An Empirical Recursive-Dynamic General Equilibrium Model of Poland's Economy

Including Simulations of the Labor Market Effects of Key Structural Fiscal Policy Reforms

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NBP
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Abstract

This publication documents the construction of an empirical recursive-dynamic computable general equilibrium (CGE) model of Poland's economy. It also describes in more detail how the social accounting matrix was assembled. Further, it presents a series of illustrative policy simulations using various model closures (neo-classical, "Keynesian" and mixed). Simulated policies were designed to enhance employment and consisted of a reduction in the tax wedge on labor compensated by a budget-neutral increase in indirect taxes (Scenario 1) or reduction in social transfers (Scenario 2). The dynamic properties of sequenced reform package implemented over the period 2006-07 was also examined.

The policy simulations suggest that the situation in the labor market in Poland can be improved considerably through such a policy mix, especially when a reduction of the tax wedge is accompanied by a reduction in transfers that enhance labor supply incentives (Scenario 2). Importantly, on average the disposable income of poor households is not negatively affected although certain transfer dependent groups (such as pensioners) are and income inequality increases in favor of non-poor households. Further, the impact of this policy mix on participation and employment of poor households is more positive than when indirect taxes are increased to offset the budget impact of a lower tax wedge.

Foreword

This report concludes approximately one year of work by a team of Polish economists, representing three institutions (National Bank of Poland, Ministry of Economy and Labor and Ministry of Finance) working together with Dr. Peter Griffin (World Bank consultant) on a joint project to build a computable general equilibrium (CGE) model for Poland.

From the very beginning, two objectives of the projects were highlighted: a multi-institutional setup, to elaborate a common modeling framework for main economic policy institutions in Poland; and to build a model that would allow for addressing key structural policy issues, including fiscal and labor market reforms.

The results of the joint work presented here confirm that both objectives have been met to a satisfactory degree. As for the first one, all together 10 participants from the institutions involved and from the World Bank took part in the project. The project involved not only training in CGE modeling but an active model building exercise followed by economic policy simulations. I am pleased to say that the National Bank of Poland economists were those who led the Polish side of the project.

As for the results, the authors present below an extensive documentation of the model, it's data-base and the results of the simulations. The model provides a quite detailed description of the economy, in terms of production sector (39 industries), labor market (3 categories of labor), households (10 groups) and also taxes (social contributions, taxes on production, CIT, PIT, VAT, excises, and import tariffs). Together with properly modeled behavior of the agents it allows for simulating the effects of important fiscal and labor market reforms.

The authors present and discuss the results of a series of simulations that are aimed at examining how different combinations of tax and social transfer reforms affect the situation in the labor market and are consistent with fiscal viability. The main result may be summarized as follows: properly designed fiscal reform can substantially improve the situation in the labor market if it affects both tax wedge on wages and social transfers, with no deterioration of public finances. This reform package is, according to the results of the model, clearly superior to the one that also introduces cuts in tax wedge but assumes balancing public finance through indirect taxes. Undoubtedly, the former reform package is more difficult to implement than the latter. The results of the simulation leave no doubt that only this difficult reform will lead to significant progress. That seems to be a very important conclusion and contribution to the current debate on labor market issues in Poland.

On behalf of the Polish institutions involved in the project let me express gratitude to the World Bank for funding most of the costs of the project. Dr Peter Griffin did a great job of simultaneously designing the main features of the model, training participants in CGE theory and practice, and learning the peculiarities of the Polish economy.

We all owe very special thanks to Thomas Laursen, Lead Economist for Central Europe and the Baltics of the World Bank. His enthusiasm and continuous support for the project, and also comments to the interim results and final editing of the report, were crucial for the success of the project.

Let me also mention that since the completion of project the model has been extensively used in the NBP, in collaboration with the Ministry of Finance, for a series of major fiscal reform simulations, e.g. flat tax reform and fiscal consolidation related to Poland's euro adoption strategy. Some results were presented at both domestic and international seminars. Flat tax reform scenarios were also discussed with economic experts during the election campaign in autumn 2005. There is ongoing work in collaboration with the Ministry of Finance on further fiscal policy simulations. We have also started the work to refine and improve the model's capabilities, especially with respect to dynamics and imperfect competition. The model has

already proved very useful and there are new plans for its application in simulating structural reforms and generating growth trajectories for the Polish economy.

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Table of Contents

An Empirical Recursive-Dynamic Computable General Equilibrium Model of Poland's Economy

Figures

PART I

Model Description

I. General Features of the CGE Model

I.1. Preamble

Economic models come in many shapes and forms. The type of model to be constructed and its adequacy depends entirely on the tasks envisaged for the model. Economic model building, and economic models in particular, are abstractions from reality. They are designed to provide a simplification of some part or parts of a highly complex economic system. Hence every model is good for addressing some issues but might be inappropriate for addressing others.

One consequence of simplification is that economic models are often criticized for their lack of detail. While it is always easy to identify what has been left out of a model, this in itself is insufficient to discredit the use of the model. The relevant test is whether the omitted factors, if included, would alter the set of conclusions drawn from the original model.

The art of model building is to incorporate sufficient detail to capture the essence of the problem under study while excluding those factors likely to be of lesser importance. It is simply not possible to include all factors likely to influence a particular outcome. Nor would it be sensible to do so. As it is seldom useful to work with maps of the scale 1:1, so there is a trade off between the amount of detail and hence the complexity of the model, and the transparency of the economic mechanisms underlying model results.

I.1. Model Design Features and Previous Applications

Computable General Equilibrium (CGE) models, as the one discussed below, are designed to address three basic types of policy problems. In particular, the model presented here/CGE models will highlight the effects on the level and composition of industry activity and employment, trading performance, household incomes and consumption, macro-economic outcomes and the government's budgetary position of:

1. policies and changes which occur at the sector - industry, commodity - level or at the household level;
2. policies and changes which occur at the macro-economic level; and
3. changes external to the domestic economy but to which it must adjust.

Examples of policies under (1) include taxes on and subsidies to industry production or exports and imports, occupational wage distortions, sector specific technical change and sector specific investments. Examples of policies under (2) include changes in the aggregate level of government spending, a change in the exchange rate, or a change in the economy-wide real wage. Examples of (3) include changes in world import and export prices or in foreign aid flows.

In terms of the academic literature on economic models, the model developed in the following sections may be classified as an economy-wide, recursive-dynamic computable general equilibrium model of the Johansen type. Each of these descriptors is considered in turn.

Economy-wide

This label means that the model is comprehensive in that, for a particular level of disaggregation, all sectors of economic activity are included. Projections from the model add up in the sense that outcomes for each sector, appropriately weighted, are equivalent to outcomes for the relevant macro-economic variables. For example, the sum of value added for each sector equals the economy's aggregate output (real GDP), the sum of commodity exports net of the

sum of commodity imports equals the balance of trade, and the sum of employment by industry and/or occupation equals total employment in the economy.

Recursive-dynamic

The term recursive-dynamic reflects that the model has two key elements: a **static model** and an **intertemporal** or **dynamic link**. The detailed static model is solved for a single period's (e.g., one year) general equilibrium, reflecting a variety of constraints and values for exogenous variables. Given the initial position of the economy represented in the data, the assumptions concerning the nature of underlying economic behavior and the specified exogenous constraints, the static equilibrium reflects an optimum solution. In this sense, the solution of the model would be replicated until some exogenous change, such as a new investment project, change in government policy, world prices and/or the composition of the labor force alters economic conditions and causes the allocation of resources to adjust.

Once a static equilibrium is achieved, the intertemporal link "updates" the data reflecting the position of the economy (e.g., capital stocks adjusted for depreciation and new investment), exogenous variables (e.g., world commodity prices) and policy parameters (e.g., changes in trade restrictions as part of a multi-year program of reform). This is accomplished with a series of linkage equations and provides the basis for solving the next period's static model.

An alternative fully dynamic specification, which takes agents' expectations into consideration in a dynamic optimizing framework, can be specified in a second stage of this modeling effort.

Computable general equilibrium

The term computable reflects the numerical specification of the model that can be solved quantitatively with a computer. The term general equilibrium refers to the inclusion of production and demand relationships, the interrelationships between them and the simultaneous determination of prices through the interaction of demand and supply in all markets. As is noted later, however, this does not mean that all markets must clear at their full employment level.

In their treatment of production and demand, CGE models incorporate the conventional features of neo-classical microeconomics. They assume optimizing behavior on the part of producers (profit maximization, cost minimization) and consumers (utility maximization) subject to various constraints in the economy such as the supply of factors of production - labor, capital, land - and restrictions on the government budget, the balance of payment and so on. The resulting equations emphasize the responsiveness of economic agents to changes in relative prices, with the degree of responsiveness imposed dependent of the values assigned to substitution elasticities.

Johansen type

This label refers to the way in which the model is solved. CGE models fall into two groups:

1. models that are solved in the levels of variables; and
2. models that are solved in logarithmic differentials or percentage changes.

There is a long running debate about the relative merits of each type of solution. The debate is to a wide extent artificial, since developments in computer software have made the two types equivalent. The choice of model type employed by modelers is primarily determined by individual tastes.

