

Public Disclosure Authorized

# Republic of South Africa Systematic Country Diagnostic

## An Incomplete Transition: Overcoming the Legacy of Exclusion in South Africa

### Background note

#### Exchange Rate Misalignment and its Relationship to Output Growth in South Africa

Ha Nguyen

# Real Exchange Rate Misalignment and its Relationship to Output Growth in South Africa

Ha Nguyen<sup>1</sup>

## Abstract

This paper establishes a simple theory-based real exchange rate (RER) Misalignment Index for countries around the world from 1950-2014, and shows that South Africa's RER has been undervalued over the last decade. For the most recent year of 2014, depending on the proxy for productivity, it is undervalued from about 15% to 18%. We find that terms of trade fluctuations explain a large part of the undervaluation.

## Introduction and motivation

A country's real exchange rate (RER) is formally defined as the relative price of a domestic consumption basket (which includes a domestic non-tradable good and the tradable good) in terms of a foreign consumption basket (which includes a foreign non-tradable good and the tradable good). A depreciated RER generally means that the country's non-tradable goods are cheap compared to the tradable goods. When a country increases its exports, it effectively reduces the domestic supply of tradable goods and hence raises their relative price in terms of non-tradable goods (i.e. undervalued RER) and stimulates the production of tradable goods. An undervalued RER is also argued to boost growth via other *indirect* channels. The first one is productivity improvement. The earliest theory to explain the productivity channel is based on the seminal paper of Lewis (Lewis, 1954). As the export sector -- mostly manufacturing -- expands due to a low RER, it attracts labor from the agricultural sector. Since labor moves from agriculture or services - relatively less productive sectors- to manufacturing- a relatively more productive sector, the economy's aggregate productivity rises, and so does output. More recently, Rodrik (2008) theorizes that, an undervalued exchange rate, or equivalently, an increase in the relative price of tradables, acts as a second-best mechanism to alleviate distortions that disproportionately hurts the tradable sector.<sup>2</sup> The distortions, he argues, come from the institutional weakness and contracting incompleteness that characterize low-income environments. Korinek and Serven (2016) present a theory based on learning-by-investing externality: the expansion of investment in the export sector increases the sector's productivity. The export-driven productivity improvement increases output over and above the direct impact that export brings about.

In this paper, we base our calculation of RER misalignment on the work of Rodrik (2008). He has a simple theory-based approach to measure RER misalignment. In his work, he has shown that undervalued real exchange rates are associated with higher output growth. He does not discuss export growth however. In the section below, we will slightly modify his approach to measure RER misalignment and will present evidence about the relationship between undervaluation and a series of economic outcomes: exports, imports, manufacturing production, TFP and output.

South Africa's RER has been undervalued in the last decade. In the most recent year of 2014 when the data are available, the RER misalignment for South Africa equals 0.15-0.18, which means that South Africa's RER was about 15-18% undervalued. In addition, on average, across all countries and all time, a 10% RER undervaluation boosts growth in real export by 1%, in real value added by 0.38%, in TFP by 0.23% and in real GDP per capita by 0.29%. However, with South Africa data, we do not see the effects. Of course, this

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<sup>1</sup> World Bank

<sup>2</sup> Weak institutions reduce the ability of private investors to appropriate the returns on their investment through a variety of mechanisms: contractual incompleteness, hold-up problems, corruption, lack of property rights, and poor contract enforcement. Rodrik argues that this problem is more severe in tradables than in nontradables because production systems tend to be more complex and roundabout in tradables. When the institutions that foster these relationships are weak, the result is to impose a higher "tax" on tradables—especially modern tradables.

finding comes with the caveat of small sample size in the within-country regression. In section 5, we provide some hypotheses for the insignificant effects.

