THE IMPLICATIONS OF HYPERBOLIC DISCOUNTING FOR PROJECT EVALUATION

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The neoclassical theory of project evaluation (Arrow and Kurz, 1970) is based on models in which agents discount the future at a constant exponential rate. There is, however, strong empirical evidence that people discount the future hyperbolically, applying larger annual discount rates to near-term returns than to returns in the distant future (Ainslie, 1992; Cropper, Portney and Aydede, 1994). In this paper we trace out the implications of hyperbolic preferences for private investment choices and public policy.

The immediate problem posed by hyperbolic discounting is that it leads to time-inconsistent plans: A person who discounts the future hyperbolically will not carry out the consumption plans he makes today. From today's perspective the discount rate between two distant periods, t and $t+1$, is a long-term low discount rate. But, when period t arrives, the individual will apply a short term high discount rate to consumption in period $t+1$. Because it makes sense to discuss investment decisions only along consumption paths that will actually be carried out, one must begin by characterizing time-consistent plans for a consumer with hyperbolic preferences. This can be done by allowing the consumer's different temporal selves to play a game and to analyze the equilibrium of this game.

In the case of a finite-lived consumer with quasi-hyperbolic preferences, the game has a unique subgame perfect equilibrium, which (as Arrow has conjectured) can be characterized by an Euler equation similar to that in the Ramsey model. The consumption path that characterizes the equilibrium of the hyperbolic consumer is thus observationally equivalent to the consumption path of a consumer who discounts the future exponentially. Moreover, the consumption rate of discount along this path should always equal the rate of return on capital. This suggests that one should discount future returns using the rate of return on capital, whether consumers have hyperbolic or exponential preferences.

This is not, however, the end of the story. As Phelps and Pollak (1968) demonstrated many years ago, the equilibrium of the game played by quasi-hyperbolic consumers is Pareto- inefficient. Consumers in all years would be better off if they each saved more, but, absent a commitment mechanism, this will never occur. This implies that there *is* a role for government policy when preferences are hyperbolic. Specifically, the government can induce Pareto improvements by subsidizing the return on capital or, equivalently, by lowering the required rate of return on investment projects. Calibration of the hyperbolic model implies that the magnitude of this subsidy should be about two percentage points annually.

We hasten to add that this conclusion does not favor environmental projects vis-a-vis other forms of investment. Hyperbolic preferences provide a motive for lowering the required return on *all* capital investment projects because of the under-saving that occurs along the hyperbolic equilibrium path, but do not favor one type of capital over another.

The remainder of the paper is organized as follows. The next section introduces the notion of hyperbolic preferences. Section 2 describes the intertemporal game played by a consumer with quasi-hyperbolic preferences and characterizes the equilibrium of that game. Section 3 adds a government to the model of section 2 and section 4 concludes.

1. Hyperbolic Discounting and Its Consequences

The neoclassical theory of optimal growth assumes that people have stationary time preferences: that the choice between two payoffs depends only on the absolute time interval separating them. There is, however, strong empirical evidence that people are more sensitive to a given time delay if it occurs closer to the present than if it occurs farther in the future (Ainslie, 1992; Cropper, Portney and Aydede, 1994). In other words the discount rate that applies to near-term consumption tradeoffs is higher than the discount rate that applies to long-term consumption tradeoffs. Loewenstein and Prelec (1992) present an axiomatic analysis of such preferences, which implies a generalized hyperbolic discount function, i.e., a function of the form

$$f(t) = (1+at)^{-?/a}\ , \qquad a,? > 0. \tag{1}$$

As $a \rightarrow 0$, $f(t)$ approaches the exponential function. When a is very large, $f(t)$ approximates a step function, implying that all periods after the first receive approximately equal weight. For $a > 0$, $f(t)$ lies below the exponential function at low t and above it at high t.