The type (1) approach produces exact solutions. It requires a flexible solution algorithm that can solve non-linear systems of equations. With the rapid development of computer software in this area this has become much less of a problem than it used to be. The type (2) approach, pioneered by Johansen (1960), is used by the Australian Impact School of CGE modelers and in

the Orani tradition (see Dixon et al. 1982). It involves totally differentiating all equations of the model to achieve a system linear in percentage change of the variables. Simple matrix manipulation methods or linear programming are then used to generate solutions. Compared with the type (1) approach, type (2) has one disadvantage - the results are only linear approximations to the non-linear system and hence are strictly valid only for small changes. Again, developments in computer software code, in particular the use of advanced extrapolation techniques, have made it possible to generate exact solutions. Furthermore, there are a number of advantages - flexible-form functions can readily be specified, and when there are several policy changes under study, the separate effects of each can be decomposed additively. Because of these advantages, the model in this study is designed around the type (2) Johansen approach.

CGE models are not without their own problems and limitations. These are well illustrated by the following quotations from (Shoven, Whalley 1992):

"Most modelers recognize the difficulties of parameter specification and the necessity for assumptions. Elasticity and other key parameter values play a vital role in all model outcomes, and no consensus exists regarding numerical values for most of the important elasticities. The choice of elasticity values is frequently based on scant empirical evidence, and evidence that is often contradictory. This limits the confidence with which model results can be held. On the other hand, there is no clearly superior alternative framework available to policy makers who base their decisions on efficiency and distribution consequences of alternative policy changes. Whether partial equilibrium, general equilibrium, or back-of-the-envelope quantification is used, key parameter values must be selected."

"CGE models are not tested in any meaningful statistical sense. Parameter specifications usually proceed using deterministic calibration, and there is no statistical test of the model specification. In determining parameter values by calibrating to a single year's data observations, equilibrium features in the data are emphasized and the model is constructed in a manner consistent with the observed economy. Econometricians, who are accustomed to thinking in terms of models whose economic structure is simple but whose statistical structure is complex, rather than vice versa, frequently find this a source of discomfort."

The first works of the Polish authors on computable general equilibrium modelling were published in the early 1990s. The pioneer CGE model in Poland was elaborated by Witold Orłowski (1992). The model was focused on tax analyses and it was used for simulating effects of various tax policies, and especially the early simulations of the VAT introduction effects that was a hot topic in Poland during this time. By the mid-1990s, several works of Polish authors in the area of CGE were published. Orłowski's 1992 dynamic (recursive multi-period) CGE model that was used several times (Orłowski 1996, 1998, 2000) to estimate the effects of Poland's accession to EU (focus on agriculture) and some projections for FAO. Roberts and Żółkiewski (1993, 1996) elaborated a series of CGE models focused on income distribution implications of transition. Żółkiewski (1995, 1996, 2001) conducted research on modelling market imperfections within a CGE framework. The model was then used for simulating the introduction of free trade between Poland and EU, EFTA and CEFTA countries. There were also attempts to use CGE models to analyse environment related problems (Kiuiła 1999).

It was during this time as well that the first attempt to elaborate a CGE model for the purpose of the central administration took place. In the context of a joint project of the Central Planning Office (CPO) of Poland and the Central Planning Bureau of the Netherlands that 1994. However, the project was suspended after the pilot version of the model was completed because CPO was dissolved. On the positive side, certain educational effects with respect to CGE modelling within the group of young economists and decision makers was achieved.

During 2001 – 2003, a joint project of the Ministries of Finance of Poland and Finland under the EU Phare scheme resulted in the elaboration of a computable general equilibrium model model for Poland (POLGEM) with an emphasis on public finance applications (Honkatukia and

Vaittinen 2003). The POLGEM model belongs to the ORANI family of computable general equilibrium models (Dixon et al. 1982). The results of the simulations with this model were presented at international conferences (eg., Honkatukia,Vaittinen, Fudała-Poradzińska and Janiak 2003).

I.2. Model Structure

The model is intertemporal in the sense that saving decisions of economic agents affect the future economic outcomes through accumulation of productive capital. Given the recursive structure of the model, the evolution over time can be described as a sequence of single period temporary equilibria. The main characteristics of these equilibria are outlined below – a more technical description of the model is given in the next section.

Production

The model includes 39 production sectors, which represent an aggregation of the whole economy:

1. Agriculture, hunting, forestry logging, and related service activities
2. Fishing, operation of fish hatcheries and fish farms
3. Mining of coal and lignite extraction of peat
4. Mining not elsewhere classified
5. Manufacture of food products and beverages
6. Manufacture of tobacco products
7. Manufacture of textiles
8. Manufacture of wearing apparel, dressing and dyeing of fur
9. Manufacture of leather and leather products
10. Manufacture of wood and wood products
11. Manufacture of pulp
12. Publishing, printing, and reproduction of recorded media
13. Manufacture of coke, refined petroleum products, and nuclear fuel
14. Manufacture of chemicals and chemical products
15. Manufacture of rubber and plastic products
16. Manufacture of other non-metallic mineral products
17. Manufacture of basic metals
18. Manufacture of fabricated metal products except machinery and equipment
19. Manufacture of machinery and equipment
20. Manufacture of office machinery and computers not elsewhere classified
21. Manufacture of electrical machinery and apparatus
22. Manufacture of radio, television, and communication equipment and apparatus
23. Manufacture of medical optical instruments, watches, and clocks
24. Manufacture of motor vehicles, trailers, and semi-trailers

25. Manufacture of other transport equipment
26. Manufacture of furniture and manufacturing not elsewhere classified
27. Electricity, gas, steam, and hot water supply
28. Collection, purification and distribution of water
29. Construction
30. Trade
31. Hotels and restaurants
32. Transport
33. Post and telecommunications
34. Financial intermediation
35. Real estate activities, renting of equipment and computer activities
36. Public administration, defense, and compulsory social security
37. Education
38. Health and social work
39. Other community social and personal service activities, recycling

All 39 sectors are assumed to operate under a constant return to scale technology, i.e. doubling all inputs would result in doubling the output of the sector. The quantities of all inputs are chosen optimally by producers in order to minimize cost given the level of sectoral demand and relative after-tax input prices. Once the optimal combinations of inputs are determined, sectoral output prices are calculated for each period assuming competitive conditions in all markets. Since each sector supplies input to other sectors, the output price of the supplying sector becomes the input price in the receiving sectors. Thus, the optimal combination of inputs is determined simultaneously in all sectors. Producers are assumed to decide whether to supply to the domestic market or to export according to domestic market prices and exogenously given foreign prices.

The assumption of competitive conditions in all markets is only made for convenience at this stage. Other types of market clearing mechanisms, such as monopolistic competition or mark-up pricing, can be incorporated in the framework.

Primary Factors

Four types of primary production factors are identified by the model:

1. Capital
2. Skilled Labor
3. Semi-skilled Labor
4. Un-skilled Labor

Labor factors are assumed to consist of 3 distinct labor types, which are used in all sectors. A single capital factor type is assumed to include all types of assets, such as land, structures and machinery, used in production activities. Institutional and productive sectors are allocated initial holding and use of capital, respectively, according to the return on the capital stock in

2002, our base year. The single capital factor type is assumed to be tradable between the sectors ensuring an equal capital rent in all sectors.

Household Consumption

For households, ten (2*5) representative consumers are specified according to their social economic status:

1. Non-poor
2. Poor

and their employment status:

1. Employees
2. Self-employed
3. Farmers
4. Pensioners
5. Other

Each of the household types is allocated an initial holding of factors. All labor income generated by economic activity is allocated to the households. Physical capital income is split between households, banks, and firms according to their shares in capital ownership. In addition, households receive a part of their income in the form of remittances from abroad as well as transfers from the government. Furthermore, capital income not retained in the banks and firms are transferred as dividends to the households and the external sectors and government according to their ownership in the banks and firms.

The ten representative consumers are allocating their disposable income optimally among goods and services, leisure and savings according to the after-tax prices.

Foreign Trade

The model differentiates between EU and non-EU external sectors. A key feature of CGE trade analysis is the identification of imports using product differentiation. The Armington specification, which stipulates that products are differentiated according to the country of origin or destination, is used to achieve this. With the Armington specification, imported commodities are treated as imperfect substitutes for domestically produced commodities of the same category. In intermediate and final consumption domestic goods and imports are split according to relative prices and preferences. These preferences are derived from a cost minimizing aggregation framework.

The product differentiation is retained on the exports side. This assumption implies that the economy faces a downward sloping demand curve for its products.

Foreign savings (current account deficit) are calculated residually in the model. The difference between the earnings from exports of goods and services, net official and private transfer payments, and outlays on imports of goods and services is equal to the savings inflow.

Government

The government collects taxes on consumer expenditure, indirect taxes on primary production factors and inputs, tariff revenues from imports and excises on some import and domestic production. Taxes influence the decisions of economic agents by changing relative prices and disposable incomes. Tax revenues are endogenous in the model since they depend on the level of economic activity. In addition to tax revenues, the government receives all official foreign transfer inflows. Government expenditure is allocated exogenously between goods and

services and transfers to other institutions. The residual between government revenue and its expenditure is allocated to government savings.