## The Real Exchange Rate

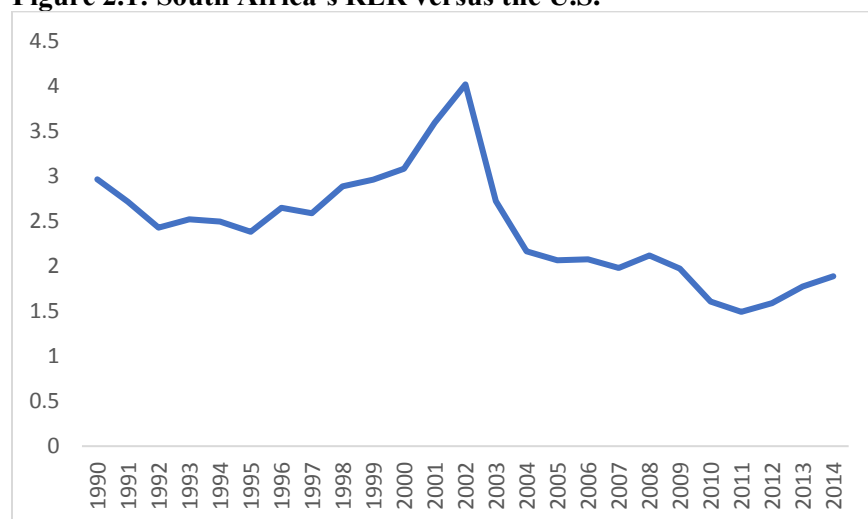
### Definition of the Real Exchange Rate

The Real Exchange Rate (RER) measures the relative purchasing power of the two currencies. The RER of the Rand versus the U.S. dollar is the purchasing power of the rand versus the dollar. It is calculated as the dollar price of the rand (the nominal exchange rate) times the dollar price of one unit of the consumption basket in the U.S. divided by the rand price of one unit of the consumption basket in South Africa.

$$RER_{c,US} = ner_{c,US} \frac{price_{USD}}{price_{Rand}} \quad (1)$$

For developing countries, the RER is usually greater than 1 because the prices in the U.S. is usually more expensive. Hence the ratio is greater than 1. This is also true for South Africa's RER. Figure 2.1 below shows the evolution of RER of the South Africa's Rand versus the U.S. dollar since 1990. Data are from the Penn World Table 9.0. The detailed Rand's RER versus the U.S. dollar are in Appendix A1.

**Figure 2.1: South Africa's RER versus the U.S.**



Why are the RER for developing countries is usually greater than 1? In other words, why prices in developing countries are generally cheaper than prices in the U.S. and other developed countries? One explanation is the Balassa-Samuelson effect. The Balassa-Samuelson effect captures the effect of an economy's productivity on its non-tradable goods' prices. In details, this can be explained as follows: We usually observe that the prices of services (like a haircut) are higher in developed countries than in developing countries, because wages are higher in developed countries. But why wages are higher in developed countries? It is because the tradable sector of developed countries has higher productivity than that in developing countries. Given the law of one price on tradable goods, this implies that wages paid to tradable-sector workers in developed countries must be higher to commensurate their high productivity. In other words, low productivity explains a large part why prices are cheaper in developing countries.

## On Purchasing Power Parities (PPP) conversion rate

We find that the PPP conversion rate is consistent with the nominal exchange rate and the real exchange rate.

From the OECD, “*Purchasing power parities (PPPs) are the rates of currency conversion that equalize the purchasing power of different currencies by eliminating the differences in price levels between countries*”. In other words, it is the rate of currency conversion between the rand and the U.S. dollars so that  $RER_{c,US} = 1$  (i.e. the purchasing power of the Rand and the U.S. dollar are equalized).

$$PPP_{c,US} \frac{price_{USD}}{price_{Rand}} = 1 \quad (2)$$

From (1) and (2), it should be:

