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Taxes and Caps as Climate Policy Instruments with Domestic and Imported Fuels

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Abstract

This paper develops a global model of climate policy, focusing on the choice between tax and cap-and-trade solutions. The analysis assumes that the world can be split into two regions, with two fuels that both lead to carbon emissions. Region A consumes all fuels, and is responsible for defining and implementing climate policy. Region B produces all of fuel 1 (oil), while fuel 2 (interpreted as coal, natural gas, or renewables) is both produced and consumed in region A. The paper studies three model variants. All involve full policy coordination in each country block, but no coordination across blocks; and all involve an optimal producer tax on fuel 1 by region B. In model 1, region A sets two fuel consumption taxes, one for each fuel. The optimal region A tax on fuel 1 then exceeds the Pigou level as defined by the region; the tax set on fuel 2 is Pigouvian. The presence of a second fuel in region A reduces region B's optimal tax on fuel 1. In model 2, region A sets a *common carbon tax,* which is lower (higher) for fuel 1 (2) than in model 1. In model 3, region A sets a *carbon emissions cap.* This enhances region B's strategic position via the trade-off between fuels 1 and 2 in region A, following from the cap. In realistic cases, this leaves region A strategically weaker under a cap policy than under a tax policy, more so the less carbon-intensive the local fuel (2) is. In conclusion, a fuel-consuming and importing region that determines a climate policy will typically prefer to set a carbon tax, instead of setting a carbon emissions cap. The main reason is that a tax is more efficient than a cap at extracting rent from fuel (oil) exporters.

This paper—a product of the Environment and Energy Team, Development Research Group—is part of a larger effort in the department to address issues related to mitigation of greenhouse gases contributing to climate change. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at jstrand1@worldbank.org.

Taxes and Caps as Climate Policy Instruments with Domestic and Imported Fuels

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1. Introduction

Most economists and policy makers today seem to find few fundamental differences between setting an emissions tax, and setting a cap on total emissions with free trade of emissions rights among emitters, as an effective policy tool for reducing global carbon emissions.[1] This paper demonstrates that such a view is generally incorrect. The two climate policy instruments are not (and often far from) equivalent when different groups of countries have antagonistic interests in fuel markets. I will below make the assumption, considered realistic, that these markets and policies are dominated by two groups of countries, with clashing interests: one group that consumes most fossil fuels, and that defines and implements climate policy; and another group that produces fossil fuels. Importantly, both producer and consumer countries tend, as groups when their within-group policies are coordinated, to behave non-competitively in the fossil fuels markets. I then show that tax solutions typically dominate cap-and-trade (c-a-t) solutions, as the most efficient and effective climate policy instrument from the point of view of fuel consuming countries.

I consider a highly stylized set-up where the world economy is divided into two blocks. The first block, called region A, consumes all fossil fuels, and defines and implements a climate policy. I assume that region A consumes two fuels: fuel 1 imported entirely from the other region (think of oil); and fuel 2 produced in its entirety within region A itself (interpreted alternatively as coal, natural gas, or renewables). Region B is assumed to produce all of fuel 1, but nothing of fuel 2, and consumes no fuel itself. The two regions have antagonistic interests in the sense that a higher importer (exporter) tax on fuel 1 lowers maximal welfare for the exporting region B (the importing region A). I assume, accordingly,

[1] In this presentation we ignore other climate gases than carbon, which represent about 80 percent of the climate effect of all greenhouse gases. Concerning our two sets of policy instruments, any differences are generally considered to exist only under uncertainty, or in terms of distribution between government and the private sector, depending e g on whether emissions rights are given away or auctioned off by governments; differences resulting from non-competitive behavior of policy makers is much less recognized.

that *the two regions do not cooperate* in setting fuel or carbon taxes, or cap policies. Instead, each block sets its policy optimally, given the policy of the other block. The natural equilibrium concept invoked is then the non-cooperative Nash Equilibrium (NE).

I also assume that *policies are fully coordinated among all countries within each block*, so that the block acts as a single decision maker. In terms of climate policy, such an ideal is easy to conceptualize: an optimal climate policy for carbon emitters must be one that coordinates all emitters' strategies optimally. Region A, which sets climate policy, can be thought of as the entire OECD, plus most of Asia, Latin America and Africa (apart from a small number of countries). These countries are overwhelmingly fuel importers, and together comprise almost 90 percent of both global energy consumption and population. These are also the countries that appear to be preoccupied with limiting greenhouse gas emissions. Region B, consisting of fuel exporters, are a much smaller group of countries, comprising principally OPEC and Russia. The view that fuel exporters coordinate their export strategies is, we argue, not far-fetched, in particular with reference to OPEC as a global force in the oil market. Our assumption that this block has no energy consumption of its own, nor little basic interest in controlling climate change, are also realistic: the block represents no more than about 10 percent of global population, and energy consumption.

In addition to "fuel 1" (oil), which is produced entirely by exporters and consumed entirely by importers, we assume that there exists a "fuel 2", produced and consumed exclusively by the importing block. The existence of this second fuel turns out to make a great difference for the analysis. Its main significance is to increase the choice set for the fuel importing region (A) in the case when region A chooses a c-a-t policy to limit carbon emissions. With an additional fuel (in addition to oil), the importing block has two fuels, instead of just one, by which the overall carbon cap can be adjusted or fulfilled. This turns out to improve the strategic position of region A versus the exporting region B, under a cap policy.

The analysis focuses, as already mentioned, on two alternative paradigms for controlling carbon emissions, namely fuel or carbon taxes versus a cap-and-trade solution, which also represents the two main competing paradigms today for addressing mitigation of climate gases. The paper extends a previous paper, Strand (2009a), where I considered a similar model with two world regions, but with only one fuel (oil) instead of two fuels as here. I then showed that the NE aggregate fuel (and carbon) tax exceeds the Pigou level. I also showed that selecting an optimal c-a-t solution is less advantageous for the importer than setting a tax. Intuitively, when the importer sets a cap, the exporter is free to extract more of the rent from the fuel market, compared to when the importer sets a tax. In the NE solution to that game, the exporter strategically adjusts its tax so as to extract maximum rent from the importer, at the given (exogenous) cap, leading to a zero equilibrium value for tradable emissions quotas in the importer region.

I here consider three different models, dealt with in Sections 3-5. In all cases region B (the fuel exporter) sets an optimal export tax (which can equally be interpreted as a domestic tax on fuel production within region B). The models differ in terms of the behavior of region A. Under model 1, I assume *NE in tax setting, with region A optimally differentiating between fuels 1 and 2,* thus setting one consumption tax for fuel 1, and another tax for fuel 2.

Under model 2, I assume that region A sets *a common carbon tax for both fuels*. This is to recognize that when taxes are differentiated between two fuels, the tax on fuel 1 could, conceivably, be interpreted not as an emissions tax, but instead as a strategic import tax, which could be difficult to defend in view of WTO regulations. A common tax per unit of carbon emissions, which also applies to a domestically-produced and -consumed fuel, is likely to be much harder to oppose on such grounds.

Model 3 implies that region A, instead of setting a tax, selects *a c-a-t solution*, fixing the level of carbon emissions from the region's consumption of the two fossil fuels combined.

Region A is assumed to be in the position to set, and enforce, a global carbon emissions cap, as the only region that consumes fossil fuels. Region B cannot directly control the amount of carbon emitted, but is assumed to still set an export tax on fuel 1.

While the strategic games in the two tax-setting cases (under models 1 and 2) are similar, and parallel to the equivalent game in the one-fuel case, the strategic game in the c-a-t case (model 3) is quite different. The exporting region B now sets its tax optimally, taking *the schedule of tradeoffs between consumption of the two fuels in region A* as given, facing a pre-set cap on carbon emissions. The importing region can here be viewed as setting an emissions quota price (dual to the cap, given that a common quota price is set for both fuels, and given that region A faces a given export tax on fuel 1 set by region B). By contrast, in my previous paper on the topic, Strand (2009a; with only one fossil fuel), setting an emissions cap was tantamount to selecting a given amount of fuel 1 imports. This strategic difference turns out to have substantial impact on the equilibrium solution, at least in some cases. This is illustrated numerically in Tables 2-4. In the limit when fuel 2 has no carbon content (and fuel 2 is interpreted as renewables), the two models turn out to be equivalent.

Section 6 provides numerical illustrations, while Section 7 concludes.

To reiterate, two major weaknesses of my previous paper are remedied here which should make this paper far more realistic and policy relevant. These remedies are both related to the feature that there are two fuels instead of one. First, a second fuel 2 serves as a substitute, albeit imperfect, for fuel 1 in region A. Secondly, the fuel-consuming region now also produces fuels: it is the only producer (and consumer) of fuel 2. Fuel 1 can be considered as oil throughout (the main internationally traded fuel). I have three alternative interpretations in mind for fuel 2. Under two of these, fuel 2 is fossil, either coal or natural gas. When fuel 2 is coal (natural gas), emissions per energy unit are higher (lower) than for fuel 1. In the third alternative, fuel 2 is renewable energy. Some renewables are likely to have emissions properties similar to those of gas; but some are also likely to have much lower (perhaps negligible) emissions. Interestingly, the model turns out to yield distinctly different, and illuminating, results under these three alternative interpretations. The outcomes under a c-a-t solution for the importer now differ substantially according to the characteristics of fuel 2, as illustrated in the simulations reported in Tables 1-3 below. The paper thus yields a set of empirically relevant predictions, in particular concerning the optimal strategy of the fuel exporter in meeting an importer-determined emissions cap.

The literature dealing with the topics of this paper is limited. A previous companion paper, Strand (2009a), considers a similar (static) model, but with only one fuel. I there showed that fuel importers gain from a tax policy instead of a cap for limiting their fuel consumption; while exporting countries always wish to set a positive fuel tax. With strategy coordination within each country block (but not across blocks), the sum of exporter and importer fuel taxes exceeds the Pigou level, reflecting two separate motives for taxation: externality correction (due to the climate impact of emissions); and a strategic rent-extraction (which for the importer may be justified as an "energy security" motive).