Firms

Part of the physical capital ownership has been allocated to firms. The firms pay corporate income taxes on their income arising from capital rents. The firms retain part of the after-tax income for investment purposes and distribute the remainder as dividends to households and the external sectors according to the ownership of the firms.

Financial institutions

A sector of financial institutions has been introduced into the model in anticipation of a future expansion of the model to incorporate financial flows in addition to the capital and current flows. The banking institution, being the major part of this sector, owns the part of the physical capital used in the banking activity. The banking sector makes and receives transfers to and from other institutions, representing interest payments on deposits and loans respectively. In addition, the banking sector demands banking services from its activity. This has been included to offset the FISIM adjustment made in the underlying dataset and thus ensure a positive return to capital in this sector.

The banking sector pays corporate income taxes on its net-transfer (net-interest) income less its demand for banking services. The remaining after-tax income is either retained for investment or distributed as dividends among the owners of the banks.

Investment

Due to our assumption of one type of non-sector specific capital, which is used in all competitive sectors of the economy, the model need not incorporate any explicit investment behavior by firms. In each period the aggregate level of investment is derived as the sum of household savings, retained earnings of firms and banks, the government net balance and the foreign inflow of capital, deflated by an aggregate price for investment goods. Aggregate investments are translated into demand for goods and services used to produce the investment goods. Investment is added to the existing stock of general capital assets used by the producers and allocated between producers equalizing all competitive sectoral profit rates.

II. Equations of the CGE Model

The SAM discussed in the previous sections provides a schematic portrait of the circular flow of income in the economy: from activities and commodities, to factors of production, to institutions, and back again to activities and commodities. The different accounts in the SAM define the boundaries of an economy-wide model. Specification of a complete model requires that the market, behavioral, and system relationships embodied in each account in the SAM be described in the model. Activity, commodity, and factor accounts all require the specification of market behavior: supply; demand; and clearing conditions. The household and government accounts embody the budget constraints of private households and the public sector budget: income equals expenditure. Finally, the capital and the rest of the world accounts represent the macro-economic requirements for internal balance (savings equal investment) and external balance (exports plus net-capital inflows equal imports).

When applying the SAM it is assumed for simplicity that all prices net of taxes, subsidies and tariffs are normalized to 1 in the base year. In this way, the benchmark data may be assembled in value terms, without the need to specify corresponding volumes.

The presentation of equations of the CGE model follows the pattern of income generation of the SAM. First, the equations describing production and value added generation are presented.

Activities manufacture goods for sale to households, other producers, government, investment or for export. They in turn pay taxes to the government and employ factors of production and intermediate inputs, both domestic and imported. Next are equations describing the mapping of value added into institutional income and equations describing institutional demand and savings. **Households** supply factors of production in return for income, consume domestic and imported goods and make resources available for investment through savings. **Government** collects taxes and borrows from abroad and, for example, invests in infrastructure or subsidizes particular agents or activities. Equations describing the import demand from the **rest of the world** follow, along with equations describing the mapping of intermediate demand into **investment** goods. The circular flow is completed by equations showing the balance between demand and supply of goods by various actors. Finally, there are a number of system equations which the model must satisfy. These include both market clearing conditions and the choice of macro-economic closure for the model.

CGE models normally employ the supply and demand equations derived from the agents' maximization problem, rather than the maximization problems directly. In order to describe the structure of the model, all the equations are initially described in level format without specifying the functional forms. The resulting model in its level format is subsequently linearized, and the advantages of a model specification using the Johansen approach are made apparent.

II.1. Notation

There are many hundreds of equations in the model. As a notational convenience, therefore, **sets** are used widely in the specification of the model. This limits the number of different *types* of equations to a manageable number. All activities are grouped into a set of activities: commodities are grouped into one set of commodities; different household types are grouped into a single household set; the factors of production into a set of factors, etc. The actual number of elements in each of these sets is determined by the dimensions of the model. The different sets used in the model are defined below. In the following presentation of equations, the underlying theoretical equations are written for convenience in general terms with subscripts and superscripts running over the full range of options. In many instances, however, the database contains zero entries so that the relevant variable will simply drop out of the model.

In Table 1, the individual elements of each set are also specified together with the index reference assigned to each set or subset. In specifying equations of the algebraic model, the index assigned to each variable will always be assigned its corresponding set.

Table 1. Definition of Sets

	Set Name	Index	Set Elements
Commodities/Activities			
Production Sectors/Commodities	COMG	m, j	1. *Agriculture, hunting, forestry logging, and related service activities* 2. *Fishing operation of fish, hatcheries, and fish farms* 3. *Mining of coal and lignite extraction of peat* 4. *etc.* … 39. *Other community social and personal service activities*
Capital Types	COMCAP	c	1. *Capital*
Labor Types	COMLAB	p	1. *Skilled Labor* 2. *Semi-skilled Labor*

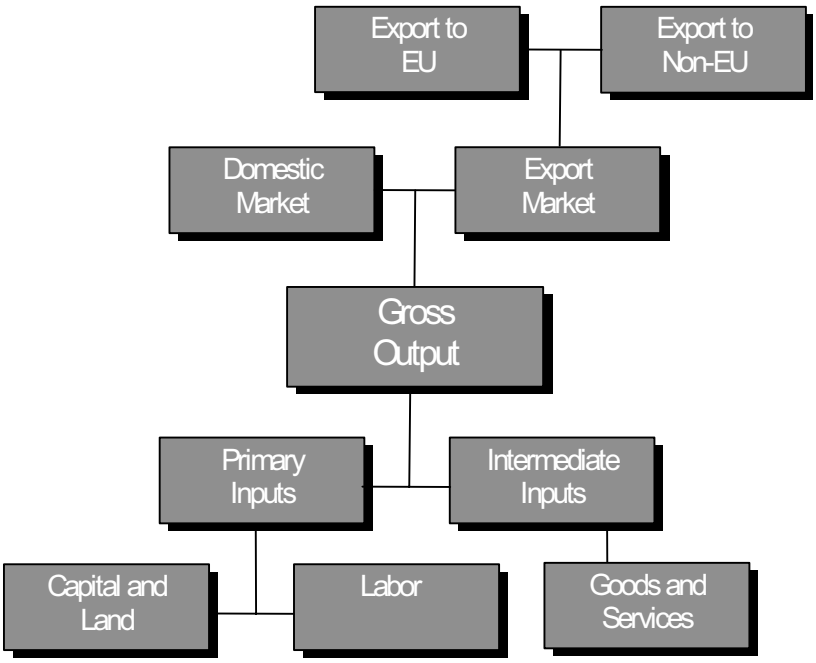
			3. *Unskilled Labor*
All Production Factors	COMF	f	COMCAP + COMLAB
All Commodities	COM	n, l	COMG + COMF
Investment Sector	COMI	k	1. *Aggregate Investment Good*
Institutions			
Households Types	INSTH	h	1. *Non-poor - Employees* 2. *Poor - Employees* 3. *Non-poor - Self-employed* 4. *Poor - Self-employed* 5. *Etc.* ... 10. *Poor - Other*
Government	INSTG	g	1. *Government*
Rest of the World	INSTF	a	1. *EU* 2. *Non-EU*
Banks	INSTB	b	1. *Banks*
Firms	INSTC	r	1. *Firms*
All Institutions	INST	i.e.	INSTH + INSTG + INSTF + INSTB + INSTC
Domestic	INSTD	d	1. *Poland*
All Countries	INSTW	w, o	INSTD + INSTF

II.2. Behavior of Activities - The Production Equations

In the competitive sectors – the activities - the quantities of all primary production factors and all intermediate inputs are chosen in order to minimize production costs given the level of sectoral demand and relative after-tax prices. The production technology has a nested CES (Constant Elasticity of Substitution) structure, which implies separability between different types of input bundles, as demonstrated below. Similarly, the output is split between different destinations - Poland, EU or non-EU markets - using a CET (Constant Elasticity of Transformation) function.

The production sub-model is depicted in its structural form below.

Figure 1. Structure of Production



With these simplifying assumptions it is possible to separate the decisions of producers into different stages. **First**, producers are assumed to choose the mix between a composite intermediate input made up of all types of intermediate goods used in production, and a composite primary input including the different types of primary factors employed. **Second**, the subdivision of the composite primary input among

different labor types and capital is determined. **Third**, the bundle of intermediate inputs is allocated between the types of intermediate goods and services. **Fourth,** once the overall activity level in each sector has been determined, the sales are split between domestic markets and export.

The **final element** in the production sub-model concerns the determination of producer prices. Once an optimal combination of inputs and the output market split are determined, sectoral output prices are calculated assuming competitive supply (i.e., zero-profit) conditions in all markets.