In what follows we approximate the hyperbolic function with a quasi-hyperbolic function, first proposed by Phelps and Pollak (1968) for intergenerational analysis and then applied by Laibson (1997) for intrapersonal analysis. Specifically, we examine a representative consumer who lives T periods and whose period-t self receives utility from the consumption sequence $(c_0,c_1,....,c_T)$ according to

$$U_t(c_0,c_1,...,c_T) = u(c_t) + ß \sum_{i=1}^{T-t} d^i u(c_{t+i}), \qquad 0 < ß,d < 1. \tag{2}$$

When $0 < ß < 1$ the discount structure in (2) mimics the qualitative properties of the hyperbolic function, while maintaining most of the analytical tractability of the exponential discount function. We shall refer to the discount factors $\{1, ßd, ßd^2, ßd^3, \ldots\ldots\}$ as quasi-hyperbolic. Figure 1 graphs the exponential discount function for $d = 0.97$, the hyperbolic discounting function with $a = 10^5$ and $? = 5x10^3$, and the quasi-hyperbolic discounting function with $ß = 0.6$ and $d = 0.99$.

To illustrate the time-inconsistency problems to which quasi-hyperbolic preferences give rise, consider what happens if self 0 chooses the consumption sequence $(c_0,c_1,....,c_T)$ to maximize (2) subject to the constraints (3) and (4),

$$0 \leq c_t \leq W_t \tag{3}$$

$$W_{t+1} = R \cdot (W_t - c_t) \tag{4}$$

where W_t is period-t wealth and R is the gross return on capital. As Strotz (1955) first pointed out, the consumption levels $(c_1,c_2,. . . .,c_T)$ chosen by self 0 will not be followed by future selves if they are free to choose their consumption levels. From self 0's perspective the discount rate between two distant periods, t and $t+1$, is a long-term low discount rate. But self t will discount consumption in period $t+1$ at a much higher rate. Self t will, therefore, consume more and save less than self 0 would have chosen for him.

To analyze a set of consumption plans that all future selves would actually follow, we examine the equilibrium of an intertemporal game first analyzed by Phelps and Pollak (1968).

2. The Equilibrium of a Game with Quasi-Hyperbolic Consumers

Consider the decision problem of a consumer with a finite lifetime ($t = 0, 1, . . ., T$). Suppose that self t of the consumer has control over the period-t consumption decision. We assume that self t observes all past consumption levels ($c_0,c_1,c_2,. . . .,c_{t-1}$) and current wealth, W_t, and chooses consumption in period t, c_t, subject to the budget constraint (3). Self $t+1$ inherits wealth W_{t+1}, according to equation (4), and similarly chooses c_{t+1}. The payoff that self t receives is given by (2), where $u(c)$ is a member of the class of constant relative risk aversion (CRRA) utility functions

$$u(c) = (c^{1-?} - 1) / (1-?), \qquad \infty > ? > 0. \tag{5}$$

Consider now the equilibria of this game. As Laibson has shown (1996), when T is finite, the game played by different temporal selves has a unique subgame perfect equilibrium. Each self's equilibrium consumption strategy is a linear function of its inherited wealth,
$c_t = ?_t W_t$, and the consumption path is characterized by

$$u'(c_t) = Rdu'(c_{t+1})[?_{t+1} (ß-1) + 1], \tag{6}$$

where $?_{t+1} = \partial c_{t+1} (W_{t+1},T)/\partial W_{t+1}$. As $T \rightarrow \infty$, equation (6) converges to

$$u'(c_t) = Rdu'(c_{t+1})[?^* (ß-1) + 1], \tag{6'}$$

where $?^*$ is the solution to the non-linear equation

$$?^* = 1 - (dR^{1-?})^{1/?}[?^*(ß-1) + 1]^{1/?}. \tag{7}$$

When ß = 1, equation (6') is identical to the condition that characterizes the optimal consumption path in the Ramsey model. Even when ß ≠ 1, there is still an observational equivalence result conjectured by Arrow, between the equilibrium of the game with quasi-hyperbolic preferences and the Ramsey model. Specifically, the consumption path corresponding to (6') is identical to the consumption path generated by a Ramsey model in which utility is discounted at the constant exponential rate $\underline{d}$ = d[$?^*$ (ß-1) + 1].

In contrast to the Ramsey model, however, the consumption path that characterizes the game with quasi-hyperbolic preferences is not Pareto-efficient. As Phelps and Pollak (1968) first pointed out, all selves would be better off if they all consumed less than the equilibrium consumption rate, but there is no mechanism to guarantee that this strategy will be followed.[1] The intuition behind this result is simple. While requiring self t to save more lowers self t's utility, requiring all other selves to save more raises self t's utility, and the second effect dominates the first. This suggests that it may be possible for the government to enact policies to increase saving that will be Pareto-improving.