$$\frac{ner_{c,US}}{PPP_{c,US}} = RER_{c,US}$$

**Figure 2.2: South Africa’s RER and NER/PPP ratio.**

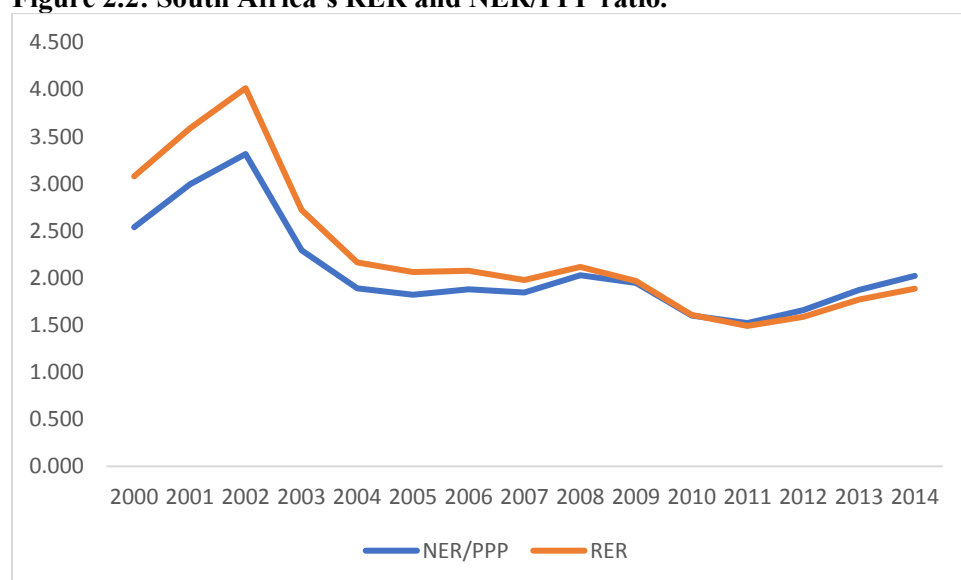


Figure 2.2 presents  $\frac{ner_{c,US}}{PPP_{c,US}}$  and  $RER_{c,US}$  where  $ner_{c,US}$  is the Rand’s nominal exchange rate versus the US dollar (data from the Penn World Table),  $PPP_{c,US}$  is the purchasing power parities (PPPs) conversion rate (data from the OECD), and  $RER_{c,US}$  is the real exchange rate versus the U.S. (data from the Penn World Table). The detailed data are in Appendix A2.

NER/PPP and RER series are similar and move in the same direction. In other words, PPP currency conversion rate provided by the OECD is consistent with the nominal and real exchange rate data by the Penn World Table.

## Data

Our main data sources are Penn World Table 9.0 and World Development Indicators 2017. The Penn World Table 9.0 covers 182 countries between 1950 and 2014. The World Development Indicators cover 217 countries between 1960 and 2016.

The following variables are from the Penn World Table 9.0:

Real Exchange Rates: A real exchange rate to the U.S. dollar is calculated as the inverse of the price level of consumption. In the PWT data, the variable name for the price level of consumption is *pl\_con*. Rodrik (2008) also uses this approach to calculate the Real Exchange Rate.

Nominal exchange rates.

Population (in millions)

Employment (in millions)

The following variables are from the World Development Indicators

Nominal GDP per capital in current \$US

Terms of trade. We use net barter terms of trade index. Net barter terms of trade index is calculated as the percentage ratio of the export unit value indexes to the import unit value indexes, measured relative to the base year 2000 (the index is normalized to 100 in 2000).

### **On South Africa's RER misalignment**

We should start by stating that it is difficult to determine if a country's exchange rate is undervalued and if so, how much it is. Currently, the most popular approach is to regress a country's real exchange rate against a large set of country's fundamentals to establish a real exchange rate norm. The gap between a country's actual real exchange rate and its norm (i.e. the residual in the regression) is considered the "misaligned" part. The most well-known research using this approach is from the IMF (Lee et al, 2006; and subsequently IMF, 2013). The study forms the basis for the IMF's work on assessing countries' RER misalignment in its Article IV papers.

The basic problem with this approach is that this is a "comprehensive" approach: researchers put many fundamental variables to the right-hand side of the regression, sometimes without a rigorous theory behind them. The approach answers the following questions: what is the "typical" exchange rate of a country as a function of its fundamentals? It does not address the question of what a country's real exchange rate should be, or what is its frictionless benchmark. In other words, the methods calibrate the "typical" rather than "normative" or "frictionless" exchange rates. There are two problems with this. First, the residual may include neglected fundamentals affecting the real exchange rate. It is impossible to come up with an exhaustive list of factors affecting productivity and consumption and saving decisions. The identification of the real exchange rate misalignment as a regression residual is likely to be very noisy, as the residual includes other things as well. Second, many variables considered "fundamentals" might contain elements that distort the real exchange rate. For example, in Lee et al (2006), government spending is considered a "fundamental".<sup>3</sup> However, there are several reasons why government consumption could be directly affected by an incentive to lower the real exchange rate. Government consumption may be incorrectly counted as a "fundamental" thereby concealing a real exchange rate misalignment. Eden and Nguyen (2012) offer more detailed criticisms of the current approaches.