The producer problem

The optimization problems of the producers are given below. First, for each time period *t*, the input mix, V_{mn}^{t}, where *n* refers to the set of commodities and factors (COM) in the production of *m* that in turn refers to the set of commodities (COMG), is chosen to minimize the cost of producing a specific amount of output, Z_m^t [1], subject to the technology represented by the production function. The assumption that the production function is of a CES type implies that in its basic form it is a linear homogenous function that take the form:

$$Z_m = \left(\sum_n \beta_{mn} V_{mn}^{-\sigma_m} \right)^{\frac{-1}{\sigma_m}} \quad m \in COMG, n \in COM$$

Where β_{mn} are cost shares of production and σ_m is the substitution elasticity measuring the ease by which an input can be substituted for another in the production.

The nested characteristic implies that the factor inputs have a different substitution elasticity from that of the intermediate inputs and that the input bundles have a third substitution elasticity as demonstrated below. First, the factors are aggregated into an aggregate factor input, ZF_m, using a CES relationship. Second, the intermediate inputs are aggregated into an aggregated input, ZG_m, and finally the two aggregated inputs are combined into the final production.

$$ZF_m = \left(\sum_n \beta_{mf} V_{mf}^{-\sigma_m^1} \right)^{\frac{-1}{\sigma_m^1}} \quad m \in COMG, f \in COMF$$

$$ZG_m = \left(\sum_n \beta_{mj} V_{mj}^{-\sigma_m^2} \right)^{\frac{-1}{\sigma_m^2}} \quad m \in COMG, j \in COMG$$

$$Z_m = \left(\alpha_1 ZF_m^{-\sigma_m^3} + \alpha_2 ZG_m^{-\sigma_m^3} \right)^{\frac{-1}{\sigma_m^3}} \quad m \in COMG$$

For simplicity we represent this production function by $Z_m^{CES}(\underline{V}^{mn})$ in the following.

<u>Input Demand : Cost Minimisation</u>

$$C_m(\underline{Q}^{mn}, Z_m) = \underset{\underline{V}^{mn}}{Min}\, C_m = \left[\sum_{n \in COM} V_{mn} Q_{mn} \middle| Z_m = A_m Z_m^{CES}(\underline{V}^{mn}) \right] \quad m \in COMG$$

[1] In the following we have omitted the time superscript *t*

Where
C_m *is the cost function for sector m*
V_{mn} *is the input demand for commodity n in sector m*
Z_m *is the gross output/activity level of sector m*
A_m *is a technological progress/factor productivity parameter for sector m*
Q_{mn} *is the prices of input n in sector m*
CES is Constant Elaticity of Substitution

The second problem facing producers is the allocation of sales. We assume that the producers are maximizing their revenue subject to the ease of switching between different destinations, represented by a CET function.

Output Supply : Revenue maximisation

$$R(\underline{P}^{mw}, Z_m) = \underset{\underline{X}^{mw}}{Max}\ R_m = \left[\sum_{w \in INSTW} X_{mw} P_{mw} \middle| Z_m = Z_m^{CET}(\underline{X}^{mw}) \right] \quad m \in COMG$$

Where
R_m *is the revenue function for sector m*
X_{wm} *is the supply of commodity m to destination w*
P_{wm} *is the prices of commodity m at destination w*
CET is Constant Elasticity of Transformation

The following first order conditions result from the two maximization problems. First, the output contingent input demand equations and, second, the output contingent supply functions are given.

First order conditions

Factor and intermediate demand:

1.a $V_{mn} = V_{mn}(\underline{Q}^{mn}) \cdot \dfrac{Z_m}{A_m} \qquad m \in COMG,\ n \in COM$

Domestic and export supplies:

2.a $X_{wm} = X_{wm}(P^{mw}) \cdot Z_m \qquad m \in COMG,\ w \in INSTW$

Where
$\underline{Q}^{mn}$ *is the vector of input prices in occupation m*
$\underline{P}^{mw}$ *is the vector of regional supply prices of commodity m*

The resulting equations show demand to be a function of all primary factor prices, all intermediate input prices, and the level of production. The output split is determined by the relative prices between domestic and foreign prices as well as the total activity level in each sector.

In addition to our assumptions about technology, given by the CES and CET equations, we assume that producers are operating under CRTS (Constant Return To Scale) conditions. With CRTS, profit maximizing behavior does not lead to a unique relationship between an output and its price. If an activity is profitable at a given set of prices, then, for example, doubling output will lead to a doubling of the profit, and there will in general be a large undetermined number of outputs compatible with a given set of prices. In order to constrain the firms we add the zero-profit conditions. This makes perfect economic sense, since activities with a positive profit would be constantly expanding while if the profit was negative they would be forced to close down. Only if the profit level is zero will the CRTS activity be in equilibrium.

Zero-profit condition:

3.a $\sum_{w \in INSTW} P_{wm} X_{wm} = \sum_{n \in COM} Q_{mn} V_{mn} \qquad m \in COMG$

The zero-profit condition shows the revenue of output, which is dependent on relative prices between domestic markets and the rest of the world, to be equal to the cost, which is dependent on all primary factor prices and all intermediate input prices only.

The model specification employed in this study utilizes the **Johansen** method. This method's strength is that no specific functional forms need to be imposed on the framework. The model is expressed in a general structural form and then log-linearized. The outcome is a model specified in relative changes of the variables, in elasticities, and in market shares. The specification method is implemented for the producer sub-model by a first order expansion of the structural forms. Relating these changes to the variables' initial level yields the variables in the form of relative changes as demonstrated below. In the algebraic model description we use *UPPERCASE* letters to identify level variables and *lowercase* letters to identify relative changes.

For example:

$$\frac{V_{jn}^{t+1} - V_{jn}}{V_{jn}} = v_{jm} \quad j \in COMG, n \in COM$$

Factor and intermediate demand equations:

$$dV_{mn} = \sum_{l \in COM} \frac{\partial V_{mn}}{\partial Q_{ml}} dQ_{ml} + \frac{\partial V_{mn}}{\partial \left(\frac{Z_m}{A_m} \right)} d\left(\frac{Z_m}{A_m} \right) \qquad m \in COMG,\ n,l \in COM$$

$\Updownarrow$

$$\frac{dV_{mn}}{V_{mn}} = \sum_{l \in COM} \frac{\partial V_{mn} Q_{ml}}{\partial Q_{ml} Vmn} \frac{dQ_{ml}}{Q_{ml}} + \frac{\partial V_{mn} \left(\frac{Z_m}{A_m} \right)}{\partial \left(\frac{Z_m}{A_m} \right) V_{mn}} \left(\frac{dZ_m}{Z_m} - \frac{dA_m}{A_m} \right) \qquad \text{given CRTS}: \frac{\partial \mathrm{V}_{\mathrm{mn}} \left(\frac{Z_m}{A_m} \right)}{\partial \left(\frac{Z_m}{A_m} \right) V_{mn}} = 1$$

$\Updownarrow$

1.b $v_{mn} = \sum_{l \in COM} \lambda_{nl}^{m} q_l^{m} + z_m - a_m \qquad m \in COMG,\ n,l \in COM$

Domestic and export supplies:

$$dX_{mw} = \sum_{o \in INSTW} \frac{\partial X_{mw}}{\partial P_{mo}} dP_{mo} + \frac{\partial X_{mw}}{\partial Z_m} dZ_m$$

$\Updownarrow$

$$\frac{dX_{mw}}{X_{mw}} = \sum_{o \in INSTW} \frac{\partial X_{mw}}{\partial P_{mo}} \frac{P_{mo}}{X_{mw}} \frac{dP_{mo}}{P_{mo}} + \frac{\partial X_{mw}}{\partial Z_m} \frac{Z_m}{X_{mw}} \frac{dZ_m}{Z_m}$$

$\Updownarrow$

2.b $x_{mw} = \sum_{o \in INSTW} \lambda_{wo}^{m} p_{om} + z_m \qquad m \in COMG, w \in INSTW$

Zero-profit conditions:

$$R_m(\underline{P^{wm}}, Z_m) = C_m(\underline{Q^{mn}}, Z_m)$$

$$\Updownarrow$$

$$\sum_{w \in INSTW} \frac{\partial R_m}{\partial P_{wm}} dP_{wm} + \frac{\partial R_m}{\partial Z_m} dZ_m = \sum_{w \in INSTW} \frac{\partial C_m}{\partial Q_{mn}} dQ_{mn} + \frac{\partial C_m}{\partial Z_m} dZ_m$$

$$\Updownarrow$$

$$\sum_{w \in INSTW} X_{mw} P_{wm} p_{wm} + \frac{R_m}{Z_m} dZ_m = \sum_{w \in INSTW} V_{mn} Q_{mn} q_{mn} + \frac{C_m}{Z_m} dZ_m$$

$$3.b \quad \sum_{w \in INSTW} \frac{X_{mw} P_{mw}}{R_m} p_{wm} = \sum_{n \in COM} \frac{V_{mn} Q_{mn}}{C_m} q_{mn} \qquad m \in COMG$$

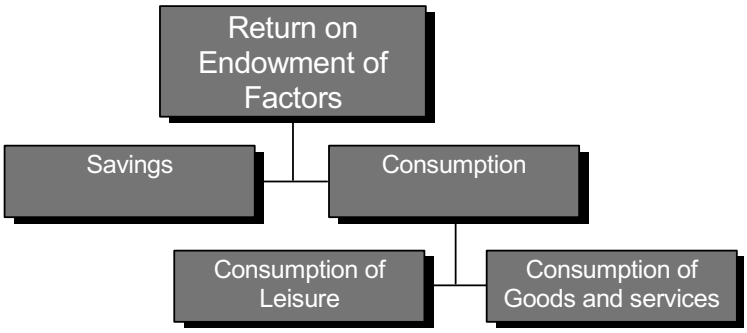
Where lowercase letters correspond to relative changes in uppercase variables
λ_{nk}^{m} = *own and cross price elasticities between input n and l in occupation m*
λ_{ow}^{m} = *own and cross price elasticities between origion o and w in activity m*
CRTS = Constant Return to Scale

The resulting relative changes in the primary factor and intermediate input demands for each sector are dependent on the sum of relative changes in factor and intermediate input prices multiplied by the own and cross price-elasticities and on the relative changes in production levels. An increase in technical progress has a negative impact on intermediate demand. It results in less inputs being needed to produce the same amount of a given level of output. The relative change in the price of output in each sector is seen to be equal to the weighted sum of changes in all factor and intermediate input prices, with weights being the inputs' cost share in the total production costs. The procedure for calibrating the price elasticities is shown in section III.