Some of the other related literature is explicitly dynamic, but ignores the two key new issues here, by focusing on one fuel, and on taxes only. An early but influential paper by Bergstrom (1982) considered strategic tax setting by independent oil importing countries who face either a competitive oil supply or a monopoly oil exporter, but ignoring environmental costs. The full burden of an importer excise tax was here shown to generally fall on the exporter.[2] Similar results were found by Karp and Newbery (1991), and Amundsen and Schöb (1999); in the latter paper the strategic and environmental tax motivations were combined,

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[2] This result rests on assumptions of exhaustibility of the fossil-fuel resource and zero extraction cost, which leads to a shift in the equilibrium future price path for the resource when an importer tax is imposed. Note that Bergstrom's assumption of no extraction cost is restrictive, and makes his results somewhat difficult to generalize. In our formulation, extraction costs play a key role, while exhaustibility is ignored.

and some conclusions are similar to ours; e g that rent capture arguments may lead fuel importers to set fuel taxes in excess of Pigou levels. More recently, Rubio (2005) considered quantitative versus price mechanisms (as we do in Section 6), but where the exporter (and not as in our model, the importer) sets a quantity. Liski and Tahvonen (2004) study a dynamic model where only importers tax fossil fuels, with conclusions similar to ours, in particular, that the importer tax may be set in excess of the Pigou level when the environmental externality is "small", but lower when the externality is "great". Most recently, Wei (2009) has studied a dynamic "end game" (for extraction of a given fossil fuel resource) between a block that consumes and imports fossil fuels (oil), and a second block that exports it, similar to Liski and Tahvonen (2004), but where the exporter has no extraction costs, and consumes part of the fuel itself. Results are quite similar to those under tax solutions in our paper: the importer sets a tax in excess of the Pigou level; and the exporter sets a positive export tax to extract part of the potential producer surplus. This analysis confirms that fuel or carbon taxation is advantageous for the importer also in a dynamic context, and then perhaps even more so than in a short-run model such as ours.[3]

Recently Eichner and Pethig (2009) have considered cap policies in a dynamic model with three regions: fuel exporters; fuel importers with a climate policy (implementing a carbon price through a c-a-t scheme); and fuel importers without any policy. They focus on the possibility of a "green paradox", conceptualized by Sinn (2008), whereby a stricter cap among climate-policy makers can lead to excessive "leakage" to the region without a policy. They find that a "green paradox" arises only in very special cases, and more likely when a cap

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[3] Wei (2009) shows that the exporter generally gains from selling some of its fuel less expensively on the home market, thus explaining the propensity of fuel exporters to subsidize domestically sold fuels. See also Strand (2009b) for a similar result.

policy is executed only in the first of two periods. Note that this paper does not study (strategic) taxation by exporters, which is a main feature here.

None of the cited papers deals with what I consider as the two main topics of this paper, namely optimal or equilibrium climate policy when fuel importers and consumers have their own fossil fuel production; and the choice between tax and c-a-t solutions in climate policy.

2. Basics

Consider a world with two fossil fuels, both of which are consumed only in one region (A). Fuel 1 (identified as oil) is produced in its entirety in region B, while fuel 2 is produced (*and* consumed) in its entirety in region A.[4] Fuel 2 can be identified with either coal natural gas or reneweables. All demand and supply functions are assumed to be linear.[5]

Global externalities (from carbon emissions when fossil fuels are burnt) are $(1+\alpha)(c_1R_1+c_2R_2)$, where R_1 and R_2 are global consumption levels for fuels 1 and 2, and c_1 and c_2 represent carbon emissions per unit of fuel for the two fuels. Normalizing the externality cost per unit of emissions for region A to one, these are also the externalities per fuel unit felt by the population in region A, while αc_1 and αc_2 are externalities felt by the population in region B. α is a coefficient indicating region B's valuation of carbon emissions, relative to that of region A.[6] We assume that $\alpha < 1$ so that "most" of the (willingness to avoid) climate damage occurs in region A. When fuel 1 is oil and fuel 2 is coal, we can assume that

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[4] To fix ideas, region A can be thought of as the OECD, most of South and East Asia, and much of Africa and Latin America; region B is OPEC and Russia. Important characteristics of the regions are then that region A cares about climate change but region B much less; region A has the bulk of population and fuel consumption; while region B has high oil output but much less output of other fuels (an exception is Russia's natural gas output).

[5] Quadratic utility and production functions, which lead to linear demand and supply functions, can be viewed as second-order Taylor approximations to the respective "true" functions. This is not a serious limitation on functional form as long as the changes in variables under consideration are small.

[6] Throughout, we abstract from local externalities due to fuel consumption.

c_2 is substantially greater than c_1. When fuel 2 instead is natural gas or renewables, c_2 can be presumed smaller than c_1.

The utility of the public in region A related to fuel consumption (not counting externality values is assumed to be given by)[7]

(1) $$V(A) = a_1 R_1 - \frac{1}{2}\gamma_1 R_1^2 + a_2 R_2 - \frac{1}{2}\gamma_2 R_2^2 - \theta R_1 R_2 - (p_1 + t_1) R_1 - (p_2 + t_2) R_2$$

where p_1 and p_2 are producer prices of fuels 1 and 2, and t_1 and t_2 are fuel taxes imposed by country A. (1) implies a simple quadratic form for the utility functions related to each of the two fuels, apart from the interaction term with θ which indicates a negative demand dependency whereby the two fuels are assumed to be substitutes in consumption for region A. Maximizing V(A) with respect to R_1 and R_2 yields the first-order conditions

(2) $$\frac{dV(A)}{dR_1} = a_1 - \gamma_1 R_1 - \theta R_2 - (p_1 + t_1) = 0$$

(3) $$\frac{dV(A)}{dR_2} = a_2 - \gamma_2 R_2 - \theta R_1 - (p_2 + t_2) = 0 .$$

Fuel 1 is produced by region B (but consumed entirely in region A), by competitive firms with collective profit function

(4) $$\Pi_1(P) = (p_1 - s) R_1 - p_{01} R_1 - \frac{1}{2}\phi_1 R_1^2$$

yielding the first-order condition

(5) $$p_1 - s - p_{01} - \phi_1 R_1 = 0 .$$

Producers of fuel 2 are located in region A, and pay no additional fuel taxes (consumer countries are already taxing consumption of this fuel). Their profit function is assumed to take the following quadratic form:

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[7] Individual private actors here have no incentive to take the emission externalities into consideration in determining their demand: thus these externality terms can be dropped when determining demands.

(6) $$\Pi_2 = p_2 R_2 - p_{02} R_2 - \frac{1}{2}\phi_2 R_2^{\ 2}$$

Their first-order condition for producers of fuel 2 is

(7) $$p_2 - p_{02} - \phi_2 R_2 = 0\,.$$

We now solve for p_1 and p_2, together with R_1 and R_2, from (2)-(3), (7), and (A2) (in the appendix).[8] The full derivation is given in the appendix. We find that consumption (and production) of one fuel is affected negatively by higher taxes on this fuel, and affected positively by higher taxes on the other fuel. The latter follows from demand substitutability. Since higher taxes on one fuel reduce consumption of the same fuel, the marginal consumption value of the other fuel is increased, and its demand increased. The direct response of production of any fuel to its own taxes (t_1 and s for fuel 1, and t_2 for fuel 2) is greater than in the case of a single fuel. This follows from the negative cross demand effect: when the tax on a given fuel (say, fuel 1) increases, region A substitutes into more of the opposite fuel (2), which at the same time reduces the demand for fuel 1.

3. Nash Equilibrium (NE) Taxes with Separate Fuel Taxes

3.1 NE taxes for region A

We now turn to deriving Nash Equilibrium (NE) levels of t_1 and t_2, set by the consuming country, and a "tax" on fuel 1, s, set by the producing country.[9] Define the objective function of the importing countries' governments as follows:

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[8] All formulas in the technical appendix, that are not stated in the main text, are numbered as (A1), (A2), etc.

[9] We need not necessarily interpret s as a "fuel tax" on fuel 1 set by the exporter. This may equally well be a production tax, charged to all fuel producers in the exporting block. It may also in principle be interpreted as a scheme for supplying fuels from publicly-owned fuel companies where a part of revenue per fuel unit is retained in exporting countries' treasuries and individual countries' fuel supplies respond to prices as assumed. Such alternative interpretations are highly relevant in practice in particular as an explicit export tax may be difficult to maintain formally, in view of WTP rules.

(8) $W(A) = a_1 R_1 - \frac{1}{2}\gamma_1 R_1^2 + a_2 R_2 - \frac{1}{2}\gamma_2 R_2^2 - \theta R_1 R_2 - p_1 R_1 - p_{02} R_2 - \frac{1}{2}\phi_2 R_2^2 - c_1 R_1 - c_2 R_2$.

(8) simply mirrors the public utility function (1) plus firms' objective (6), corrected for government tax receipts. R_1, R_2, p_1 and p_2 are determined simultaneously by t_1, t_2 and s, the two former of which are set by region A, and the latter by region B. NE tax setting in region A implies that s is taken as given when setting the t_i. The first-order conditions for optimal t_1 and t_2 for given s, by region A are derived in the appendix. Define $t_1^* = t_1 - c_1$, $t_2^* = t_2 - c_2$. We find the following solutions for t_1^* and t_2^* in terms of R_1:

(9) $$t_1^* = \phi_1 R_1$$

(10) $$t_2^* = 0 .$$

Using (5), (9) can alternatively be written as

(9a) $$t_1^* = p_1 - p_{01} - s .$$

(9) has the same basic form as in the one-fuel case, from Strand (2009a). Obviously, $t_1^* > 0$. The optimal tax on the foreign-produced fossil fuel, set by the importer, then *exceeds* its marginal environmental damage cost for the importer. Moreover, introducing an additional, domestically produced and consumed, fuel does not alter the basic tax-setting rule by the consuming region (A), for the internationally traded fuel involving strategic tax setting, given that the fuel importer is able to optimally differentiate the taxes on the two fuels. This is the case regardless of the substitution relationship between the two fuels (represented by the degree of substitutability parameter θ).