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[1]This can be seen as follows. Write self t's utility as a function of $?^*$, the fraction of wealth consumed in the long run:

$$U_t(?^*) = u(?^*W_t) + ßdu(?^*(1- ?^*)RW_t) + ßd^2u(?^*(1- ?^*)^2R^2W_t) + ßd^3u(?^*(1- ?^*)^3R^3W_t)+. .$$

Phelps and Pollak showed that $\partial U_t(?^*)/\partial ?^* < 0$.

3. Pareto-Improving Government Policies in a Quasi-Hyperbolic World

In a world in which consumers can be made better off by lowering the fraction of wealth that they consume, there are two ways that the government can improve welfare. One is to subsidize interest rates (raise R to R); the other is to penalize consumption. Laibson (1996) focuses on both strategies. Here we consider only interest rate subsidies.

We model the government as a sequence of players {0,1,2,. . . .,T} who can tax consumers and use the proceeds to subsidize interest rates. An essential feature of each government is that it can implement policies only with a lag: due to delays in the budget process, government t picks the lump-sum tax in period $t+1$, t_{t+1}, and $R_{t+1}-R$, the interest rate subsidy in period $t+1$.[2] The effect of this assumption is to give the government a commitment technology. The time t government cannot instantaneously overturn the policies of the time $t-1$ government. In this way, the government is able to overcome the self control problem that plagues consumers.

The goal of the government at time t is to maximize the well-being of self t. The government's policy instrument influences marginal tradeoffs between periods $t+1$ and $t+2$. Between $t+1$ and $t+2$ the time-t government would ideally like the consumption path to be:

$$u'(c_{t+1}) = dRu'(c_{t+2}), \tag{8}$$

where R is the unsubsidized marginal rate of transformation. Note that ß does not appear in this equation, as d is the relevant discount factor between period $t+1$ and $t+2$ from government t's

[2]Since the consumers in this economy are not liquidity-constrained, the timing of lump-sum taxes is irrelevant. We therefore focus on the choice of interest rate subsidy.

perspective. Equation (8) implies

$$(c_{t+2}/c_{t+1})^{?} = dR. \quad (9)$$

The time t government can implement this path by choosing an interest subsidy $R = R_{t+1}$ such that the generalized Euler equation (with subsidized interest rate) is consistent with the government's desired consumption path,

$$(c_{t+2}/c_{t+1})^{?} = d\, R\, [?^{*}(R)(ß-1) + 1], \quad (10)$$

where $?^{*}(R)$ is the value of $?^{*}$ implied by equation (7) when R is replaced by R. Equations (9) and (10) jointly imply that the time t government picks R such that

$$dR = d\, R\, [?^{*}(R)(ß-1) + 1]. \quad (11)$$

This argument holds for all governments (i.e., for all times t). So, in equilibrium $R_{t+1} = R$ for all $t \geq 0$.

To solve explicitly for the interest rate subsidy as a function of model parameters, we rely on the fact that $c_{t+s} = ?^{*}(R)W_{t+s}$, for all $s \geq 1$. Together with (4) this implies that

$$c_{t+s+1}/c_{t+s} = (1- ?^{*}(R))\, R. \quad (12)$$

Inserting (12) into (9) yields $?^{*}(R)$ as a function of R and R,

$$[\ (1- ?^{*}(R))\ R]^{?} = dR, \quad (13)$$

which, together with (11) yields the interest rate subsidy as a function of model parameters,

$$R\text{-}R = [(1\text{-}ß)(1\text{-}(dR^{1\text{-}?})^{1/?}] \cdot R/ß. \qquad (14)$$

To illustrate the magnitude of the interest subsidy, we consider plausible values for the parameters of the quasi-hyperbolic discounting function, ß and d, the gross returns on capital, R, and the elasticity of marginal utility with respect to consumption, ?. Suppose that ? = 3 and R = exp(.04), i.e., the gross return on capital is 4%. The values ß = 0.6 and d = 0.99 pictured in Figure 1 are roughly consistent with empirical evidence on intertemporal choices presented by Ainslie (1992). Together, these parameters imply an interest rate subsidy of over two percentage points (R-R = 0.021).