II.3. Behavior of Institutions - Household/Consumer Groups

Each of the household types is given an initial allocation of physical capital, of ownership of firms, and of the three different types of labor. For each household type, a representative consumer is assumed to allocate his disposable income optimally among goods and services, leisure and savings. The structure of household demand is depicted in Figure 2.

Figure 2. Structure of Household Demand



The consumption or savings decisions are static. Savings are determined by a fixed propensity to save out of disposable income. Given the level of disposable income devoted to consumption, consumers choose an optimal mix of leisure and goods and services. At this stage, the model of consumer demand is sufficiently flexible to allow for substitution as well as different income elasticities across leisure and goods and services.

The optimization problems of the households are given below. The demand mix, D_{hn} , is chosen to maximize the utility for a specific level of disposable income net of savings, EY_h . The demand vector runs over the whole commodity index COM indicating that own consumption of labor, leisure, is included in the demand mix.

The consumer problem

Commodity Demand : Utility Maximization

$$V_h(\underline{Q^{hn}}, EY_h) = \underset{\underline{D}^{hn}}{Max}\, U_h = \left[U_h^{CES}(\underline{D}^{hn}) \middle| EY_h = \sum_{n \in COM} Q_{hn} D_{hn} \right] \quad h \in \text{INSTH}, n \in \text{COM}$$

Where
V_h *is the indirect utility function of household h*
$\underline{Q^{hn}}$ *is the vecor of after - tax consumer prices facing household h*
U_h *is the standard utility function of household h*
$\underline{D^{hn}}$ *is the vector of household h' s demand for commodities and leisure*

Below the first order conditions give the household demand functions:

First order conditions

Household commodity and leisure demand equations:

4.a $D_{hn} = D_{hn}(\underline{Q^{hn}}, EY_h) \quad h \in INSTH, n \in COM$

In order to calculate the part of the income available for consumption, the household taxable income is defined below.

Household taxable income:

5.a $YT_h = \sum_{f \in COMF} W_{hf} S_{hf} + \sum_{i \in INST} PTR_{ih} \quad h \in INSTH$

Where
YT_h *is the taxable income of household h*
W_{hf} *is the after - social contribution wage rates of factor f paid to household h*
S_{hf} *is the supply of factor f sold by household h*
PTR_{if} *is the transfers paid by institution i to household h*

The taxable income is given as the sum of labor and capital incomes (net of social contributions) and transfers from other institutions – e.g. remittances from abroad, interest on deposits, dividend payments, government transfers not dependable of household *h*'s employment status, etc.

The monetary disposable income of household *h* is defined as:

Household monetary disposable income:

6.a $YD_h = (1 - IT_h) YT_h - \sum_{i \in INST} PTR_{hi} \quad h \in INSTH$

Where
YD_h *is the monetary disposable income of household h*
IT_h *is the income tax rate facing household h*
PTR_{hi} *is the transfers paid by household h to institution i*

Household savings are assumed to be given as an exogenous saving rate times the monetary disposable income.

Household savings:

7.a $TS_h = SS_h YD_h \quad h \in INSTH$

Where
TS_h *is the savings of household h*
SS_h *is the saving rate of houshold h out of the monetary disposable income*

The disposable income including the value of own consumption of labor factors, i.e. leisure, is defined as the sum of the monetary disposable income and the after-tax wage rate times the leisure demand:

Household disposable income (incl. leisure value):

8.a $ENY_h = \sum_{p \in COMLAB} W_{hp} D_{hp} + YD_h \quad h \in INSTH$

Where
ENY_h *is household h's disposable income, including value of own consumption of labor facotrs (leisure)*
W_{hp} *is the after-tax wage rate of factor p for houshold h*
D_{hp} *is household h's demand for leisure of labor type p*

The disposable income net of saving available for consumption is defined below.

Total household consumption:

9.a $EY_h = ENY_h - TS_h \quad h \in INSTH$

The household labor supply is given as the labor endowment of each type of labor for each household less its own voluntary consumption and involuntary consumption (unemployment) of labor:

Household labor supply:

10.a $S_{hd} = L_{hp} - (D_{hp} + LU_{hp}) \quad h \in INSTH, p \in COMLAB$

Where
L_{hp} *is household h's endowment of labor type p*
LU_{hp} *is household h's unemployment of labor type p*

The resulting equations for household demand are functions of commodity prices and the disposable income available for consumption. These equations show income to be generated through the sale of primary factors and transfers. Savings are fixed shares of the households' disposable incomes. The linear specification of the household sub-model is performed in a fashion similar to the procedure used in the production sub-model.

Household demand equations:

$$dD_{hn} = \sum_{l \in COM} \frac{\partial D_{hn}}{\partial Q_{hl}} dQ_{hl} + \frac{\partial D_{hn}}{\partial EY_h} dEY_h \qquad h \in INSTH,\ n,l \in COM$$

$$\Updownarrow$$

$$\frac{dD_{hn}}{D_{hn}} = \sum_{l \in COM} \frac{\partial D_{hn} Q_{hl}}{\partial Q_{hl} D_{hn}} \frac{dQ_{hl}}{Q_{hl}} + \frac{\partial D_{hn} EY_h}{\partial EY_h D_{hn}}$$

$$\Updownarrow$$

4.b $d_{hn} = \sum_{l \in COM} \varepsilon_{nl}^h q_{hl} + \gamma_{hn} ey_h \qquad h \in INSTH, n \in COM$

Where
ε_{nl}^h = *household h's own and cross price elasticities between commodity n and l*
γ_{hn} = *household h's income elasticity for commodity n*

The resulting changes in demand are dependent on the relative changes in after-tax consumer prices (multiplied by the own and cross price elasticities) and the relative income changes (multiplied by the income elasticities).

Household taxable income:

$$dYT_h = \sum_{f \in COMF} (W_{hf} dS_{hf} + dW_{hf} S_{hf}) + \sum_{i \in INST} dPTR_{ih} \quad h \in INSTH$$

$$\Updownarrow$$

$$\frac{dYT_h}{YT_h} = \sum_{f \in COMF} \frac{W_{hf} S_{hf}}{YT_h} \left(\frac{dS_{hf}}{S_{hf}} + \frac{dW_{hf}}{W_{hf}}\right) + \sum_{i \in INST} \frac{PTR_{ih}}{YT_h} \frac{dPTR_{ih}}{PTR_{ih}} \quad h \in INSTH$$

$$\Updownarrow$$

5.b $$yt_h = \sum_{f \in COMF} \frac{W_{hf} S_{hf}}{YT_h} (s_{hf} + w_{hf}) + \sum_{i \in INST} \frac{PTR_{ih}}{YT_h} ptr_{ih} \quad h \in INSTH$$

Household monetary disposable income:

$$dYD_h = d(1 - IT_h) YT_h + d(1 - IT_h) dYT_h - \sum_{i \in INST} dPTR_{hi} \quad h \in INSTH$$

$$\Updownarrow$$

$$dYD_h = (1 - IT_h) YT_h \left(yt_h - \frac{IT_h}{(1 - IT_h)} it_h\right) - \sum_{i \in INST} PTR_{hi} ptr_{hi}$$

6.b $$yd_h = \frac{(1 - IT_h) YT_h}{YD_h} \left(yt_h - \frac{IT_h}{(1 - IT_h)} it_h\right) - \sum_{i \in INST} \frac{PTR_{hi}}{YD_h} ptr_{hi}$$

The relative changes in monetary disposable incomes are given as the weighted sum of the after-tax relative changes in the value of the households' factor supplies and foreign transfers.

Household savings:

$$dTS_h = dSS_h + dYD_h \qquad h \in INSTH$$

$$\Updownarrow$$

$$\frac{dTS_h}{TS_h} = \frac{SS_h}{TS_h} \frac{dSS_h}{SS_h} + \frac{YD_h}{TS_h} \frac{dYD_h}{YD_h}$$

$$\Updownarrow$$

7.b $$ts_h = \frac{SS_h}{TS_h} ss_h + \frac{YD_h}{TS_h} yd_h$$

The relative changes in savings are equal to the relative change in income and saving rates.