For fuel 2, produced and consumed domestically by region A, the NE tax equals marginal environmental damage cost for the region ($t_2^* = 0$ implies the Pigou rule $t_2 = c_2$). The intuition is simple: Region A behaves strategically when taxing the foreign-produced fuel 1 only. Fuel 2 is domestically produced and consumed, and invokes no strategic response in the "first-best" solution for this region.

3.2 NE tax for region B

For a unified region B government, policy is limited to setting an excise tax s per unit of the exported fuel, assuming as before that this government faces a competitive supply from independent producers; and that the fuel is wholly exported (region B has no independent fuel demand). The objective function of region B can be expressed as

(11) $$W(B) = p_1 R_1 - p_{01} R_1 - \frac{1}{2}\phi_1 R_1^2 - \alpha c_1 R_1 - \alpha c_2 R_2$$

The first-order condition for the regional government in region B, derived in the appendix, is

(12) $$s^* = \left(\gamma_1 - \frac{\theta^2}{\gamma_2 + \phi_2}\right) R_1 - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2,$$

where $s^* = s - \alpha c_1$ is the unit export tax on fuel 1 in excess of the externality per unit of fuel 1 consumption as valued by the exporter.

To interpret (12), note first that the exporter tax rule can then be said to be Pigouvian if $s^*=0$. The expression for s* has one positive term (the first main term on the right-hand side), and one negative term (the second main term).[10] Both terms have straightforward interpretations. The first is a *strategic trade term*: s is set high (above the Pigou level) in order for the exporter of fuel 1 to extract more rent than otherwise from consumers of fuel 1, who all reside in region A. This is similar to the result in Strand (2009a); except that there the export tax took the simpler form $s^* = \gamma_1 R_1$. This expression is now modified downward by the second term inside the bracket on the right-hand side of (12). The modification is due to a strategic trade effect for region B, which in turn is due to the presence of an additional fuel 2, produced and consumed by region A. The existence of fuel 2 makes region A "less dependent" on fuel 1, as this additional fuel to some degree substitutes fuel 2 for fuel 1 when

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[10] Note that, from our basic assumption $\gamma_1\gamma_2 - \theta^2 > 0$, the first bracket on the right-hand side of (12) must be positive.

the price of fuel 1 increases. This effect is stronger when the degree of fuel substitution, represented by the substitution parameter θ, is greater. This effect to some degree deters exporter taxation of fuel 1.

The second, negative, term on the right-hand side of (12) is new. It represents the effect of taxation of fuel 1 by region B, on welfare in the same region, through the effect on production and consumption of fuel 2 in region A. When s increases, region A substitutes fuel 2 for fuel 1, and thus increases its consumption of fuel 2. This increase results in increased carbon emissions. This is an externality for region B, of size αc_B per unit increase in fuel 2 consumption, since region B has no direct control over the consumption of fuel 2. This effect discourages taxation of the home-produced fuel in region B.

Arguably, these additional effects are small. First, the correction of the first term depends on θ (representing the degree of substitutability of the two fuels in region A) which could be small.[11] More significantly, the marginal climate damage effect αc_2 for region B is likely to be small. As also argued above, the coefficient α (representing marginal climate damage caused to region B, relative to that caused to region A) is reasonably small: first, as the population of region B is much smaller than that of region A (and thus the valuation of climate damages, when aggregated over all individuals in a region, much smaller for region B given equal valuation by each individual); and secondly, as the preference for avoiding climate damages per individual, as expressed through the political process, could be weaker in region B.

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[11] This may be reasonable at least when fuel 2 is interpreted as coal or natural gas, where the substitution possibilities against oil (used mainly in transport) are moderate. When fuel 2 is renewable (in particular, biofuels), our assumption here could however be less realistic. A recent meta study by Stern (2009) indicates that, while highly uncertain and variable, there is in general a high degree of substitutability between fuel types at the macro level. Average typical substitution elasticities, between oil and coal, and between oil and natural gas, are around unity; although there are many examples of estimated elasticities close to or even below zero.

3.3 Overall NE tax setting

We now derive the overall equilibrium tax levels t_1 and s (given $t_2^* = 0$ from (10)), jointly with the solution for R_1. In the appendix we show that the solutions for t_1 and s are

$$(13) \quad t_1 = \frac{[(2\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - 2\theta^2]c_1 + \phi_1(\gamma_2 + \phi_2)(a_1 - p_{01} - \alpha c_1) - \phi_1\theta(a_2 - p_{02} - c_2)}{2[(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2]}$$

$$(14) \quad s = \frac{[(\gamma_1 + 2\phi_1)(\gamma_2 + \phi_2) - \theta^2]\alpha c_1 + [\gamma_1(\gamma_2 + \phi_2) - \theta^2]\left(a_1 - p_{01} - c_1 - \frac{\theta}{\gamma_2 + \phi_2}(a_2 - p_{02} - c_2)\right)}{2[(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2]} - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2$$

For the aggregate (consumer plus producer) tax, $z_1 = t_1 + s$, on fuel 1 we find the simpler expression

$$(15) \qquad z_1 = \frac{1}{2}(c_1 + \alpha c_1 + a_1 - p_{01}) - \frac{1}{2}\frac{\theta}{\gamma_2 + \phi_2}(a_2 - p_{02} - c_2) - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2 .$$

The aggregate tax on fuel 1 is reduced when θ increases (when there is a higher degree of substitutability between the two fuels), for two reasons. The first effect, identified by the second main term on the right-hand side of (15), is for substitutability of fuel 2 for fuel 1. Higher inter-fuel substitutability leads to a greater demand response for fuel 1 to own taxation (as a greater part of demand is replaced by the other fuel). In response, both t_1 and s are optimally reduced. The second, represented by the last main term in (15), is the externality impact on welfare in region B of increased substitution of fuel 2 for fuel 1 by region A (which is the only fuel-consuming region). As we have remarked, region B has an interest in avoiding consumption of fuel 2 in region A, due to a negative "consumption externality" αc_2 (which represents the negative climate effect of this consumption) on region B. Holding back on s induces more consumption of fuel 1, and thus less consumption of fuel 2, and less of an externality for region B.

In conclusion, the second fuel in region A reduces somewhat the power of region B to manipulate the price of its exported fuel 1 (oil), and leads to more rent being transferred to region A. This is seen directly from expression (12) for the optimal s. In addition, exporter taxing power is eroded also by the externality caused by carbon emissions from the domestic sector 2 in the importing region (A). The exporter controls this effect only indirectly, through the export price on fuel 1, which is in consequence set lower.

Another new effect is that this latter effect is stronger, the greater the coefficient c_2 is; or in other words, the "dirtier" the second fuel is (since, we recall, c_2/c_1 is the relative carbon content of fuels 2 and 1). The point is that when region A has a "dirty" fuel with which to substitute out fuel 1 (oil) when the tax on the latter is increased, emissions are increased "a lot" when such substitution takes place. This is disliked by region B, and serves to reduce region B's export tax. We however stress that this effect is could to be weak, since region B might have little general concern for climate effects of emissions.

Overall, the total tax on fuel 1 is lower here than in the one-fuel case, independent of the carbon intensity of fuel 2.

For comparison with other cases below, we consider the special case where fuel demands are independent ($\theta = 0$; on which we will concentrate in the following sections). t_1 and s then have simplified forms, as follows:

(13a) $$t_1^* = t_1 - c_1 = \frac{1}{2}\phi_1^*(a_1 - p_{01} - c_1 - \alpha c_1)$$

(14a) $$s^* = s - \alpha c_1 = \frac{1}{2}\gamma_1^*(a_1 - p_{01} - c_1 - \alpha c_1)\,,$$

where we have defined $\gamma_1* = \dfrac{\gamma_1}{\gamma_1 + \phi_1}, \phi_1* = \dfrac{\phi_1}{\gamma_1 + \phi_1}$, so that $\gamma_1 * + \phi_1 * = 1$.

The sum of taxes on fuel 1, in excess of the Pigou level, is in this case[12]

(16) $$z_1* = t_1 * + s* = \frac{1}{2}(a_1 - p_{01} - c_1 - \alpha c_1) .$$

z_1* is here always positive. Optimal NE taxation, by two antagonistic blocks as here, thus always leads to overall taxation in excess of the Pigou level.[13]

4. NE Taxation with a Common Emissions Tax in Region A

4.1 Introduction

The previous section was based on the assumption that region A could differentiate its "emissions tax" optimally between the two sectors. But it might then be inappropriate to call the resulting tax on fuel 1 an "emissions tax", but instead rather a combination of an emissions tax for both fuels, and a specific import tax for fuel 1. This could be problematic, as it may clash with WTO rules. We will thus in this section consider an alternative case where region A is confined to setting a common emissions tax for both sectors: This can more easily be defended as a "pure" emissions tax, and thus more difficult to attach on WTO grounds. Overall revenue T to region A from such a tax is

(17) $$T = (c_1 R_1 + c_2 R_2)q$$

where q is a common tax rate per unit of carbon emissions set by fuel consumers, where c_1 and c_2 are still the carbon contents per fuel unit of fuels 1 and 2 . The tax rate per unit of

[12] Note here that t_1* and s* are defined relative to the respective regional externalities, the sum of which comprise global externalities from consuming fuel 1. Accordingly, z_1* is defined as the tax in excess of the global externality caused by fuel 1.

[13] It should be noted that this is not a fully general result; it follows in particular from the assumption that all functional forms, for demand and supply functions, are linear. See e g Liski and Tahvonen (2004) for a discussion.

output is then c_1q for fuel 1, and c_2q for fuel 2. The expressions for the R_i and p_i are derived in the appendix.