### **Balassa-Samuelson effect**

In this part, we base our calculation of RER misalignment on the work of Rodrik (2008). He has a simple theory-based approach to measure RER misalignment. In his work, he has shown that undervalued real exchange rates are associated with higher output growth. He did not discuss export growth however. In the section below, we will slightly modify his approach to measure RER misalignment and will present evidence about the relationship between RER undervaluation on other economic outcomes, namely, growth in exports, imports, manufacturing and labor productivity. We are aware that the approach is simplistic and might not capture other true fundamentals.

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<sup>3</sup> In IMF (2013), government spending was tried but ultimately not used because the estimated coefficient is not significant or has an opposite sign to theoretical priors.

In the first step, we measure an RER misalignment index after controlling for the Balassa-Samuelson effect. The Balassa-Samuelson effect captures the effect of an economy's productivity on its non-tradable goods' prices. Intuitively, this can be explained as follows: We usually observe that the prices of services (like a haircut) are higher in developed countries than in developing countries. This is because wages are higher in developed countries. But why wages are higher in developed countries? It is because the tradable sector in developed countries has higher productivity than that in developing countries. Given the law of one price of tradable goods, this implies that wages paid to tradable-sector workers in developed countries must be higher to commensurate their higher productivity. In other words, low productivity explains a large part why tradable /non-tradable good price ratio (i.e. the real exchange rate) in developing countries is larger than that in developed countries. After the Balassa-Samuelson effect is captured, the remaining residual is considered the misaligned part.

We capture the Balassa-Samuelson effect with four different variations of the Rodrik regression.

$$\ln RER_{i,US,t} = \beta_0 + \beta \ln y_{i,t} + fe_t + u_{i,t}^1 \quad (1)$$

$$\ln RER_{i,US,t} = \beta_0 + \beta (\ln y_{i,t} - \ln y_{US,t}) + fe_t + u_{i,t}^2 \quad (2)$$

In the first two variations, a country's productivity is proxied by its nominal output per capita. Equation (1) is the original Rodrik regression, where  $\ln RER_{i,US,t}$  is the log of real exchange rate of country i relative to the US;  $\ln y_{i,t}$  is log of country i's nominal output per capita in US\$ at time t and  $fe_t$  is a time fixed effect. Note that we do not use country fixed effects. Coefficient  $\beta$  captures the Balassa-Samuelson effect with an expected negative sign. The idea is that per Balassa-Samuelson effect, a country RER, at any given time, is larger if its output per capita (a proxy for productivity) is smaller.  $u_{i,t}^1$  will be our RER misalignment variable of country i where  $\widehat{u}_{i,t}$  is the residual of the regression. A positive  $u_{i,t}^1$  implies an undervalued RER. That is, the RER is larger (more depreciated) beyond the explanation of the Balassa-Samuelson effect.

In equation (2), rather than log output per capita, productivity is proxied by the difference in country i's log output per capita and the U.S.' ( $\ln y_{US,t}$  is the U.S.'s nominal output per capita at time t). Since RER is a relative concept, we add the U.S. output per capita to the right-hand side of the equation to generate output differential, which is a relative concept as well.

$$\ln RER_{i,US,t} = \beta_0 + \beta \ln lpro_{i,t} + fe_t + u_{i,t}^4 \quad (3)$$

$$\ln RER_{i,US,t} = \beta_0 + \beta (\ln lpro_{i,t} - \ln lpro_{US,t}) + fe_t + u_{i,t}^5 \quad (4)$$

Equations (3) and (4) are similar to (1) and (2). The only difference is instead of using log output per capita as a proxy for productivity, we use nominal labor productivity in US\$.  $lpro_{i,t}$  is calculated as total nominal output divided by total employment. Equation (4) will be the baseline results of this paper because labor productivity is closer to the concept of productivity (compared to GDP per capita). This is the first key difference to Rodrik.

