Other authors, however, suggest that illicit activities and money laundering can have a positive impact on economic growth, especially for developing economies, if they generate new resources that can enter the economy and fund productive investment.

This paper contributes to the economic analysis of illicit activities and money laundering in two interrelated ways. First, it presents a theoretical model of long-run growth that explicitly considers illicit workers, activities, and income, alongside a licit private sector and a functioning government. Second, it generates estimates of the size of illicit income and provides simulated and econometric estimates of money laundering in the Colombian economy.

On the theoretical contribution, the paper presents an overlapping-generation growth model with both licit and illicit activities. In the model, earnings from illicit activities can be in part "laundered" into the economy by consumption of licit goods and investment of physical capital. The model restricts attention to only one asset, capital. The licit sector, which operates in a perfectly competitive environment, produces a licit good using capital and labor through a standard neoclassical production function. The illicit sector, which operates in an imperfectly competitive environment, is composed of two different activities. The first produces an illicit good (e.g., illicit drugs) that nonetheless is valuable in the market. The second activity does not produce anything new or add value to the economy but consists of illicit appropriation of somebody else's income (e.g., robbery, kidnapping, and fraud). In a purely economic sense, the first illicit activity can be characterized as productive, while the second one as redistributive.

In the model, young individuals self-select into working in either the licit or illicit sector depending on their subjective moral loss of undertaking illicit activities. Young individuals are endowed with a unit of time to work, receive at the end of their first period of life an inheritance in the form of

[1] See Boghean, C., (2001), Reuter and Truman (2004), Gilmore, W.C., (2006), Walker and Unger (2009).
[2] Tanzi (1996), Prokhorov (2001A, 2001B), Masciandaro (2001). Ardizzi et al.(2013).

capital from their adult parents, and consume the licit good. Old individuals do not work, leave a bequest of capital for their offspring, and consume the licit good. The volume of illicit income is either consumed or saved by illicit households (young and old) much in the same way as licit households do. An important difference, however, is that a part of illicit income is detected and confiscated by the government with a given probability, while the rest is either consumed or saved. Some of the savings from illicit income are funneled to fund capital investment, which can then be integrated into the licit economy through the bequests from the old to the young.

The last agent in the model is the government, which raises fiscal revenues from both taxes (from licit income) and confiscation proceeds (from illicit activities). It then uses this revenue to produce public goods for all households and to fund a police and judicial system that can detect illicit activities and confiscate their income. We solve the model, showing that it has a short-run equilibrium with illicit activities as well as a steady-state or long-run equilibrium. We perform a set of comparative-static exercises to explore how changes in key parameters and exogenous variables affect licit and illicit activities, their corresponding income, and key variables like public good provision, social welfare and aggregate savings for an economy.

On the empirical contribution of the paper, we generate estimates of illicit income derived from drug trafficking and from common crime for the early 1980s to the early 2010s in Colombia. This period was characterized by a substantial increase in drug trafficking in the 1980s and 1990s, and a considerable decline after 2000, coinciding with the country's economic recovery and the implementation of Plan Colombia..Moreover, we use two quantitative methods to estimate the stock of laundered assets in the country's economy. We first use a calibration method based on a parameterization of the solution equation for laundered assets in the theoretical model. Next, we use a Kalman filter method for the estimation of an unobserved variable, which is an innovation in the crime literature.

To implement these quantitative methods, we develop a macroeconometric system of equations that is derived directly from the theoretical model. The system includes two equations: The first one is a transition equation that specifies the dynamic nature of asset laundering in the economy in terms of its determinants. The second is a measurement equation that represents the mechanisms through which asset laundering affects the stock of physical capital in the economy. The estimable system is applied to data for Colombia in the period 1985 to 2013 in order to estimate the volume of assets and money laundered in this economy. We see our contribution as a first step to measure and quantify these elusive unobservable but relevant variables for any economy.

On a normative perspective, we expect that this study can contribute to improve the country's regulatory, preventive and anti-criminal systems by providing an economic analysis of the incentives, mechanisms, and effects of illicit activities. Likewise, it can contribute by quantifying the size and effects of illicit income and corresponding money laundering in the economy. Finally, the model and empirical methods can be used later on to incorporate new data as it becomes available in Colombia, and more broadly, they can be used in other countries facing similar conditions.

The rest of the article is organized as follows. The second section presents a brief literature review on money laundering, while the third section presents a conceptual framework in order to guide the theoretical modelling. The fourth section develops the theoretical economic growth model with illicit activities. The fifth section presents some comparative statics on key parameters of the model, while the sixth section develops the macroeconometric system used in estimation. The seventh section reviews the methods used to estimate the trajectory of the volume of assets laundered in the Colombian economy between 1985 and 2013. The eighth section introduces the data while the ninth section presents the empirical results. The final section concludes.

2 Literature Review

This section presents a brief literature review on money laundering. We present this literature review in four parts: i) general concepts on the issues around money laundering as well as anti-money laundering policies, ii) theoretical models that have been developed to understand the main drivers of money laundering, iii) empirical and econometric estimations of shadow/underground economies and volumes of money laundering using various methods and finally iv) a review of the case of Colombia.

2.1 General Concepts

Kumar (2012) defines money laundering as the process by which large amounts of illegally obtained money (from drug trafficking, terrorist activity or other serious crimes) is given the appearance of having originated from legitimate sources. The effects of money laundering and of anti-money laundering policies (AMLP) on an economy are a subject of ongoing debate. Some economists argue that AMLP generate negative effects on developing countries, since these countries could benefit from illicit funds being laundered in their economies. Geiger and Wuensch (2006), for example, argue that complying with AMLP can become so costly that it backfires, increasing delinquency and therefore increasing the volume of money laundered. These authors believe that the benefits of AMLP are lower than the costs for society and thus question whether these regulations should be pursued in the first place.

Other authors, such as Kumar (2012) and Bartlett and Ballantine (2002), argue that money laundering has significant negative impacts on the development of a country. Kumar (2012), for example, argues that money laundering is a criminal activity against governments and countries, and notes that since the events of September 11 of 2001, it has been targeted as an activity that threatens national security because of its link with the financing of terrorism. In general, the authors note that negative effects of money laundering occur in three main sectors of the economy:

i) The financial sector, which becomes vulnerable to the risks of harboring illicit funds and in turn has its reputation eroded with the banking system,

ii) The real sector, which is adversely affected as illicit funds are channeled to less efficient sectors, leading to a distortion of exchange rates, inflation rates, money supply and eventually lowering the economic growth rate of a country. The real sector is also negatively impacted as corruption and delinquency increase, charging an implicit tax.

iii) The international commerce sector and capital markets, which are affected since illicit funds distort a country's imports and exports as well as capital flows in and out of the country.

According to these authors asset laundering involves highly complex financial operations around the world from those countries where the assets are generated to those countries that lack sufficient regulation. Moreover, it generates powerful criminal organizations that also have the political power to curb government policies for their benefit.

Slim (2011), meanwhile, argues that money laundering has a different expected impact on an open economy than on a closed one at the macro level. The author argues that in a closed economy it generates an increase in money supply in the economy, an increase in consumption and economic activity in general, an increase in inflation and fiscal revenue for the government and the appearance of real estate bubbles. On the other hand, he argues that the effects on an open economy vary. If the economy produces drugs and receives money laundered, then remittances increase, which increases the balance of payments. If the economy receives the illicit proceeds only temporarily, there is an increase in the current account deficit due to an increase in imports which also revalues the currency. Finally, if the economy receives illicit proceeds permanently then there is a current account surplus and a revaluation of the currency which increases the inflow of foreign capital.

Another section of the literature focuses on understanding how the shadow economy (SE) works and on estimating its size within a given economy. For example, Eilat and Zinnes (2000) find that in less developed economies the SE represents more than 50% of the economy, which suggests that it plays a key role in economic growth. The authors also argue that the SE acts as a cushion when the official GDP decreases, by increasing opportunities to generate income in times of recession. This would imply a negative correlation between the size of the SE and GDP during a business cycle. The authors find that the strength of this relation is not symmetric and depends on the business cycle, which could have inertia or hysteresis in the creation and destruction of the SE. The authors also find that the SE is associated with negative effects on private as well as public investments and seems to be positively related with greater monetary instability and inefficiency in the allocation of resources. Furthermore, they argue that it disintegrates social norms, official institutions and the rule of law. The authors identify three types of policies that can influence or are affected by the SE: (i) policies with multiple benefits: those that enhance economic liberalization, maintain macroeconomic stability, enhance regulation, generate greater transparency and public participation, greater decentralization and a stronger rule of law, (ii) policies that attack the SE directly: tax cuts, better regulation and greater temporal policy consistency; and (iii) policies affected by the presence of the SE: monetary and fiscal policy that ignore the size and the way the SE operates. Overall, the authors conclude that the concept of an SE is intrinsically vague and difficult to estimate due to noisy data, which does not allow us to disentangle simultaneity issues between causes and effects.

Tanzi (1999) also studies issues related to the SE and methods for measuring it. The author argues that the SE has been growing through time in most economies, which has made the official licit measures of macroeconomic variables, such as national accounts of GDP, unemployment rates and the size of tax evasion, less precise. These variables in turn influence and decrease precision of other macro economic variables, such as the fiscal deficit, public debt and fiscal goals. In addition, Tanzi argues that the relationship between unemployment and the SE is ambiguous since the SE is more labor intensive than capital intensive due to the illicit or informal nature of the production of illicit goods and due to the fact that workers can labor in both the licit and the illicit SE. Finally, Tanzi emphasizes that tax evasion increases with the SE which in turn generates a negative impact on the fiscal revenues of a government. Spiro (1996) applies Tanzi's methodology to estimate the shadow economy in Canada. He finds, counter intuitively, that for Canada, recessions were not a key factor for the expansion of the underground economy.

Finally, money laundering has been historically linked with drug trafficking in the literature. There is a commonly held view among economists and researchers, such as Taylor (1992), that drugs are the fundamental illicit good produced that enhance asset and money laundering. Taylor argues that economic liberalization of markets as well as financial markets across borders, such as the European Union, Canada or the United States, can increase drug trafficking and thus money laundering. This problem is even more relevant today, more than 20 years later, given the issues around drug trafficking on the Mexico and United States border.

2.2 Theoretical Models

One of the first authors to construct a microeconomic model of money laundering which considers regulation is Masciandaro (1999, 2007). We present the basic model from the 2007 paper since it is more developed and pertinent for our purposes. Masciandaro builds a microeconomic model of money laundering in order to assess the impact of regulation. The author starts by assuming the existence of a criminal organization defined as a group of individuals that collaborate to exchange and produce illegal goods and services in the economy. The organization accumulates resources through its illegal activity and eventually faces a laundering decision. Following the principles of Beckerian crime models, Masciandaro proposes that money laundering by a professional launderer is a decision that depends on two key factors: (i) the probability of detection and apprehension by law enforcement authorities, denoted $p \in (0,1)$ and (ii) the penalty in case of being detected, denoted $T(Y) = tY^2$ for $t \in (0,1)$. The expected utility of a criminal organization that is risk neutral is then defined as $U(Y) = (1-p)[B(Y) - C(Y)] + p[-C(Y) - T(Y)]$, where $B(Y) = (1+r)Y$ are the benefits of laundering, where r is the licit return on the investment of the volume Y of money laundered, and $C(Y) = cY$ represents the costs of laundering money in either eventuality where $c \in (0,1)$. Finally, the model assumes that if there is no money to be laundered, then the utility is normalized to zero. In this static environment the optimal amount of money to be laundered is simply found by maximizing the utility with respect to Y, yielding $Y^* = \frac{(1+r)(1-p)-c}{2pt}$. This is a decreasing function in p, c and t yet increasing in r. The author then develops a macroeconomic model based on these microeconomic foundations in which money laundering is a fraction of criminal organizations' income in the economy, i.e. $AFI = m * ACI$, where AFI represents illegal funds laundered, ACI represents criminal organizations' incomes and $m \in (0,1)$. Moreover, total investment in the economy is composed of licit investment, ARL, and illegal investment, which is money laundered from the illegal sector AFI and depends on illegal income ACI. Hence, in this model, money laundered from criminal organizations increases the investment in the economy which positively affects economic growth.

Silva et al. (2012) develop a simple dynamic model with no microeconomic foundations in order to relate crime, money laundering and anti-crime policies. They postulate a simple dynamic equation in which criminal activities are a function of their past level as well as repressive and preventive policies to counteract crime, which in turn depend positively on public resources and a stochastic shock. Since income from criminal activities must be laundered so that it can be used in the licit economy, the authors postulate a money laundering function in which it similarly depends on its past value and on criminal activities and a stochastic shock. A policy maker must choose the level of repressive and preventive policies as well as the level of crime that society will tolerate so as to minimize a quadratic loss function. A linear policy function is obtained that generates the fundamental policy result: the level of repressive and preventive anti-crime policies increases with the volume of money laundered.

Walker and Unger (2009) use a gravity model developed by Walker to estimate the volume of money laundered in Australia in 2004. The proposed model is a variation of the Newton gravity model which postulates that the flow of goods between countries is related as in a Newtonian gravity model that does not have microfoundations. The model assumes that: i) crime generates income in all countries, ii) criminal income depends on the prevalence of different types of crimes and on the average profit per criminal offense, iii) organized crime is more productive than simple crime, iv) crime is more profitable in higher income countries, v) income inequality allows the existence of a criminal class even in poor countries and vi) not all criminal profit is laundered. The authors estimate the benefits of crime for the Australian economy to be between \$2.8 and \$6.3 billion Australian dollars for 2004.

Using a methodology similar to Ingram et al. (1997) and Busato et al. (2006), Argentiero et al. (2008) calibrate a dynamic general equilibrium model to measure money laundering in Italy. They argue that econometric estimations have limitations since the key variable is unobservable leading to technical statistical issues. The authors develop a model that has a formal licit sector and an informal-criminal sector in which three main agents interact: i) licit firms that produce licit goods using capital and labor and illicit firms that produce illicit goods using labor and land; ii) households that pay taxes, demand both goods, demand money and supply labor and capital to both sectors of the economy; and iii) a government which collects fiscal revenue in the form of taxes from households and firms, and includes a central bank that supplies money to the economy. The authors find that money laundering depends optimally on labor in both sectors, capital, prices and quantities of both goods. They calibrate the model and find that money laundered as a proportion of Italian GDP has increased over time. They also find a negative correlation between money laundering and GDP. In Argentiero et al. (2009), they apply their methodology to the United States and 15 countries in the European Union.

2.3 Empirical and Econometric Estimations

Unger (2009) describes a series of methods that have been used to assess the volume and mechanisms behind the phenomenon of asset laundering. These include: i) field studies that help understand the way individuals and institutions behave in the presence of crime and asset laundering; ii) interviews and surveys of individuals that work in the different sectors in which asset laundering occurs, which help understand the different mechanisms through which asset laundering operates; iii) analyses of suspicious or unusual transactions that help understand some of the mechanisms used by criminals to launder their illicit income; iv) analyses of statistical discrepancies at the macro level in the balance of payments or money supply-demand that help understand the volume of the SE in which asset laundering operates; and v) the latent variable approach, in which asset laundering is viewed as an unobservable variable to estimate indirectly using statistical methods such as Multiple Indicator-Multiple Causes (MIMIC) and Dynamic MIMIC. These methods consist of using the software LISREL to estimate a system of linear structural equations of at least two types: i) a *transition* equation that relates the unobservable variable, in this case asset laundering, to its determinants, and ii) *measure* equations that relate observable indicator variables, e.g. money supply, GDP, capital stock, to the unobservable variable, which can be used to measure the impact of the unobservable variable. Unger (2009) also discusses other methods that have been used to estimate the volume of asset laundering like a Gravity Model as in Walker and Unger (2009). For an overview of structural linear models and MIMIC see Manzano and Zamora (2009), Schumacker and Lomax (2010), Kline (2011).

Using the DYMIMIC methodology Schneider (2007) estimates the volume of assets laundered and its time series trajectory between 1995 and 2006 for 20 OECD countries. The conceptual framework of the illegal economy used distinguishes between the SE, in which illegal goods are produced, and the underground or criminal economy, in which all types of criminal offenses are considered. All illegal income that comes from these sectors is potentially laundered. Schneider uses a transition equation which includes criminal activities and income distribution as determinants of asset laundering, while confiscated illegal income, number of offenders in the judicial system, and income per capita, are used as measure or indicator variables. The author finds that for economies like Italy, France and Great Britain, the SE decreased in size over the study period while the underground economy grew rapidly. An overview of some previous results by Schneider with his several coauthors is provided in Frey and Schneider (2000).

Like Schneider, Prokhorov (2001) considers the underground economy to be composed of a SE and a criminal economy where asset laundering is included in the latter. The author argues that a functioning licit economy cannot be separated from illicit activities and therefore argues that there must be an optimal level of illegality such that if surpassed society should coordinate to make it smaller. Prokhorov uses a MIMIC methodology where fiscal and non-fiscal regulation, inflation, population below the poverty line, real income and unemployment rate are determinants of the transition equation for the ratio of the size of the underground economy to GDP. The indicator variables affected by this latent unobserved variable include the monetary base, M2, foreign investment, crime rate and energy consumption over GDP. The estimation is carried out for Russia in the periods 1992-2001 and 2000-2004.

Tedds and Giles (2000), following Giles (1999), similarly use the MIMIC methodology to estimate the shadow economy of Canada and New Zealand using as indicator variables the growth rate of the economy, M3 and other monetary measures of money in circulation and male labor participation. As determinants of the SE they use self-employment income, crime rates, unemployment rates, tax rates relative to GDP and the inflation rate among others. The authors find that these SEs have been growing over time. However, this type of methodology has been criticized by Breusch (2005).

2.4 The Case of Colombia

Colombia is an ideal country for studying asset and money laundering as it is one of the main drug producing countries in the world, particularly of marijuana and cocaine. It is also a country with high crime rates, particularly kidnapping for ransom and theft. Not surprisingly then, the domestic literature on Colombia has been dominated by the role of drugs in its economy which is considered the main predicate offense of asset laundering. For an overview of drug trafficking in Colombia up to 2001 see Thoumi (2002).

Holmes and Gutiérrez de Piñeres (2006) argue that the drug trade has had negative effects on the economy through: i) higher unemployment rates, (ii) an accelerating increase in the cost of real estate, (iii) a substitution from licit agricultural crops to illicit crops, like coca, (iv) an increase in smuggling that has increased asset laundering in the country, (v) lower investment levels in licit firms, (vi) an increase in land concentration in order to produce drugs, and (vii) an increase in long run inequality, among other things. Moreover, drugs have brought much violence to the country, generating forced displacement of people, the creation of illegal guerrillas and the proliferation of paramilitary groups involved in massacres and kidnapping since the 1980s.

Several approaches have been used to understand and estimate asset laundering as well as the shadow/underground activities of the Colombian economy. For example, Schneider and Hametner (2007) use a demand approach to estimate the SE in Colombia between 1976 and 2000. The authors find that the SE grew between 1976-1985 and 1992-2000 and argue that this occurred because of an increase in taxes, higher unemployment and economic crises. They also find a positive relationship between the overall economic growth of the economy and the SE. Arango et al. (2006) uses a latent variable approach to estimate the SE in Colombia using the Kalman Filter and an optimizing algorithm with transition and measure equations. They find that the money supply in the Colombian economy has been affected positively by the SE. Other studies like Amaya and Caballero (2011) try to estimate the aggregate volume of asset laundering in Colombia with basic accounting principles under certain adhoc assumptions.

Our approach combines several of the methodologies used in the literature. We develop a general equilibrium overlapping generations growth model to study the main determinants of asset laundering and the way that this activity affects the capital stock of an economy. On this basis, we build a two linear structural equation macroeconometric model with a transition equation, which specifies the determinants of asset laundering, and a measure or indicator equation for an observable variable affected by asset laundering, in this case, the capital stock of the economy. This linear system is then taken to the data for the Colombian economy between 1985 and 2013 using two empirical methods: Calibration and Kalman Filter, where asset laundering is understood as a latent variable to be estimated.

The next section builds the conceptual framework on asset laundering to be used in the theoretical model.

3 Conceptual Framework on Asset Laundering

In the literature on asset laundering, there appears to be no consensus on the difference between the terms laundered assets and laundered money. In some cases they are considered to be synonymous while in others the former contains the latter. Most definitions, though, focus mostly on criminal aspects, accounting aspects or on the illicit nature of the funds generated through illicit activities. In general, the process of laundering illicit income is the process of converting a wide range of proceeds of crime into apparently legitimate income, concealing its origin. While a small portion of laundered funds are intended to be hidden for some period of time, the ultimate purpose is to use them in licit activities. In this paper we distinguish between money and asset laundering according to the purpose for which it is undertaken by the launderer. As shown in the figure below, we consider that while undetected illicit incomes can be used to consume licit goods and services or to invest in licit assets, only the latter is part of the asset laundering process. We do not consider confiscated illicit income as laundered, since it was detected and thus did not achieve the purpose of legitimizing proceeds of crime. The following graph presents the conceptual framework.

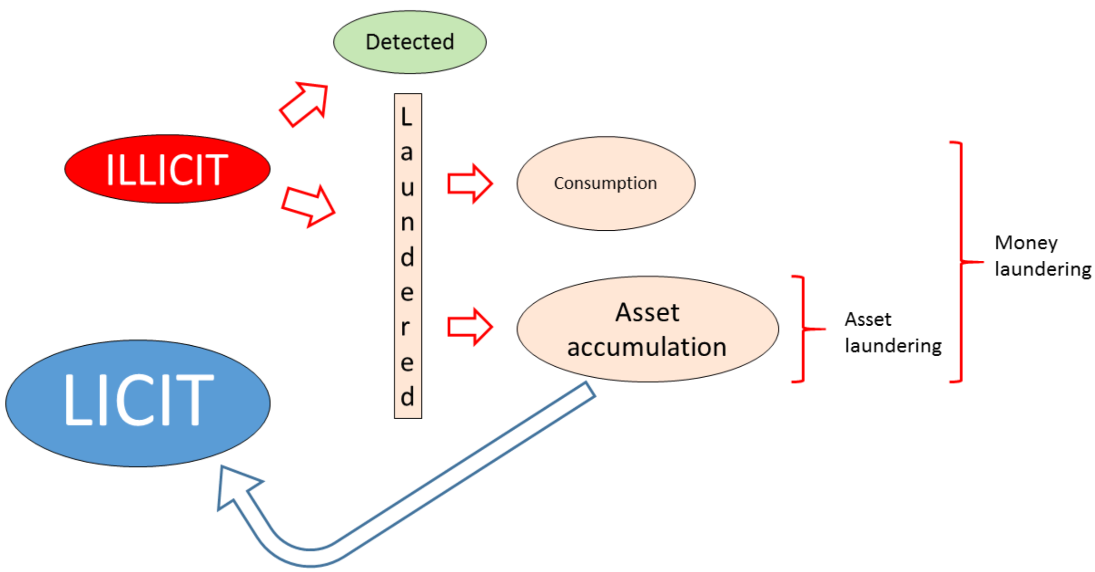

Figure 1

Note that our conceptual framework does not directly capture the potential for undetected illicit incomes to be reinvested in illicit activities. However, it is captured indirectly, since all illicit income enters the licit economy either through consumption or investment eventually. Specifically, undetected illicit income that is reinvested in illicit activities requires labor and other inputs from the licit economy. When these inputs are paid for, money becomes income that workers as well as providers use to buy goods and services in the licit economy.