4.2 NE taxes for regions A and B

To derive NE tax setting in this case, we again start by considering region A, which now only sets one tax, namely the common rate q per unit of carbon emissions. For region B the situation is simple: the same analysis as in Section 3 applies, and the basic formula (12) still applies. Analytical derivations are also here given in the technical appendix. The most important features of this solution, and how it deviates from the differentiated-tax solution in Section 3, can be discussed for a simplified case with no demand interaction ($\theta = 0$), on which we focus from now on. The solutions for q and s are (as shown in the appendix):

$$c_1 q = \frac{c_{r1}\phi_1 * (a_1 - p_{01} - \alpha c_1) + (2 - \phi_1 *)c_1}{2 - \phi_1 * \left(1 - c_{r1}\right)} \quad (18)$$

$$s = \frac{(1 - \phi_1 *)(a_1 - p_{01} - c_1) + (1 + c_{r1}\phi_1 *)\alpha c_1}{2 - \phi_1 * (1 - c_{r1})}. \quad (19)$$

c_1q^* (the importer tax per unit of fuel 1 in excess of the Pigou level), while still positive, is smaller in (18) than the equivalent tax t_1^* in model 1, from (13a). s* (the exporter tax in excess of the Pigou level) is however greater in (19) than in model 1, from (14a). Region A now sets the emissions tax as a compromise between the two individual-fuel taxes (one of which, for fuel 2, was Pigouvian in the individual-tax case), which implies a lower tax for fuel 1 than when taxes are set individually. Region B reacts to this (in Stackelberg fashion) by setting the export tax on good 1 higher. Thus, in this way, setting an optimal “compromise” emissions tax transfers some market power from region A to region B, which the latter region exploits by upping its own export tax.

The aggregate tax on fuel 1 is in this case

(20) $$z_1 = c_1 q + s = \frac{[1-\phi_1 *(1-c_{r1})](a_1 - p_{01}) + c_1 + \alpha c_1}{2-\phi_1 *(1-c_{r1})}.$$

z_1 is lower in (20) than in (16) when region A sets optimal sector-differentiated taxes (model 1).[14] The reason is that region B, as a (Stackelberg) follower, while increasing its tax in response to the reduced tax on fuel 1, this increase is less than the original reduction in t_1.

5. Cap Policy in Region A

I will in this section study the case where region A imposes a cap on overall carbon emissions, in aggregate for the two fuels. (Region B is in no position to impose an overall emissions cap, since it has no command of fuel 2, nor any fossil-fuel consumption whatsoever.) This is a case of particular interest as it corresponds to a mitigation policy recommended by many observers and analysts. The given cap on total carbon emissions can be defined by C, as follows:

(21) $$C = c_1 R_1 + c_2 R_2 .$$

With a common quota price per unit of emissions, quotas (assumed to be fully auctioned away by the government in region A, and traded freely among emitters) will in total be worth T given by (17). the quota price per unit of fuel will be $q_1 = c_1 q$ for fuel 1, and $q_2 = c_2 q$ for fuel 2, equivalent to the tax on fuel 1 in Section 4.

Consider each of the two regions separately, as in the tax-setting case. The only difference from Sections 3-4, in terms of actual policy, is that region A now sets a cap on overall GHG emissions; instead of setting a tax as in Section 4 (or a set of taxes as in Section 3). I assume as noted that any cap solution is implemented within region A through a system of freely tradable emissions quotas that must be purchased from the government. Within region A, this

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[14] This follows readily from our assumption that $a_1 - p_{10} > c_1 + \alpha c_1$ (which must hold for production of fuel 1 to be at all socially efficient).

defines a common price on emissions from the two fuels (fuel 1, and fuel 2; the price per unit of fuel will vary with emissions intensity).

Under NE behavior, region A chooses an optimal cap given the tax s set by region B. Region B on its side chooses an optimal tax s given a cap set (optimally) by region A. From the point of view of region A, the quantity and price setting are dual problems. As a consequence, for region A the problem of choosing a quantity limit is strategically equivalent to that of choosing an emissions price q implementing the same given emissions level for given s.[15] This implies that the maximization problem for region A can be formulated as one of choosing an internal trading price q for emissions quotas within the region.

In consequence, the problem for region A can, analytically, be set up as one of maximizing the regional objective function (8) with respect to q, and taking s as exogenous, in the same way as the problem solved in Section 4. The analysis for region A in that section thus applies also here.

For region B, the strategic situation is now quite different from the previous two models. This region can now be viewed as facing the constraint (21) whereby R_1 and R_2 are related, with given C. Region B correspondingly maximizes W(B) with respect to s, subject to a constant C from (21). It can be shown (in the appendix) that this gives rise to the following price response for the fuel importer, when the exporter tax is increased:

(22) $$\frac{dp_1}{ds} = 1 - \phi_1 * (1 - c_{r1}) .$$

(22) reveals that p_1 is more sensitive to changes in s here than under a carbon tax (model 2 in Section 4); in the latter case (from expression (A10) in the appendix), $dp_1/ds = \gamma_1 *$.

[15] One way to see this, is to realize that a given cap with c-a-t and full auctioning can, full certainty and perfect competition in the quota market, be implemented by the importer in two ways: by setting the cap directly and auctioning the corresponding emissions rights, thus inducing a given and a priory known quota price; or by setting the same quota price directly and let agents purchase emissions rights freely at this given price, thus inducing quota sales equivalent to the given cap.

Moreover, this sensitivity is greater, the greater is c_{r1} (and thus the greater is c_1 relative to c_2). (A low c_2 implies little scope for substitution of good 2 for good 1 when s increases for a given cap on overall GHG emissions; the effect of a higher s is then mainly to reduce the quota price q.) Correspondingly, p_1 is less sensitive to changes in s when c_2 is high: a relatively small substitution out of sector 1 and into sector 2 is then required to retain the carbon emissions constraint when the exporter tax s is increased (leading to reduced demand for good 1).

The problem can now be formulated as maximizing (11) with respect to s, under the constraint (21) with C as given (this exercise is presented in the appendix). Since a cap solution is chosen by region A, region B takes overall emissions, C, as given in its own optimization. From the point of view of region B, then, its own tax policy has no influence on carbon emissions. In consequence, the two last terms in (11) then drop out. The solutions for the price of emissions quotas per unit of fuel 1, c_1q^* (= $c_1(q-1)$), and the exporter tax on fuel 1, s^* (= $s-\alpha c_1$) are found (in the appendix) as

(23)
$$c_1 q^* = \phi_1 * c_{r1} \frac{1-c_{r1}}{2-c_{r1}-\phi_1 *(1-c_{r1})^2}(a_1-c_1-p_{01})$$

(24)
$$s^* = \frac{1-\phi_1 *(1-c_{r1})}{2-c_{r1}-\phi_1 *(1-c_{r1})^2}(a_1-c_1-p_{01})-\alpha c_1$$

For the sum of the exporter tax and the quota price per unit of fuel 1 in the importing block (which, together, comprise the effective "tax" on fuel 1), we find

(25)
$$z_1 * = c_1 q * + s* = \frac{1-\phi_1 *(1-c_{r1})^2}{2-c_{r1}-\phi_1 *(1-c_{r1})^2}(a_1-p_{01}-c_1)-\alpha c_1 .$$

We may study the expressions (23)-(25) under different parametric assumptions. Consider then first the case where α is negligible: this is the case where region B puts a negligible value on damage caused by carbon emissions. In this case, the overall effective net "tax" (in excess of the Pigou level; as the sum of the exporter tax and the equilibrium emissions quota price in region A) from (25) is then always greater than the equivalent sum from (20) in the case of a common emissions tax in region A. Comparing to the sum of fuel 1 taxes in the case of optimal tax differentiation, from (16) in Section 3, we find that this sum is greater from (25) than from (16), given that

(26) $$c_{r1} > \phi_1 * (1 - c_{r1})^2$$

which holds whenever c_{r1} is not too small: e.g., when $\phi_1 * = ½$, (26) holds for all c_{r1} greater than approximately ¼. c_{r1} is here less than one half when the carbon content per value unit is lower for fuel 1 than for fuel 2 (as would be the case when fuel 1 is oil, and fuel 2 is coal). In this case, the trade-off between fuel for the importer, for a given emissions budget, implies that demand for the traded fuel (oil) is highly elastic, which for the exporter translates into a relatively low market power, resulting in a moderate optimal exporter tax.

For small α, c_1q^* is always lower in (23) than in (18), while s* is always greater in (24) than in (19). Thus when the exporter has low aversion to carbon emissions, the exporter of fuel 1 will be more aggressive in its export price policy when the importer sets a carbon cap, than when the importer sets a common emissions tax. To understand why, note that the only reason why the export tax would be lower in the tax than the cap case case, is that the exporter has great aversion against increased carbon emissions; this is the case only when α is high.

When α is larger, these conclusions may change.[16] Remember again that α represents region B's marginal valuation of damages due to GHG emissions. When this valuation is high, it has an appreciable impact on the export tax in the tax-setting cases of Sections 3-4, as the export tax is then set higher to limit emissions, and thus damages, as felt by region B.

[16] We argued that this case is implausible as the exporter's population must be small and due to practical political factors (actually today, major fuel exporters do not seem to worry much about the prospect of climate change).

When region A sets a cap, by contrast, region B faces a level of emissions which is independent of its own tax, and its export tax is perceived as having no influence whatsoever on emissions. Thus, region B does not consider climate damages in setting s. This factor contributes to a lower export tax in the cap case. In principle, this could lead to a higher exporter tax in the case where the importer also sets a tax, instead of a cap.

For the importer (region A) the situation is opposite: this region has no intrinsic preference attached to the term containing α, and takes this term into consideration only indirectly, through its effect on the tax set by the exporter. But as noted a high α increases the exporter tax in the tax case compared to in the cap case; the importer reacts to this in opposite manner (since its optimal reaction in terms of own tax is a falling function of the other side's tax: importer and exporter taxes are strategic substitutes). This factor contributes to a higher importer "tax" (carbon trading price) in the cap case (when the exporter's tax is reduced by this factor) than in the tax case.