The second difference is our use of annual data, as opposed to the 5-year average data as in Rodrik (2008). We argue that unlike the slow-moving impacts of other traditional explanations for growth (such as education or institutions), the impact of RER misalignments on export growth is faster. Hence, annual data are probably more suitable than 5-year average data.

The third difference is the use of nominal GDP. While Rodrik (2008) uses PPP GDP per capita, we argue that market-value GDP per capita is more precise to proxy for the tradable productivity. Appendix A4 presents a model to show exactly why that is the case. Intuitively, the difference between the PPP GDP and market-value GDP rests with the non-tradable sector: while market-value GDP takes different prices of non-tradable goods in different countries, PPP GDP equates the prices of non-tradable goods to

international PPP prices. Since the non-tradable price gap reflects the tradable productivity gap, market-value GDP is more precise.

Table 3.1 presents the result of the regression with Penn World Table 9.0 data. The coefficient of the GDP gap is negative and highly significant, suggesting that Balassa-Samuelson effect is in effect. The coefficient of -0.16 implies that if the labor productivity gap with the U.S. improves by 1%, the real exchange rate with the \$US on average goes down by 0.21%. The residuals of this regression,  $u_{i,t}^4$ , are considered the RER misalignments for countries. They are a component of the real exchange rate not explained by the Balassa-Samuelson effect.

**Table 3.1: On the Balassa-Samuelson effect**

VARIABLES	(1) Real exchange rate
Log(labor productivity)	-0.214*** [0.003]
Constant	1.848*** [0.035]
Year fixed effects	Y
Observations	6,954
R-squared	0.746

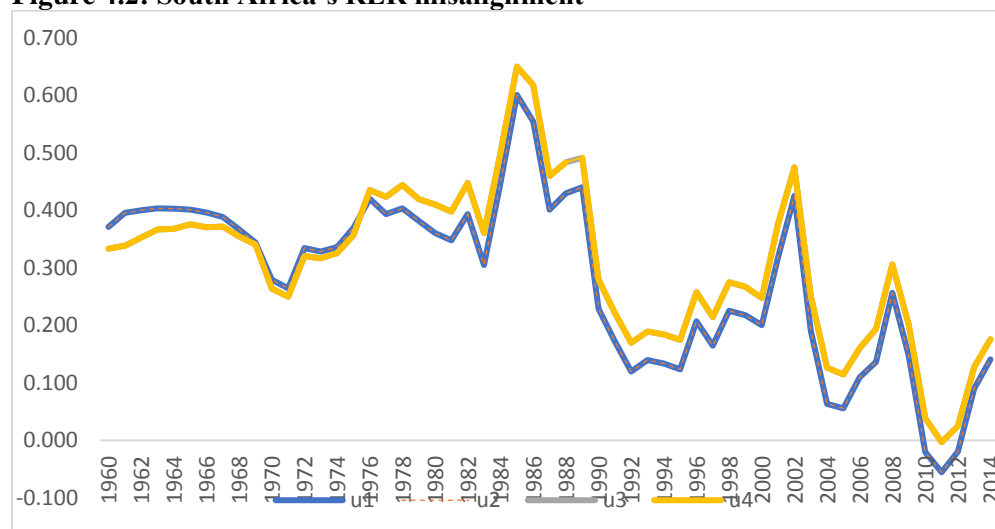
Robust standard errors in brackets

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

#### On South Africa's RER misalignment

Figure 3.2 plots the residuals  $u_{i,t}^4$  for South Africa since 1960 until 2014. This is South Africa's RER misalignment. As shown in Figure 3.2, South Africa's RER was mostly undervalued but the trend is going down, suggesting prices are getting more expensive. The magnitude of RER undervaluation is quite consistent across specifications. With the baseline results from equation (4), as of 2014, the level of undervaluation is about 15%. See Appendix A3 for the country's detailed misalignment values from 1950 until 2014.