We consider two types of illicit activities: those that produce an illicit good, such as drugs (cocaine in the Colombian case), and common crimes, in which criminals appropriate licit workers' income. In purely economic terms, the first activity can be characterized as productive while the second one as redistributive. In the macroeconomic model that we develop below we assume that households directly attempt to launder their illicit income by investing in the only asset in the economy, called capital. After households place their funds in the financial system to purchase capital law enforcement authorities detect the illicit origins of the funds with a certain probability. If detected, the illicit income is confiscated and used to fund the government. Asset laundering occurs when adult parents (old members of a household) leave an inheritance of capital to their young offspring. Once illicit capital is transferred from adult parents to their offspring this capital is fully integrated into the licit economy and the financial sector lends these funds to licit firms so that they can replace and expand their capital stock.

4 An Economic Growth Model with Illicit Activity and Money Laundering

As previously described, part of the literature estimates asset laundering using simple empirical models, which make a number of ad hoc assumptions not clearly founded in theory. Though some theoretical models have been constructed and calibrated, none have done so in the context of a growth model. We believe that a growth model is necessary for understanding the main mechanisms through which asset laundering, an unobservable variable, affects the capital accumulation of an economy. Moreover, using a dynamic model can help the macroeconometrics required to estimate the volume of assets laundered in an economy given that it allows identification of structural parameters.

We consider an overlapping generations model similar to Diamond (1965) since it is a dynamic time discrete model which is not as simple as the Solow model nor as complex in terms of continuous time structure as the Ramsey model. Moreover, it is a structure that allows us to model the transmission of wealth from generation to generation through which assets are laundered and illicit assets are integrated into the licit economy which we believe is a key aspect of asset laundering in an economy. Finally, the model's structure allows us to attain the Markov property used in empirical specifications of transition equation and measure equations that lend themselves to apply econometric techniques.

Next, we present an overview of the economy and some details of the environment which constitute the building blocks for the growth model to be developed. The economy is composed of four actors and activities.

i) Households, made up by a young and an adult generation that overlap in any given period. The young generation is born with a moral cost type for illicit activities as well as a time endowment that is offered inelastically in the labor market either to work in licit or illicit activities, but not both. At the end of the period, the young generation receives licit or illicit income according to the type of work chosen at the beginning of the period and an inheritance in the form of capital from the adult (parent) generation. The young generation has to choose how much to save and consume of the licit good out of total wealth at the end of their first period of life. The adult generation at the beginning of the period puts savings (capital) from the previous period into the financial sector and obtains a return at the end of the period. At the end of the period the adult generation decides the amount to transfer out of total wealth as an inheritance to their offspring and the amount to consume in licit goods. The young generation receives these savings and invests them in capital (the only asset in the economy), obtaining a return on them at the end of the period. We assume households have perfect foresight and that the only heterogeneity among households is their moral cost when young of working in illicit activities.

ii) Licit firms, that produce licit consumption goods, demanded by households when young and when adult. These firms operate in a perfectly competitive environment using a constant returns to scale technology that uses capital and licit labor and have perfect foresight. To simplify the analysis we assume that licit firms in the model are not corruptible.

iii) Illicit activities are of two types: i) those that produce an illicit good, such as drugs, for which the price and demand is determined by world markets, and ii) those that redistribute income, i,e. common crimes such as burglary, larceny, theft, motor vehicle theft, arson, shoplifting, robbery, kidnapping and extortion, all of which generate insecurity in the economy and aim to apprehend income from licit workers. Both types of illicit activities use a techonology that is labor intensive, consistent with the assumption that capital accumulation in this sector generates perfect detection from the government. Drug trafficking operates in an imperfect competition environment where workers are not paid according to their marginal productivity but by a sales commission based on average production compatible with income sharing. Illicit labor markets are segmented in the sense that drug trafficking first hires the labor it needs to meet foreign demand and the remaining illicit workers are allocated to common criminal activities. Illicit income is not subject to income tax.

iv) A government that raises fiscal revenues from both taxes (from licit income) and confiscation proceeds (from illicit activities). It then uses this revenue to produce public goods for all households and to fund a police and judicial system that can detect illicit activities and confiscate their income, such that it maximizes a utilitarian social welfare function under a balanced budget in every period.

In the next section we outline the precise mathematical assumptions for each of these participants in the economy, and derive their corresponding optimal behavior, specifying the variables that each of these agents take as given.

4.1 Households

We first describe licit and illicit households in order to characterize their behavior in terms of their optimal savings (in assets) and consumption decisions. Then we study the way they self select into illicit and licit activities. Assume that there are H_t households in period t and that the population grows at the exogenous rate $n > 0$ such that $H_{t+1} = (1+n) H_t$. Each member of a household lives two periods, one as a young person and one as an adult. The two generations overlap in any given period of time such that the young in period t have only one parent that is adult in period t and was born in period $t-1$. Since in each household in every period there are two individuals, then H_t also represents the young population in period t and therefore n is the fertility rate of youths that are born in every period.

Households are of two types, licit and illicit, where the former type works in licit activities while the latter works in illicit activities. Young individuals are born with a moral type and one unit of labor that they offer inelastically in their first period of life to work either in the licit or illicit sector. Working decisions are based on moral types and potential incomes and are irreversible decisions in the sense that once a worker decides to offer their unit of time to work in the licit or illicit sector they cannot go back to the other sector. At the end of their first period of life, young individuals receive a bequest in the form of capital (the only asset in the economy) from their adult parent, which is not subject to confiscation regardless of legality of origin. [3]

4.1.1 Preferences and Lifetime Utility

A generation consumes the licit good when young (denoted C_t^y) and when adult (denoted C_{t+1}^a) and leaves a bequest to their offspring at the end of period $t+1$ denoted h_{t+1}. Adults do not work and discount at rate of $\beta \in (0,1)$ their utility in period $t+1$. The variable ζ represents the moral cost of working in illicit activities and is assumed to be heterogenous in the population such that in any given period, it takes two values $\zeta \in \left\{\underline{\zeta}, \bar{\zeta}\right\}$ where $\bar{\zeta} > \underline{\zeta} \geq 1$ such that fraction ϕ of households have $\zeta = \underline{\zeta}$ while the complement $1-\phi$ have $\zeta = \bar{\zeta}$. This reflects a time invariant heterogeneity in the moral cost of entering the illicit sector. We assume that the type ζ is bequeathed by the adult to the young generation at the beginning of the period in which the young are born. Lifetime preferences for a generation that lives period t as young and period $t+1$ as an adult are represented by the following log utility function

$$U_{t,t+1} = \ln C_t + \beta \ln C_{t+1} + \beta \ln h_{t+1} + \eta \ln g_{t,t+1} - I \ln \zeta \tag{1}$$

where I takes the value one if the young generation in period t works in illicit activities and zero otherwise and $g_{t,t+1}$ denotes a government transfer that a generation in both periods would receive which represents social investments from the government in public goods and services benefitting all households alike. The weight on the utility of these public goods and services for a household relative to the unity utility weight on consumption goods is $\eta \in (0,1)$.

[3] Ideally one would want to include more than one asset in the economy, say housing and a financial asset like bonds, which combined could make up the capital of the economy. This would allow one to analyze the various incentives for illicit households to place their income in different types of assets that could also have different detection and confiscation probabilities attached to them. However, doing this is beyond the scope of this article because we are interested in modelling the basic aggregate mechanisms involved in asset laundering related to economic growth.

4.1.2 Lifetime Wealth

The young generation born in period t is endowed with moral type ζ and with one unit of time to be used to work in a licit or illicit activity in the period, and which yields a disposable income. Licit labor income is equal to disposable wage income after tax and crime i.e. $(1-\tau_t)(1-\rho)\,w_t$ where $\tau_t \in (0,1)$ denotes the income tax rate while $\rho \in (0,1)$ denotes the fraction of income apprehended by common crime, which is assumed to be proportional to disposable licit income. Moreover, the young generation inherits h_t^I for $I = 1,0$ of capital at the end of period t, which it receives as a bequest from the adult generation (parent). Therefore licit wealth for a young individual at the end of period t is defined as $W_t^0 \equiv h_t^0 + (1-\tau_t)(1-\rho)\,w_t$.

A young individual that becomes an illicit worker can work in either of two illicit activities in the economy, one that produces drugs in the economy or one that appropriates income from licit workers, i.e. common crimes. Wealth for an illicit worker at the end of period t is thus the bequest received plus the illicit income from one of these two illicit activities which we denote as Y_t^i for $i = D, R$, where D stands for the illicit goods sector that produces drugs to export and satisfy a foreign demand while R stands for robbery or common crime.[4] Illicit incomes are detected and confiscated with probability q_t in period t. Hence expected wealth for an individual that chooses to enter the illicit sector is $W_t^1 \equiv h_t^1 + (1-q_t)\,Y_t^i$, for $i = D, R$.

We assume that bequests that come from the proceeds of illicit activity in the previous period are no different to ones that come from a licit source in the sense that they are not subject to detection and confiscation by the government in period t. This is because adults that invest illicit income in the financial system in period t are not detected and therefore can bequest their wealth to their offspring at the end of period t. Hence, bequests do not face the risk of detection and confiscation even though they retain for our analytic purposes their illicit nature.[5] In our model, then, this is the way by which illicit assets are fully integrated and laundered in the economy. The adult generation in period t are the owners of the capital in the economy, do not work and live off what they saved when young plus the interest it earns.

4.1.3 Budget Constraint

The budget constraint for period t for a young individual is therefore

[4] In this paper we refer to "drugs" as the illicit good being produced. This is because in the empirical section we use cocaine as the main illicit good produced in the Colombian economy and exported to foreign countries. Nonetheless, the theoretical model is general enough to incorporate any type of illicit good or service produced in the domestic economy for exportation. Other examples of illicit goods include weapons, illegal mining and human trafficking.

[5] This assumption is plausible for generations of 25 to 30 years, the length of time per generation to which the model corresponds. Even if laundered assets were to be detected after 20 years we assume reasonably that this wealth cannot be confiscated by law enforcement authorities. In this sense once bequests are transferred from parents to sons/daughters then no confiscation can be implemented for laundered assets even if detected. This assumption works well for our theoretical purposes, however we relax this assumption in the empirical section and allow with certain probability that laundered assests be confiscated even after being integrated into the licit economy.

$$C_t^{I,y} + a_t^I = W_t^I \quad \text{for } I = 0, 1 \tag{2}$$

while in period $t+1$ the individual when adult consumes $C_{t+1}^{I,a}$ and leaves a bequest of h_{t+1}^I from the savings plus the accrued interest

$$C_{t+1}^{I,a} + h_{t+1}^I = (1 + i_{t+1})\, a_t^I \text{ for } I = 0, 1 \tag{3}$$

where i_{t+1} is the real interest rate on one-period loans between periods t and $t+1$. Combining these two restrictions yields the lifetime budget constraint

$$C_t^{I,y} + \frac{C_{t+1}^{I,a} + h_{t+1}^I}{1 + i_{t+1}} = W_t^I \text{ for } I = 0, 1 \tag{4}$$

4.1.4 Time Line

Figure 2 shows the time line of household decisions during periods t and $t+1$. At the beginning of period t, each household includes an adult and a young person born that period. The adult generation does not work and puts their savings (invests capital) into the financial system. The young generation has one unit of time endowment and chooses to work in the licit or illicit sector depending on their moral type. At the end of period t, the adult generation gets a return on their investment (savings) and divides their wealth into a bequest for their offspring and consumption of the licit good. The young generation at the end of period t receives labor income, either licit or illicit, and the bequest in the form of capital from their adult parent, which is also licit or illicit in nature. The young generation then chooses the amount to save out of total wealth, which is transferred to the next period, while the rest is used to consume the licit good. This is repeated in period $t+1$, as the young generation of period t becomes the adult generation of $t+1$, which invests its savings in the financial system while a new young generation is born in period $t+1$ and has to choose to work in the licit or illicit sector. This dynamic continues indefinitely into the future.

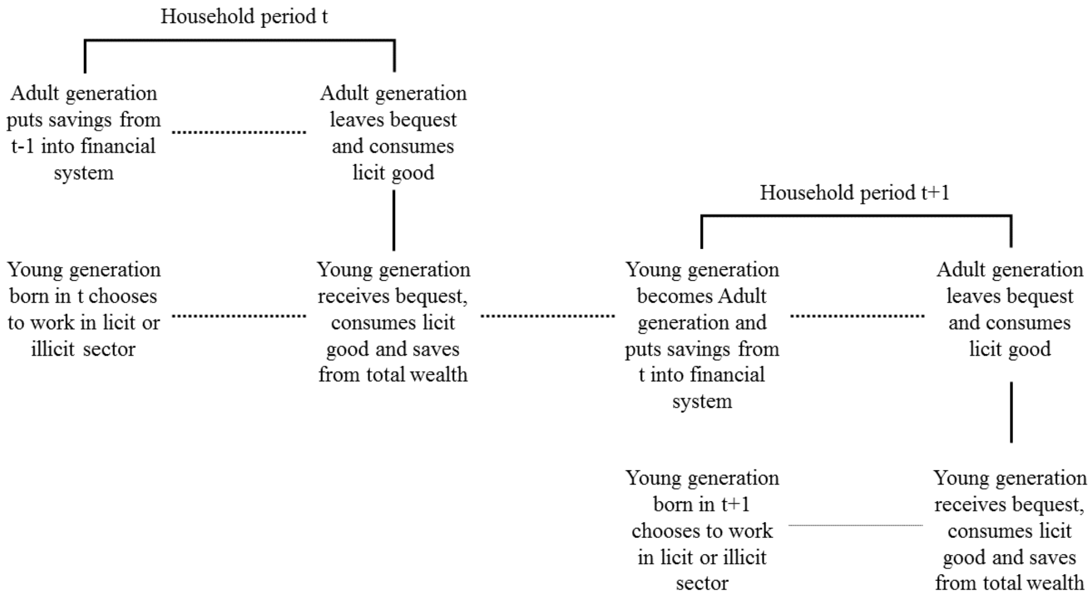

Figure 2 Time Line

4.1.5 Behavior of a Generation

An individual born in period t solves the same problem, regardless of whether they work in the licit or illicit sector. They must choose $C_t^{I,y}$, $C_{t+1}^{I,a}$ and h_{t+1}^I in order to maximize equation (1) subject to equation (4) taking as given $g_{t,t+1}$, i_{t+1} and W_t^I

$$\max_{C_t^I, C_{t+1}^I, h_{t+1}^I} \ln C_t^{I,y} + \beta \ln C_{t+1}^{I,a} + \beta \ln h_{t+1}^I + \eta \ln g_{t,t+1} - I \ln \zeta$$

$$s.t. \quad : \quad C_t^{I,y} + \frac{C_{t+1}^{I,a} + h_{t+1}^I}{1 + i_{t+1}} = W_t^I \quad \text{for } I = 0,1$$

Since the objective function is strictly quasi-concave then we can replace $C_t^{I,y}$ in the objective function from the budget constraint to get the following first order conditions which are necessary and sufficient for the optimal choices the individual makes in his lifetime

$$\frac{C_{t+1}^{I,a}}{C_t^{I,y}} = \beta \left(1 + i_{t+1}\right); \quad C_{t+1}^{I,a} = h_{t+1}^I \quad \text{for } I = 0,1$$

These conditions jointly with the budget constraint yield the following demand and bequest functions

$$C_{t+1}^{I,a} = h_{t+1}^I = \frac{\beta W_t^I \left(1 + i_{t+1}\right)}{1 + 2\beta}; \quad C_t^{I,y} = \frac{W_t^I}{1 + 2\beta} \quad \text{for } I = 0,1 \tag{5}$$

while from restriction (3) we get the savings function

$$a_t^I = s\left(\beta\right) W_t^I \quad \text{for } I = 0,1 \tag{6}$$

where $s(\beta) \equiv \frac{2\beta}{1+2\beta} \in \left[0, \frac{2}{3}\right)$ denotes the constant marginal propensity to save given out of total wealth, where $s'(\beta) > 0$ and $s''(\beta) < 0$.

Replacing these demand functions in the utility function gives us the indirect utility function for an individual in his lifetime which, after simplifying terms, becomes

$$\begin{aligned} U^I_{t,t+1} &= (1+2\beta)\ln W^I_t + 2\beta \ln(1+i_{t+1}) \\ &\quad + \eta \ln g_{t,t+1} + \xi - I \ln \zeta \quad \text{for } I = 0,1 \end{aligned} \tag{7}$$

where $\xi \equiv 2\beta \ln \beta - (1+2\beta)\ln(1+2\beta)$.

4.1.6 Decision for Young Workers to Enter Licit or Illicit Sector

Young individuals self select between the licit and the illicit sector. A young individual born in period t has to decide either to work in the licit sector or the illicit sector. If he chooses to work in the illicit sector, he enters drug trafficking with probability ε_t , which yields illicit income Y^D_t, or common crime with probability $1-\varepsilon_t$, which yields illicit income Y^R_t stolen from licit workers. We assume that each type of illicit income is confiscated with the same probability q_t and confronts the same penalty. Therefore the net expected illicit income becomes $(1-q_t)\left(\varepsilon_t Y^D_t + (1-\varepsilon_t) Y^R_t\right)$.

A young worker chooses to work in the illicit sector rather than the licit one if and only if the indirect expected utility of the former activity is no less than the indirect utility of the latter

$$U^1_{t,t+1} \geq U^0_{t,t+1}$$

Evaluating (7) at $I = 1$ and $I = 0$, canceling common terms and substituting for the wealth levels W^1_t and W^0_t, this inequality reduces to

$$\zeta \leq \left[\frac{h^1_t + (1-q_t)\left(\varepsilon_t Y^D_t + (1-\varepsilon_t) Y^R_t\right)}{h^0_t + (1-\tau_t)(1-\rho) w_t}\right]^{1+2\beta} \equiv \widehat{\zeta}_t \tag{8}$$

The threshold variable $\widehat{\zeta}_t$ is the moral cost such that a worker is indifferent between entering the licit or illicit sector of the economy. Condition (8) says that a worker with low enough moral cost (lower than threshold value $\widehat{\zeta}_t$) would choose to work optimally in the illicit sector in period t, and otherwise would not. Note that $\widehat{\zeta}_t > 0$ given that all variables involved are positive. Interestingly, the threshold value is decreasing in q_t and w_t yet increasing in ρ, Y^D_t, Y^R_t and τ_t. By assumption, individuals are heterogenous in the moral cost of entering the illicit sector such that there are two types $\zeta \in \left\{\underline{\zeta}, \bar{\zeta}\right\}$ where in each period of time there is fraction $\phi \in (0,1)$ of the population that has parameter $\zeta = \underline{\zeta}$ while fraction $1-\phi$ has parameter $\zeta = \bar{\zeta}$.[6] A sufficient condition for the existence

[6] Ideally the heterogeneity would be a continuum such that the threshold type would determine whether a worker enters the licit or illicit sector, and changes in economic incentives would change the share of each in the population in a continuous manner. In this scenario individuals would not simply have either good or bad moral types but rather a more complex continuum of types ranging in moral cost. We use a simple binary set up for this heterogeneity to make the model tractable.

of a positive fraction of illicit workers is that $Y_t^e \equiv (1-q_t)\left(\varepsilon_t Y_t^D + (1-\varepsilon_t) Y_t^R\right)$ be sufficiently large such that $\bar{\zeta} > \widehat{\zeta}_t > \underline{\zeta}$ so that not all workers enter the illicit sector. In the section on the short run equilibrium we determine a sufficient condition for this to hold which then partitions young workers between illicit activities in period t given by $L_t^1 = H_t \phi$ and licit activities given by the complement $L_t^0 = H_t(1-\phi)$ such that $L_t^1 + L_t^0 = H_t$ for all t. It is important to point out that illicit activities generate costs to the economy through the moral costs of young individuals that enter this sector and also generate a negative externality to licit households through crime.[7]

4.2 Licit Firms

Licit firms in the economy produce the licit good (C) using capital and licit labor. The capital used by firms in period t comes from asset holdings offered as savings by adults in that period, which under perfect competition, is equal to the real interest rate i_t for each unit of capital. These firms operate in a perfectly competitive environment with perfect foresight and produce the licit good demanded by young and adult generations in period t through a Cobb-Douglas technology $X_t = BK_t^\alpha N_t^{1-\alpha}$ where $B > 0$, $\alpha \in (0,1)$, K denotes capital and N denotes licit labor hired. Since the technology has constant returns to scale in capital and licit labor, then the representative firm produces $X_t = H_t C_t$ for $H_t C_t^a$ adults and $H_t C_t^y$ youths in the economy in period t. Under perfect competition the real cost of using a unit of physical capital is i_t and w_t is the real wage. The representative firm maximizes profits in a competitive environment taking as given the real interest rate and the real wage

$$\max_{K_t, N_t} \Pi_t^C = BK_t^\alpha N_t^{1-\alpha} - i_t K_t - w_t N_t$$

Perfect competition implies that licit firms have to satisfy the following first order conditions

$$\frac{\partial \Pi_C}{\partial K_t} = 0 \Rightarrow \alpha B K_t^{\alpha-1} N_t^{1-\alpha} = i_t$$

$$\frac{\partial \Pi_C}{\partial N_t} = 0 \Rightarrow (1-\alpha) B K_t^\alpha N_t^{-\alpha} = w_t$$

We assume that licit labor is distributed between the real sector that produces the licit good and the government sector such that in either sector licit households earn w_t. Define $P_t \equiv \theta_t L_t^0$, where $\theta_t \in (0,1)$, as the amount of licit labor hired by the government as policemen and judges in order to generate a detection and confiscation probability, while the complement $N_t \equiv (1-\theta_t) L_t^0$ is the licit labor hired by licit firms. Moreover, define capital per licit worker as $k_t \equiv \frac{K_t}{L_t^0}$. Replacing these

[7] We do not model the disutility that crime generates to licit households besides the income they lose, for example due to the anxiety and fear it can generate in innocent victims. We do not think these utility costs are unimportant but rather choose to focus on the most simple set up in order to discuss the relevant issues concerning asset and money laundering. This approach, the standard approach in the economics of crime literature, is restrictive in this aspect but we believe it to be fruitful for our purposes.

definitions in the first order conditions yields

$$i_t = \alpha B \frac{(1-\theta_t)^{1-\alpha}}{k_t^{1-\alpha}}; \quad w_t = \frac{(1-\alpha)\, B k_t^{\alpha}}{(1-\theta_t)^{\alpha}} \tag{9}$$

Note that as the fraction θ_t of policemen and judges hired increases, then i_t decreases while w_t increases.