One clear result here is that region A loses from a cap solution relative to a tax solution, apart from exceptional and unrealistic cases (in particular, where region B has a very high degree of aversion to carbon emissions). This is simply a consequence of the region B tax on fuel 1 being higher in the cap case, and that optimized region A welfare is a decreasing function of the region B tax.

6. Numerical Illustrations

This section provides numerical illustrations of the various cases and models, for equilibrium taxes t_1 (alternatively, carbon trading prices, c_1q^*) and s, and their sum z_1. Throughout we consider a very simplified example where all γ and φ coefficients are identical (demand and supply curves are assumed to have equal slopes, and the same slopes in both sectors). We consider two different values for region B's valuation of environmental damage (relative to the valuation of region A, and represented by the parameter α): either a "low" value, $\alpha = 0$; or a "high" value, $\alpha = ½$.[17] We also consider three alternative values for c_2 relative to c_1, to represent our three prototype fuel categories, as follows:

$c_2 = 2c_1$. The carbon content per energy (and economic value) unit of this fuel is consequently twice that of oil. This is a reasonable description of the average properties of coal; correspondingly, we in this case identify fuel 2 with coal.

$c_2 = ½ c_1$. The carbon content per value and energy unit for fuel 2 is now half of that of oil. This is a realistic description of average properties of natural gas, which which fuel 2 is identified in this case.

$c_2 = 0$. There are now no net carbon emissions whatsoever arising from the consumption of fuel 2. We will associate fuel 2 with renewable energy in this case (realizing, of course, that zero carbon emissions are practically unrealistic for many types of renewable energy).

Table 1 deals with model 1 only, where we assume that region A sets individual taxes for each fuel. We consider three alternative values for the demand dependence parameter θ, namely $\theta = 0$ (demand independence), $\theta = 0.1$ ("weak" demand dependence), and $\theta = 0.25$ ("stronger" demand dependence).

In Table 1, the overall tax z_1 ($= t_1 + s$) in all cases exceeds the respective Pigou levels (either 1 for $\alpha = 0$, or 1.5 for $\alpha = 0.5$), but is generally lower when θ is larger. A noticeable feature is that all taxes are reduced when θ is increased, but s more so than t_1. Interestingly also, when fuel 2 has a lower carbon content (as when fuel 2 is renewables), the tax rate for fuel 1 is reduced by more when θ increases. This is, from (15), related to the overall social value of fuel 2, which is greater when the marginal carbon externality it causes is smaller; this value is induced by the substitution of fuel 2 for fuel 1 when $\theta > 0$ and as a result deters high

[17] Arguably, $\alpha = ½$ is an "extreme upper-end" case, in particular since the population of region B is only a fraction of that in region A.

taxation of fuel 1. Another observation to be made is that, for the range of parameter values chosen, the variation in tax rates is rather moderate.

Table 1: Numerical Illustrations of Taxes on Fuel 1 in Model 1, Different Values of α and θ

	Tax parameter	θ = 0		θ = 0.1		θ = 0.25	
		α = 0	α = 0.5	α = 0	α = 0.5	α = 0	α = 0.5
Fuel 2 is coal ($c_2 = 2c_1$)	t_1	1.5	1.38	1.48	1.35	1.47	1.34
	s	0.5	0.88	0.47	0.72	0.41	0.54
	z_1	2	2.25	1.95	2.07	1.88	1.88
Fuel 2 is gas ($c_2 = c_1/2$)	t_1	1.5	1.38	1.44	1.32	1.37	1.23
	s	0.5	0.88	0.44	0.75	0.32	0.65
	z_1	2	2.25	1.88	2.07	1.69	1.88
Fuel 2 is renewables ($c_2 = 0$)	t_1	1.5	1.38	1.43	1.30	1.34	1.20
	s	0.5	0.88	0.42	0.77	0.31	0.68
	z_1	2	2.25	1.85	2.07	1.63	1.88
Global Pigou tax, fuel 1	z_1	1	1.5	1	1.5	1	1.5

Tables 2-4 compare models 1-3, in all cases assuming demand independence between the two fuel sectors (θ = 0; which is the only case studied in Sections 4-5). In all tables 2-4, results do not differ between the three tables for model 1 (individual tax setting for the two fuels in region A), as there is no interaction between the two fuels in this case. Such interaction however occurs under models 2-3. In all cases, z_1 *exceeds* its Pigou level: there is *too much* taxation of fuel 1 at the respective Nash Equilibria. Correspondingly, output of fuel 1 is inefficiently low. This results from our strong assumption with respect to coordination of regional strategies, and the fact that a strategic tax motive adds to an externality-correcting motive. Note also that while tax rates differ according to α (region B's valuation of climate damages) under tax-setting solutions for region A (models 1-2), they are independent of such valuation under a c-a-t solution (model 3). The reason, as pointed out above, is that region B, in facing a c-a-t solution, takes overall emissions as given, and is not influenced by any climate implications in setting its sector 1 tax, s. When region A sets a tax or taxes, by contrast, region B will in general perceive such an influence, as overall carbon emissions will then be a function of the exporter tax, s, set by region B.

Table 2 compares the three models when $c_2 = 2c_1$, where fuel 2 is interpreted as coal (remember that fuel 1 is interpreted as oil throughout). We find in Table 2 that taxes on fuel 1 are generally more moderate under models 2-3 than under model 1 (where region A differentiates the taxes on fuels 1 and 2 optimally). The fuel 1 tax is more moderate in model 2 than in model 1, as the tax rate set by region A in model 2 is a common carbon tax for both fuels, which is in this case influenced heavily by the tax on the emissions-intensive fuel 2 (interpreted as coal), and this reduces the tax on fuel 1 in the direction of the Pigou level (while still remaining above this level). Correspondingly, the fuel 2 tax is higher under models 2-3 than under model 1 (when the tax on fuel 2 is "Pigouvian" for region A).

Table 2: Numerical Examples of Taxes for Models 1-3, for $\theta = 0$ and $c_2 = 2c_1$ (Fuel 2 is Coal)

Variable	Model 1: Region A differentiates sectoral taxes optimally		Model 2: Region A selects a single carbon tax		Model 3: Region A selects c-a-t solution for emissions	
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$
t_1	1.5	1.375	1.13	1.09	1.11	1.11
s	0.5	0.875	0.63	1	0.81	0.81
z_1	2	2.25	1.76	2.09	1.92	1.92
Global Pigou tax, fuel 1	1	1.5	1	1.5	1	1.5
t_2	2	2	2.25	2.18	2.22	2.22
Global Pigou tax, fuel 2	2	3	2	3	2	3

For fuel 2 the tax situation differs from that for fuel 1 when region B values damages negatively ($\alpha = ½$): then the region B tax t_2 on fuel 2 is then everywhere below the (global)

Pigou level (= 3). This result would however clearly be overturned for α values closer to zero.[18]

An interesting feature of equilibrium when fuel 2 has a high carbon content (and interpreted as coal), is that the "overall tax" (the sum of the exporter tax s and the c-a-t quota price in importer countries) does not differ much under the c-a-t solution (model 3) from the case of a unified carbon tax (model 2). The exporter (region B) is in this case restrained in its tax setting due to the importer's (region A's) extensive substitution possibilities for a given cap facing the fuel importer. In addition, remember, under the c-a-t solution region B does not consider any effects of overall carbon emissions from its fuel pricing decision; its tax setting is thus in no way affected by climate considerations. This factor moderates tax setting in the c-a-t case, and makes it more favorable to the importing block (region A) which of course prefers a lower region B tax on fuel 1. In fact, from Table 2, when region B has a "high" aversion to carbon emissions (α = ½), this tax-moderating factor outweighs the monopoly-enhancing effect of the c-a-t scheme on region B taxation, and makes the tax t_2 *less aggressive* (lower) in the c-a-t case, model 3, than in the tax-setting case, model 2). In this particular case, the importing block favors setting a cap instead of a tax. This is however the only case where this result arises; it is arguably unrealistic as the value of α (region B's aversion to additional carbon emissions, relative to that of region A) is unrealistically high in this case.

Table 3 deals with cases when fuel 2 has a positive but low carbon content, ($c_2 = ½ c_1$), with fuel 2 interpreted as natural gas. The main difference from the numerical examples in Table 2 is that the exporter is now much more aggressive in its tax setting in the c-a-t case (model 3). The reason is, as noted above, that region A's trade-off between the two fuels (for

[18] Arguably, one half is a very high value for α to take. Remember that, in our model, region B consumes no fuel; logically this must imply that its population is very small (fuel exports are concentrated to a narrow population group globally). If marginal climate damage valuations were equal across populations globally, the total marginal valuation by region B, relative to region A, will simply equal the ratio of the two populations.

any chosen carbon cap) is now much more favorable for region B, as the demand for fuel 1 is now much less responsive to an ex post increase in the price of this fuel. This leads to more aggressive taxation by region B. In response, the NE tax (or, rather, carbon quota price) for fuel 1 in region A is then set lower in the c-a-t case (model 3) than in the carbon tax case (model 2). The overall tax is also found to be higher in the c-a-t case, and fuel consumption and emissions lower. In this case the c-a-t solution is far less favorable than a carbon tax solution for region A, due to the much higher tax set by region B (which, as a result, extracts a much greater share of the rent). The difference between the two solutions is found to be greater when region A has no value of carbon emissions reductions ($\alpha = 0$) than when it has such value ($\alpha = ½$), as s is lower in the former case under model 2.