We note that this subsidy puts the economy on a Pareto-efficient path. The equilibrium path that arises in the game with government is identical to the consumption path that would be chosen by self 0 if self 0 could commit all future selves. Note that on the equilibrium path $u'(c_t) = dRu'(c_{t+1})$ for all t > 0. This is exactly the equilibrium path self 0 would like to implement. Hence, the equilibrium path is Pareto-efficient, as any perturbation to the path would make self 0 worse off.

4. Conclusions

When agents discount future utility of consumption using a quasi-hyperbolic rather than an exponential function, the equilibrium consumption path in the economy is no longer Pareto efficient. All consumers would be better off if they each saved more, but there is no way to coordinate this behavior. This suggests a role for government policy: By subsidizing interest rates (reducing the required return on investment projects) the government can help to overcome the self-control problem that results from hyperbolic preferences.

As we noted in the introduction, however, this is not a pro-environment result. It does not justify applying a lower discount rate to an environmental project (e.g., a reforestation project) than would be applied to the building of a steel mill. Are there assumptions that would justify such a practice? The answer is yes. In a world of quasi-hyperbolic preferences one can justify applying a lower discount rate to environmental projects under the following conditions: (1) The production of environmental services (Y^E) from an environmental capital stock (E), e.g., a forest, is a separate process from the production of private output (Y) from private capital (K) (separability in production). (2) The environmental consumption good (c^E) and the private consumption good (c) are imperfect substitutes in the utility function (separability in consumption). (3) The government controls the rate of consumption of the environmental good and, as is assumed above, the government can act only with a lag--in period t it chooses the amount of the environmental good that will be consumed in period t+1 (c^E_{t+1}).[3]

Under these conditions one can show that the government will choose to consume a lower

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[3]Formally, suppose that :

$Y_t = A_t K_t^a$	$0 \le c_t \le K_t$	$K_t = (1-d)K_{t-1} + Y_t - c_t$
$Y^E_t = A_t E_t^a$	$0 \le c^E_t \le E_t$	$E_t = (1-d)E_{t-1} + Y^E_t - c^E_t$

The preferences of self t and government t are given by $v(c^E_t,c_t) + \text{ß}d \mathop{S}\limits_{i=1}^{T-t} d^i v(c^E_{t+i},c_{t+i})$.

fraction of the environmental capital stock than consumers will choose to consume out of private capital, and that the steady-state rate of return on environmental capital will lie below the return on private capital. Thus, a lower discount rate should be applied to environmental projects than to private investments. The intuition behind this result is as follows: As long as the government can act only with a lag, it is prevented from over-consuming the environmental good, as consumers are tempted to do in the case of a private good. Furthermore, because of the assumed lack of substitutability between the environmental good and other goods, both in production and in consumption, consumers cannot undo the government's choices.

This result is, however, a fragile one: it will fail to hold if any one of the three assumptions listed above is violated. In particular, if there is substitutability in production or consumption between the environmental good and other goods, the same rate of return will apply to both environmental and non-environmental capital. This underscores the main point of this paper. While hyperbolic discounting provides a rationale for lowering the required rate of return on investment projects, it does not provide justification for those who seek to treat environmental projects differently from other investment projects.

References

Ainslie, George W. (1992) *Picoeconomics*, Cambridge: Cambridge University Press.

Arrow, Kenneth J. and Mordecai Kurz. (1970) *Public Investment, The Rate of Return and Optimal Fiscal Policy*, Baltimore: Johns Hopkins University Press.

Cropper, Maureen L., Sema K. Aydede and Paul R. Portney. (1994). " Preferences for Life Saving Programs: How the Public Discounts Time and Age." *Journal of Risk and Uncertainty*, 8, 243-265.

Laibson, David I. (1996) "Hyperbolic Discount Functions, Undersaving, and Savings Policy," NBER Working Paper 5635, June 1996.

Laibson, David I. (1997) "Golden Eggs and Hyperbolic Discounting." *Quarterly Journal of Economics*, 112, 443-477.

Loewenstein, George, and Drazen Prelec. (1992) "Anomalies in Intertemporal Choice: Evidence and an Interpretation." *Quarterly Journal of Economics* 57, 573-598.

Phelps, E.S., and R.A. Pollak (1968) "On Second-Best National Saving and Game-Equilibrium Growth." *Review of Economic Studies*, 35, 185-199.

Strotz, Robert H. (1955) "Myopia and Inconsistency in Dynamic Utility Maximization." *Review of Economic Studies*, 23, 165-180.