4.3 Illicit Activities

There are two types of illicit activities: i) organizations that produce drugs (e.g. cocaine) in order to satisfy a foreign (exogenous) demand using a technology that is labor intensive and has positive but marginally decreasing returns; and ii) common crime activities that prey on licit workers in order to steal a fraction of their disposable wage after taxes. We assume that labor markets in this sector are segmented in the following way: illicit labor is allocated first to the production of drugs in order to satisfy the exogenous foreign demand,and the rest of the supply of illicit labor is residually allocated to common criminal activities. Drug production is assumed to be more profitable than common crime in every period.[8]

4.3.1 Drug Trafficking

The main driver of the production of drugs is not a domestic market but overseas foreign markets. In particular, an economy like Colombia produces drugs mainly for export to other countries, specially to the United States and to Europe. For simplicity, we abstract from domestic drug demand and consider only that the exogenous foreign drug demand $\widetilde{D}_t$ motivates drug trafficking in the domestic economy. Furthermore, we assume that the foreign price of drugs d_t is determined by world markets where drug trafficking organizations produce drugs D_t with a technology that is labor intensive $D_t = A\left(l_t^{1,D}\right)^{\lambda}$, the demand of illicit labor is denoted as $l_t^{1,D}$, and $A > 0$, $\lambda \in (0,1)$ are technological parameters. We assume that workers in this sector act as vendors and obtain a commission on total sales i.e. $\nu_t d_t D_t$ where $\nu_t \in (0,1)$ is the common sales commission.

Since all drug trafficking organizations are assumed to be the same, consider a representative organization which is not subject to criminal predation by common criminals. Given that the production of drugs is realized to satisfy a foreign exogenous demand $\widetilde{D}_t$, the technology implies that the demand for illicit labor in this sector is given by $l_t^{1,D} = \left(\frac{\widetilde{D}_t}{A}\right)^{\frac{1}{\lambda}}$ which is perfectly inelastic. Moreover, total revenue for the sector is given exogenously by $d_t\widetilde{D_t}$ and, given that each drug trafficker obtains $\nu_t d_t \widetilde{D}_t$,

[8] There is a huge literature on the "shadow economy" as an analytical concept that is measured using different estimation methods. We do not use this concept, because even though the production of illicit goods could be seen as pertaining to a "shadow economy" criminal activities that generate violence and predatory behavior should not be included in this concept. The nature of these criminal activities makes the concept of the shadow economy an inadequate term to use in our conceptual framework. A more reasonable term used would be "underground economy".

total costs are given by $CT_t^D\left(\widetilde{D}_t\right)=\frac{\nu_t d_t \widetilde{D}_t^{\frac{1}{\lambda}+1}}{A^{\frac{1}{\lambda}}}$. Hence, total profits are

$$\Pi_t^D = d_t\widetilde{D}_t - \frac{\nu_t}{A^{\frac{1}{\lambda}}} d_t \widetilde{D}_t^{\frac{1}{\lambda}+1} \tag{10}$$

Since both the quantity and the price are given for the representative organization, the common commission v_t under an egalitarian income sharing rule is determined such that profits are zero i.e. $\Pi_t^D = 0$ for all t. Hence we get $\nu_t^* = \left(\frac{A}{\widetilde{D}_t}\right)^{\frac{1}{\lambda}}$ and income per illicit worker in this activity is given by

$$Y_t^D \equiv \frac{IT_t^D}{l_t^{1,D}} = \frac{d_t A^{\frac{1}{\lambda}}}{\widetilde{D}_t^{\frac{1-\lambda}{\lambda}}} \tag{11}$$

Labor market segmentation is such that foreign demand for drugs determines the fraction of illicit labor hired into drug trafficking. Therefore, the fraction of illicit households that work in this sector is $\widetilde{\varepsilon}_t = \min\left\{1, \frac{l_t^{1,D}}{H_t\phi}\right\}$. Under perfect foresight expectations, this corresponds to the probability that an illicit household finds a "job" in the drug trafficking sector i.e. $\widetilde{\varepsilon}_t = \varepsilon_t$.[9]

4.3.2 Common Crime

The second type of illicit activity is common crime[10] that preys on licit workers[11] for pecuniary reasons. The representative criminal activity does not produce a good in the economy but generates a *negative externality* called robbery (R) and only uses illicit labor, denoted $l_t^{1,R}$. Total income from these activities is given by $IT_t^R = H_t(1-\phi)(1-\tau_t)\rho w_t$, where $H_t(1-\phi)$ represents the total number of licit workers in the population that earn disposable income $(1-\tau_t)w_t$.[12] Hence, illicit workers in these activities obtain a fraction $(1-\tau_t)\rho w_t$ of licit workers' disposable income. Assuming that each

[9] Even though profits are zero which determines v_t^* in each period, this does not correspond to a competitive solution since illicit labor is not paid according to its marginal productivity based on a publicly known wage as in a competitive environment. Instead, it is paid according to an egalitarian income sharing rule based on average product. To see this, note that perfect competition would entail the following optimization problem

$$\max_{l_t^{1,D}} d_t D_t - \varpi_t l_t^{1,D}$$
$$s.t. \quad : \quad D_t = A\left(l_t^{1,D}\right)^{\lambda}$$

where ϖ_t would be the so called competitive illicit wage in the sector. Since the technology has decreasing returns in the only factor, profit would not necessarily be zero in a competitive equilibrium. Moreover, solving the problem would yield a standard demand for illicit labor given by

$$l_t^{1,D} = \left(\frac{\lambda d_t A}{\varpi_t}\right)^{\frac{1}{1-\lambda}}$$

The total supply of illicit labor is perfectly inelastic and is given by $L_t^1 = H_t\phi$ where fraction $\widetilde{\varepsilon}_t H_t\phi$ of illicit workers would work in drug trafficking. Hence, equalizing demand and supply of illicit labor would yield $\varpi_t^* = \frac{\lambda d_t A}{(\widetilde{\varepsilon}_t H_t \phi)^{1-\lambda}}$. Since our solution $v_t^* = \left(\frac{A}{\widetilde{D}_t}\right)^{\frac{1}{\lambda}}$ is a sales commission then $\frac{\varpi_t^*}{d_t \widetilde{D}_t}$ would be comparable with v_t^*. Note that $\frac{\varpi_t^*}{d_t \widetilde{D}_t} \neq v_t^*$, showing that the solution derived above is not the solution under perfect competition.

[10] This type of crime is composed of property crimes as well as violent crimes with a pecuniary motive like kidnaps, robbery, larceny, burglary and extortion.

[11] For simplicity, we abstract from predation on illicit workers.

[12] We are implicitly assuming that only young licit individuals are victims of crime since they are the licit workers in the economy that have labor income. Hence, common crime is assumed to prey on licit labor income and not capital income from assets.

criminal has the same level of ability and so captures the same quantity of income from licit workers, illicit income from crime per worker is given by

$$Y_t^R \equiv \frac{IT_t^R}{l_t^{1,R}} = \frac{(1-\tau_t)\,\rho\,(1-\phi)\,w_t}{\phi\,(1-\varepsilon_t)} \tag{12}$$

where the total number of illicit workers in these activities is given residually by $l_t^{1,R} = H_t\phi\,(1-\varepsilon_t)$ such that $l_t^{1,R} + l_t^{1,D} = H_t\phi$ for all t. Note that Y_t^R is strictly decreasing in ϕ since more workers in common crime activities yield less income per illicit worker.

As argued above, the expected (average) illicit income for a young individual when entering the illicit sector is given by $Y_t^e \equiv (1-q_t)\left[\varepsilon_t Y_t^D + (1-\varepsilon_t)\,Y_t^R\right]$ where under perfect foresight the probability ε_t equals the fraction of illicit jobs available in drug trafficking $\widetilde{\varepsilon}_t$ while the complement $1-\varepsilon_t$ equals the fraction of illicit "jobs" available in common crime activities $1-\widetilde{\varepsilon}_t$. Hence, Y_t^e can be thought of as the average illicit income a delinquent in the sector earns in period t. The segmentation of the labor market is consistent with rational choices of illicit workers under the sufficient assumption that $Y_t^D > Y_t^R$ for all t and that demand for illicit labor in drug trafficking is perfectly inelastic.

Note that average real illicit undetected income is then

$$Y_t^e = (1-q_t)\left[\frac{d_t\widetilde{D}_t}{H_t\phi} + H_t\,(1-\tau_t)\,\rho\,(1-\phi)\,w_t\right]$$

which is strictly decreasing in ϕ, τ_t and q_t while strictly increasing in ρ, $\widetilde{D}_t$ and d_t.

4.4 Government

The government has two functions in the economy. Its first function is to generate a detection and confiscation probability q_t of illicit incomes and its second is to provide a certain amount of public goods $g_{t,t+1}$. The probability of detection and confiscation q_t depends on a technology that hires licit workers and pays the same as licit firms while the amount of public goods $g_{t,t+1}$ provided is determined jointly with q_t. The government chooses these two levels in every period so as to maximize the social welfare function of licit households in period t, subject to a balanced budget constraint.

In order to do this, the government must also choose the licit income tax rate τ_t that determines part of its fiscal revenue. Though there are several ways to define a social welfare function, we consider a simple utilitarian social welfare function of licit households which the government maximizes. To build this function we aggregate the indirect utility function (7) evaluated at $I = 0$ for all $H_t\,(1-\phi)$ licit households giving

$$SW_t^u = H_t\,(1-\phi)\,U_{t,t+1}^0 \tag{13}$$

where the key assumption is that social welfare is only sensitive to the indirect utility of licit households.

Let us build the revenue and expenditure functions for the government. In terms of cost, the government hires $P_t \equiv \theta_t L_t^0$ workers at wage w_t as policemen and judges in order to detect and confiscate illicit incomes with the purpose of generating probability q_t in the economy according to technology $q_t = \gamma\sqrt{\frac{P_t}{H_t}}$, where $\gamma \in (0,1)$ is a positive efficiency parameter. This technology captures the intuitive idea that a greater number of law enforcement officers generates a greater marginal probability of detection and confiscation but with marginal decreasing returns.[13] Note that since $L_t^0 \equiv H_t(1-\phi)$ and $\theta_t \in (0,1)$, replacing $P_t \equiv \theta_t L_t^0$ in the technology yields $q_t = \gamma\sqrt{\theta_t(1-\phi)}$ which shows that $q_t \in (0,1)$ and that q_t is a strictly increasing function of fraction θ_t and a decreasing one in ϕ.

The government must choose P_t so as to minimize cost $w_t P_t$ subject to $\overline{q_t} = \gamma\sqrt{\frac{P_t}{H_t}}$ for a given level of $\overline{q_t} \in (0,1)$ that is desired to be implemented. Since each licit worker is costly, to generate probability $\overline{q_t}$, the government needs to hire $\overline{P}_t = \frac{H_t\overline{q_t}^2}{\gamma^2}$ workers. Hence, the cost function of generating $\overline{q_t}$ is given by $\frac{H_t w_t \overline{q_t}^2}{\gamma^2}$ which is an increasing convex function of w_t and $\overline{q_t}$, as expected. Furthermore, the government provides a given level of public goods in the economy denoted as $\overline{g_{t,t+1}}$, which gives us the total expenditure function of the government

$$E\left(\overline{q_t}, \overline{g_{t,t+1}}\right) \equiv \frac{H_t w_t}{\gamma^2}\overline{q_t}^2 + \overline{g_{t,t+1}}. \tag{14}$$

The government's fiscal revenue comes from income taxes paid by licit workers in the economy[14] i.e. $H_t(1-\phi)\overline{\tau_t}w_t$, for a given level of taxes, $\overline{\tau_t} \in (0,1)$, as well as the amount of illicit income confiscated from illicit workers i.e. $H_t\phi\overline{q_t}\left(\varepsilon_t Y_t^D + (1-\varepsilon_t)Y_t^R\right)$. This gives total fiscal revenue as

$$R\left(\overline{\tau_t}, \overline{q_t}\right) \equiv H_t(1-\phi)\overline{\tau_t}w_t + H_t\phi\overline{q_t}\left(\varepsilon_t Y_t^D + (1-\varepsilon_t)Y_t^R\right). \tag{15}$$

We assume that the government satisfies a balanced budget constraint in every period, $R\left(\overline{\tau_t}, \overline{q_t}\right) = E\left(\overline{q_t}, \overline{g_{t,t+1}}\right)$. Therefore, in order to generate probability of confiscation q_t, public good g_t and tax rate level τ_t, the government solves the following problem

$$\begin{aligned} &\max_{\tau_t, q_t \cdot g_t \in [0,1]^2 \times \mathbb{R}_+} \quad H_t(1-\phi)U_{t,t+1}^0 \\ &s.t. \quad : \quad R(\tau_t, q_t) = E(q_t, g_{t,t+1}) \end{aligned} \tag{16}$$

Replacing the values for the term $\varepsilon_t Y_t^D + (1-\varepsilon_t)Y_t^R$ from equations (11), (12) and $\varepsilon_t = \frac{\left(\frac{\widetilde{D}_t}{A}\right)^{\frac{1}{\lambda}}}{H_t\phi}$ yields $\varepsilon_t Y_t^D + (1-\varepsilon_t)Y_t^R = \frac{d_t\widetilde{D}_t}{H_t\phi} + \frac{(1-\tau_t)\rho(1-\phi)w_t}{\phi}$. Substituting into the balanced budget constraint and rearranging gives

[13] Empirically one can think of there being different values of q_t for each illicit activity on the grounds that society can have preferences for policy makers to target these two types of illicit activities differently. However, for simplicity, in the theoretical part we consider q_t to be unique in the problem that the government faces, which is equivalent to assuming that all illicit activities are targeted in the same way. We relax this assumption in the empirical section, considering different probabilities of detection and confiscation for the two types of illicit activity in the Colombian economy.

[14] Note that the government does not charge income tax on capital earnings. For an economy like Colombia this assumption is reasonable since income tax comes mainly from labor and not from capital.

$$g_{t,t+1} = H_t (1-\phi) \tau_t w_t + H_t q_t \left(\frac{d_t \widetilde{D}_t}{H_t} + (1-\tau_t) \rho (1-\phi) w_t \right) - \frac{H_t w_t}{\gamma^2} q_t^2 \tag{17}$$

which shows that $g_{t,t+1}$ is determined once τ_t and q_t are determined. The optimization problem in (16) can be written more explicitly by replacing the indirect utility $U^0_{t,t+1}$ from equation (7) evaluated at $I = 0$ in the objective function subject to equation (17) where we avoid variables and constants taken as given by the government

$$\begin{aligned} &\max_{\tau_t, q_t \in [0,1]^2} (1+2\beta) \ln \left(h_t^0 + (1-\tau_t)(1-\rho) w_t \right) + \eta \ln g_{t,t+1} \\ s.t. \quad : \quad & g_{t,t+1} = H_t (1-\phi) \tau_t w_t + H_t q_t \left(\frac{d_t \widetilde{D}_t}{H_t} + (1-\tau_t) \rho (1-\phi) w_t \right) - \frac{H_t w_t}{\gamma^2} q_t^2 \end{aligned} \tag{18}$$

Replacing the constraint in the objective function yields an unconstrained optimization problem in the control variables (τ_t, q_t) that the government chooses. Note that the control variables (τ_t, q_t) belong to $[0,1]^2$ which is a compact set of $\mathbb{R}^2$ and since the objective function is a continuous function in (τ_t, q_t), the Weierstrass theorem[15] guarantees the existence of a solution $(\tau_t^*, q_t^*) \in [0,1]^2$ to problem (18). Moreover, consider the following first order conditions for problem (18)

$$\begin{aligned} \frac{(1+2\beta)(1-\rho)}{h_t^0 + (1-\tau_t)(1-\rho) w_t} &= \frac{H_t (1-\phi) \eta (1-\rho q_t)}{g_{t,t+1}} \\ \frac{\gamma^2}{2} \left(\frac{d_t \widetilde{D}_t}{H_t w_t} + (1-\tau_t) \rho (1-\phi) \right) &= q_t \end{aligned} \tag{19}$$

while the second order condition for an optimum $(\tau_t^*, q_t^*) \in [0,1]^2$ is satisfied given that the Hessian matrix of the objective function, once it is evaluated at the first order conditions, becomes a negative definite matrix[16] under the sufficient assumption that $\frac{H_t w_t}{g_{t,t+1}} \geq \frac{\rho^2 \gamma^2}{(1-\rho)(1-(2-\sigma)\rho)}$ for all t. Hence, we have

[15]The **Weierstrass** Theorem states that a continuous real-valued function on a compact set S achieves a minimum and a maximum in S.

[16]The first order derivatives of the objective function in problem (18) with respect to τ_t and q_t are

$$\begin{aligned} \frac{\partial SW}{\partial \tau_t} &= \frac{-(1+2\beta)(1-\rho)}{h_t^0 + (1-\tau_t)(1-\rho) w_t} + \frac{\eta H_t (1-\phi)(1-\rho q_t) w_t}{g_t} \\ \frac{\partial SW}{\partial q_t} &= \frac{\eta}{g_t} \left(H_t \left(\frac{d_t \widetilde{D}_t}{H_t} + (1-\tau_t) \rho (1-\phi) w_t \right) - \frac{2 H_t w_t}{\gamma^2} q_t \right) \end{aligned} \tag{20}$$

while the Hessian matrix of the objective function evaluated at the τ_t^*, q_t^* is

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 SW}{\partial \tau_t^2} & \frac{\partial SW}{\partial \tau_t \partial q_t} \\ \frac{\partial SW}{\partial q_t \partial \tau_t} & \frac{\partial^2 SW}{\partial q_t^2} \end{pmatrix}_{\tau_t^*, q_t^*} = \begin{pmatrix} -\frac{(1+2\beta)(1-\rho)^2 w_t}{\left(h_t^0 + (1-\tau_t^*)(1-\rho) w_t\right)^2} - \frac{\eta H_t^2 (1-\phi)^2 \left(1-\rho q_t^*\right)^2 w_t^2}{2 g_t^2} & -\frac{\eta H_t \rho (1-\phi) w_t}{2 g_t} \\ -\frac{\eta H_t \rho (1-\phi) w_t}{g_t} & -\frac{\eta H_t w_t}{\gamma^2 g_t} \end{pmatrix}.$$

The Hessian matrix is negative definite given that $\frac{\partial^2 SW}{\partial \tau_t^2} < 0$, $\frac{\partial^2 SW}{\partial q_t^2} < 0$ and $\frac{\partial^2 SW}{\partial \tau_t^2} \frac{\partial^2 SW}{\partial q_t^2} - \left(\frac{\partial SW}{\partial q_t \partial \tau_t} \right)^2 = \frac{(1+2\beta)(1-\rho)^2 w_t}{\left(h_t^0 + \left(1-\tau_t^*\right)(1-\rho) w_t\right)^2} + \frac{\eta H_t w_t (1-\phi)^2}{2 g_t} \left(\frac{H_t w_t}{g_t} - \frac{\rho^2 \gamma^2}{\left(1-\rho q_t^*\right)^2} \right)$ is strictly positive if $\frac{H_t w_t}{g_t} \geq \frac{\rho^2 \gamma^2}{(1-\rho)^2}$ for all t. This inequality is satisfied if $\frac{H_t w_t}{g_t} > \frac{\rho^2 \gamma^2}{(1-\rho)(1-(2-\sigma)\rho)}$ given that $\frac{\rho^2 \gamma^2}{(1-\rho)(1-(2-\sigma)\rho)} \geq \left(\frac{\rho \gamma}{1-\rho} \right)^2$ since $\sigma \in (0,1)$ which is what we assume.

found necessary and sufficient conditions for a solution $(\tau_t^*, q_t^*) \in [0,1]^2$ to problem (18). Nonetheless, the uniqueness of the solution is not guaranteed since the problem is non-linear and the objective function is not globally concave. In Appendix A we establish sufficient conditions for the uniqueness of the solution $(\tau_t^*, q_t^*) \in [0,1]^2$ to problem (18).

From equation (17) and the second equation of (19) the optimal (positive) level of public good provision chosen by the government is given by

$$g_{t,t+1}^* = \left((1-\phi)\,\tau_t^* + \left(\frac{q_t^*}{\gamma}\right)^2 \right) H_t w_t. \tag{21}$$

Note that $g_{t,t+1}^*$ is positive and strictly increasing in q_t^* and τ_t^*, given that these allow the government to increase its fiscal revenue. Moreover, $g_{t,t+1}^*$ is strictly decreasing in ϕ which shows that the greater fraction of workers dedicated to illicit activities in the economy, the more the provision of public goods in the economy deteriorates. Furthermore, from the technology to generate the probability of detection and confiscation we can determine $\theta_t^* = \frac{q_t^{*2}}{\gamma^2(1-\phi)}$ as well as the number of workers hired by the government as policemen and judges, $P_t^* \equiv \theta_t^* L_t^0$.

Finally, note that although we here assume that the government maximizes the utilitarian social welfare of licit households only, as in equation (13), the solutions would be the same if the government had maximized the social welfare of all workers defined as $H_t U_{t,t+1}^0$ i.e. as if all workers were dedicated to licit activities.

4.5 Capital Accumulation

The financial system channels all savings from young individuals at the end of period t into the licit economy in order to match the investments of licit firms that expand the capital in period $t+1$. To derive the equation for capital accumulation we follow the time line of Figure 1 and note that capital is the only asset in the economy, therefore aggregate bequests h_t, that the adult generation gives the young generation at the end of period t, must equal net capital used, partly depreciated at the end of period t, plus the real interest that the capital generates

$$h_t = (1 + i_t - \delta)\, K_t \tag{22}$$

where aggregate bequests in period t correspond to $h_t \equiv H_t (1-\phi)\, h_t^0 + H_t \phi h_t^1$ and $\delta \in (0,1)$ denotes the depreciation rate of capital. Moreover, aggregate wealth W_t at the end of period t, according to the time line, is composed of aggregate bequests (capital) plus aggregate labor income (licil and illicit)

$$W_t = h_t + Y_t \tag{23}$$

where aggregate wealth in period t is defined as $W_t \equiv H_t (1-\phi)\, W_t^0 + H_t \phi W_t^1$ and aggregate labor income is defined as $Y_t \equiv H_t (1-\phi)(1-\tau_t)(1-\rho)\, w_t + H_t \phi Y_t^e$. Furthermore, according to the time line in Figure 1, wealth in assets after consumption at the end of period t is defined as

$$a_t = W_t - C_t \tag{24}$$

where aggregate assets of licit and illicit origin correspond to the definition $a_t \equiv H_t (1-\phi) a_t^0 + H_t \phi a_t^1$ while aggregate consumption is defined as $C_t \equiv H_t (1-\phi) \left[C_t^{0,a} + C_t^{0,y}\right] + H_t \phi \left[C_t^{1,a} + C_t^{1,y}\right]$.