Table 3: Numerical Examples of NE Taxes for Models 1-3, $\theta = 0$ and $c_2 = ½\, c_1$ (Fuel 2 is Gas)

Variable	Model 1: Region A differentiates sectoral taxes		Model 2: Region A selects a single carbon tax		Model 3: Region A selects c-a-t solution for emissions	
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$
t_1	1.5	1.375	1.42	1.31	1.14	1.14
s	0.5	0.875	0.53	0.90	1.52	1.52
z_1	2	2.25	1.95	2.21	2.66	2.66
Global Pigou tax, fuel 1	1	1.5	1	1.5	1	1.5
t_2	0.5	0.5	0.71	0.66	0.57	0.57
Global Pigou tax, fuel 2	0.5	0.75	0.5	0.75	0.5	0.75

Table 4 deals with the case where fuel 2 consumption leads to no carbon emissions (and fuel 2 is interpreted as a non-carbon renewable energy source). Here the contrast between models 2 and 3 is even starker than in Table 3. Region A, in setting its carbon cap, must now achieve the given cap fully on the basis of fuel 1 (oil) consumption, and no ex post fuel substitution is possible. This gives region B maximum monopoly power in setting the tax s on its exported fuel. This tax is now set to eliminate the quota trading price in region A, which is zero in consequence. In this case also, models 1 and 2 collapse to one as setting a carbon tax is the same as setting a tax on fuel 1 given that this fuel is the only source of carbon emissions; in either case, the effective region A tax on fuel 2 is zero.[19]

Table 4: Numerical Examples of NE Taxes for Models 1-3, $\theta = 0$ and $c_2 = 0$ (Fuel 2 is Renewables)

Variable	Model 1: Region A differentiates sectoral taxes		Model 2: Region A selects a single carbon tax		Model 3: Region A selects c-a-t solution for emissions	
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$
t_1	1.5	1.375	1.5	1.375	0	0
s	0.5	0.875	0.5	0.875	2	2
z_1	2	2.25	2	2.25	2	2
Pigou tax, fuel 1	1	1.5	1	1.5	1	1.5
t_2	0	0	0	0	0	0
Pigou tax, fuel 2	0	0	0	0	0	0

7. Conclusions

I have in this paper considered a modeling framework where two fuels are produced and consumed in the world market, and the world is divided into two main regions (A and B). Region A consumes all fuels, and produces one of them (fuel 2), while region B produces all of fuel 1. The two regions are involved in three alternative Nash Equilibrium (NE) games, analyzed in models 1-3, corresponding to alternative ways in which climate policy is

[19] This result can be interpreted as a limit result as c_2 tends to zero. It then replicates, and generalizes, a parallel result in Strand (2009a), for a similar one-fuel model.

conducted. In models 1-2, the fuel exporter and importer both set taxes. In model 1, the fuel-importing region (A) sets two taxes, one for each fuel; while in model 2, region A sets one common carbon tax. The former is the preferable strategy for region A. We however consider also model 2, in particular since a carbon tax may, less than individual fuel taxes, be subject to WTO (or other retaliatory) sanctions. In model 3 the importer sets a carbon emissions cap. In all three models, the fuel exporter (region B) is assumed to select an optimal export (or domestic producer) tax on fuel 1.[20] We assume a negative demand dependence between the two fuels for region A (represented by a positive interaction parameter θ); this feature is explicitly considered only in the context of model 1 (in our discussion of models 2-3, we abstract from demand dependence and assume $\theta = 0$).

Under model 1, the importer (A) *sets two taxes, one for each fuel*, taking the exporter tax on fuel 1 as given. The tax on the imported fuel, t_1, is motivated by two factors: an externality effect due to carbon emissions, and an import-strategic motive. This tax is set above the Pigou level (echoing corresponding results in other papers including Amundsen and Schöb (2000), Liski and Tahvonen (2004), and a companion paper to the current one, Strand (2009a)). For fuel 2, which is both produced and consumed by region A, there is no strategic motive in tax setting; region A sets this tax at the Pigou level.

For the exporting region, the optimal (NE) export tax in model 1 is adjusted down relative to the level found in Strand (2009a) where only one fuel was assumed. First, the presence of fuel 2 in region A now puts that region in a more powerful strategic position relative to region B, as consumption of fuel 2 can be substituted for fuel 1 in region A in response to a higher import price on fuel 1, given a negative demand dependence between fuels (the interaction parameter θ is positive). This factor tends to discourage fuel taxation by region B. The second

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[20] Note again that these two taxes here are equivalent as region B's entire output of fuel 1 is exported.

new factor relative to the one-fuel case is the externality imposed on region B, when carbon emissions from fuel 2 are increased in response to the export tax on fuel 1; This factor reduces the optimal fuel 1 tax set by region B. Overall, relative to the one-fuel case in Strand (2009a), the exporter fuel tax is reduced. The region A tax may in response be either higher or lower than in the one-fuel case, but most likely lower.

In model 2 (dealt with in Section 4) region A sets *a carbon tax common for both fuels*. Region B sets, as before, an export tax on fuel 1. The carbon tax is now set as a “best compromise” for both fuels jointly. It is a weighted sum of individual optimal taxes under model 1 (where, we note, the optimal tax on fuel 1 exceeded the Pigou level, while the optimal tax on fuel 2 was at the Pigou level). The optimal exporter tax on fuel 1, as a best response to the importer tax, is now somewhat higher than under model 1; while the sum of (export and import) taxes on fuel 1 is lower than under model 1.

Model 3 deals with a fundamentally different type of climate policy. Region A now sets an *overall cap on carbon emissions* from both fuels combined, and implements the cap within the region through a unified emissions quota price for both fuels. The main strategic difference from models 1-2 lies in the behavior of region B, which now faces a c-a-t scheme instead of a tax or set of taxes, as under models 1-2. For region B, the strategic situation is altered in two ways, relative to the tax case. First, region B is now strategically more powerful as it recognizes that a reduction in demand for fuel 1, prompted by an increase in the exporter tax on this fuel, must be met through an increased consumption of fuel 2, and thus a substitution of fuel 2 for fuel 1, which may be ex post expensive for region A. This factor *increases* the optimal tax set by region B. Secondly, region B now has no incentive to consider adverse effects of changes in carbon emissions, as the cap ascertains that the emissions level is a constant. Under tax setting in region A, by contrast this second factor usually leads region B to set a higher export tax on fuel 1, in order to limit region A’s consumption of fuel 1 and thus on carbon emissions.[21] The absence of such a factor in the c-a-t case then tends to make the optimal region B tax lower, when region A selects a c-a-t policy.

The exact nature of this tradeoff turns out to be important for the overall tax-setting behavior of region B. In particular, the greater carbon content that fuel 2, has relative to that of fuel 1, the more elastic the demand for fuel 1 will be in response to an export tax from region B, and the less powerful will region B be when region A sets an emissions cap. The first factor noticed above (which serves to raise the region B tax) is then weaker. Conversely, when fuel 1 has a high carbon content relative to that of fuel 2, and a given emissions tax is set, the strategic position of region B is very powerful. For the second factor (tending to reduce the optimal region B tax in the c-a-t case) the situation is opposite: when fuel 2 has a high (low) carbon content, this factor matter a lot (little) for the region B tax setting. In consequence, for a "low carbon content" fuel 2, the tax set by region B is increased a lot in the c-a-t case relative to the tax cases; for a "high carbon content" fuel 2 the region B tax is increased less or perhaps not at all.

These principles are illustrated in the numerical calculations in Tables 2-4, by considering three possible levels for the carbon content of fuel 2 relative to that of fuel 1 (oil): high-carbon (coal); low-carbon (natural gas); and zero-carbon (renewable energy). These numerical examples verify the theoretical results in showing that the exporter is more powerful, and sets its export tax more aggressively, when fuel 2 has a lower carbon content. In the limit case as the carbon content of fuel 2 goes to zero, the c-a-t solution approaches unity, with maximal

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[21] Note, however, that when the interaction effect in fuel demand, represented by the interaction coefficient θ, is sufficiently strong, this effect is weakened, or even eliminated, as consumption of fuel 2 increases in response to the higher export tax on fuel 1.

monopoly power in tax setting to region B, and no power to region A (where, in the limit, the carbon quota price is zero).

To recapitulate, the c-a-t solution is, in "almost all" cases, less attractive to the importing (and climate policy defining) block than either of the tax solutions. This leads the fuel importer (and main consumer) to prefer tax solutions over a c-a-t solution in climate policy, in a wide set of circumstances. The only exceptions are found to be when ex post substitution possibilities under a cap solution are "very good" for the importer (when fuel 2 has a "very high" carbon content), and the fuel exporter at the same time has high aversion (perhaps, unrealistically high) against carbon emissions.

The immediate policy conclusion is that major fuel-importing and -consuming countries should, on balance, choose tax solutions over cap solutions. The main problem with choosing a c-a-t solution is that it leaves them more vulnerable to adverse strategic manipulation of fuel prices by monopolistic exporters. Other implications may be less obvious. One might in particular be tempted to conclude that the importing block ought to maintain a substantial output of high-carbon fuel (coal), so as to diminish the strategic pricing power of fuel exporters, thus making a c-a-t solution "optimal". But this would also have unwanted side effects including that of increasing overall carbon emissions. While not studied here, a further analysis of such cases is clearly warranted for future work.

This paper takes my previous analysis of tax and cap behavior by major fuel-consuming and exporting blocks, in Strand (2009a), several steps ahead. Many issues are however left unaddressed. First, the model is rigid in its sharp separation between fuel consumers and exporters; and in assuming full cooperation within each block of countries and no cooperation across blocks. My initial conjecture is that relaxing these assumptions need not alter the model fundamentally; in particular, it should not to alter the derived balance between tax and cap solutions.[22] Without full policy coordination, NE tax and cap solutions will however be affected when these assumptions are altered. In particular, equilibrium taxes will be set lower, which is more realistic and potentially also globally more efficient (when overall fuel taxes exceed the Pigou level, as here, they are globally excessive, and unrealistically so). A further step toward realism would entail importer countries themselves producing some of fuel 1 (oil). I conjecture that such extensions would have fewer fundamental implications, although it remains to be studied.