Household disposable income (incl. leisure value):

$$dENY_h = \sum_{p \in COMLAB} (D_{hp} dW_{hp} + W_{hp} dD_{hp}) + dYD_h \quad h \in INSTH$$

$$\Updownarrow$$

$$\frac{dENY_h}{ENY_h} = \sum_{p \in COMLAB} \frac{W_{hp} D_{hp}}{ENY_h} \left(\frac{dW_{hp}}{W_{hp}} + \frac{dD_{hp}}{D_{hp}}\right) + \frac{YD_h}{ENY_h} \frac{dYD_h}{YD_h}$$

$$\Updownarrow$$

8.b $$eny_h = \sum_{p \in COMLAB} \frac{W_{hp} D_{hp}}{ENY_h} (w_{hp} + d_{hp}) + \frac{YD_h}{ENY_h} yd_h$$

The relative change in the disposable income including leisure is given as the weighted sum of changes in monetary disposable income and changes in values of own consumption of labor.

Total household consumption:

$$dEY_h = dENY_h - dTS_h \quad h \in INSTH$$

$$\Updownarrow$$

$$\frac{dEY_h}{EY_h} = \frac{ENY_h}{EY_h}\frac{dENY_h}{ENY_h} - \frac{TS_h}{EY_h}\frac{dTS_h}{TS_h}$$

$$\Updownarrow$$

9.b $$ey_h = \frac{ENY_h}{EY_h} eny_h - \frac{TS_h}{EY_h} ts_h$$

The change in total consumption expenditure is given as the weighted sum of changes in the disposable income and changes in savings.

Household labor supply:

$$dL_{hp} = dS_{hp} + dD_{hp} + dLU_{hp} \quad h \in INSTH, p \in COMLAB$$

$$\Updownarrow$$

10.b $$l_{hp} = \frac{S_{hp}}{L_{hp}} s_{hp} + \frac{D_{hp}}{L_{hp}} d_{hp} + \frac{LU_{hp}}{L_{hp}} lu_{hp}$$

Changes in the household supply of labor is given residually as the weighted changes of the endowment of labor less weighted changes in voluntary and involuntary own consumption of labor.

The final element in the specification of the household/consumer sub-model is the determination of the elasticities. The income elasticities have to be estimated. Given the assumptions on the demand structure, the own and cross-price elasticities can be specified further. The procedure for calculating the price elasticities in the household demand equations is shown below in section III.

II.4. Behavior of Institutions – Firms

Part of the physical capital ownership has been allocated to an institution named firms. The firms derive their income from capital rents and pay corporate income taxes on this income.

Firm taxable income:

11.a $$YT_r = \sum_{f \in COMF} W_{rf} S_{rf} \quad r \in INSTC$$

Firm disposable income:

12.a $$YD_r = (1 - TI_r) YT_r + \sum_{i \in INST} PTR_{ir} \quad r \in INSTC$$

Where
YD_r *is the disposable income of firm r*
IT_r *is the income tax rate of firm r*
W_{rf} *is the after - factor tax (social - contributions) factor wage rate of firm r*
S_{rf} *is the supply of firm r of factor f*
PTR_{ir} *is transfers of institution i to firm r*

The firms retain part of the after-tax income for investment purposes and distribute the remainder as dividends to households and the external sectors according to the ownership of the firms as well as net-interest payments to banks (in the model represented by transfers).

Firm retained earnings:

13.a $TS_r = YD_r - \sum_{i \in INST} PTR_{ri} \quad r \in INSTC$

The linearized equations used in the model are derived below.

Firm taxable income:

$$dYT_r = \sum_{f \in COMF} (dW_{rf} S_{rf} + W_{rf} dS_{rf}) \quad r \in INSTC$$

$\Updownarrow$

11.b $yt_r = \sum_{f \in COMF} \frac{W_{rf} S_{rf}}{YT_r} (w_{rf} + s_{rf})$

$$dYD_r = dYT_r(1 - TI_r) + YT_r d(1 - TI_r) + \sum_{i \in INST} dPTR_{ir} \quad r \in INSTC$$

$\Updownarrow$

12.b $yd_r = \sum_{f \in COMF} \frac{YT_r(1 - TI_r)}{YD_r} (yt_r - \frac{TI_r}{(1 - TI_r)} ti_r) + \sum_{i \in INST} \frac{PTR_{ir}}{YD_r} ptr_{ir} \qquad r \in INSTC$

Firm retained earnings:

$$dTS_r = dYD_r - \sum_{i \in INST} dPTR_{ri} \quad r \in INSTC$$

$\Updownarrow$

13.b $ts_r = \frac{YD_r}{TS} yd_r - \sum_{i \in INST} \frac{PTR_{ri}}{TS} tr_{ri}$

II.5. Behavior of Institutions – Banks

A banking sector has been introduced in to the model in anticipation of a future expansion of the model to incorporate financial intermediation and flows in addition to the capital and current flows. The banking institution owns the part of the physical capital used in the banking activity. The banking sector makes and receives transfers to and from other institutions, representing interest payments on deposits and loans respectively. In addition, the banking sector demands banking services from the banking activity.

The banking sector pays corporate income taxes on its net-transfer (net-interest) and capital income arising from domestic non-governmental sources. The banking sector does not pay taxes on foreign earnings or on payments from the government. The banking sector demands banking services. The remaining after-tax income is either retained for investment or distributed as transfers (interest payments or dividends) among the owners of the banks.

Bank taxable income:

14.a $YT_b = \sum_{f \in COMF} W_{bf} S_{bf} + \sum_{i \in INSTHBC} (PTR_{ib} - PTR_{bi}) \quad b \in INSTB$

Bank disposable income:

15.a $YD_b = (1 - IT_b) YT_b + \sum_{a \in INSTF} PTR_{ab} + \sum_{g \in INSTG} PTR_{gb} \quad b \in INSTB$

Where
YD_b *is the disposable income of bank b*
IT_b *is bank b's the income tax rate*
W_{bf} *is bank b's wage rate for factor f*
S_{bf} *is the bank b's supply of factor f*
PTR_{ib} *is transfers from households, firms and banks i to bank b*
PTR_{ab} *is transfers from country a to bank b*
PTR_{gb} *is transfers from government g to bank b*
PTR_{bi} *is transfers from bank b to households, firms and banks i*

Bank retained earnings:

16.a $$TS_b = YD_b - \sum_{i \in INST} PTR_{bi} - \sum_{m \in COMG} CP_m D_{bm} \quad b \in INSTB$$

Where
TS_b *is the savings of bank b*
CP_m *is the price of commodity m*
D_{bm} *is the bank b demand for commodity m*

The linearized equations used in the model are given below.

Bank taxable income:

$$d\,YT_b = \sum_{f \in COMF} (dW_{bf} S_{bf} + W_{bf} dS_{bf}) + \sum_{i \in INSTHBC} dPTR_{ib} - dPTR_{bi} \quad b \in INSTB$$

$\Updownarrow$

14.b $$yt_b = \sum_{f \in COMF} \frac{W_{bf} S_{bf}}{YT_b} (w_{bf} + s_{bf}) + \sum_{i \in INSTHBC} \left(\frac{PTR_{ib}}{YT_b} ptr_{ib} - \frac{PTR_{bi}}{YT_b} ptr_{bi} \right) \quad b \in INSTB$$

Bank disposable income:

$$dYD_b = dYT_b(1 - TI_b) + YT_b d(1 - TI_b) + \sum_{a \in INSTF} dPTR_{ab} + \sum_{g \in INSTG} dPTR_{gb} \quad b \in INSTB$$

$\Updownarrow$

15. b $$yd_b = \frac{YT_b(1 - TI_b)}{YD_b} \left(yt_b - \frac{TI_b}{(1 - TI_b)} ti_b \right) + \sum_{a \in INSTF} \frac{PTR_{ab}}{YD_b} ptr_{ab} + \sum_{g \in INSTG} \frac{PTR_{gb}}{YD_b} ptr_{gb}$$

Bank retained earnings:

$$dTS_b = dYD_b - \sum_{i \in INST} dPTR_{bi} - \sum_m (dCP_m D_{mb} + CP_m dD_{mb}) \quad b \in INSTB$$

$\Updownarrow$

$$ts_b = \frac{YD_b}{TS_b} yd_b - \sum_{i \in INST} \frac{PTR_{bi}}{TS_b} ptr_{bi} - \sum_m \frac{CP_m D_{mb}}{TS_b} (cp_m + d_{mb})$$

$\Updownarrow$

16.b $$ts_b = \frac{YD_b}{TS_b} yd_b - \sum_{i \in INST} \frac{PTR_{bi}}{TS_b} ptr_{bi} - \sum_m \frac{CP_m D_{bm}}{TS_b} (cp_m + d_{bm})$$

II.6. Behavior of Institutions - Government

In this framework, the government sector has two principal functions: the collection of revenues and the determination of the level and pattern of public expenditures in the shape of consumption of goods and services, subsidies to activities, subsidies to leisure consumption, and other transfers to households and other institutions. Revenues are typically derived through

taxes on factor incomes, indirect taxes on goods and services, including inter alia, VAT and excise taxes, and import tariffs. The incentive effects of the tax regime are reflected through their influence on the decisions of economic agents by changing relative prices and disposable incomes. The level of tax revenues is endogenous in the model as it depends on the resulting (equilibrium) level of economic activity.