Capital used for production of the licit good during period $t+1$ must equal aggregate assets of licit and illicit origin at the end of period t

$$K_{t+1} = a_t. \tag{25}$$

Hence, replacing (24) in (25) and then replacing equations (22) and (23), we get

$$K_{t+1} = (1 + i_t - \delta) K_t + Y_t - C_t \tag{26}$$

which gives us a familiar equation for accumulation of capital where aggregate net investment defined as $K_{t+1} - (1-\delta) K_t$ is equal to aggregate income minus consumption defined as $Y_t + i_t K_t - C_t$.

4.6 Short Run Equilibrium and Steady State

We do not define a competitive equilibrium because illicit activities do not operate in a competitive environment. Instead we define a short-run equilibrium with illicit activities for each period as a situation in which every agent chooses their corresponding control variables optimally such that workers self-select into licit and illicit activities when young, illicit labor markets are segmented while licit markets clear and prices and quantities are either determined outside the economy, say by world markets, or are determined as a function of capital per licit worker for each period. In Appendix A we show that there is a short-run equilibrium (not necessarily unique) with illicit activities for each period and a given level of capital per licit worker.

We define a long-run equilibrium or steady state as a short-run equilibrium with illicit activities in which capital per licit worker is constant and positive over time i.e. $k_t = \overline{k} > 0$ for all t. From equation (25) and after replacing $a_t \equiv H_t (1-\phi) a_t^0 + H_t \phi a_t^1$ and the optimal levels of a_t^I for $I = 0, 1$ from equation (6), we get

$$K_{t+1} = H_t (1-\phi) s(\beta) W_t^0 + H_t \phi s(\beta) W_t^1$$

where $s(\beta) \equiv \frac{2\beta}{1+2\beta} \in (0,1)$ is the marginal propensity to save out of total wealth. Substituting for wealth levels using $h_t \equiv H_t (1-\phi) h_t^0 + H_t \phi h_t^1$ and equation (22) yields

$$\begin{aligned} K_{t+1} &= s(\beta)(1 + i_t - \delta) K_t + H_t (1-\phi) s(\beta)(1-\tau_t)(1-\rho) w_t \\ &\quad + H_t \phi s(\beta)(1 - q_t \sigma) \left(\varepsilon_t Y_t^D + (1-\varepsilon_t) Y_t^R\right) \end{aligned}$$

Dividing both sides of this last equation by $H_{t+1}(1-\phi)$, capital per licit worker is defined as $k_t \equiv \frac{K_t}{H_t(1-\phi)}$, and using $H_{t+1} = (1+n) H_t$ for all t, we get the fundamental capital accumulation equation in terms of k_t

$$\begin{aligned} \frac{(1+n)}{s(\beta)} k_{t+1} &= (1+i_t-\delta) k_t + (1-\tau_t)(1-\rho) w_t \\ &+ \frac{\phi(1-q_t\sigma)\left(\varepsilon_t Y_t^D + (1-\varepsilon_t) Y_t^R\right)}{1-\phi} \end{aligned} \tag{27}$$

Note that in a short-run equilibrium, the licit real wage (w_t) and the real interest rate (i_t) are determined as functions of capital per licit worker by equation (9). Moreover, the optimal income tax rate (τ_t) is determined by equation (19), which is implicitly a function of the licit real wage, w_t and the optimal detection and confiscation probability, q_t^{**}, which is given by an implicit function $q_t^{**} = q(k_t; \Omega_t, \Gamma_{t-1})$ where Γ_{t-1} corresponds to predetermined variables in period t and constant parameters such that

$$\Gamma_{t-1} \equiv \Gamma(k_{t-1}; \rho, \beta) = \left(h_{t-1}^0(k_{t-1}), \tau_{t-1}(k_{t-1}), w_{t-1}(k_{t-1}); \rho, \beta\right).$$

and where Ω_t corresponds to variables exogenous variables that the government takes as given, i.e. $\Omega_t \equiv (H_t, d_t\widetilde{D}_t; \rho, \gamma, \eta, \beta, \phi, \sigma, \delta)$.

Hence, q_t^{**} is a function of k_t and k_{t-1} which implies that $\theta_t = \frac{(q_t^{**})^2}{\gamma^2(1-\phi)}$ and τ_t are also functions of k_t and k_{t-1}. Furthermore, we have that $Y_t^R \equiv \frac{(1-\tau_t)\rho(1-\phi)w_t}{\phi(1-\varepsilon_t)}$ which shows that it is also a function of k_t and k_{t-1} while ε_t and Y_t^D are independent variables of capital per licit worker. Finally, since h_{t-1}^0, τ_{t-1} and w_{t-1} are themselves functions of $\Gamma_{t-2} \equiv \Gamma(k_{t-2}; \rho, \beta)$, by backward iteration it must be the case that they are functions of all values of k from k_0 up to k_{t-1}, which we denote as $\mathbf{k}_{t-1} = (k_{t-1}, .., k_0)$. Hence equation (27) is a t order non-linear difference equation in capital per licit worker. Importantly, the vector $\mathbf{k}_{t-1}$ directly affects only q_t^{**}, θ_t and τ_t and these are all variables bounded in the interval $[0,1]$, which is key to proving the existence of a steady state. The following proposition, which is proved Appendix A, establishes that the model has a long-run equilibrium or steady state (not necessarily unique).

Proposition 1 *There is at least a steady state $\overline{k} > 0$ that satisfies equation (27).*

5 Comparative Statics

In this section we study the short-run effects on aggregate savings, public good provision, and social welfare when the following parameters are changed: a) efficiency in the technology of common criminals (ρ), b) price of drugs (d), c) efficiency of the government to generate the probability of detection and confiscation (γ) and d) efficiency of the licit sector to produce the licit good (B). We keep the optimal income tax fixed in these short run comparative static exercises since we believe that in the short run income taxes are difficult to change and would adjust slowly to any of the changes studied. The following table summarizes the main comparative effects for these parameter changes in a short-run equilibrium with illicit activities which is shown to hold in the appendix.

Variable	Efficiency of common crime	International drug cocaine	Efficiency of government	Productivity of private licit sector
Delinquent incentives	+	+	-	-
Aggregate savings	-	+	-	+
Public goods	+	+	+	+
Social welfare	-	+	+	+

Table 1

In the following subsections we report the intuition for these comparative analysis results.

5.1 Efficiency of Common Crimes

An increase in the efficiency of common crimes i.e. $\Delta\rho > 0$ increases illicit incomes from these activities. It does not alter the allocation of illicit labor across both illicit activities given that production of cocaine is still assumed to be more profitable for a young individual that opts to work in the illicit sector in a short run equilibrium. However it does increase in principle the potential incentives to enter the illicit sector because the average returns for young workers increase. We discuss "potential" effects here rather than actual effects because the binary moral cost set up, which simplifies the analysis, has the restrictive nature of not allowing a smooth change in the fractions of licit and illicit workers. Though a "continuous" shift of workers from licit to illicit activities is not possible, the model allows for the possibility of a quantum change, from no illicit activity to some, and vice versa. An increase in the efficiency of common crimes also increases the optimal probability of detection and confiscation which would in principle generate an ambiguous effect for delinquent incentives. Nonetheless the more likely case is that the net effect should be positive for a low level of the probability of detection and confiscation which seems the relevant case to consider. Illicit incomes from common crimes as well as average incomes in the whole illicit sector increase, while licit incomes decrease, generating a redistribution of income from licit to illicit households and since there are more licit households in the economy then the net effect is a decrease in national savings (affecting capital accumulation adversely in the next period). Furthermore, since a greater efficiency of common crimes triggers an increase in the probability of detection and apprehension then there is an increase in public goods provided in the economy since this is financed by confiscation of illegal incomes. Finally, the more likely case is that an increase in common crime efficiency reduces social welfare since this is sensitive to the decrease in licit incomes in the economy which should more than compensate for the increase in public goods provided.

5.2 Price of Cocaine

An increase in the price of cocaine, i.e. $\Delta d_t > 0$, which is determined by foreign markets, leads to an increase in illicit incomes from drug trafficking. The increase in average illicit income would increase potential incentives for young workers to enter the illicit sector. Again the optimal probability of detection and confiscation is increased in this situation since it responds positively to an increase in d_t which would in principle generate an ambiguous effect on delinquent incentives. Again the more likely case to arise is a positive net effect on delinquent incentives for economies with low levels of detection and confiscation. Furthermore, an increase in the price of illicit drugs would generate a type of *Dutch disease*, since an increase in d_t is equivalent to increasing the relative price of the illicit good (cocaine) with respect to the price of the licit good which has been normalized to one. Hence, an increase in the relative price of drugs generates higher potential incentives that attract more labor resources into the inefficient illicit sector deteriorating the labor resources for the licit sector. On the other hand public goods provided increase with d_t since this change triggers an increase in the optimal probability of detection and confiscation and therefore more resources would the government have from illicit activities. Moreover, given that licit disposable incomes in a short run equilibrium are unaffected with an increase in d_t while illicit incomes from cocaine trafficking are increased then national savings are increased with d_t which benefits capital accumulation in the next period. Finally, social welfare is increased since licit households benefit from the increase in public goods provided.

Note that a simultaneous increase in common crime efficiency and the international price of cocaine would generate an ambiguous effect on national savings which shows that money laundering in this model does not necessarily generate positive incentives for capital accumulation.

5.3 Efficiency of Government

An increase in the efficiency of the government, i.e. $\Delta\gamma$, increases the optimal probability of detection and confiscation which decreases illicit incomes and therefore the delinquent incentives to enter the underground sector in the economy. Since public goods provision depends positively on the optimal probability of detection and confiscation then we would have an increase in the optimal level of public goods provision. Moreover, licit disposable incomes are unaffected and since social welfare is increasing in the provision of public goods then social welfare would increase. Furthermore, aggregate savings would decrease since licit incomes are unaffected, given that the government does not give back to households confiscated income, while illicit incomes are decreased.

5.4 Productivity of Licit Firms

An increase in the productivity of licit firms, i.e. ΔB, increases the wage in the economy and therefore of disposable licit incomes. It also generates a decrease in the optimal probability of detection and confiscation since licit labor is more expensive and a larger polic force is less sustainable. Hence, fiscal revenue increases which then generates an overall increase in the provision of public goods. There is also an increase in both the licit disposable income and average illicit income which generates an increase in aggregate savings. Importantly, an increase in the efficiency of licit firms deteriorates the potential incentives to enter the illicit sector, since licit wages are increased more than average illicit income is. The increase in disposable income as well as in the provision of public goods generates an increase in social welfare. This explains the signs reported in the table 1.

6 Macroeconometric Model

This section builds the macroeconometric model as a two equation system derived directly from the conceptual framework of the OLG model developed above. This macroeconometric model is built in order to estimate the volume of asset laundering in the Colombian economy in a given period of time. The first equation in the macroeconometric model, called the *transition* equation, specifies the dynamic nature of asset laundering activity in the economy in terms of its determinants. The second equation, called a *measure* equation, represents the mechanism through which accumulation of laundered assets affects capital accumulation in the licit sector of the economy.

6.1 Transition and Measure Equations

As conceptualized in the OLG model developed above, asset laundering is the part of total capital that is *placed* and *integrated* into the licit economy and thus assists in the production of the licit good. Recall that by assumption young individuals inherit their moral type and capital as a bequest from their adult parent. This means that illicit adult parents bequest capital to their young children that was accumulated illicitly in the previous period during which the adult parent was a young illicit worker. Hence, bequests that come from illicit activities at the end of period t constitute the net stock of laundered assets, in the form of capital, after adjusting for partial depreciation at the end of period t and any return on it, measured by i_t

$$H_t \phi h_t^1 = (1 + i_t - \delta)\, AL_t \tag{28}$$

where AL_t denotes the stock of capital of illicit origin laundered by the end of period t, which is assumed to depreciate at the same rate $\delta \in (0, 1)$ as capital from licit sources. Wealth of illicit origin at the end of period t is composed of bequests (capital) from illicit activities plus illicit labor income

$$W_t^1 = h_t^1 + Y_t^e \tag{29}$$

while wealth in assets from illicit sources after consumption at the end of period t is defined as

$$a_t^1 = W_t^1 - C_t^1 \tag{30}$$

where consumption by illicit households at the end of period t is $C_t^1 \equiv C_t^{1,a} + C_t^{1,y}$. Hence, the volume of assets laundered and used as licit capital for the production of the licit good during period $t+1$ must equal aggregate assets of illicit origin plus the capitalization of these assets' return

$$AL_{t+1} = H_t \phi a_t^1. \tag{31}$$

Hence, replacing (30) in (31) and then using equations (28) and (29) we get the *transition* equation of asset laundering lagged by one period

$$AL_t = (1 + i_{t-1} - \delta) AL_{t-1} + H_{t-1}\phi \left(Y_{t-1}^e - C_{t-1}^1\right). \tag{32}$$

where $C_t^1 \equiv C_t^{1,a} + C_t^{1,y}$ and $Y_t^e \equiv (1 - q_t)\left(\varepsilon_t Y_t^D + (1 - \varepsilon_t) Y_t^R\right)$.

Asset laundering affects the capital accumulation of the licit sector of the economy, which generates the mechanism through which the latent variable AL_t affects the accumulation of capital in the economy. Recall equation (26) that corresponds to the dynamic equation for the accumulation of capital

$$K_{t+1} = (1 + i_t - \delta) K_t + Y_t - C_t$$

where Y_t represents the aggregate labor income (not including the returns to capital) from licit and illicit sources. Substituting $H_t(1-\phi) h_t^0 + H_t \phi h_t^1 = (1 + i_t - \delta) K_t$ from equation (22) into this last equation, using the definition $H_t \phi h_t^1 = (1 + i_t - \delta) AL_t$ from equation (28) and replacing $H_t(1-\phi) h_t^0 = \mu (1 + i_t - \delta) K_t$ for some $\mu \in (0, 1)$, we get the following *measure* equation which we lag by one period

$$K_t = \mu (1 - \delta + i_{t-1}) K_{t-1} + Y_{t-1} - C_{t-1} + (1 - \delta + i_{t-1}) AL_{t-1} \tag{33}$$

The intuition for this equation is simple in that the capital stock in period t is a function of the capital stock in period $t-1$, which corresponds to the licit part and its net return $\mu(1 - \delta + i_{t-1}) K_{t-1}$, plus illicit capital laundered plus its net return $(1 - \delta + i_{t-1}) AL_{t-1}$, plus the aggregate savings in the economy (licit and illicit) which correspond to $Y_{t-1} - C_{t-1}$.

6.2 Linear Equation System

The empirical model to be constructed follows from the two linear equations (32) and (33). We assume that $C_{t-1}^1 = (1 - s(\beta)) Y_{t-1}^e$, i.e. that illicit consumption of the licit good C_{t-1}^1 is equal to a fraction of illicit undetected income Y_{t-1}^e. Since illicit young workers have the same preferences as licit young workers, this fraction is the marginal propensity to consume $1 - s(\beta)$, where $s(\beta) = \frac{2\beta}{1+2\beta}$. This method is used because the consumption of illicit households is not directly observable. Substituting these into the transition equation (32) and rearranging common terms yields the following transition equation

$$AL_t = (1 + i_{t-1} - \delta) AL_{t-1} + s(\beta) H_{t-1} \phi Y^e_{t-1}. \tag{34}$$

Note that assets laundered (AL) is lagged in the measure equation (33) which complicates the estimation process we use later on. To avoid this, we substitute for $AL_{t-1}(1 + i_{t-1} - \delta)$ in (33) using $(1 + i_{t-1} - \delta) AL_{t-1} = AL_t - s(\beta) H_{t-1} \phi Y^e_{t-1}$from the transition equation (34), yielding an equation with AL_t in period t as the key unobservable to estimate

$$K_t = \mu (1 - \delta + i_{t-1}) K_{t-1} + Y_{t-1} - C_{t-1} + AL_t - s(\beta) H_{t-1} \phi Y^e_{t-1}$$

We also replace in this last equation the definition of aggregate *labor* income

$$Y_{t-1} \equiv H_{t-1} (1 - \phi) (1 - \tau_{t-1}) (1 - \rho) w_{t-1} + H_{t-1} \phi Y^e_{t-1}$$

which yields, after rearranging terms

$$\begin{aligned} K_t &= \mu (1 - \delta) K_{t-1} + (1 - \rho) \left((1 - \tau_{t-1}) H_{t-1} (1 - \phi) w_{t-1} + \frac{\mu}{1 - \rho} i_{t-1} K_{t-1} \right) \\ &\quad + (1 - s(\beta)) H_{t-1} \phi Y^e_{t-1} - C_{t-1} + AL_t \end{aligned} \tag{35}$$

In order to obtain an estimable linear system, we must define measures of theoretical variables and assume values for some parameters. In what follows we present the empirical variables used to approximate the theoretical counterparts involved in the linear system in equations (34) and (35).

7 Data

The period of analysis for the empirical section is 1985 to 2013, during which we observe variables at an *annual* frequency for the Colombian economy. Several of the empirical variables needed for the estimation, however, are not directly observable, particularly those related to illicit activities. The structural equations (34) and (35) suggest a way to create these variables using observable ones.

Firstly, we observe the amount of cocaine *confiscated* by Colombian authorities and estimate the aggregate amount of projected cocaine *production* every year for the Colombian economy. The estimated projected production of cocaine is a proxy variable for actual production, based on a measure of the land size used to grow coca. From this, we obtain an estimate of the probability of detection and confiscation of cocaine for period t

$$\widetilde{q}_t = \frac{[\text{Confiscated Kgs of Cocaine}]_t}{[\text{Projected Kgs of Cocaine Produced}]_t}. \tag{36}$$

We assume in the case of cocaine production that once it is detected by law enforcement authorities, no illicit income is generated from it, which is consistent with the idea that cocaine production is destroyed once it is detected.

Overall illicit income earned by drug lords in Colombia from cocaine trafficking can be estimated using the estimated projected production and price of cocaine. The relevant cocaine price however is not clear, as local prices of cocaine are quite low while international prices from the United States during the period of study are too high. Caulkins and Reuter (2010) report cocaine prices from raw production in Colombia up to the retail prices in the United States for the years 1997, 2000 and 2007. They note that the export cocaine price in the ports of Colombia is around one-third of the mid-level wholesale price reported in the United States while the import cocaine price in the ports of the United States corresponds roughly to two-thirds of the mid-level wholesale price reported inside the United States. Assuming that somewhere between the ports of Colombia and the United States, Colombian drug lords deliver their production to others that continue the trafficking in the United States, the cocaine price relevant for estimating the illicit income of Colombian drug lords would be between these two bounds. Accordingly, we use a price of half the mid-level wholesale cocaine price per kilogram reported for the United States by UNODC, as in the following measure

$$\left[\varepsilon_t \widetilde{\phi H_t Y_t^D}\right] = \frac{\begin{array}{c}[\text{Projected Kgs Cocaine Produced}]_t \\ \times \frac{1}{2}\,[\text{Mid-level whole sale cocaine Price in New York per kg in dollars}]_t \\ \times\,[\text{Nominal Exchange Rate}]_t\end{array}}{\text{Consumer Price Index}_t^{\text{base 2005}}}$$

Note that we use the consumer price index with base year 2005 to get illicit aggregate income from cocaine trafficking in pesos in real terms. Hence, the aggregate real undetected illicit income from cocaine production per year $RUICOC_t$ is given by the following proxy variable

$$RUICOC_t \equiv (1 - \widetilde{q}_t) \times \varepsilon_t \widetilde{\phi H_t Y_t^D} \tag{37}$$

Similarly, we do not observe total potential income from common crimes but only income from *reported crimes*. We use National Police Department statistics for the period under study on the income stolen through common crimes against property which includes 56 offenses, such as robbery, theft, burglary, motor vehicle theft, larceny etc., and excludes drug trafficking from aggregate values. This allows us to obtain the following measure of reported income stolen

$$\left[\kappa_t q_t \widetilde{(1 - \varepsilon_t)}\, \phi H_t Y_t^R\right] = \begin{bmatrix}\text{Reported Stolen Income} \\ \text{from Common Crimes}\end{bmatrix}_t$$

where $\kappa_t q_t$ measures the probability of detection and confiscation of income from common crimes for some $\kappa_t > 0$. This reflects the idea that empirically (though not in the theoretical model, where q_t is determined to be the same for both illicit sectors) the probability of detection and confiscation of income from cocaine can be different than for common crimes.

Importantly, this empirical variable only includes *reported* crimes while we also need to consider undetected crimes. To adjust these values we must take into account the reporting rates for these types of crimes. According to Soares (2004a, 2004b) there is a huge difference between official statistics on reported crime and what is found in victimization surveys. For Colombia, this problem is particularly acute, as Soares finds that less than 1% of offenses are actually reported to law enforcement authorities. Soares also argues that as a country develops, that is as its GDP increases, citizens are more likely to report stolen goods. Hence, reporting rates are positively correlated with economic growth rates for underdeveloped economies like Colombia. Undetected incomes stolen by common crimes are thus defined as

$$RUIC_t = \left[\widetilde{(1-\varepsilon_t)\,\phi} H_t Y_t^R\right] \equiv \left[\begin{array}{c}\text{Reported Income Apprehended}\\ \text{from Common Crimes}\end{array}\right]_t \times adjr_t \tag{38}$$

where $adjr_t$ represents[17] the reporting adjustment factor which takes into account the reporting rate for Colombia and grows with the country's licit GDP in the period under study.

Therefore, we can obtain a measure of aggregate real undetected illicit income generated in period t defined as

$$RUII_t = RUICOC_t + RUIC_t \tag{39}$$

which aggregates real illicit undetected income from cocaine trafficking and common crimes for each period.