**Figure 4.2: South Africa's RER misalignment**



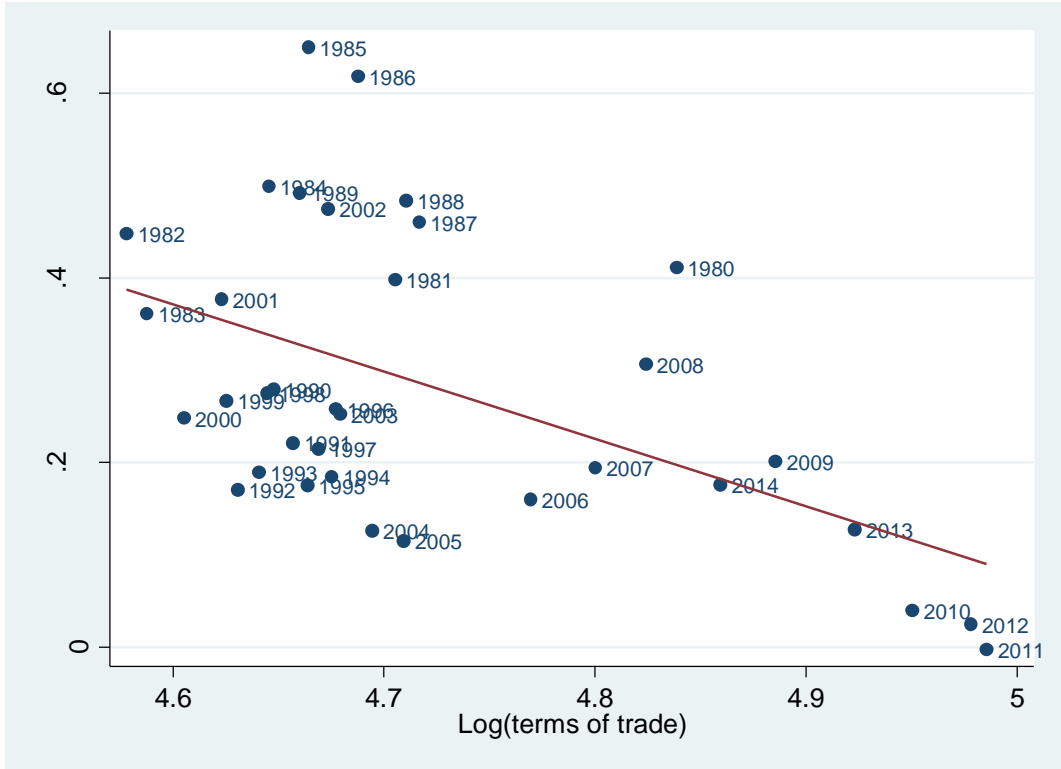
Note: u1,u2 represent RER misalignment derived from equations (1) (2); u4, u5 and u6/s represent RER misalignment derived from equations (4), (5) and (6); u8 and u9/s represent RER misalignment derived from equations (8) and (9). u9/s is our benchmark series. A positive u implies RER undervaluation.

### **Explaining South Africa misalignment**

The most important candidate is South Africa's terms of trade. Regression result indicates that terms of trade explain 26% the variation of South Africa's RER misalignment (see table 5.1) Log terms of trade and RER misalignment is strongly negatively correlated as shown in the scatter plot below. Net barter terms of trade index is calculated as the percentage ratio of the export unit value indexes to the import unit value indexes, measured relative to the base year 2000. What this means is higher terms of trade are associated with episodes where RER misalignment is low (i.e. appreciating RER). They are the for example the years of 2010, 2011, 2012. In other words, an improvement of terms of trade causes RER to appreciate, *above and beyond the effect of productivity*.<sup>4</sup>

<sup>4</sup> One could also include terms of trade as an additional explanatory variable in addition to productivity (see equations 1,2,3,4). However, given the limit of terms of trade data, we could not do so. The reason is that terms of trade in all available sources are normalized to 100 for year 2000 for all countries. In other words, countries' terms of trade are normalized to have the same value of 100 in 2000, which prevents us from including it as an explanatory variable in a cross-country regression.