The government is constrained by its budget equation given below.

Government budget equation:

$$
\begin{aligned}
TS_g = & \sum_{m \in COMG} \sum_{n \in COM} CP_n V_{mn} (IR_{mn} - SU_{mn}) \\
& + \sum_{w \in INSTW} \sum_{j \in COMG} \sum_{t \in TCOM} P_{wj} M_{wj} MT_{twj} - \sum_{w \in INSTF} \sum_{j \in COMG} \sum_{s \in SCOM} P_{wj} M_{wj} SUBMT_{swj} \\
& + \sum_{i \in INST} \sum_{f \in COMF} ST_{if} S_{if} CP_f - \sum_{h \in INSTH} \sum_{p \in COMLAB} SUBST_{hp} W_{hp} (D_{hp} + LU_{hp}) \\
& + \sum_{r \in INSTC} IT_r YT_r + \sum_{b \in INSTB} IT_b YT_b + \sum_{h \in INSTH} IT_h YT_h \\
& - \sum_{g \in INSTG} \sum_{j \in COMG} CP_j D_{gj} + \sum_{i \in INST} (PTR_{ig} - PTR_{gi})
\end{aligned}
\qquad g \in \text{INSTG} \tag{17.a}
$$

Where

TS_g *is Government Gross Savings (Operational Balance)*

IR_{mn} *is the tax rate on input n in occupation m*

SU_{mn} *is the subsidy rate to input n in occupation m*

MT_{twj} *is the tax rate of type t on commodity j from source w*

$SUBMT_{saj}$ *is subsidies of type s to commodity j from source w*

ST_{if} *is taxes on factor f from supplier i*

$SUBST_{hp}$ *is subsidies to labor type p in consumption of household h*

IT_i *is income tax rate for institution i*

D_{gj} *is government consumption of commodity j*

Two new sets have been introduced for convenience in the definition of commodity taxes and subsidies:

	Set Name	Index	Set Elements
Taxes levied on commodities	TCOM	t	1. Import Tariffs 2. Excise Taxes 3. Value Added Taxes
Subsidies to commodities	SCOM	s	1. Subsidies

Government expenditures are allocated between the purchases of goods and services for the provision of public goods, and direct resource transfers (e.g., subsidies to sectors or to particular types of households). The government budget equation is given below using the **Johansen** specification.

17.b

$$TS_g = \frac{1}{TS_g}\left[\begin{array}{l} \sum_{m\in COMG}\sum_{n\in COM} CP_n V_{mn}((IR_{mn} - SU_{mn})(cp_n + v_{mn}) + IR_{mn} ir_{mn} - SU_{mn} su_{mn}) \\ + \sum_{w\in INSTW}\sum_{j\in COMG}\sum_{t\in TCOM} P_{wj} M_{wj} MT_{twj}(p_{wj} + m_{wj} + mt_{twj}) \\ - \sum_{w\in INSTF}\sum_{j\in COMG}\sum_{s\in SCOM} P_{wj} M_{wj} SUBMT_{swj}(p_{wj} + m_{wj} + submt_{swj}) \\ + \sum_{i\in INST}\sum_{f\in COMF} ST_{if} S_{if} CP_f(st_{if} + s_{if} + cp_f) \\ - \sum_{h\in INSTH}\sum_{p\in COMLAB} SUBST_{hp} W_{hp}((D_{hp} + LU_{hp})(subst_{hp} + w_{hp}) + D_{hp} d_{hp} + LU_{hp} lu_{hp}) \\ + \sum_{r\in INSTC} IT_r YT_r(it_r + yt_r) + \sum_{b\in INSTB} IT_b YT_b(it_b + yt_b) + \sum_{h\in INSTH} IT_h YT_h(it_h + yt_h) \\ - \sum_{g\in INSTG}\sum_{j\in COMG} CP_j D_{gj}(cp_j + d_{gj}) + \sum_{i\in INST}(PTR_{ig} ptr_{ig} - PTR_{gi} ptr_{gi}) \end{array}\right] \quad g \in INSTG$$

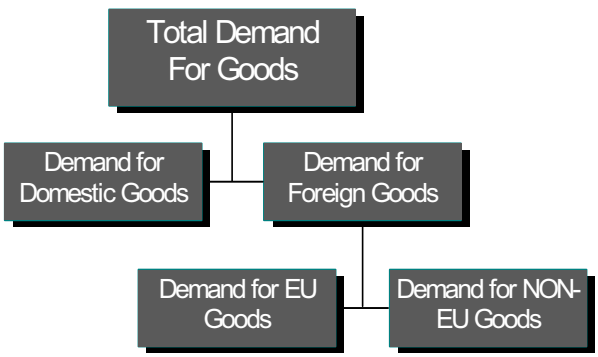
A number of different approaches may be taken within this framework with respect to fiscal policies. The focus of this model is typically on the medium-long term real effects. Here, government savings adjust to compensate for variation in the budget caused by changed revenues due to changes in the level of economic activity.

II.7. Behavior of Institutions – External Sectors

Foreign trade - imports

A feature often employed in the analysis of foreign trade using the CGE framework is the identification of traded goods by degree of product differentiation. The **Armington specification** is used to achieve this result. Imported commodities are treated as less than perfect substitutes for the domestically produced commodities of the same type. The differentiation between foreign and domestic goods in the model is done at the national demand level. Total domestic demand is divided between domestic and foreign sources of supply by applying a CRTS cost minimization specification, similar to the specification used for the production sectors. The relationship is depicted in Figure 3.

Figure 3. Import Demand



Domestically produced and imported goods of the same category are aggregated into a composite commodity which is subsequently divided between the different components of total demand. The degree of responsiveness to price differentials between domestic and foreign goods is determined by the elasticity of substitution between domestic and foreign goods. In order to achieve a responsiveness associated with the very small open economy of Poland, the substitution elasticities between imports and domestic goods are kept well above unity.

The domestic demand categories for each commodity (government and consumer demand, investment demand, and intermediate demand) are added to create the total domestic demand. This demand is carried over in a CRTS - CES relationship. The cost minimization of this

relationship yields the demand equations for commodities from each of the countries of origin. The prices of the composite commodities are calculated in a way similar to the zero-profit equations in the production sub-model. They are equated to the unit cost of producing composite commodities.

Total demand:

18.a $TD_j = \sum_{m \in COMG} V_{mj} + \sum_{i \in INST} D_{ij} + \sum_{k \in COMI} D_{kj} \quad j \in COMG$

The Armington split of demand:

19.a $M_{wj} = M_{wj}(\underline{Q}^{oj})TD_j \quad j \in COMG,\ w,o \in INSTW$

Market prices:

20.a $CP_j = CP_j(\underline{Q}^{oj}) \quad j \in COMG,\ o \in INSTW$

Domestic goods equilibrium:

21.a $M_{md} = X_{md} \quad m \in COMG,\ d \in INSTD$

Where

TD_j *is total demand for commodity j*
M_{wj} *is demand for commodity j from source w*
CP_j *is price of composite commodity j*
$\underline{Q}^{wj}$ *is the vector of domestic and foreign prices for commodity j*

The change in total demand is given as the weighted sum of the relative changes in the individual demand components. The weights equal each component's share of total demand. The relative changes in the demand for either domestically produced or imported goods are given as functions of the relative changes in prices multiplied by the own and cross price elasticities and the relative changes in total demand. The relative changes in the composite prices are calculated as the weighted sum of the domestic producer prices and the after-tariff import prices. The weights in this price aggregation are the shares of the value of total demand attributed to each country or source of origin.

Total demand:

18.b $td_j = \sum_{m \in COMG} \frac{V_{mj}}{TD_j} v_{mj} + \sum_{i \in INST} \frac{D_{ij}}{TD_j} d_{ij} + \sum_{k \in COMI} \frac{D_{kj}}{TD_j} d_{kj} \quad j \in COMG$

Armington split of demand:

19.b $m_{wj} = \sum_{o \in INSTW} \varepsilon^j_{wo} q_{oj} + td_j \quad w \in INSTW,\ j \in COMG$

Where ε^j_{wo} *is the own or cross-price elasticity between supply source w and o in the total demand of commodity j.*

Market prices:

20.b $cp_j + td_j = \sum_{w \in INSTW} \frac{Q_{wj} M_{wj}}{CP_j TD_j} (q_{wj} + m_{wj}) \quad j \in COMG$

Domestic goods equilibrium:

21. b $m_{md} = x_{md} \quad m \in COMG,\ d \in INSTD$

Note that all domestic demand is for the composite good, which is a CES aggregation of imported and domestic goods. Thus, the input-output matrix required corresponds to the usual "total" (domestic plus imported) fixed coefficients matrix of input-output analysis. The calculation of the own and cross price elasticities are shown in section III.