More crucial is perhaps the assumption of a static model. Dynamism can be introduced in at least two ways: through an inter-temporal budget constraint of exporters (who have a fixed total amount of fossil fuels to be sold); and through an inter-temporal carbon emissions constraint whereby a given emissions cap is assumed to be valid for an extended period of time. As noted in the introduction some of the related literature is dynamic; first, the seminal Bergstrom (1982) paper; and later, Rubio and Escriche (2001), Salo and Tahvonen (2001), Liski and Tahvonen (2004), and Wei (2009). Generally, these papers consider dynamic games of rent extraction between unified exporting and importing blocks, for a given amount of fuel (typically interpreted as oil) to be extracted in finite time. The general conclusion from these papers is that importer taxes are highly efficient in extracting exporter rents, and more than in a static context. Wei's (2009) model contains an extension to a case where the exporter sells part of its fuel in its own domestic market. He finds that an optimal strategy for the exporter,

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[22] One extension has already been addressed in another companion paper, Strand (2009b), where I assume that the exporter consumes part of its own fuel production. This is shown to not lead to substantial changes in the model, given that the exporter is allowed to differentiate the fuel tax between exports and domestic consumption. The own-consumption tax is then Pigouvian, and the export tax takes the same basic form as derived in the current paper. When the exporter cannot differentiate its own tax on a given fuel, the solution will be affected more fundamentally, as the exporter will then set the tax on fuel exports lower than otherwise. This could be of significance in particular when the importer chooses a c-a-t solution; a more complete analysis will be left for future work.

in response to an optimal importer tax, is to induce excessive domestic fuel consumption (through a lower domestic fuel price), in order to optimally increase the fuel export price.[23]

These existing papers, dealing with dynamic aspects, however do not study the two main new issues treated in the current paper, namely the inclusion of a second fuel, and analysis of a c-a-t policy for the fuel importer. An obvious topic for future research is to embed these two extensions into a dynamic framework. Without fully prejudging results, I conjecture that the main result here, that taxes are preferable over c-a-t solutions for fuel importers, survives; there seems immediately to be no way for the importing block to effectively extract producer rent when a c-a-t solution is chosen, even in a dynamic context.

Several features related to the strategic model setup could also conceivably be altered in future work. One would be to assume that the fuel importer (here, region A) maintains a cap on overall carbon emissions, but in addition sets a tax on imported fuels so as to extract part of the producer surplus.[24] Then the fuel tax however becomes a very visible strategic import tax, and its objective clear, namely as a device for rent extraction; this could for obvious reasons clash with WTO rules.[25] Another setup would be to consider the exporter as a Stackelberg leader in the tax-setting game (as is common when one thinks of OPEC as the main strategic fuel exporter), and let the importer react by setting either a (set of) tax(es) or a carbon cap. Alternatively, "OPEC" could be given further strategic powers by being allowed to set both export price and quantity independently; this is not totally unrealistic when most of OPEC's output stems from state-owned oil companies. Results are then different: Strand (2009a) the in particular shows that more of the rent can then be extracted by the exporter.

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[23] See Strand (2009b) for a related static model formulation.

[24] This point was suggested to me by Michael Toman.

[25] One could here of course argue that the import tax simply retaliates against the export tax and thus cannot itself be sanctioned. But a problem is that the export tax may be hidden via producer taxes or other exporter rent extraction schemes, that may make it difficult to verify by outside observers.

Another relevant extension would be to assume, as done by Eichner and Pethig (2009) among others, that not the entire world, and thus not all global emissions, are covered by either emissions taxes or caps. This would open up for discussing policy-relevant issues such as interregional carbon leakage, and scope for offset policies, within my basic model framework.

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Technical Appendix: Analytics

Basics

Utility for region A related to fuel consumption, not counting externality values, is given by

(1) $$V(A) = a_1R_1 - \frac{1}{2}\gamma_1 R_1^{\,2} + a_2R_2 - \frac{1}{2}\gamma_2 R_2^{\,2} - \theta R_1R_2 - (p_1 + t_1)R_1 - (p_2 + t_2)R_2$$

where p_1 and p_2 are producer prices of fuels 1 and 2, and t_1 and t_2 are taxes on these fuels imposed by country A. Maximizing V(A) with respect to R_1 and R_2 yields the following set of first-order conditions:

(2) $$\frac{dV(A)}{dR_1} = a_1 - \gamma_1 R_1 - \theta R_2 - (p_1 + t_1) = 0$$

(3) $$\frac{dV(A)}{dR_2} = a_2 - \gamma_2 R_2 - \theta R_1 - (p_2 + t_2) = 0\,.$$

(2)-(3) lead to the following solutions for R_1 and R_2 in terms of the p_i and t_i:

(A1) $$R_1 = \frac{1}{\gamma_1\gamma_2 - \theta^2}[\gamma_2(a_1 - p_1 - t_1) - \theta(a_2 - p_2 - t_2)]$$

(A2) $$R_2 = \frac{1}{\gamma_1\gamma_2 - \theta^2}[\gamma_1(a_2 - p_2 - t_2) - \theta(a_1 - p_1 - t_1)]\,.$$

A basic stability condition is $\gamma_1\gamma_2 - \theta^2 > 0$, which amounts to the interaction term in (1) not dominating direct quadratic terms.

Fuel 1 is produced by region B (but consumed entirely in region A), by competitive firms with collective profit function

(4) $$\Pi_1(P) = (p_1 - s)R_1 - p_{01}R_1 - \frac{1}{2}\phi_1 R_1^{\,2}$$

Maximizing (4) with respect to R_1 yields the first-order condition

(5) $$p_1 - s - p_{01} - \phi_1 R_1 = 0\,.$$

Producers of fuel 2 are located in region A, and are assumed to pay no fuel taxes (all taxation of this fuel is on consumption). Their collective profit function takes the form

$$\Pi_2 = p_2 R_2 - p_{02} R_2 - \frac{1}{2}\phi_2 R_2^{\,2} \tag{6}$$

Their first-order condition is, maximizing Π_2 with respect to R_2:

$$p_2 - p_{02} - \phi_2 R_2 = 0\,. \tag{7}$$

We now solve for p_1 from (A1) and (5), and for p_2 from (A2) and (7). We then arrive at the following two-equation system, which solves for R_1 and R_2:

$$(\gamma_1\gamma_2 + \gamma_2\phi_1 - \theta^2)R_1 - \theta\phi_2 R_2 = \gamma_2(a_1 - s - t_1 - p_{01}) - \theta(a_2 - t_2 - p_{02}) \tag{A3}$$

$$(\gamma_1\gamma_2 + \gamma_1\phi_2 - \theta^2)R_2 - \theta\phi_1 R_1 = \gamma_1(a_2 - t_2 - p_{02}) - \theta(a_1 - s - t_1 - p_{01}) \tag{A4}$$

We find the following solutions:

$$R_1 = \frac{(\gamma_2 + \phi_2)(a_1 - s - t_1 - p_{01}) - \theta(a_2 - t_2 - p_{02})}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2} \tag{A5}$$

$$R_2 = \frac{(\gamma_1 + \phi_1)(a_2 - t_2 - p_{02}) - \theta(a_1 - s - t_1 - p_{01})}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2}\,, \tag{A6}$$

as well as the following derivatives:

$$\frac{dR_1}{dt_1} = \frac{dR_1}{ds} = -\frac{\gamma_2 + \phi_2}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2} \tag{A7}$$

$$\frac{dR_1}{dt_2} = \frac{dR_2}{dt_1} = \frac{dR_2}{ds} = \frac{\theta}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2} \tag{A8}$$

$$\frac{dR_2}{dt_2} = -\frac{\gamma_1 + \phi_1}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2}\,. \tag{A9}$$

Producer prices are found, from (5), (7), (A6) and (A7), as follows (noting that consumer prices are given by $p_i + t_i$):

(A10) $$p_1 = \frac{(\gamma_2+\phi_2)\phi_1(a_1-t_1)+[\gamma_1(\gamma_2+\phi_2)-\theta^2](s+p_{01})-\phi_1\theta(a_2-t_2-p_{02})}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A11) $$p_2 = \frac{-\phi_2\theta(a_1-s-t_1-p_{01})+(\gamma_1+\phi_1)\phi_2(a_2-t_2)+[\gamma_2(\gamma_1+\phi_1)-\theta^2]p_{02}}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}.$$

Nash Equilibrium Taxes for Region A

Define the objective function of region A:

(8) $$W(A) = a_1R_1 - \frac{1}{2}\gamma_1R_1^2 + a_2R_2 - \frac{1}{2}\gamma_2R_2^2 - \theta R_1R_2 - p_1R_1 - p_{02}R_2 - \frac{1}{2}\phi_2R_2^2 - c_1R_1 - c_2R_2 .$$

Region A here takes s as given in setting the t_i to maximize W(A). The two resulting first-order conditions for region A have the forms:

(A12) $$\frac{dW(A)}{dR_1}\frac{dR_1}{dt_i} + \frac{dW(A)}{dR_2}\frac{dR_2}{dt_i} + \frac{dW(A)}{dp_1}\frac{dp_1}{dt_i} + \frac{dW(A)}{dp_2}\frac{dp_2}{dt_i} = 0; i = 1,2.$$

Using (2), (3) and (7), we find the following partial derivatives of W(A):

(A13) $$\frac{dW(A)}{dR_1} = t_1 - c_1; \frac{dW(A)}{dR_2} = t_2 - c_2; \frac{dW(A)}{dp_1} = -R_1; \frac{dW(A)}{dp_2} = 0 .$$

The two relations (A12) give rise to the two-equation system

(A14) $$-(t_1-c_1)(\gamma_2+\phi_2)+(t_2-c_2)\theta + R_1\phi_1(\gamma_2+\phi_2) = 0$$

(A15) $$(t_1-c_1)\theta-(t_2-c_2)(\gamma_1+\phi_1) - R_1\phi_1\theta = 0 .$$

Define $t_1^* = t_1 - c_1$, $t_2^* = t_2 - c_2$. (A14)-(A15) now solve for t_1^* and t_2^* in terms of R_1 as follows:

(9) $$t_1^* = \phi_1 R_1$$

(10) $$t_2^* = 0 .$$

Using (5), (9) can be written as

(9a) $$t_1^* = p_1 - p_{01} - s .$$

Nash Equilibrium Tax for Region B

The objective function of region B is

(11) $$W(B) = p_1 R_1 - p_{01} R_1 - \frac{1}{2}\phi_1 R_1^{\,2} - \alpha c_1 R_1 - \alpha c_2 R_2$$