[17] Let R_1 represent stolen goods reported to authorities and R_2 represent stolen goods not reported to authorities in any given period of time. Let V_1 represent the value of the goods stolen and reported and V_2 represent the value of stolen goods not reported. Total income stolen by common criminals in a given period of time is defined as

$$S \equiv R_1 V_1 + R_2 V_2$$

We define the reporting rate of stolen goods as

$$r = \frac{R_1}{R_1 + R_2}$$

which according to Soares (2004) is close to zero for an economy like Colombia. In line with Soares we assume that this rate grows at the same rate as licit GDP for Colombia in the period under study. Moreover, define the value rate of stolen goods as

$$\nu \equiv \frac{V_1}{V_1 + V_2}$$

which should be close to one since in less developed economies the value of goods stolen and reported is high since in these economies robberies are mainly for highly valuable goods and not petty crimes. We assume that this rate is 0.95 and decreases over time at a rate of 0.1% per year. From these equations we have

$$\begin{aligned} R_2 &= R_1\left(\frac{1}{r}-1\right) \\ V_2 &= V_1\left(\frac{1}{\nu}-1\right) \end{aligned}$$

And therefore

$$R_2 V_2 = R_1 V_1 \left(\frac{1}{r}-1\right)\left(\frac{1}{\nu}-1\right)$$

which, replaced in the first equation, yields

$$S = R_1 V_1 \left(1 + \left(\frac{1}{r}-1\right)\left(\frac{1}{\nu}-1\right)\right).$$

Hence the adjustment factor rate is defined as

$$adjr \equiv \left(1 + \left(\frac{1}{r}-1\right)\left(\frac{1}{\nu}-1\right)\right).$$

We use reported Colombian GDP in constant 2005 pesos as a proxy for licit aggregate income that includes both private as well as public income ($RGDP$) in the following way

$$RGDP_t \equiv \frac{[\text{Nominal Gross Domestic Product}]_t}{\left[\text{Consumer Price Index}^{\text{base 2005}}\right]_t}. \tag{40}$$

We also use an estimate of aggregate real consumption in the Colombian economy, an observable quantity, obtained from National accounts

$$RCONS_t = \frac{[\text{Nominal Aggregate Consumption}]_t}{\left[\text{Consumer Price Index}^{\text{base 2005}}\right]_t}. \tag{41}$$

We use a measure of the real stock of capital (KS) for the Colombian economy, for which more than one estimate is available. We use the measure of real aggregate capital stock in each period from the Departamento Administrativo Nacional de Estadísticas (DANE), Colombia's National Bureau of Statistics

$$KS_t = [\text{Aggregate Capital Stock in constant pesos 2005}]_t\,. \tag{42}$$

Finally, for our calibration measure of laundered assets we use the real interest rate which can be obtained by using the Fisher identity which says that the real interest rate (RIR) plus one is equal to the nominal interest rate plus one divided by the inflation rate plus one, yielding the following measure

$$RIR_t \equiv \frac{1 + [\text{Nom. Interest Rate bond 90 days in December}]_t}{1 + [\text{Inflation Rate}]_t} - 1. \tag{43}$$

All of the data used were collected by the Unidad de Información y Análisis Financiero (UIAF) from September 2013 to March 2014.[18] All macroeconomic time series come from official sources, mainly from the Banco de la República of Colombia, National Bureau of Statistics DANE, National Office of Taxes and Customs DIAN, National Planning Department DNP and Treasury Department. Data on crimes and drugs come from the Ministry of Defense, National Police Department, National Institute of Prisons INPEC, United Nations Office on Drugs and Crime UNODC, Ministry of Justice and the District Attorney's Office. Although the data were initially to be collected quarterly for the period 1999 to 2013, data limitations made this unfeasible. Hence, data were collected at an annual frequency for 1985 to 2013, which affected the size of the time series. Not all of the data collected were used since the theoretical model guided the final data necessary for the application of the estimation methods. Descriptive statistics for the variables used in the estimation are reported in Table 2.

[18] We thank the support of the Director of the UIAF Luis Edmundo Suárez and deputy director Javier Gutiérrez. We also thank especially Angela Hurtado from the UIAF for her superb assistance in constructing the data set used in this article. The UIAF did an excellent job in finding the best data available for the development of the empirical part of this project and assisted us in selecting the necessary information to construct the main time series required at the annual frequency and to obtain the relevant literature on asset laundering from national and international sources.

	Mean	Std Dev	Source
KS (billion pesos 2005)	898,037	266,207	DANE
RUII (billion pesos 2005)	14,124	7,207	National Police and UIAF
RUICOC (billion pesos 2005)	10,747	7,451	National Police and UIAF
RUIC (billion pesos 2005)	3,376	958	National Police and UIAF
RGDP (billion pesos 2005)	304,488	88,417	Banco de la Republica
RIR (%)	0.045	0.038	Banco de la Republica

Table 2 Summary Statistics

The key constructed variable used in the macroeconometric model is illicit undetected incomes $RUII_t$. Figure 3 shows the constructed series $RUICOC_t$ and $RUIC_t$ from equation (39), as well as their sum, $RUII_t$, relative to $RGDP_t$. As shown, $RUICOC_t$ is usually greater in magnitude than $RUIC_t$ for the period, which is consistent with the maintained assumption in the theoretical model that drug trafficking is more profitable than common crime activities. Undetected illicit income is an unobservable yet key quantity for any economy. We believe that these proxy variables, constructed with the help of the theoretical model, are thus a contribution to the literature in this regard.

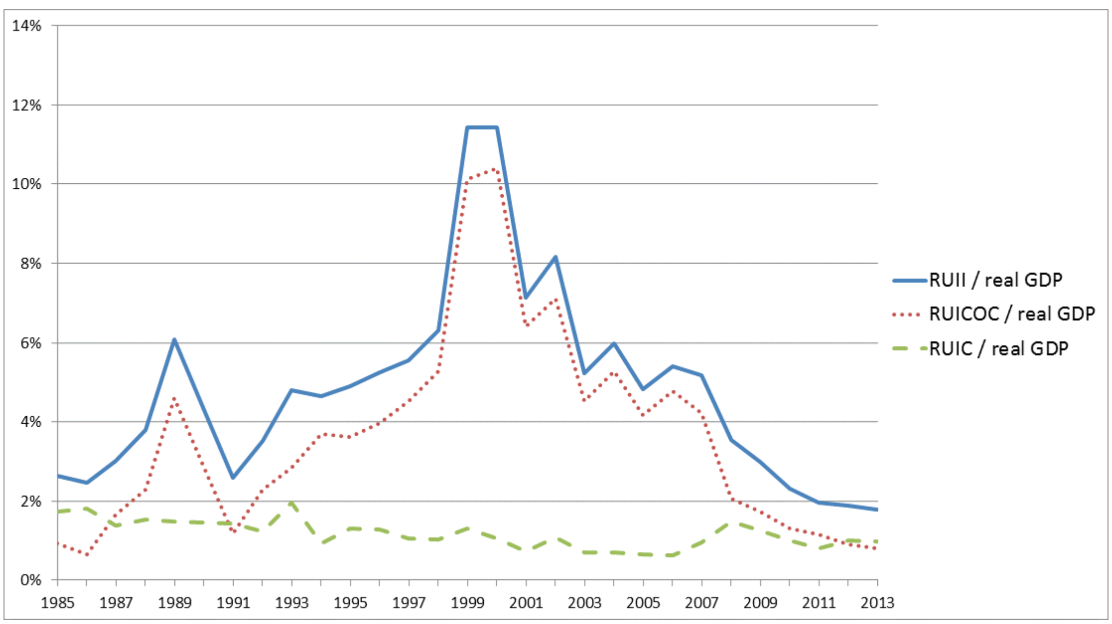

Figure 3

**$RUII$ refers to real undetected illicit income, i.e. money laudering per period, $RUICOC$, to real undetected income from drug trafficking per period and $RUIC$, to real undetected income from common crime per period.*

Figure 3 also shows that undetected income from cocaine rose from 1985 to 1999 and then decreased steadily until 2013. Undetected income from common crime follows a different path, which is more or less stable but follows a slight negative trend over the period. These series coincide with historical evidence that during the 1990s drug exports dominated the illicit sector of the Colombian economy, while in the 2000s, these activities lost steam due to other competitors like Mexico. Figure 3 also shows that aggregate undetected illicit income has been around 4.7% of real GDP in the period under study, reaching a peak in 1999 and 2000 of 11.4% of real GDP. Up to the beginning of the 2000s, cocaine trafficking was mainly driving undetected illicit incomes. However, at the end of the period, undetected income from common crime made up a greater proportion of total undetected illicit income than that from cocaine trafficking.

Figure 4 shows that the measure variable, capital stock KS, used in the empirical estimation of asset laundering, has been around 3 times the real GDP of the Colombian economy.

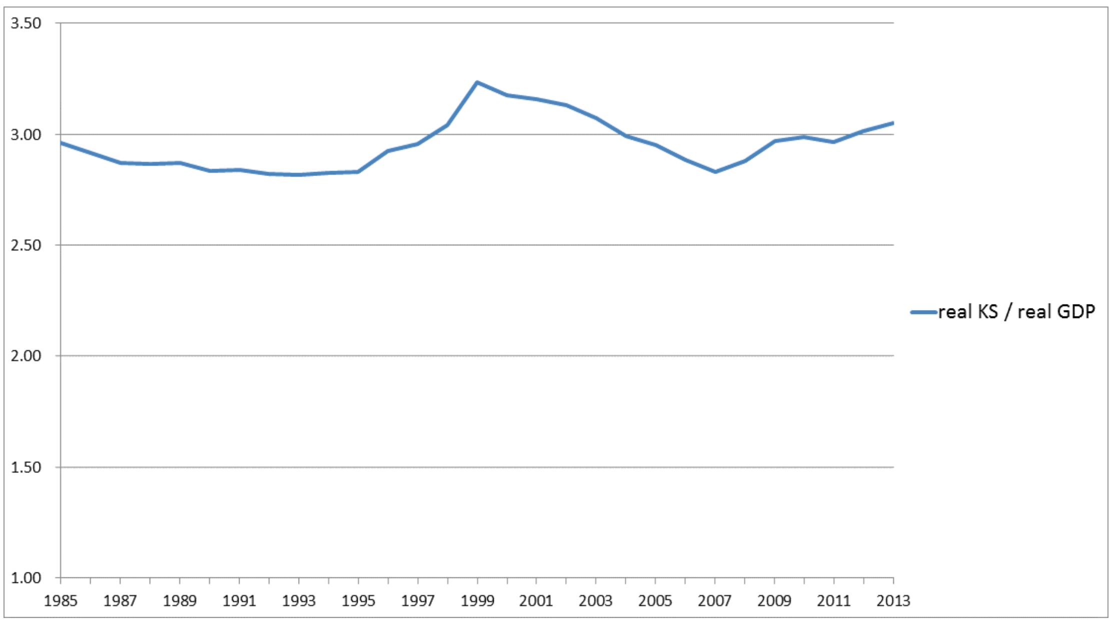

Figure 4

Before this project was undertaken over two years ago, the UIAF believed that money laundering in Colombia represented approximately 3% of real GDP. This estimate is not far from our finding that on average, aggregate undetected illicit income made up 4.7% of real GDP between 1985 and 2013.

8 Estimation of Stock of Laundered Assets

In principle there are several methodologies that could be used to estimate the volume of assets laundered in the Colombian economy in the period 1985 to 2013. In this section we first describe a macroeconometric estimable linear system of equations derived from the linear equation system and the proxy variables defined above. Then, we present and describe two different estimation methodologies:

i) calibration and ii) Kalman filter with an optimizing algorithm. The first method relies on the transition equation while the second takes both the transition and the measure equation into account.

8.1 Estimable Linear System

The proxy variables defined in the previous section can be used to construct a macroeconometric estimable linear system with error forms for the structural equations (34) and (35). The error form of the equations comes from noting that the observable variables used in the structural equations are given at the country level and thus, may include aggregated measurement errors. This implies a stochastic error term in the macroeconometric linear system given that we must assume the existence of *conditional* distributions of the dependent variables with respect to the conditioning independent variables such that all variables have finite first and second moments, which yields

$$AL_t = \alpha_0 + \alpha_1 RUII_{t-1} + \alpha_2 AL_{t-1} + u_t^{AL} \tag{44}$$

$$\begin{aligned} KS_t &= \mu_0 + \mu_1 KS_{t-1} + \mu_2 RGDP_{t-1} + \mu_3 RCONS_{t-1} \\ &\quad + \mu_4 RUII_{t-1} + \mu_5 AL_t + u_t^K \end{aligned} \tag{45}$$

for $t = 1985, ..., 2013$. Reduced form parameters are defined in terms of deep structural parameters

$$\begin{aligned} \alpha_0 &= 0;\ \alpha_1 = \frac{2\beta}{1+2\beta};\ \alpha_2 = 1 + E(i) - \delta; \\ \mu_0 &= 0;\ \mu_1 = \mu(1-\delta);\ \mu_2 = 1-\rho; \\ \mu_3 &= -1;\ \mu_4 = \frac{1}{1+2\beta};\ \mu_5 = 1 \end{aligned} \tag{46}$$

The errors u_t^{AL} and u_t^K of equations (44) and (45) are defined as

$$u_t^{AL} \equiv AL_t - E\left[AL_t \mid AL_{t-1}, RUII_{t-1}\right];$$

$$u_t^K \equiv KS_t - E\left[KS_t \mid AL_t, Z_{t-1}^K\right]$$

where

$$Z_{t-1}^K = \left[KS_{t-1}, RGDP_{t-1}, RCONS_{t-1}, RUII_{t-1}\right]$$

is the conditioning vector of variables in the measure equation and the homoskedastic variances of the two equations are denoted as σ_{AL}^2 and σ_K^2. The definition of the errors reflects the assumption that we have a complete set of conditioning variables such that no omitted relevant variable problem arises in the linear equation system.

8.2 Calibration Method

First we use the transition equation in equation (34), calibrate the parameters $s(\beta)$ and δ and use the variables $RUII_{t-1}$ and RIR_{t-1} from equations (39) and (43) as proxy variables for $\phi H_{t-1} Y_{t-1}^{e}$ and i_{t-1} respectively. The marginal propensity to save out of income for Colombia is around 0.17. This value is obtained by regressing aggregate real consumption on real GDP (both in 2005 pesos) and a constant, which yields a marginal propensity to save of 0.83. Under log utility specification, which we use in the theoretical part, the complement $1 - 0.83 = 0.17$ is an estimate of the marginal propensity to save out of income. Assuming that this is equal to the theoretical marginal propensity to save out of wealth, we find that $s(\beta) = 0.17$. The depreciation rate is assumed to be $\delta = 0.05$, as is standard in the growth literature. To implement the calibration method we need an initial value of the capital stock laundered for $t - 1 = 1984$. According to UNODC (2011), 75% of illicit undetected income enters the licit economy as laundered assets. This belief is supported to an extent by the UIAF of Colombia (see UIAF, 2014). On this basis we assume that 75% of average illicit income from 1984 to 1988 relative to average licit income in the same period is proportional to assets laundered in 1984 relative to the capital stock of the economy averaged from 1984 to 1988. Hence, a simple estimate of total illicit capital laundered is given by the following equation

$$AL_0 \equiv \left[\frac{0.75 * \left(\sum_{t=1984}^{1988} RUII_t \right)}{\sum_{t=1984}^{1988} RGDP_t} \right] \times \sum_{t=1984}^{1988} KS_t \tag{47}$$

which assumes that AL_0 is proportional to average KS. Recall that in the theoretical model periods correspond to generations, thus covering 20-30 years and that the way illicit income is laundered in capital investments is through bequests from parents to their offspring. The model assumes that after 30 years there is no confiscation of assets even if their illicit origins have been detected. However, in 1996, a judicial reform in Colombia gave law enforcement authorities the ability to confiscate assets that were laundered years before. Thus, since yearly data is available, we allow for confiscation of assets laundered in previous periods in the empirical estimation of assets laundered and take this into account in the empirical calibration.

For 2010 to 2014, we observe the amount of laundered assets confiscated by Colombian authorities. Nonetheless, we do not observe this amount from 1996 to 2009, despite this being after the judicial reform of 1996. For this time period, we assume that the volume of confiscated assets is proportional to illicit income $RUII$ in each period

$$\text{Assets Confiscated}_t \equiv AC_t = \xi_t \times RUII_t \tag{48}$$

where ξ_t is equal to zero from 1985 to 1997, given that the 1996 judicial reform was not implemented until 1997. After that year, we assume that ξ_t grows at the same rate as the probability of detection and confiscation of cocaine from equation (36), which can be seen as a proxy of gains in the efficiency of law enforcement authorities. The final value of ξ_t for 2013 is found using the average amount of assets confiscated in the Colombian economy from 2010 to 2014, an observable quantity, divided by the average illicit income from cocaine and common crime, as in the $RUII$ measure of equation (39), giving $\xi_{2013} = 0.2$. We then work backwards using the growth rate of $\widetilde{q_t}$ from 1997 to 2013 to impute the values of the fraction of assets confiscated by Colombian authorities between 1998 and 2012.

The calibrated transition equation comes from equation (34) where we replace the depreciation rate and the marginal propensity to save and subtract the assets confiscated by Colombian authorities after 1997. This gives us the following dynamic equation for aggregate asset laundering

$$\widetilde{AL}_t^{Calibrated} = 0.95AL_{t-1} + RIR_{t-1}AL_{t-1} + 0.17 \times RUII_{t-1} - AC_{t-1}$$

Substituting equation (48) into this last equation and regrouping terms we get

$$\begin{aligned} \widetilde{AL}_t^{Calibrated} &= 0.95AL_{t-1} + RIR_{t-1}AL_{t-1} + \left(0.17 - \xi_{t-1} \cdot 1_{1998}\right) \times RUII_{t-1} \\ \text{for} \quad t &= 1985, .., 2013 \end{aligned} \tag{49}$$

where 1_{1998} is an indicator variable that takes the value one for 1998 onwards and zero otherwise.

This equation is similar to a perpetual inventory equation for capital accumulation of the form $K_t = (1 - \delta) K_{t-1} + I_t$ where I_t denotes gross investment in period t while K_t is the capital stock. Assets laundered depreciate at the same rate as licit capital which is similar to the term $(1 - \delta) K_{t-1}$ while the term I_t is equal to the amount of income generated from illicit assets laundered $RIR_{t-1}AL_{t-1}$ plus the term $\left(0.17 - \xi_{t-1} \cdot 1_{1998}\right) \times RUII_{t-1}$ that represents the amount of undetected illicit income saved by illicit households in the economy in period $t-1$ and which enters the licit economy in period t, increasing the laundered capital stock in that period.

We can also obtain a decomposition of assets laundered from undetected income from cocaine and from common crime using equation (49) and replacing total illicit undetected income $RUII_t$ with the variables $RUICOC_t$ or $RUIC_t$ from equations (37) and (38) respectively and assuming that ξ_t is the same for both types of illicit activities.

Calibration in itself is closely attached to theoretical deep structural parameters of a given model and since AL_t is an unobservable quantity we cannot check whether the calibration works in the sense of reproducing an observed time series. Nonetheless it has the appeal of being consistent with the theory and can give us a trajectory consistent with the fundamentals of the Colombian economy.

8.3 Kalman Filter

The empirical methodology of the Kalman filter uses not only the transition equation (44) but also the measure equation (45) based on a state-space time invariant representation. In what follows we present the state-space representation that is used in this empirical method following the literature on time series as in Harvey (1994), Hamilton (1995), Engle and McFadden (1999) and Durbin and Koopman (2005). The state-space representation of the system consists of the transition equation which can be written using the standard matrix notation in this literature

$$\widetilde{\alpha}_t = T\widetilde{\alpha}_{t-1} + \gamma W_t + \eta_t \tag{50}$$

for $t = 1985, .., 2013$, where $\widetilde{\alpha}_t = AL_t$, $W_t = [RUII_{t-1}]$, $\eta_t = u_t^{AL}$, $T = \alpha_2$ and $\gamma = \alpha_1$ under the restriction that the intercept is zero according to the theoretical model, with $E(\eta_t) = 0$ and variance $\Sigma_\eta = \sigma_{AL}^2$. The measure equation can be represented as the following state-space representation for $t = 1985, .., 2013$ in its standard notation in the literature, following Harvey (1994)

$$Y_t = Z\widetilde{\alpha}_t + \lambda D_t + \varepsilon_t \tag{51}$$

where the indicator variable is $Y_t = KS_t$, the explanatory variables are denoted as

$$D_t = \begin{bmatrix} KS_{t-1} \\ RGDP_{t-1} \\ RCONS_{t-1} \\ RUII_{t-1} \end{bmatrix},$$

according to equation (45) and the parameters are defined as $Z = [\mu_5]$ and

$$\lambda = \begin{bmatrix} \mu_1 & \mu_2 & -1 & \mu_4 \end{bmatrix}$$

where the stochastic variable $\varepsilon_t = \left[u_t^K\right]$ is such that $E(\varepsilon_t)$ is a constant and the variance is given by $\Sigma_\varepsilon = \left[\sigma_K^2\right]$. Note that in the estimation method of the Kalman filter we impose the following restrictions according to definitions in equation (46): $\alpha_0 = \mu_0 = 0$ and $\mu_3 = -1$. With these three restrictions and 29 observations for the period 1985 to 2013 we have only 8 parameters to estimate which gives us at least 21 degrees of freedom. We use the same initial value A_0 for AL_{1984} as in equation (47). The initial values for the parameters T, γ, Z and λ were obtained by peforming 1200 replications of uniform distributions on intervals that corresponded to the theoretical values expected. In Appendix B we describe the recursive optimizing algorithm that is used in combination with the Kalman filter to estimate the system of equations (50) and (51).

9 Empirical Results

In this section we report the main trajectories for the key unobservable AL_t for the two different methods used to estimate this stock for the Colombian economy from 1985 to 2013.

9.1 Calibrated Estimates

Figure 5 reports the calibrated estimates of equation (49). In absolute terms laundered assets follows a steep upward trajectory from 1985 to 2001 and then decreases steadily until 2013. The figure shows the decomposition of laundered assets into those that come from illicit incomes from drug trafficking and those that come from common crime, showing that the former surpasses the latter from 1994 onwards.

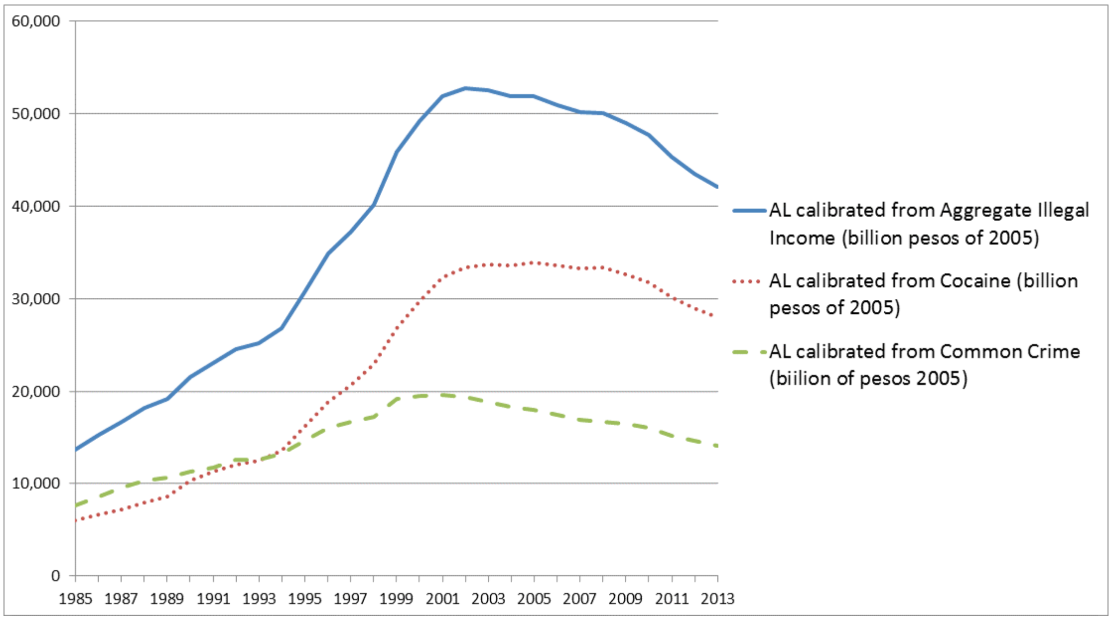

Figure 5

To get a sense of the magnitudes we divide the different calibrated estimates in Figure 6 by aggregate real capital stock and by real GDP of the Colombian economy, shown in Figures 6 and 7. Figure 6 reveals that the percentage participation of AL relative to the total capital stock of the Colombian economy rose from 2.6% in 1985 to 5.7% in 2002 and then started to decline steadily, reaching 2.9% in 2013. The portion of AL due to cocaine trafficking rose from 1.2% in 1985 to 3.6% in 2003 and fell to 1.9% by 2013. Finally the portion of AL due to common crimes stayed around 2% from 1985 to 2003 and then fell to 1% by 2013.