Recently, the role of terms of trade is larger. Since 1990s, terms of trade explain 39% of the variation in South Africa’s RER misalignment (table 5.1). This reflects the increasing trade integration of South Africa to the world economy. As can be seen in Figure 5.1, terms of trade have strong predictive power in 2010s.

**Table 5.1: Terms of trade and RER Misalignment**

VARIABLES	1980-2014 RER Misalignment	1990-2014 RER Misalignment
logtot	-0.731*** [0.146]	-0.538*** [0.116]
Constant	3.735*** [0.699]	2.749*** [0.556]
Observations	35	24
R-squared	0.260	0.398

Robust standard errors in brackets  
 \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

## **Conclusion**

The goal of this short paper is three-fold. First, it provides intuition why the PPP conversion rate provided by the OECD is consistent with the nominal and real exchange rate data by the Penn World Table. Second, it establishes the level of RER misalignment for South Africa, based on the method by Rodrik (2008). It finds that South Africa's RER is consistently undervalued during the last few years. In 2014, the value of RER undervaluation is about 15-18%. The magnitude is robust across specifications. Finally, the paper examines terms of trade as a factor that drives RER misalignment.

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## Appendix A1: Rand's Real exchange rate since 1990

Sources: Penn World Table 9.0

South Africa	1990	2.96217
South Africa	1991	2.714271
South Africa	1992	2.426224
South Africa	1993	2.519301
South Africa	1994	2.492247
South Africa	1995	2.378418
South Africa	1996	2.646778
South Africa	1997	2.586339
South Africa	1998	2.881728
South Africa	1999	2.956967
South Africa	2000	3.08014
South Africa	2001	3.589817
South Africa	2002	4.015152
South Africa	2003	2.720636
South Africa	2004	2.163257
South Africa	2005	2.061976
South Africa	2006	2.074863
South Africa	2007	1.976473
South Africa	2008	2.115962
South Africa	2009	1.969394
South Africa	2010	1.606678
South Africa	2011	1.490731
South Africa	2012	1.588317
South Africa	2013	1.771206
South Africa	2014	1.885916

## Appendix A2: Comparing NER/PPP and RER

	PPP	Nominal exchange rate (NER)	NER/PPP	RER
2000	2.733	6.940	2.539	3.080
2001	2.877	8.609	2.992	3.590
2002	3.179	10.541	3.316	4.015
2003	3.297	7.565	2.294	2.721
2004	3.419	6.460	1.889	2.163
2005	3.493	6.359	1.821	2.062
2006	3.601	6.772	1.880	2.075
2007	3.818	7.045	1.845	1.976
2008	4.075	8.261	2.027	2.116
2009	4.348	8.474	1.949	1.969
2010	4.569	7.321	1.602	1.607
2011	4.774	7.261	1.521	1.491
2012	4.946	8.210	1.660	1.588
2013	5.158	9.655	1.872	1.771
2014	5.369	10.853	2.021	1.886

### Appendix A3: South Africa's RER misalignment

Year	u1	u2	u3	u4
1959				
1960	0.372	0.372	0.333	0.333
1961	0.396	0.396	0.339	0.339
1962	0.400	0.400	0.353	0.353
1963	0.404	0.404	0.367	0.367
1964	0.403	0.403	0.368	0.368
1965	0.401	0.401	0.376	0.376
1966	0.396	0.396	0.371	0.371
1967	0.389	0.389	0.372	0.372
1968	0.367	0.367	0.355	0.355
1969	0.344	0.344	0.341	0.341
1970	0.280	0.280	0.264	0.264
1971	0.265	0.265	0.250	0.250
1972	0.335	0.335	0.321	0.321
1973	0.328	0.328	0.317	0.317
1974	0.337	0.337	0.326	0.326
1975	0.370	0.370	0.357	0.357
1976	0.420	0.420	0.435	0.435
1977	0.394	0.394	0.424	0.424
1978	0.404	0.404	0.444	0.444
1979	0.382	0.382	0.420	0.420
1980	0.361	0.361	0.411	0.411
1981	0.348	0.348	0.398	0.398
1982	0.394	0.394	0.448	0.448
1983	0.305	0.305	0.361	0.361
1984	0.446	0.446	0.499	0.499
1985	0.601	0.601	0.650	0.650
1986	0.555	0.555	0.618	0.618
1987	0.402	0.402	0.460	0.460
1988	0.430	0.430	0.484	0.484
1989	0.440	0.440	0.492	0.492
1990	0.229	0.229	0.279	0.279
1991	0.172	0.172	0.221	0.221
1992	0.120	0.120	0.170	0.170
1993	0.140	0.140	0.189	0.189
1994	0.134	0.134	0.184	0.184
1995	0.124	0.124	0.175	0.175
1996	0.207	0.207	0.258	0.258
1997	0.165	0.165	0.215	0.215
1998	0.225	0.225	0.275	0.275
1999	0.218	0.218	0.267	0.267