The first-order condition for the region B with respect to s has the form

(A16) $$\frac{dW(B)}{dR_1}\frac{dR_1}{ds} + \frac{dW(B)}{dR_2}\frac{dR_2}{ds} + \frac{dW(B)}{dp_1}\frac{dp_1}{ds} + \frac{dW(B)}{dp_2}\frac{dp_2}{ds},$$

where (using (5))

(A17) $$\frac{dV(B)}{dR_1} = s - \alpha c_1; \frac{dV(B)}{dR_2} = -\alpha c_2; \frac{dV(B)}{dp_1} = R_1; \frac{dV(B)}{dp_2} = 0.$$

From (A7)-(A8), (A10)-(A11), and (A16)-(A17),

(12) $$s^* = \left(\gamma_1 - \frac{\theta^2}{\gamma_2 + \phi_2}\right) R_1 - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2.$$

Overall NE Tax Setting in Model 1

The overall equilibrium tax levels t_1 and s (recognizing that $t_2^* = 0$) are determined jointly with R_1 from (A5), (9) and (12). We find:

(A18) $$R_1 = \frac{1}{2}\frac{(\gamma_2 + \phi_2)(a_1 - p_{01} - c_1 - \alpha c_1) - \theta(a_2 - p_{02} - c_2)}{(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2}$$

(13) $$t_1 = \frac{[(2\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - 2\theta^2]c_1 + \phi_1(\gamma_2 + \phi_2)(a_1 - p_{01} - \alpha c_1) - \phi_1\theta(a_2 - p_{02} - c_2)}{2[(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2]}$$

(14) $$s = \frac{[(\gamma_1 + 2\phi_1)(\gamma_2 + \phi_2) - \theta^2]\alpha c_1 + [\gamma_1(\gamma_2 + \phi_2) - \theta^2]\left(a_1 - p_{01} - c_1 - \frac{\theta}{\gamma_2 + \phi_2}(a_2 - p_{02} - c_2)\right)}{2[(\gamma_1 + \phi_1)(\gamma_2 + \phi_2) - \theta^2]} - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2$$

For the aggregate tax $z_1 = t_1 + s$ we find

(15) $$z_1 = \frac{1}{2}(c_1 + \alpha c_1 + a_1 - p_{01}) - \frac{1}{2}\frac{\theta}{\gamma_2 + \phi_2}(a_2 - p_{02} - c_2) - \frac{\theta}{\gamma_2 + \phi_2}\alpha c_2 .$$

Nash Equilibrium Taxation With a Common Carbon Tax in Region A (Model 2)

The general expressions for the R_i and p_i are in this case

(A19) $$R_1 = \frac{-[(\gamma_2+\phi_2)c_1 - \theta c_2]q + (\gamma_2+\phi_2)(a_1 - s - p_{01}) - \theta(a_2 - p_{02})}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A20) $$R_2 = \frac{-[(\gamma_1+\phi_1)c_2 - \theta c_1]q + (\gamma_1+\phi_1)(a_2 - p_{02}) - \theta(a_1 - s - p_{01})}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A21) $$p_1 = \frac{-[(\gamma_2+\phi_2)c_1 - \theta c_2]\phi_1 q + (\gamma_2+\phi_2)\phi_1 a_1 - \phi_1\theta(a_2 - p_{02}) + [\gamma_1(\gamma_2+\phi_2)-\theta^2](s + p_{01})}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A22) $$p_2 = \frac{-[(\gamma_1+\phi_1)c_2 - \theta c_1]\phi_2 q - \phi_2\theta(a_1 - s - p_{01}) + (\gamma_1+\phi_1)\phi_2 a_2 + [\gamma_2(\gamma_1+\phi_1)-\theta^2]p_{02}}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2} .$$

Effects of changes in the common region A emissions tax q, valid for both sectors, are

(A23) $$\frac{dR_1}{dq} = -\frac{(\gamma_2+\phi_2)c_1 - \theta c_2}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A24) $$\frac{dR_2}{dq} = -\frac{(\gamma_1+\phi_1)c_2 - \theta c_1}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A25) $$\frac{dp_1}{dq} = -\frac{[(\gamma_2+\phi_2)c_1 - \theta c_2]\phi_1}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2}$$

(A26) $$\frac{dp_2}{dq} = -\frac{[(\gamma_1+\phi_1)c_2 - \theta c_1]\phi_2}{(\gamma_1+\phi_1)(\gamma_2+\phi_2)-\theta^2} .$$

To study tax setting, start with region A, which now sets one common tax rate q per unit of carbon emissions. W(A) is given by (8), while the first-order condition has the form

(A27) $$\frac{dW(A)}{dR_1}\frac{dR_1}{dq}+\frac{dW(A)}{dR_2}\frac{dR_2}{dq}+\frac{dW(A)}{dp_1}\frac{dp_1}{dq}+\frac{dW(A)}{dp_2}\frac{dp_2}{dq}=0$$

In this case,

(A28) $$\frac{dW(A)}{dR_1}=c_1q-c_1;\frac{dW(A)}{dR_2}=c_2q-c_2;\frac{dW(A)}{dp_1}=-R_1;\frac{dW(A)}{dp_2}=0$$

The first-order condition for region A can now be found as

(A29) $$(-c_1q^*+R_1)[(\gamma_2+\phi_2)c_1-\theta c_2]-c_2q^*[(\gamma_1+\phi_1)c_2-\theta c_1]=0,$$

where q* = q-1 is the tax per unit of emissions in excess of the Pigou level of unity.

Consider in the continuation the simplified case with no demand interaction (θ = 0). The two equations solving for q and s then simplify to

(A30) $$(1+\phi_1{}^*c_{r1})c_1q+\phi_1{}^*c_{r1}s=\phi_1{}^*c_{r1}(a_1-c_1-p_{01})+c_1$$

(A31) $$(1-\phi_1{}^*)c_1q+(2-\phi_1{}^*)s=(1-\phi_1{}^*)(a_1-c_1-p_{01})+\alpha c_1,$$

where we have defined $c_{r1}=\frac{c_1c_1{}^*}{c_1c_1{}^*+c_2c_2{}^*}$, with $c_i{}^*=\frac{c_i}{\gamma_i+\phi_i}$, for i = 1,2.

The solutions for q and s are

(18) $$c_1q=\frac{c_{r1}\phi_1{}^*(a_1-p_{01}-\alpha c_1)+(2-\phi_1{}^*)c_1}{2-\phi_1{}^*\left(1-c_{r1}\right)}$$

(19) $$s=\frac{(1-\phi_1{}^*)(a_1-p_{01}-c_1)+(1+c_{r1}\phi_1{}^*)\alpha c_1}{2-\phi_1{}^*(1-c_{r1})}.$$

The aggregate tax on fuel 1 is in this case

(20) $$z_1=c_1q+s=\frac{[1-\phi_1{}^*(1-c_{r1})](a_1-p_{01})+c_1+\alpha c_1}{2-\phi_1{}^*(1-c_{r1})}.$$

Nash Equilibrium With Cap Policy in Region A

Region B can in this case be viewed as facing the constraint

(21) $$C=c_1R_1+c_2R_2$$

whereby R_1 and R_2 are related, for given C. Region B maximizes W(B) with respect to s, subject to (21), facing a constant C. (21) takes the form

(A32) $$c_1 \frac{a_1 - s - c_1 q - p_{01}}{\gamma_1 + \phi_1} + c_2 \frac{a_2 - c_2 q - p_{02}}{\gamma_2 + \phi_2} = C$$

Differentiating (A32) with respect to s yields

(A33) $$\frac{dc_1 q}{ds} = -c_{r1}.$$

We now find, using (A33):

(A34) $$\frac{dR_1}{ds} = -\frac{1}{\gamma_1 + \phi_1}(1 - c_{r1}), \frac{dR_2}{ds} = \frac{1}{\gamma_2 + \phi_2}\frac{c_2}{c_1}c_{r1}$$

(A10) in this case (with θ=0) takes the form

(A10a) $$p_1 = \frac{\phi_1 (a_1 - c_1 q) + \gamma_1 (s + p_{01})}{(\gamma_1 + \phi_1)}.$$

Differentiating (A10a) yields

(22) $$\frac{dp_1}{ds} = 1 - \phi_1 * (1 - c_{r1}).$$

Maximizing (11) with respect to s now yields

(A35) $$-(p_1 - p_{01} - \phi_1 R_1)\frac{1}{\gamma_1 + \phi_1}(1 - c_{r1}) + R_1 \frac{dp_1}{ds} = 0$$

Since the cap C on emissions is chosen by region A, region B takes C as a constant in its own optimization. Thus region B's policy has no influence on carbon emissions. The two last terms in (11) then drop out. We derive the following condition:

(A36) $$\left[1 - \phi_1 * (1 - c_{r1})\right] c_1 q * + \left[1 + (1 - \phi_1 *)(1 - c_{r1})\right] s = \left[1 - \phi_1 * (1 - c_{r1})\right](a_1 - c_1 - p_{01}).$$

(A30) and (A36) now solve for c_1q^* (= $c_1(q-1)$) and s* (= s-αc_1), as follows:

(23) $$c_1 q^* = \phi_1 * c_{r1} \frac{1 - c_{r1}}{2 - c_{r1} - \phi_1 * (1 - c_{r1})^2}(a_1 - c_1 - p_{01})$$

(24) $$s^* = \frac{1-\phi_1^*(1-c_{r1})}{2-c_{r1}-\phi_1^*(1-c_{r1})^2}(a_1 - c_1 - p_{01}) - \alpha c_1$$

The sum of the exporter tax and the importer quota price (the "effective tax" on fuel 1) is

(25) $$z_1^* = c_1 q^* + s^* = \frac{1-\phi_1^*(1-c_{r1})^2}{2-c_{r1}-\phi_1^*(1-c_{r1})^2}(a_1 - p_{01} - c_1) - \alpha c_1 .$$