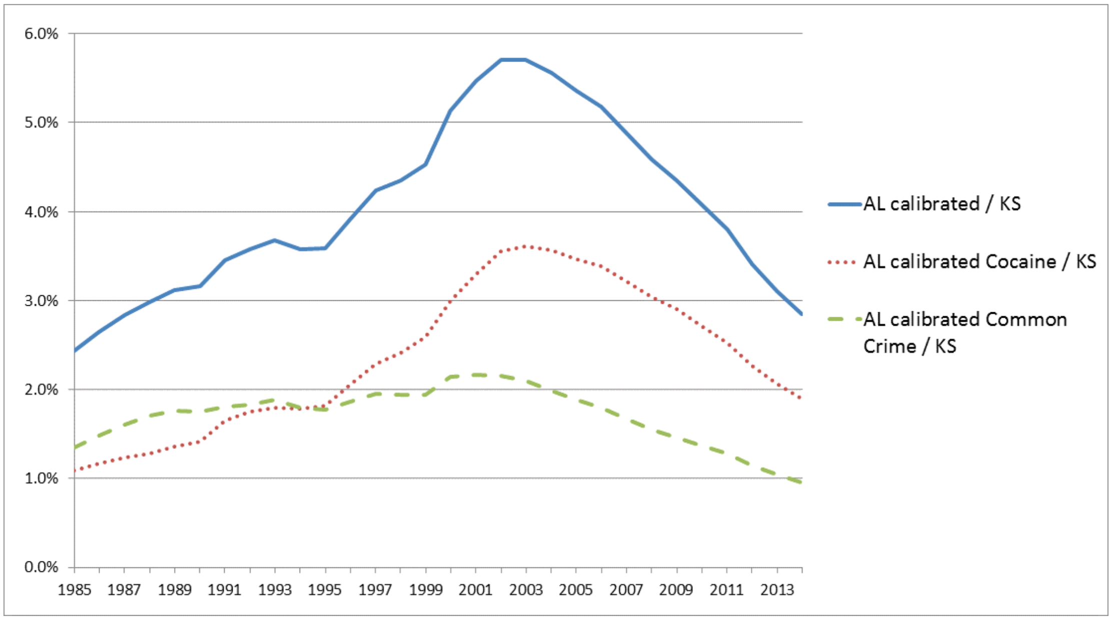

Figure 6

Figure 7 shows that total laundered assets from aggregate illicit income are much higher relative to real GDP than relative to the real capital stock shown in Figure 6, though it follows a similar trajectory. It starts at 7% of real GDP in 1985 and increases steadily to 18% in 2001, falling steadily afterwards to 8.6% in 2013. Assets laundered from illicit income from the two sources also generate similar trajectories, whereby after 1994 laundered assets that came from illicit cocaine income were greater than those from common crime activities.

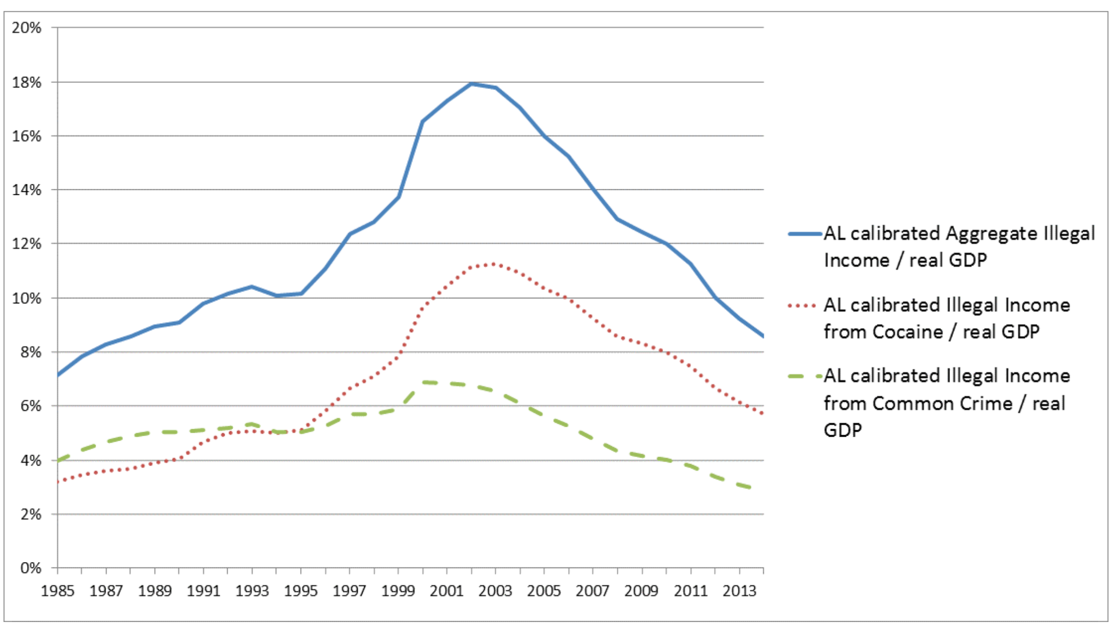

Figure 7

9.2 Kalman Filter Results

The Kalman filter estimates of the transition and measure equations in (44) and (45) with initial condition (47) are the following, based on 1,200 replications for $t = 1985, .., 2013$:

$$\begin{aligned} \widetilde{AL}_t^{Kalman} &= 0.1458 \times RUII_{t-1} + 0.9510 \times AL_{t-1} \\ \widehat{\sigma}^2_{AL} &= 37922288 \end{aligned} \tag{52}$$

$$\begin{aligned} \widetilde{KS}_t^{Kalman} &= 0.9199 \times KS_{t-1} + 1.083 \times RGDP_{t-1} - RCONS_{t-1} \\ &\quad + 0.0084 \times RUII_{t-1} + 1.1974 \times AL_t \\ \widehat{\sigma}^2_{K} &= 315458 \end{aligned} \tag{53}$$

where the optimizing procedure converges more than 90% of the time in the 1,200 replications. Recall that the only restrictions imposed in the estimation procedure was that the coefficient associated with $RCONS$ be fixed at -1 and zero values for the intercepts.

All of the estimated coefficients of the Kalman filter are positive, which is consistent with the structural deep parameter values in equation (46). In terms of magnitudes, the results are consistent with the theory since the values of the estimated coefficients are quite close to the expected theoretical values. For example, in equation (44) the parameter α_2 associated with AL_{t-1} should theoretically be close to a value just less than one, corresponding to values of the depreciation rate and the expected real interest rate in (46). The value obtained using the Kalman filter is 0.951. Furthermore, the structural model predicts that the parameter μ_5 associated to AL_t in the measure equation (45) should be equal to 1. The Kalman filter obtains the value 1.1974, also quite close to the expected theoretical value. Finally, the coefficient μ_2 in the measure equation (45) is expected to be less than one. The Kalman filter obtains the value 1.083 ,which is slightly larger than one. So overall the signs and magnitudes of the reduced form parameters are close to the expected theoretical structural counterparts.

The magnitude of the coefficients associated with $RUII$ in the transition equation and the measure equations are quite interesting. On the one hand, the coefficient in the transition equation is 0.1458, which is quite close to 0.17, the calibrated coefficient associated with this variable in equation (49). On the other hand, the coefficient associated with $RUII$ in the measure equation is rather low, 0.0084. This shows that $RUII$ is not a strong determinant of the capital stock of the Colombian economy yet does strongly affect the accumulation of laundered assets.

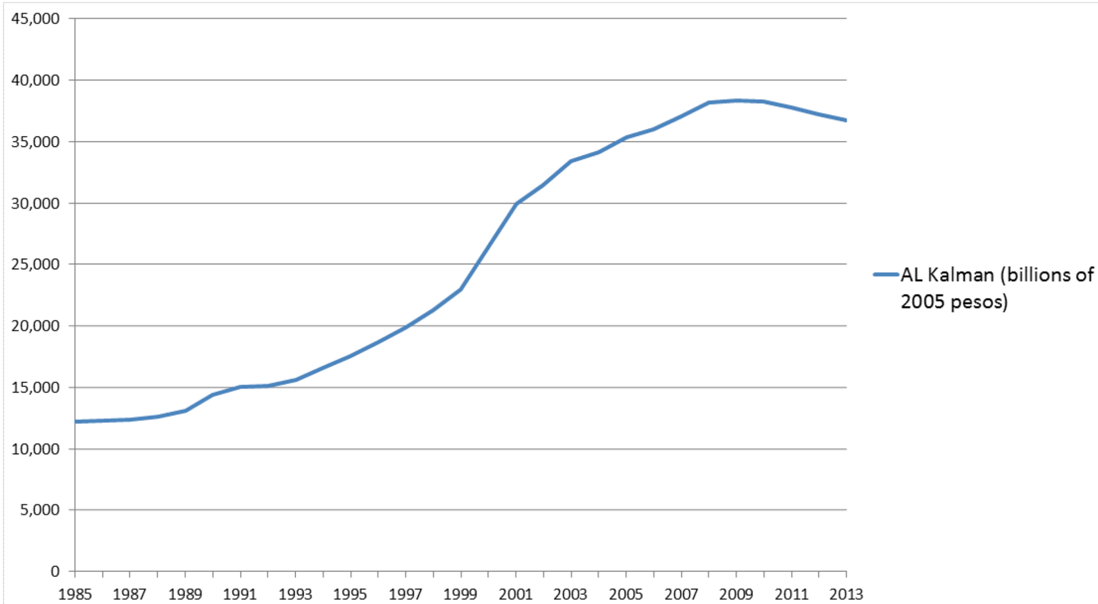

Figure 8

Figure 8 reports the trajectory of AL in billion of 2005 pesos according to the Kalman estimates obtained using equations (52) and (53), while Figure 9 reports the Kalman filter results as a percentage of real capital stock and real GDP for the Colombian economy.

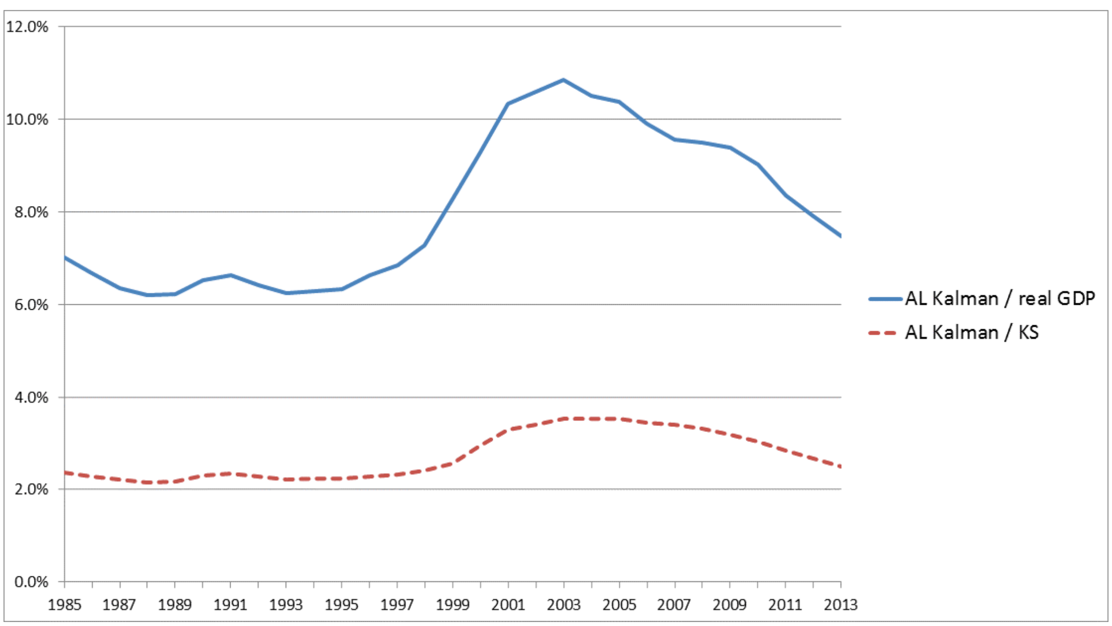

Figure 9

According to Figure 8 the volume of assets laundered in the Colombian economy rose from 1985 to 2007 and then started to decrease steadily up to 2013. Figure 9 shows that in relative terms the volume of assets laundered in the Colombian economy started at 2.4% of the capital stock of the economy in 1985, increased to 3.5% in 2003 and then decreased steadily up to 2013 when it was 2.5% of the capital stock. With respect to real GDP, the volume of assets laundered in the Colombian economy was 7.1% in 1985, decreased somewhat to 6.2% in 1989, rose to 10.8% in 2003 and then declined again steadily to 7.5% by 2013.

9.3 Comparing Estimates

The trajectories for AL estimated using the calibration method and the Kalman filter tell a similar story: starting in 1985 the volume of laundered assets in the Colombian economy rose for between 17 to 24 years and then started to decrease until the end of the period under study. In level terms, the calibrated trajectory is greater than the Kalman filter trajectory.

Since asset laundering by its very nature is an unobservable quantity and both methods produce similar predictions, we propose that an average of the two predicted estimates provides the best estimate of the true trajectory of laundered assets in Colombia. These averaged estimates in absolute and relative terms are reported in Figures 10 and 11, respectively.

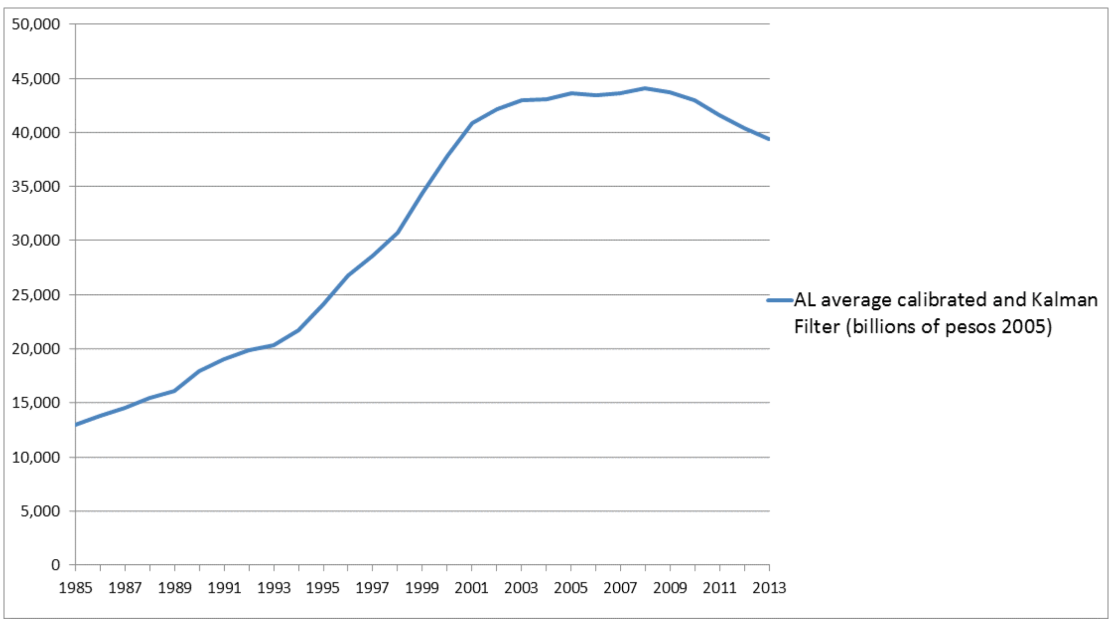

Figure 10

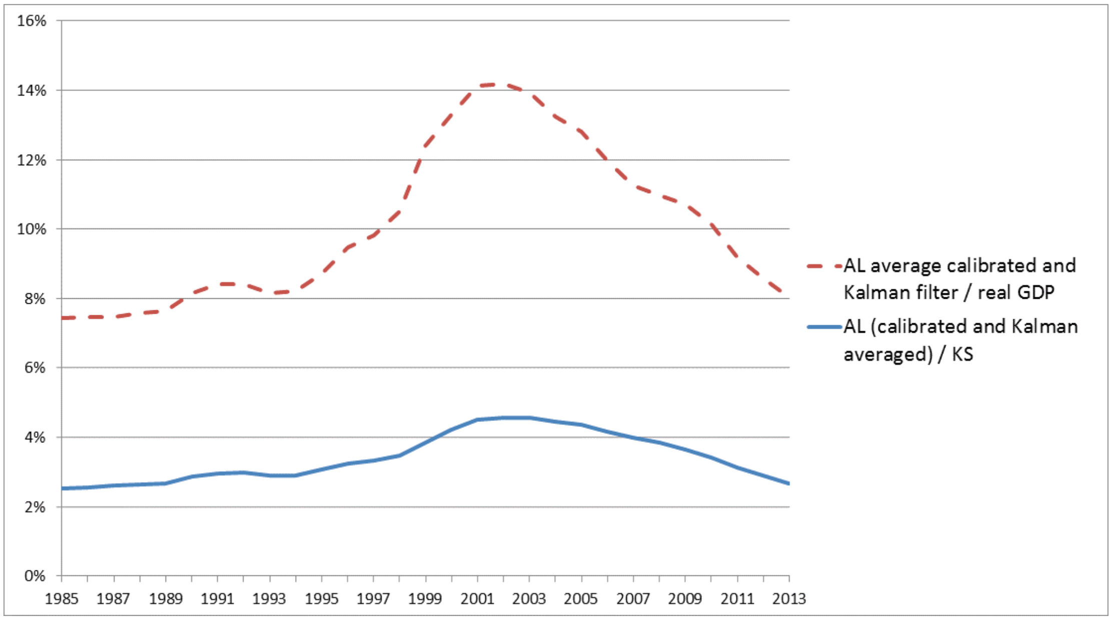

Figure 11

According to our best estimate, the volume of assets laundered in the Colombian economy in absolute terms rose steadily from 1985 to 2008 and then declined until 2013. In relative terms, the volume of assets laundered started in 1985 at 2.4% of the capital stock (7.1% of real GDP) then rose steadily until 2002, reaching a peak of 4.6% of the capital stock (14.2% of real GDP), and then decreased every year, falling to 2.7% of the capital stock (8% of real GDP) by 2013.

10 Conclusions

This paper contributes to the economic analysis of the determinants and effects of illicit activities and money laundering in two important ways. First, it presents a theoretical model of long-run growth that explicitly considers illicit workers, activities, and income, alongside a licit private sector and a functioning government. Second, it generates estimates of the size of illicit income and provides simulated and econometric estimates of asset laundering in the Colombian economy.

On the theoretical contribution, the paper presents an overlapping-generation growth model with both licit and illicit activities. The licit sector produces a consumption good using capital and labor through a standard neoclassical production function. The illicit sector is composed of two different activities. The first, produces an illicit good (e.g., illicit drugs) and adds value to the economy, while the second does not add value but consists of illicit appropriation of another person's income (e.g., robbery, kidnapping, and fraud). Earnings from illicit activities can be in part "laundered" into the economy by consumption of licit goods and investment in physical capital, the only asset in this economy. The accumulation of these licit assets in the economy from investments with an undetected illicit income behind them constitutes the process of asset laundering.

Using the model, we perform some comparative statics to trace the effects of exogenous changes in the "efficiency" of common crime, the price of cocaine, the efficiency of the government in fighting crime, and the productivity of licit firms. The summarized results are as follows:

a) "Efficiency" of common crime: an increase in the "efficiency" of common crime increases illicit income from this activity, without altering the allocation of illicit labor across the two types of illicit activity given that production of illicit drugs is assumed to be more profitable. The increase in the efficiency of common crimes generates an increase in expected illicit income, which increases the potential incentives for young people to enter the illicit sector. Moreover, it generates a redistribution of income from licit to illicit agents, has a negative effect on capital accumulation as it decreases the incentives to save and has a negative effect on the level of public goods provided by the government reducing overall social welfare.

b) Price of illicit drugs: an increase in the price of illicit drugs increases illicit income from drug trafficking. The increase in average illicit incomes increases the potential incentives to enter the illicit sector. It has an ambiguous impact on capital accumulation, positive through asset laundering but negative through the disincentive to save due to common crime. An increase in the price of illicit drugs generates a type of *Dutch disease* since it increases the relative price of the illicit good (cocaine) with respect to the price of the licit good which generates potential incentives that attract more labor resources to enter the inefficient illicit sector, thus deteriorating potential labor resources available to the efficient licit sector.

c) Government efficiency: an improvement in government efficiency increases the probability of detection and confiscation while allowing a decrease in the optimal tax rate. Under certain conditions, the level of public goods increases since the additional revenue from the confiscation of illicit income compensates for the decline in tax revenue. Moreover, social welfare improves since both disposable licit income and public goods provided increase in the economy. Furthermore, aggregate savings increase since the increase in licit disposable income more than compensates for the decrease in illicit income. Finally, the improvement of government efficiency does not alter the allocation of labor across the two illicit activities but decreases the potential incentives to enter the illicit sector for young workers.

d) Productivity of licit firms: an improvement in the productivity of licit firms generates an increase in licit wages, which turns into higher disposable licit income even though the optimal income tax rate rises. This creates higher incentives for licit rather than illicit activities. Moreover the provision of public goods increases as do aggregate savings and capital accumulation leading to an improvement in social welfare.

Overall, the effects of asset laundering on savings and social welfare are ambiguous. If asset laundering comes mainly from drug trafficking, then it may have a positive effect on aggregate savings and social welfare in the economy, while if it comes mainly form common crime then it has a clear negative effect on aggregate savings and social welfare. Hence, the common belief of some economists that asset laundering activities are always good for economic growth does not hold. On the other hand the opposite belief of some regulators that consider asset laundering activities to be always harmful for social welfare is also not always the case.

On the empirical contribution of the paper, we generate estimates of the size of illicit income for cocaine exports and common crime and provide simulated and econometric estimates of asset laundering in the Colombian economy. For this purpose, a macroeconometric specification is derived from the theoretical model described above.

An important contribution is the development of a data set for this purpose, in which the key components are estimates of illicit income from drug trafficking and common crime. Illicit income from drug trafficking is calculated as the volume of cocaine production (from UNODC) times the average of the point-of-export and US point-of-import price of the drug, minus the portion confiscated by the authorities. Income from common crime is computed as the value of reported property crime (from police sources), adjusting by the rates of official underreporting.