2000	0.200	0.200	0.248	0.248
2001	0.319	0.319	0.377	0.377
2002	0.425	0.425	0.475	0.475
2003	0.192	0.192	0.252	0.252
2004	0.063	0.063	0.126	0.126
2005	0.056	0.056	0.115	0.115
2006	0.110	0.110	0.160	0.160
2007	0.136	0.136	0.194	0.194
2008	0.257	0.257	0.306	0.306
2009	0.148	0.148	0.201	0.201
2010	-0.021	-0.021	0.039	0.039
2011	-0.055	-0.055	-0.003	-0.003
2012	-0.020	-0.020	0.025	0.025
2013	0.090	0.090	0.127	0.127
2014	0.141	0.141	0.176	0.176

This appendix presents the RER misalignment indices based on equations (1) to (4). The first two columns show the RER misalignment indices using nominal GDP per capita as the proxy for productivity. The last two columns show the indices using nominal labor productivity as the proxy for productivity. The last column (column 4) is our benchmark result.

## Appendix A4: Model

Consider a 2 country world: country 1 (Developing) and the country 2 (Advanced). Countries 1 and 2 have the same population normalized to 1. For simplicity, we assume the tradable- nontradable division is the same: each country has the same fraction of the workforce working in the tradable sector (assumption 1). For the developing country, volume output of tradable sector is  $Ak_1^\alpha l_1^{1-\alpha}$ , output of non-tradable sector is  $1 - l_1$ .  $A$  is TFP of the tradable sector. Non-tradable sector production is linear and has no capital. For the advanced country, volume output of the tradable sector is  $nAk_2^\alpha l_2^{1-\alpha}$ , output of non-tradable sector is  $1 - l_2$ . The productivity of non-tradable sector in the two countries are the same. TFP of the advanced country is higher, i.e  $n > 1$ . Note of the assumption 1,  $l_1 = l_2$ . Equalization of the marginal product of capital gives:

$$\alpha A \left( \frac{k_1}{l_1} \right)^{\alpha-1} = \alpha n A \left( \frac{k_2}{l_2} \right)^{\alpha-1}$$

Hence  $k_2 = n^{\frac{1}{1-\alpha}} k_1$ , which means country 2 accumulate more capital than 1. After some calculations, the marginal products of labor (i.e. wages) are:  $MPL_2 = n^{\frac{1}{1-\alpha}} MPL_1$ . Normalize tradable price to 1, and denote wage of country 1 is  $w_1$ , wage of country 2 is  $w_2$ . Note that  $w_2 = n^{\frac{1}{1-\alpha}} w_1$ . The market price GDP of country 1 is

$$Y_1 = Ak_1^\alpha l_1^{1-\alpha} + (1 - l_1)w_1$$

The market price of GDP of country 2 is:

$$Y_2 = nAk_2^\alpha l_2^{1-\alpha} + (1 - l_2)w_2$$

Replacing  $k_2 = n^{\frac{1}{1-\alpha}} k_1$  into the output, and note that  $l_1 = l_2$ , we obtain

$$Y_2 = n^{\frac{1}{1-\alpha}} (Ak_1^\alpha l_1^{1-\alpha} + (1 - l_1)w_1)$$

In other words,  $Y_2 = n^{\frac{1}{1-\alpha}} Y_1$ . If we consider the MPL as the productivity of the countries, the ratio of the two market-price output is precisely the productivity ratio.