Estimates of the volume of laundered assets in the Colombian economy in the period 1985 to 2013 are developed using two empirical methodologies. These methodologies are based on a macroeconomic system of equations that is derived directly from the theoretical model. The system includes two equations. The first is a "transition" equation that specifies the dynamic nature of asset laundering in the economy in terms of its determinants and the second is a "measurement" equation that represents the mechanism through which asset laundering affects the capital stock of the economy. The methodologies used are: i) a calibration based on a parameterization of the transition equation and ii) a Kalman filter method based on both the transition and the measurement equation.

The estimates of illicit income, money laundering, and laundered assets are reasonable according to the history of Colombia and are consistent with the dominant role cocaine trafficking played in the 1990s in the country until the implementation of Plan Colombia in the early years of the new century. Figure 12 presents a summary of the results.

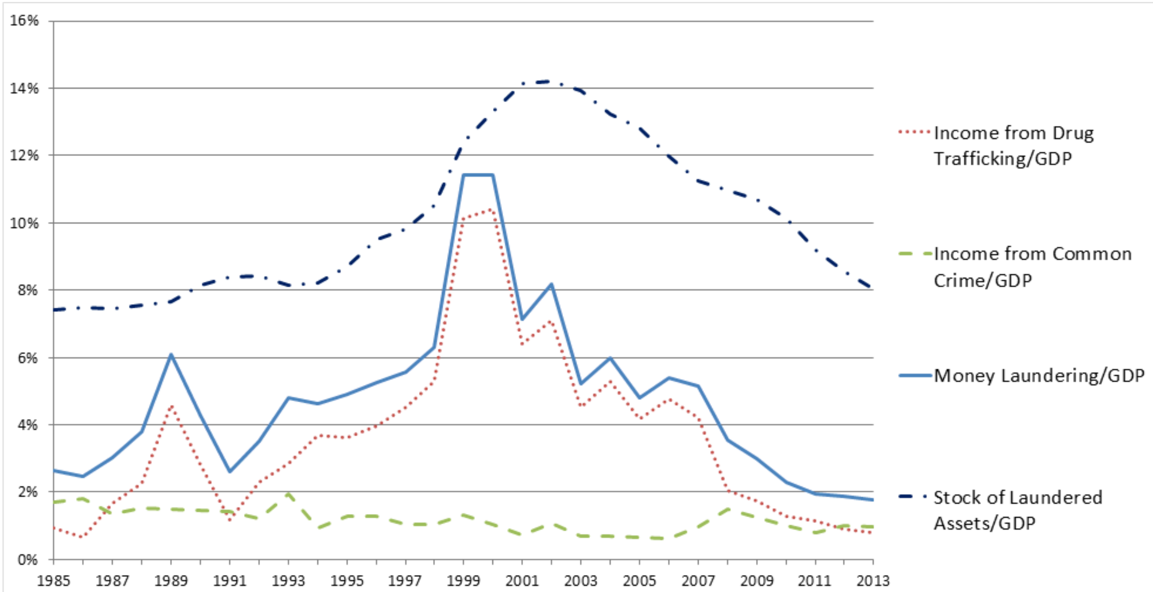

Figure 12

**"Income from Drug Trafficking" and "Income from Common Crime" refer to real undetected income. "Money Laundering" corresponds to the sum of income from these activities.*

Illicit incomes and money laundering increased drastically during the 1990s until 2000, reaching a peak of nearly 12% of GDP and then decreasing to less than 2% by 2013. Though asset laundering and money laundering are related, our conceptual framework proposes a differentiation. Specifically, while money laundering is the process by which undetected illicit income is integrated into the licit economy when it is used either for consumption of licit goods and services or invested in licit assets, asset laundering refers only to the latter. Moreover, unlike laundered assets which accumulate over time as a stock measure, we conceptualize money laundering as a flow variable. This helps explain the differences in the trajectories between money laundering and laundered assets. The stock of laundered assets depends not only on the inflow of (saved) illicit income but also on the net return of these assets. These returns respond, at least in part, to general economic conditions such as macroeconomic stability and business profitability, which were rather favorable in Colombia in the last couple of decades.

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Appendix

In the appendix we prove some of the propositions of the model.

Appendix A

Uniqueness of the Solution $(\tau_t^*, q_t^*) \in (0,1]^2$.

There is a unique solution $(\tau_t^*, q_t^*) \in (0,1]^2$ *to problem (18) if* $\rho \in \left(0, \frac{1}{2-\sigma}\right)$ *and for all* t

$$\frac{H_t w_t}{g_t} > \max\left\{\frac{\rho^2\gamma^2}{(1-\rho)^2}, \frac{2(1+2\beta)}{(1-\rho)(1-\phi)\eta}\right\}. \tag{54}$$

Proof: We know that the first order and second order conditions of problem (18) are necessary and sufficient for a solution (τ_t^*, q_t^*) under the assumption $\frac{H_t w_t}{g_t} > \frac{\rho^2\gamma^2}{(1-\rho)^2}$ which is satisfied by construction here. Hence a solution (τ_t^*, q_t^*) must satisfy the two first order conditions from equation (19) which we reproduce here as

$$\begin{aligned}\frac{(1+2\beta)(1-\rho)}{h_t^0 + (1-\tau_t^*)(1-\rho)w_t} &= \frac{H_t(1-\phi)\eta(1-\rho q_t^*)}{g(\tau_t^*, q_t^*)} \\ \frac{\gamma^2}{2}\left(\frac{d_t\widetilde{D}_t}{H_t} + (1-\tau_t^*)\rho(1-\phi)w_t\right) &= w_t q_t^*\end{aligned} \tag{55}$$

where

$$g(\tau_t^*, q_t^*) = \left((1-\phi)\tau_t^* + \left(\frac{q_t^*}{\gamma}\right)^2\right)H_t w_t. \tag{56}$$

From the second of the first order conditions we have that q_t^* is an explicitly decreasing function of τ_t^* which is given by

$$q_t^* \equiv q(\tau_t^*) = \min\left\{1, \frac{\gamma^2}{2}\left(\frac{d_t\widetilde{D}_t}{H_t w_t} + (1-\tau_t^*)\rho(1-\phi)\right)\right\}. \tag{57}$$

where the min operator restricts the optimal probability from being greater than one. This can be substituted into the first of the first order conditions of equation (55), generating a unique equation with only τ_t^* unknown

$$\frac{(1+2\beta)(1-\rho)}{h_t^0 + (1-\tau_t^*)(1-\rho)w_t} = \frac{H_t(1-\phi)\eta(1-\rho q(\tau_t^*))}{g(\tau_t^*, q(\tau_t^*))}. \tag{58}$$

Note that the left hand side is a strictly increasing function of τ_t^* while the right hand side could be an increasing or decreasing function of τ_t^*. To determine the sign of the derivative of the right hand side of equation (58) note first that the derivative of $g(\tau_t^*, q_t^*)$ with respect to τ_t^* taking into account $q(\tau_t^*)$ yields

$$\frac{dg\left(\tau_t^*, q\left(\tau_t^*\right)\right)}{d\tau_t^*} = g_\tau + g_q \frac{dq_t^*}{d\tau_t^*} \quad (59)$$
$$= H_t\left(1-\phi\right) w_t \left[1-\rho q\left(\tau_t^*\right)\right]$$

which is strictly positive. Moreover, the right hand side of equation (58) is a strictly decreasing function of τ_t^* if its derivative is negative which yields the condition

$$\frac{-H_t\left(1-\phi\right)\eta\rho q\left(\tau_t^*\right)\frac{dq_t^*}{d\tau_t^*}}{g\left(\tau_t^*, q\left(\tau_t^*\right)\right)} - \frac{H_t\left(1-\phi\right)\eta\left(1-\rho q\left(\tau_t^*\right)\right)\left[g_\tau + g_q\frac{dq_t^*}{d\tau_t^*}\right]}{g\left(\tau_t^*, q\left(\tau_t^*\right)\right)^2} < 0.$$

We can substitute for $\frac{dq_t^*}{d\tau_t^*} = -\frac{\gamma^2}{2}\rho\left(1-\phi\right) < 0$ and $g_\tau + g_q\frac{dq_t^*}{d\tau_t^*}$ from equation (59) in this last inequality to get

$$\frac{\rho^2\gamma^2}{\left(1-\rho\right)^2} < \frac{H_t w_t}{g\left(\tau_t^*, q\left(\tau_t^*\right)\right)}$$

which is satisfied under restriction (54). Hence, we conclude that the right hand side of equation (58) is a strictly decreasing function of τ_t^* under the assumptions we maintain.

Finally, if the left hand side of (58) is greater than the left hand side at $\tau_t^* = 0$ then we can conclude that there is a unique $\tau_t^* \in (0,1]$ that satisfies equation (58). The corresponding inequality that must be satisfied for this to be the case is

$$\frac{\left(1+2\beta\right)\left(1-\rho\right)}{\left[h_t^0 + \left(1-\rho\right)\right]\left(1-\rho q\left(0\right)\right)\left(1-\phi\right)\eta} < \frac{H_t w_t}{g\left(0, q\left(0\right)\right)}. \quad (60)$$

Moreover note that

$$\frac{\left(1+2\beta\right)\left(1-\rho\right)}{\left[h_t^0 + 1-\rho\right]\left(1-\rho q\left(0\right)\right)\left(1-\phi\right)\eta} < \frac{\left(1+2\beta\right)}{\left(1-\rho\right)\left(1-\phi\right)\eta}$$

since $q\left(0\right) \in [0,1]$ and $h_t^0 > 0$ for all t, which implies that inequality (60) is satisfied under restriction (54). Hence, we can conclude that there is a unique $\tau_t^* \in (0,1]$ that satisfies equation (58) and from equation (57) there must be a unique corresponding $q_t^* = q\left(\tau_t^*\right) \in (0,1]$.

Short Run Equilibrium

We show the existence of a short run equilibrium with illicit activities defined as a situation in which every agent chooses their corresponding control variables optimally (except the central bank which accommodates its money supply to equal aggregate money demand) such that workers self-select into licit and illicit activities when young, illicit labor markets are segmented while licit markets clear and prices and quantities are either determined outside the economy, say by world markets, or are determined as a function of capital per licit worker for each period.

From the analysis of the government we know that under certain additional assumptions there is a unique optimal solution to its problem represented by the pair $\left(\tau_t^*, q_t^*\right) \in (0,1]^2$ as shown earlier in the appendix.Importantly, the first order conditions in equation (19) evaluated at $\left(\tau_t^*, q_t^*; \Delta_t\right)$ are satisfied as an identity, where g_t^* corresponds to equation (21) and $\Delta_t = \left(w_t, h_t^0; \Omega_t\right)$ corresponds to variables the government takes as given, endogenous variables $\left(w_t, h_t^0\right)$ and exogenous variables $\Omega_t \equiv \left(H_t, d_t\widetilde{D}_t; \rho, \gamma, \eta, \beta, \phi, \sigma, \delta\right)$. By the Implicit Function Theorem[19] we have that there exist unique continuously differentiable functions $\tau_t^* = \Upsilon\left(w_t, h_t^0; \Omega_t\right)$ and $q_t^* = Q\left(w_t, h_t^0; \Omega_t\right)$ which satisfy the first order conditions (19) for values $\Delta_t^{'}$ in a neighborhood of Δ_t.[20] From the second of the first order conditions in equation (19) we know that the optimal confiscation probability is a one to one function with respect to the optimal tax rate which implies that without loss of generality we can study only the determination of the optimal probability q_t^* as a function of capital per licit worker. From equation (5) and the definition of wealth W_{t-1}^0 we have that $h_t^0 = \frac{\beta\left(h_{t-1}^0 + (1-\tau_{t-1})(1-\rho)w_{t-1}\right)(1+i_t)}{1+2\beta}$ where i_t according to equation (9) depends on k_t and θ_t. Hence, we have that $h_t^0 = h^0\left(i\left(k_t, \theta_t\right); \Gamma_{t-1}\right)$ where $\Gamma_{t-1} \equiv \left(h_{t-1}^0, \tau_{t-1}, w_{t-1}; \rho, \beta\right)$ are predetermined variables in period t and constant parameters. Moreover from equation (9) we have that w_t is a function of k_t and θ_t while from the technology $q_t = \gamma\sqrt{\theta_t(1-\phi)}$ we have that $\theta_t = \frac{q_t^2}{\gamma^2(1-\phi)}$. All these can be replaced in Q which yields the following fixed point equation in q_t^*

$$q_t^* = Q\left(w\left(k_t, \frac{\left(q_t^*\right)^2}{\gamma^2(1-\phi)}\right), h^0\left(i\left(k_t, \frac{\left(q_t^*\right)^2}{\gamma^2(1-\phi)}\right); \Gamma_{t-1}\right); \Omega_t, \Gamma_{t-1}\right). \tag{61}$$

Given that the function Q is defined with respect to q_t^* from $[0,1]$ into $[0,1]$ and is a continuous function for all $q_t^* \in [0,1]$ then by Brouwer's fixed point theorem[21] we have that there exists $q_t^{**} \in [0,1]$ such that the fixed point equation (61) is satisfied for all k_t and Ω_t, Γ_{t-1}. This implies that there exists an implicit function $q_t^{**} = q\left(k_t; \Omega_t, \Gamma_{t-1}\right)$ which is a function of k_t.

Given this result the following variables are also functions of capital per licit worker k_t, constant parameters and predetermined variables, namely, $\theta_t^* = \frac{\left(q_t^{**}\right)^2}{\gamma^2(1-\phi)} \in [0,1)$, $h_t^0 = \frac{\beta\left(h_{t-1}^0 + (1-\tau_{t-1})(1-\rho)w_{t-1}\right)(1+i_t^*)}{1+2\beta}$, $i_t^* = \frac{\alpha B\left(1-\theta_t^*\right)^{1-\alpha}}{k_t^{1-\alpha}} > 0$, $w_t^* = \frac{(1-\alpha)Bk_t^\alpha}{\left(1-\theta_t^*\right)^\alpha} > 0$ and $P_t^* = \theta_t^* L_t^0 > 0$. Moreover, from equation (19) we get an optimal licit income tax rate

$$\tau_t^* = \max\left\{0, 1 - \frac{2}{\rho(1-\phi)}\left[\frac{q_t^{**}}{\gamma^2} - \frac{d_t\widetilde{D}_t}{H_t w_t^*}\right]\right\} \tag{62}$$

as a function that only depends on k_t and predetermined or exogenous variables contained in Ω_t, Γ_{t-1}.

On the other hand, in terms of prices, recall that the illicit price of drugs d_t is determined by world markets while the price of the licit good is normalized to one. Illicit drug incomes are determined by

[19] Implicit Function Theorem: Let $f: D \to \mathbb{R}^n$ be a continously differentiable function, where D is an open set in $\mathbb{R}^n \times \mathbb{R}^m$. Let $\left(x^0, z^0\right) \in D$ for which $f\left(x^0, z^0\right) = 0$. If the Jacobian matrix evaluated at $\left(x^0, z^0\right)$ i.e. $f_x\left(x^0, z^0\right)$ is non singular, then there exists a unique continuously differentiable function ϕ such that $x^0 = \phi\left(z^0\right)$ and $f\left(\phi\left(z^0\right), z^0\right) = 0$ for all z that are in a neighborhood of z^0.

[20] The Jacobian matrix of the first order conditions with respect to (τ_t, q_t), evaluated at the solution (τ_t^*, q_t^*), is a non singular matrix given that the Hessian matrix evaluated at (τ_t^*, q_t^*) is a negative definite matrix under $\frac{H_t w_t}{g_t} \geq \frac{\rho^2\gamma^2}{(1-\rho)^2}$ for all t.

[21] Brouwer Fixed Point Theorem: Let S be a non empty, compact and convex set in $\mathbb{R}^n$. If $\varphi: S \to S$ is a continuous function then $\varphi(\cdot)$ has at least a fixed point i.e. $\varphi(x^*) = x^*$ for $x^* \in S$.

the foreign demand for the illicit good $\widetilde{D}_t$ which implies that $Y_t^D = \frac{d_t A^{\frac{1}{\lambda}}}{\widetilde{D}_t^{\frac{1-\lambda}{\lambda}}}$ is determined independently of k_t. Nonetheless, illicit income from common crime is determined as a function of k_t since $Y_t^{R*} = \frac{(1-\tau_t^*)\rho(1-\phi)w_t^*}{\phi(1-\varepsilon_t)}$ depends on τ_t^* and w_t^* and given that $\varepsilon_t = \frac{l_t^{1,D}}{H_t\phi} = \frac{\left(\frac{\widetilde{D}_t}{A}\right)^{\frac{1}{\lambda}}}{H_t\phi}$ is determined by $\widetilde{D}_t$ and H_t, which are determined independently of k_t. Hence, we have that the expected illicit income in period t given by $Y_t^{e*} = (1 - q_t^{**}\sigma)\left(\varepsilon_t Y_t^D + (1-\varepsilon_t) Y_t^{R*}\right)$ is a function of k_t and (Ω_t, Γ_{t-1}).

The following proposition proved below establishes the sufficient conditions under which young individuals self select into illicit and licit activities given that $\bar{\zeta} > \widehat{\zeta}_t > \underline{\zeta}$ and the condition to sustain market segmentation in illicit labor markets $Y_t^D > Y_t^R$ for all t.

Lemma: *If threshold value $\bar{\zeta}$ and the price of the illicit good d_t are sufficiently large, then the population of young individuals is partitioned between the licit and illicit sectors of the economy i.e. $L_t^1 > 0$, $L_t^0 > 0$ and the condition $Y_t^D > Y_t^R$ is satisfied which supports the segmentation of illicit labor markets.*

Proof: The condition $Y_t^D > Y_t^R$ is satisfied for all t if $d_t > \frac{(1-\tau_t^*)\rho(1-\phi)w_t^*\widetilde{D}_t^{\frac{1-\lambda}{\lambda}}}{A^{\frac{1}{\lambda}}\phi - \frac{(\widetilde{D}_t)^{\frac{1}{\lambda}}}{H_t}} \equiv \widetilde{d}(k_t; \Omega_t, \Gamma_{t-1})$ which implies that illicit labor markets are segmented. From equation (5) and the definition of wealth W_{t-1}^1 we have that $h_t^1 = \frac{\beta\left(h_{t-1}^1 + Y_t^{e*}\right)(1+i_t^*)}{1+2\beta}$ which is a function of k_t through Y_t^{e*} and i_t^* i.e. $h_t^1 = h^1(k_t; \Omega_t, \Gamma_{t-1})$. On the other hand, young workers self-select between both licit and illicit sectors if the critical type $\widehat{\zeta}_t$ satisfies equation (8) i.e. $\bar{\zeta} > \left[\frac{h_t^1(k_t;\Omega_t,\Gamma_{t-1}) + (1-q_t^{**}\sigma)\left(\varepsilon_t Y_t^D + (1-\varepsilon_t)Y_t^R\right)}{h^0(i(k_t,\theta_t^*);\Gamma_{t-1}) + (1-\tau_t^*)(1-\rho)w_t^*}\right]^{1+2\beta} > \underline{\zeta}$. This is satisfied if the following inequality is satisfied

$$
\begin{aligned}
d_t &> \frac{H_t\phi^2}{\widetilde{D}_t(1-q_t^{**}\sigma)}\left[\begin{array}{c} \underline{\zeta}^{\frac{1}{1+2\beta}} h^0(k_t; \Omega_t, \Gamma_{t-1}) \\ -h^1(k_t; \Omega_t, \Gamma_{t-1}) \end{array}\right] + \left[\frac{\phi}{1-q_t^{**}\sigma} - 1\right]\frac{H_t\phi}{\widetilde{D}_t}(1-\tau_t^*)(1-\rho)w_t^* \\
&\equiv \widehat{d}(k_t; \Omega_t, \Gamma_{t-1}).
\end{aligned}
$$

and if $\bar{\zeta}$ is sufficiently large, say $\bar{\zeta} \to \infty$. Under these conditions the young population of workers self selects into licit and illicit activities such that $L_t^0 = \frac{H_t}{2}(1-\phi) > 0$ and $L_t^1 = \frac{H_t}{2}\phi > 0$. Hence, if we define $\overline{d}(k_t) \equiv \max\left\{\widetilde{d}(k_t; \Omega_t, \Gamma_{t-1}), \widehat{d}(k_t; \Omega_t, \Gamma_{t-1})\right\}$ then both conditions are satisfied if d_t is sufficiently large in the sense that $d_t > \overline{d}(k_t)$ for all t.

Finally, the aggregate demand for the licit good is a function of k_t and k_{t-1} since from equation (5) we obtain

$$
\begin{aligned}
C_t^* &= H_t\phi\left[C^{1,y}(k_t, \Psi_t) + C^{1,a}(k_t, k_{t-1}; \Psi_t, \Psi_{t-1})\right] \\
&\quad + H_t(1-\phi)\left[C^{0,y}(k_t, \Psi_t) + C^{0,a}(k_t, k_{t-1}; \Psi_t, \Psi_{t-1})\right]
\end{aligned}
$$

where $\Psi_t \equiv (\Omega_t, \Gamma_{t-1})$ and

$$\begin{aligned}
C^{1,y}(k_t, \Psi_t) &= \frac{h^1(k_t; \Omega_t, \Gamma_{t-1}) + Y_t^{e*}}{1+2\beta}, \\
C^{1,a}(k_t, k_{t-1}; \Psi_t, \Psi_{t-1}) &= \frac{\beta\left(h^1(k_{t-1}; \Omega_{t-1}, \Gamma_{t-2}) + Y_{t-1}^{e*}\right)(1+i_t^*)}{1+2\beta} \\
C^{0,y}(k_t, \Psi_t) &= \frac{h_t^0(k_t) + (1-\tau_t^*)(1-\rho)w_t^*.}{1+2\beta}, \\
C^{0,a}(k_t, k_{t-1}; \Psi_t, \Psi_{t-1}) &= \frac{\beta\left(h_{t-1}^0(k_{t-1}) + \left(1-\tau_{t-1}^*\right)(1-\rho)w_{t-1}^*\right)(1+i_t^*)}{1+2\beta}
\end{aligned}$$

In a short run equilibrium with illicit activities we must have that aggregate demand for the licit good is be satisfied which simply means that $C_t^* \leq X_t^*$ i.e. constant returns to scale technology produces at least what the economy demands $X_t^* = \frac{BN_t^* k_t^\alpha}{(1-\theta_t^*)^\alpha}$ where $N_t^* = (1-\theta_t^*)L_t^0$. This is satisfied if B is sufficiently large, something we assume such that the licit market clears in every period t. Finally, the monetary supply of money must equal aggregate demand C_t^* according to the cash in advance constraint i.e. $M_t^s = C_t^*$ which is satisfied since the central bank accommodates its monetary supply to equate aggregate money demand by assumption.

Steady State

This section studies the existence of a long run equilibrium or steady state of the model defined as a short run equilibrium with illicit activities such that capital per licit worker is constant and positive over time i.e. $k_t = \overline{k} > 0$ for all t. The following proposition establishes the existence of a steady state.

Proposition 1 *There is at least a steady state $\overline{k} > 0$ that satisfies equation (27).*

Proof: Equation (27) can be written as

$$\begin{aligned}
\frac{(1+n)}{s(\beta)}k_{t+1} &= \left(1-\delta+\frac{\alpha B\left(1-\theta(k_t, \mathbf{k}_{t-1})\right)^{1-\alpha}}{k_t^{1-\alpha}}\right)k_t \\
&+\frac{(1-\tau(k_t, \mathbf{k}_{t-1}))(1-\rho)(1-\alpha)Bk_t^\alpha}{(1-\theta(k_t, \mathbf{k}_{t-1}))^\alpha} \\
&+\left(\frac{\phi(1-\sigma q(k_t, \mathbf{k}_{t-1}))}{1-\phi}\right)\left(\begin{array}{c}\varepsilon_t Y_t^D + \\ \frac{(1-\tau(k_t, \mathbf{k}_{t-1}))\rho(1-\phi)(1-\alpha)Bk_t^\alpha}{(1-\theta(k_t, \mathbf{k}_{t-1}))^\alpha}\end{array}\right)
\end{aligned} \tag{63}$$

where $\tau(k_t, \mathbf{k}_{t-1}) = \max\left\{0, 1-\frac{2}{\rho(1-\phi)}\left[\frac{q(k_t, \mathbf{k}_{t-1})}{\gamma^2} - \frac{d_t \widetilde{D}_t (1-\theta(k_t, \mathbf{k}_{t-1}))^\alpha}{H_t(1-\alpha)Bk_t^\alpha}\right]\right\}$. A steady state must satisfy

$k_t = \overline{k}$ for all t. Define the following continuous function in $\left(\overline{k}\right)$

$$\begin{aligned} \lambda\left(\overline{k}\right) &= \left[\frac{(1+n)}{s(\beta)} - \left(1-\delta+\frac{\alpha B\left(1-\theta\left(\overline{k},\overline{\mathbf{k}}\right)\right)^{1-\alpha}}{\overline{k}_t^{1-\alpha}}\right)\right]\overline{k} \\ &\quad - \left(1-\rho+\phi\rho\left(1-\sigma q\left(\overline{k},\overline{\mathbf{k}}\right)\right)\right)\frac{\left(1-\tau\left(\overline{k},\overline{\mathbf{k}}\right)\right)(1-\alpha)B}{\left(1-\theta\left(\overline{k},\overline{\mathbf{k}}\right)\right)^{\alpha}}\overline{k}^{\alpha} \\ &\quad - \frac{\phi\left(1-\sigma q\left(\overline{k},\overline{\mathbf{k}}\right)\right)\varepsilon_t Y_t^D}{1-\phi} \end{aligned} \tag{64}$$

given that all functions inside $\lambda\left(\overline{k}\right)$ are continuous functions in $\overline{k}$. We are looking for a $\overline{k}^* > 0$ such that $\lambda\left(\overline{k}^*\right) = 0$. First note that $\lambda(0) < 0$ since $\varepsilon_t Y_t^D > 0$ and q_t^{**}, θ_t and τ_t are all variables bounded in the interval $[0,1]$. Moreover, $\lim_{\overline{k}\to\infty}\lambda\left(\overline{k}\right) = +\infty$. To see this note that the first term in equation (64) goes to plus infinity since $\frac{(1+n)}{s(\beta)} - (1-\delta) > 0$ which holds given that $\frac{(1+n)}{s(\beta)} > 1 > 1-\delta$ for all $s(\beta) \in (0,1)$. On the other hand, the second term can tend to zero if $\lim_{\overline{k}\to\infty}\tau\left(\overline{k}\right) = 1$ or to minus infinity if $\lim_{\overline{k}\to\infty}\tau\left(\overline{k}\right) < 1$. Finally, the third term is bounded since $\lim_{\overline{k}\to\infty} q\left(\overline{k}\right) \in [0,1]$ by construction. Nonetheless, the first term dominates the second term since it grows faster to $+\infty$ than the second term tends to $-\infty$ given that $\alpha \in (0,1)$. Hence by continuity of function $\lambda(\cdot)$ there must exist $\overline{k}^* > 0$ such that $\lambda\left(\overline{k}^*\right) = 0$.

Comparative Analysis

a) Efficiency of Common Crimes

We study the short run comparative static effect of an increase in the efficiency of common crimes taken as given the optimal income tax. Consider the economy in a short run equilibrium with illicit activities and suppose that the efficiency of common crimes increases ($\Delta\rho > 0$) which could be due to an enhancement of criminals' capacity and productivity when apprehending income from licit workers. This increases illicit income from common crime since $Y_t^R = \frac{(1-\tau_t^*)\rho(1-\phi)w_t^*}{\phi(1-\varepsilon_t)}$ is increasing in ρ. According to the second first order condition in equation (19) we have for a given level of $\tau_t^* \in (0,1)$ and w_t^* that the optimal probability increases since $\frac{\partial q_t^*}{\partial \rho} = \frac{\gamma^2}{2}(1-\tau_t^*)(1-\phi) > 0$. Moreover $\frac{\partial Y_t^e}{\partial \rho} = (1-q_t^*)(1-\varepsilon_t)\frac{\partial Y_t^R}{\partial \rho} - \frac{Y_t^e}{1-q_t^*}\frac{\partial q_t^*}{\partial \rho}$ is ambiguous since the first term is positive while the second is negative. Hence, an increase in the efficiency of common crime does not imply necessarily an increase in the incentives to enter the illegal sector since the probability of detection and apprehension responds positively with the increase in ρ. Nonetheless, the more likely scenario is the net effect to be positive since the increase in q_t^* should be dominated by the increase in illicit incomes for low values of q_t^*. On the other hand, licit disposable income in a short run equilibrium with illicit activities defined as $Y_t^l = (1-\tau_t^*)(1-\rho)w_t^*$ is affected negatively with an increase in ρ for a given level of τ_t^* and w_t^*. In terms of the provision of public goods, an increase in ρ according to equation (21), for a given level of τ_t^* and w_t^*, generates in the short run an increase in public goods provision since

$g^*_{t,t+1} = \frac{2H_t w_t q^*_t}{\gamma^2} > 0$.Now in terms of aggregate savings in period t, an increase in the efficiency of common crimes ρ generates

$$\frac{\partial a_t}{\partial \rho} = H_t s(\beta)\left[\frac{\partial Y^l_t}{\partial \rho} - \phi\left(\frac{\partial Y^l_t}{\partial \rho} - \frac{\partial Y^e_t}{\partial \rho}\right)\right]$$

where we have used $a^0_t = s(\beta)\left(h^0_t + Y^l_t\right)$ and $a^1_t = s(\beta)\left(h^1_t + Y^e_t\right)$ from equation (6) and the definitions of licit and illicit wealth. Since an increase in ρ generates a redistribution of income then the term $\frac{\partial Y^l_t}{\partial \rho} - \frac{\partial Y^e_t}{\partial \rho} = 0$ and the net effect on savings is negative. This implies, according to equation (25), that capital accumulation in the next period is less than or equal to what it would have been in the absence of an increase in ρ. We thus conclude that an increase in the efficiency of common crime has a weakly adverse impact on the capital accumulation of an economy.

Social welfare only responds positively to the utility of licit households. Therefore, an increase in the efficiency of common crimes ρ has more likely a negative impact on social welfare. To see this, substitute the indirect utility of licit households from equation (7) evaluated at $I = 0$ into equation (13) and differentiate with respect to ρ in order to get

$$\frac{\partial U^0_{t,t+1}}{\partial \rho} = \frac{(1+2\beta)}{W^0_t}\frac{\partial Y^l_t}{\partial \rho} + \frac{\eta}{g^*_t}\frac{\partial g^*_{t,t+1}}{\partial \rho}$$

which is more likely negative given that $\frac{\partial Y^l_t}{\partial \rho} < 0$ should dominate $\frac{\partial g^*_{t,t+1}}{\partial \rho} > 0$ for η relatively small. Hence, we conclude that social welfare in an economy is reduced when there is an increase in the efficiency of common crime.

b) International Price of Drugs

We study the short run comparative static effect of an increase in ghe international price of drugs when τ^*_t is taken as given in a short run equilibrium. An increase in the international price of drugs ($\Delta d_t > 0$) increases the revenue of the activity and therefore the illegal income for workers of the sector $\frac{\partial Y^D_t}{\partial d_t} = \frac{A^{\frac{1}{\lambda}}}{\widetilde{D}_t^{\frac{1-\lambda}{\lambda}}} > 0$ while Y^R_t is unaffected in the short run. Moreover, the increase in d_t does not reallocate illegal labor since the foreign demand is given. From the second first order condition for the government in equation (19), for a given income tax level, the probability of detection and apprehension increases with d_t since $\frac{\partial q^*_t}{\partial d_t} = \frac{\gamma^2 \widetilde{D}_t}{2H_t w^*_t} > 0$. Hence the expected (average) illicit income for a young individual when entering the illicit sector is ambiguous in principle since

$$\frac{\partial Y^e_t}{\partial d_t} = (1-q^*_t)\,\varepsilon_t \frac{\partial Y^D_t}{\partial d_t} - \frac{\partial q^*_t}{\partial d_t}\frac{Y^e_t}{1-q^*_t}$$

is the difference between two positive values. Nonetheless the more likely case is the first term to dominate the second for low levels of q^*_t. On the other hand, licit disposable income in a short run equilibrium with illicit activities defined as $Y^l_t = (1-\tau^*_t)(1-\rho)\,w^*_t$ is unaffected with an increase in d_t for a given level of τ^*_t and w^*_t. In terms of the provision of public goods, an increase in d_t according to equation (21), for a given level of τ_t^* and w_t^*, in the short run generates $\frac{\partial g_{t,t+1}^*}{\partial d_t} = \frac{2q_t^*}{\gamma^2}\frac{\partial q_t^*}{\partial d_t} H_t w_t^*$ which is positive. Now in terms of aggregate savings defined as $a_t \equiv \frac{H_t}{2}(1-\phi) a_t^0 + \frac{H_t}{2}\phi a_t^1$ in period t an increase in the international price of drugs generates $\frac{\partial a_t}{\partial d} = H_t s(\beta)\phi \frac{\partial Y_t^e}{\partial d_t}$ where we have used $a_t^0 = s(\beta)\left(h_t^0 + Y_t^l\right)$ and $a_t^1 = s(\beta)\left(h_t^1 + Y_t^e\right)$ from equation (6). The sign is positive since $\frac{\partial Y_t^e}{\partial d_t} > 0$ is the more likely case to arise. Furthermore since social welfare only responds positively to the utility of legal households then an increase in d_t has a positive effect on social welfare since it increases public goods provision while not affecting licit incomes directly. To see this replace the indirect utility of legal households from equation (7) evaluated in $I = 0$ in equation (13) and differentiate with respect d_t in order to get

$$\frac{\partial U_{t,t+1}^0}{\partial d_t} = \frac{\eta}{g_{t,t+1}^*}\frac{\partial g_{t,t+1}^*}{\partial d_t}$$

which is positive since $\frac{\partial g_{t+1}^*}{\partial d_t} > 0$ and $g_{t,t+1}^* > 0$.

c) Efficiency of Government

We study the short run comparative static effect of increasing government efficiency when τ_t^* is taken as given in a short run equilibrium. An increase in the efficiency of the government is reflected in $\Delta\gamma > 0$, which increases the probability of detection and confiscation according to the second first order condition in equation (19), for a given level of the income tax rate, which yields

$$\frac{\partial q_t^*}{\partial \gamma} = \gamma\left(\frac{d_t \widetilde{D}_t}{H_t w_t^*} + (1-\tau_t^*)\rho(1-\phi)\right) > 0.$$

This generates in turn a decrease in the delinquent incentives to enter the illegal sector since

$$\frac{\partial Y_t^e}{\partial \gamma} = -\frac{\partial q_t^*}{\partial \gamma}\left(\frac{Y_t^e}{1-q_t^*}\right) < 0.$$

Now in terms of the provision of public goods we have according to equation (21) an increase in public goods provided since $\frac{\partial g_{t,t+1}^*}{\partial \gamma} = \left(\frac{2q_t^*}{\gamma^2}\frac{\partial q_t^*}{\partial \gamma}\right) H_t w_t^* > 0$. Moreover licit disposable income is unaffected by this change for a given income tax level since it is defined as $Y_t^l = (1-\tau_t^*)(1-\rho) w_t^*$. Now in terms of aggregate savings we have $\frac{\partial a_t}{\partial \gamma} = H_t s(\beta)\phi\left(\frac{\partial Y_t^e}{\partial \gamma}\right) < 0$ since $\frac{\partial Y_t^e}{\partial \gamma} < 0$. Finally in terms of social welfare we have that $\frac{\partial U_{t,t+1}^0}{\partial \gamma} = \frac{\eta}{g_{t,t+1}^*}\frac{\partial g_{t,t+1}^*}{\partial \gamma} > 0$ since all terms are positive.

d) Efficiency of Licit Firms

We study the short run comparative static effect of an increase in the efficiency of the private sector for a given level of the income tax rate. Consider an economy in a short run equilibrium with illicit activities and assume that the efficiency of the technology for producing the licit good increases ($\Delta B > 0$). From the first order conditions for private licit firms we see that an increase in B increases the wage since $\frac{\partial w_t^*}{\partial B} = (1-\alpha) K_t^\alpha N_t^{-\alpha} > 0$. From the second first order condition in equation (19) we get that the the optimal probability q_t^* decreases with w_t^* for a given level of τ_t^* since $\frac{\partial q_t^*}{\partial B} = -\frac{\gamma^2}{2}\frac{d_t \widetilde{D}_t}{H_t (w_t^*)^2}\frac{\partial w_t^*}{\partial B} < 0$. The illegal income Y_t^D from drug trafficking would be unaffected while the illegal income of common crime would increase since $\frac{\partial Y_t^R}{\partial B} = \frac{(1-\tau_t^*)\rho(1-\phi)}{\phi(1-\varepsilon_t)}\left(\frac{\partial w_t^*}{\partial B}\right) > 0$ for a given level of τ_t^* in the short run. This generates an increase in the expected income of entering the illegal sector since we have

$$\frac{\partial Y_t^e}{\partial B} = -\frac{\partial q_t^*}{\partial B}\left(\frac{Y_t^e}{1-q_t^*}\right) + (1-q_t^*)(1-\varepsilon_t)\frac{\partial Y_t^R}{\partial B} > 0$$

which would seem to generate higher incentives for delinquency. Nonetheless the legal disposable income increases with w_t^* since $\frac{\partial Y_t^l}{\partial B} = (1-\tau_t^*)(1-\rho)\frac{\partial w_t^*}{\partial B} > 0$ for a given level of τ_t^* in the short run. Hence the threshold value $\widehat{\zeta}_t$ in equation (8) does not necessarily increase with w_t^*. For economies with high levels of capital the net effect is more likely to be that $\widehat{\zeta}_t$ decreases which lowers the incentives to enter the illegal sector. We conclude that the more likely case is that an increase in B implies a decrease in the incentives for delinquency. Furthermore, from equation (21) we get that public goods provision is affected ambiguously with an increase in B since

$$\frac{\partial g_{t,t+1}^*}{\partial B} = H_t\left(\frac{2w_t^* q_t^*}{\gamma^2}\frac{\partial q_t^*}{\partial B} + \frac{g_{t,t+1}^*}{w_t^*}\frac{\partial w_t^*}{\partial B}\right)$$

which has an ambiguous sign since $\frac{\partial q_t^*}{\partial B} < 0$ while $\frac{\partial w_t^*}{\partial B} > 0$. Again for economies with high levels of capital the more likely effect is that the second term dominates the first. Hence we conclude that the more likely case is that in the short run an increase in B increases the provision of public goods in the economy. Furthermore, aggregate savings increase since $\frac{\partial a_t}{\partial B} = H_t s(\beta)\left[(1-\phi)\left(\frac{\partial Y_t^l}{\partial B}\right) + \phi\left(\frac{\partial Y_t^e}{\partial B}\right)\right] > 0$ given that all terms are positive. Finally, we find that social welfare increases with B since $\frac{\partial U_{t,t+1}^0}{\partial B} = \frac{(1+2\beta)}{W_t^0}\frac{\partial Y_t^l}{\partial B} + \frac{\eta}{g_{t,t+1}^*}\frac{\partial g_{t,t+1}^*}{\partial B}$ is more likely positive given that the first term is strictly positive and the second is more likely to be positive as argued above.

Appendix B

Kalman Filter and the Optimizing Algorithm

In principle there exist a number of algorithms to solve the full system in equations (50) and (51), without imposing $\lambda = 0$ as in MIMIC equations, in terms of parameter estimates of Z, λ, T and $\Sigma_\varepsilon, \Sigma_\eta$ as well as in terms of the unobservable key variable of asset laundering AL_t. As Harvey (1994) mentions the main algorithm to do this is the Kalman filter. The Kalman filter is a recursive iterative method that allows one to obtain an optimal estimator of the parameter vector at every point in time based on the information available at that moment and update the vector when new information comes in. Clar et al. (1998) illustrate the inherent recursive procedure that the Kalman filter uses as in Figure B1, for a given set of initial values where a_{t-1} is the optimal estimator of $\widetilde{\alpha}_{t-1}$ based on the available information at that moment, which includes Y_{t-1} and P_{t-1} and where in terms of expected values we have that $P_{t-1} = E\left[(\widetilde{\alpha}_{t-1} - a_{t-1})(\widetilde{\alpha}_{t-1} - a_{t-1})'\right]$.

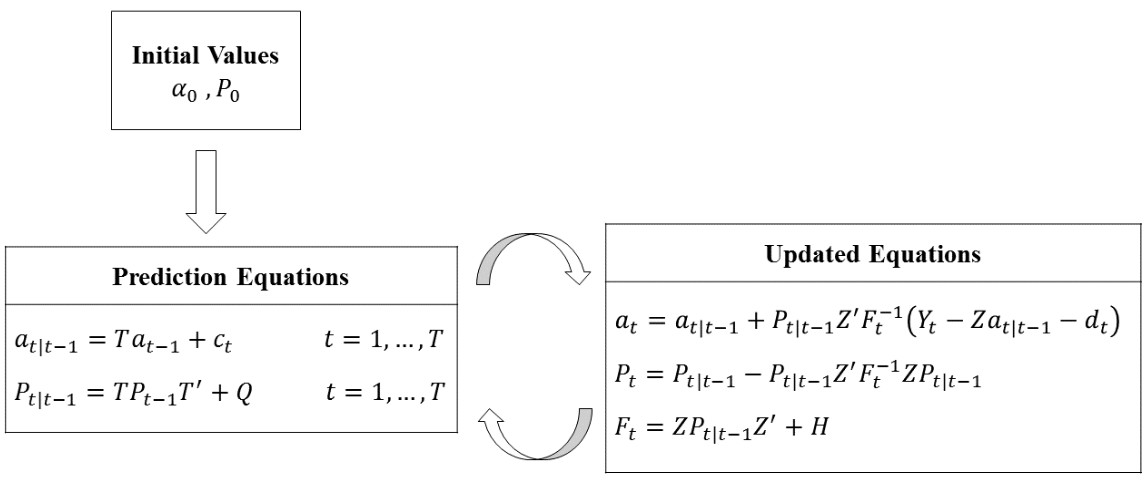

Figure B1

The classic maximum likelihood estimation theory is applied to obtain the estimation of the state vector θ that includes all parameters of the system Z, λ, T, γ and variances. Assuming that stochastic perturbations ε_t and η_t follow a normal distribution then Y_t conditional on all available information up to $t-1$, denoted as $\mathcal{F}_{t-1}$, would also follow a normal distribution

$$
\begin{aligned}
Y_t | \mathcal{F}_{t-1} &\sim N\left(Z a_{t|t-1} + \lambda D_t, Z P_{t|t-1} Z' + \Sigma_\eta\right) \\
\mathcal{F}_{t-1} &\equiv \{Y_{t-1}, .., Y_1\}
\end{aligned}
$$

with likelihood function

$$\begin{aligned}
\log L &= -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|G_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' G_t^{-1} v_t \qquad (65)\\
v_t &= Y_t - \widehat{Y}_{t|t-1} \quad t = 1,..,T\\
G_t &= Z P_{t|t-1} Z' + \Sigma_\eta
\end{aligned}$$

As Clar et al. (1998) argue the expression in equation (65) is usually too complex to get the analytic values of the parameters. This difficulty can be overcome by an optimizing numeric algorithm, as shown in Figure B2, that follows Cuthbertson et al. (1995).

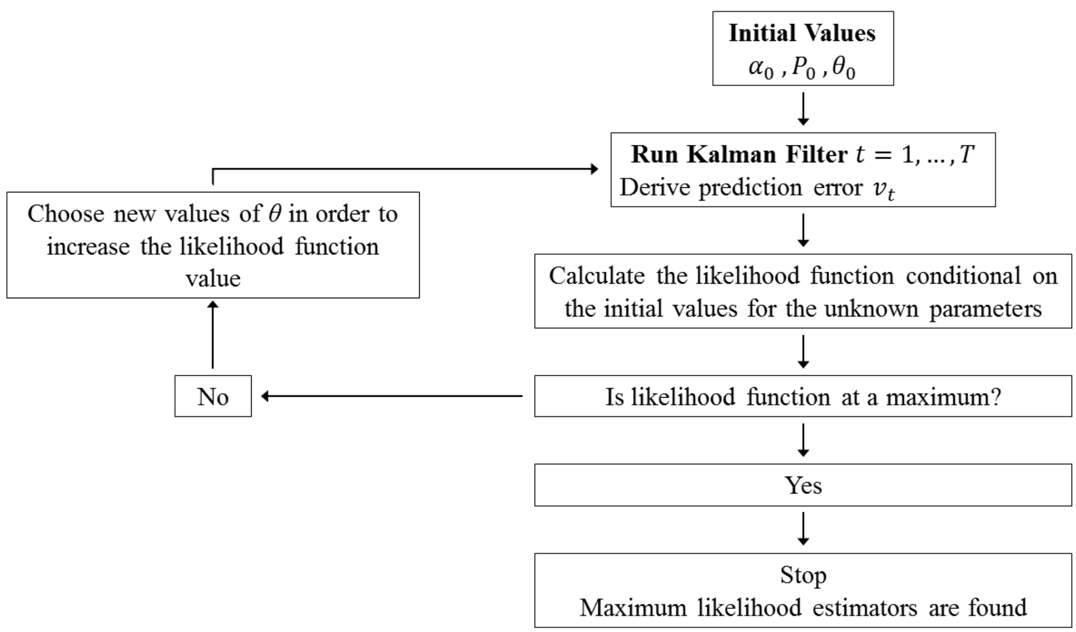

Figure B2

There is then an iterative process that can be summarized in the following steps:

- Identification of the system in its state-space representation and initial values
- Generation of perturbations v_t from the Kalman filter as in Figure B2
- Determination of the value of the logarithmic likelihood function
- Iterative process finishes when a maximum is obtained, otherwise the recursive search process continues.