Foreign savings

The part of total savings arising from foreign sources is calculated residually in the model. The difference between total imports and total exports plus net non-official and official payments constitutes the foreign contribution to domestic savings. The relationship is shown below in its structural form.

Foreign savings:

22.a $$TS_a = \sum_{j \in COMG} P_{aj} M_{aj} + \sum_{f \in COMF} Q_{af} S_{af} - \sum_{j \in COMG} P_{aj} X_{aj} + \sum_{i \in INST} (PTR_{ia} - PTR_{ai}) \quad a \in INSTF$$

Where

PTR_{ai} *is transfer payments* <u>*from*</u> *country a to institution i*
PTR_{ia} *is transfer payments* <u>*to*</u> *counrty a from institution i*

External transfers payments are given as:

23.a $$PTR_{ai} = FPTR_{ai} EXR_a$$

Where

$FPTR_{ai}$ *is the trasfers form country a to institution i in foreign curreny*
EXR_a *is the exchange rate of country a*

The foreign saving equations in the relative change specification are shown below.

Foreign savings (balance of payments):

22. b $$ts_a = \frac{1}{TS_a} \left[\begin{array}{l} \sum_{j \in COMG} P_{aj} M_{aj} (p_{aj} + m_{aj}) \\ + \sum_{f \in COMF} Q_{af} S_{af} (q_{af} + s_{af}) \\ - \sum_{j \in COMG} P_{aj} X_{aj} (p_{aj} + x_{aj}) \\ + \sum_{i \in INST} (PTR_{ia}\, ptr_{ia} - PTR_{ai}\, ptr_{ai}) \end{array} \right] \quad a \in INSTF$$

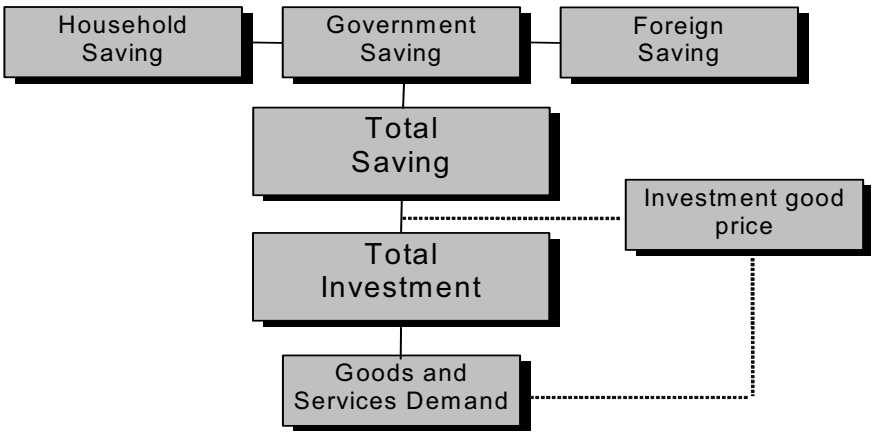
Official foreign transfers are paid in foreign currency and need to be converted into domestic currency in the model:

23.b $$ptr_{ai} = fptr_{ai} + exr_a$$

II.8. Capital Formation

A critical element of the dynamic character of the model is the determination of the level of total savings and their allocation to investment across sectors. The standard CGE model does not incorporate explicit investment behavior by firms, either at the sectoral or aggregate level. In each period the total level of savings available for investment is defined as the sum of household savings, the government sector's net balance and the net inflow of foreign capital given by the balance of payments.

Figure 4. Structure of Capital Formation



It is assumed that an investment good is produced by means of a CES production function with intermediate goods as arguments, but excluding primary factors. The price of the investment good is equal to the unit cost of the CES aggregation.

Total investment:

24.a $$TI_k = \frac{\sum_{i \in INST} TS_i}{PI_k} \quad k \in COMI$$

Intermediate investment demand:

25.a $$D_{kj} = D_{kj}(\underline{CP^m}) \cdot TI_k \quad m,j \in COMG,\ k \in COMI$$

Total investment price aggregation:

26.a $$PI_k = UC_k(\underline{D^{kj}}) \quad k \in COMI$$

Where

TI_k *is total investment in investment good k*
D_{kj} *is demand for intermediate input j in transformation to investment good k*
PI_k *is composite price of investment good k*
$\underline{CP^m}$ *is vector of intermediate input prices*

The relative change in investment demand is given as the weighted sum of the relative changes in the institution's savings. The relative changes in the intermediate demands are generated by the change in the investment demands. The price of the investment good is given as the weighted sum of the input prices of the corresponding intermediate goods used in its production.

Investment:

24.b $$ti_k = \sum_{i \in INST} \frac{TS_i}{\sum_{e \in INST} TS_e} ts_i - pi_k \quad k \in COMI$$

Intermediate investment demand:

25.b $$d_{kj} = \sum_{m \in COMG} \lambda^k_{jm} cp_m + ti_k \quad j,m \in COMG,\ k \in COMI$$

Where

λ_{jm}^{k} *is cross or own price elasticity between input j and m in the production of asset k*

Investment price aggregation:

26.b $pi_k = \sum_{j \in COMG} \frac{CP_j D_{kj}}{PI_k TI_k} cp_j \quad k \in COMI, j \in COMG$

It is assumed that capital investment in all sectors has the same structure as average investment. This simplification eliminates the possibility of affecting the pattern of final demand in the model through investment allocation - since all capital goods have the same composition, the allocation pattern does not matter. The specification of the own and cross price elasticities are given in section III.

II.9. Price Definitions

It is assumed for simplicity that all prices net of taxes, subsidies and tariffs are normalized to 1 in the base year. In this way, the benchmark data may be assembled in value terms, without the need to specify corresponding volumes.

Basic price definitions:

P_{om} *is before - tax domestic producer price for commodity m*	$m \in COMG, o \in INSTD$
P_{am} *is before - tax foreign price for commodity m*	$m \in COMG, a \in INSTF$
FP_{am} *is before - tax foreign currency price for commodity m*	$m \in COMG, a \in INSTF$
CP_n *is before - tax domestic market price for commodity n*	$n \in COM$
PI_k *is price for investment good k*	$k \in COMI$

The assumption that the price of a factor is the same across sectors in the initial period is in contradiction with the reality of the dispersion of wage and rental rates for labor and other primary factors. However, quantities are implicitly measured in units that differ from observed physical ones. Any difference in factor quantities across sectors corresponds to differences in adjusted magnitudes, which are measured in **efficiency units**. In taking this approach the implicit assumption is that observed differences in relative factor prices reflect underlying differences in factor efficiencies.

Intermediate and factor prices:

27.a $Q_{mn} = (IR_{mn} - SU_{mn} + 1) CP_n \qquad m \in COMG, n \in COM$

Factor wages:

28.a $W_{if} = (-ST_{if} + 1) CP_f \qquad i \in INST, f \in COMF$

Final consumption prices for goods:

29.a $Q_{im} = CP_m \quad i \in INST, m \in COMG$

Consumer prices for leisure:

30.a $Q_{hp} = (1 - SUBST_{hp}) W_{hp} \quad h \in INSTH, p \in COMLAB$

Wholesale prices:

31.a $Q_{wm} = (\sum_{t \in TCOM} MT_{twm} - \sum_{s \in SCOM} SUBMT_{swm} + 1) P_{wm} \quad w \in INSTW, m \in COMG$

Foreign prices in domestic currency:

32.a $P_{am} = EXR_a \cdot FP_{am} \quad a \in INSTF, m \in COMG$

Where

Q_{mn} *is the after - production - tax price for intermediate input or factor n facing activity m*
IR_{mn} *is the tax rate on intermediate input or factor n in activity m*
SU_{mn} *is the subsidy rate to intermediate input or factor n in activity m*
W_{if} *is after - tax wage rate for factor f paid to institution i*
ST_{if} *is tax rate on wages for factor f paid to institution i*
Q_{im} *is the after - consumtion - tax price on instition i consumption of commodity m*
Q_{hp} *is the after - subsidy price of household h' s consumption leisure type p*
$SUBST_{hp}$ *is the subsidy to household h' s consumption leisure type p*
Q_{wm} *is after - commodity - tax price of commodity m from source w*
MT_{twm} *is the commodity - tax rate of type t on commodity m from source w*
$SUBMT_{swm}$ *is the commodity - subsidy rate of type s on commodity m from source w*

The specification of the model explicitly includes the sources of price distortions such as income taxes, commodity taxes, tariffs and subsidies (the impacts of quantitative restrictions, such as import quotas, may also be introduced). Since the relevant prices guiding producer and consumer behavior are the after-tax prices at which transactions take place, it is necessary to clearly distinguish between before- and after-tax prices. In addition, the foreign prices of traded goods need to be converted to domestic currency using the exchange rate.

In the present context, all taxes, tariffs, and subsidies are defined in ad-valorem terms. The exports and competing foreign market prices are grouped together for notational convenience. The linearized price system is shown below.

Intermediate and factor prices:

27.b $q_{mn} = ir_{mn} \dfrac{R_{mn}}{(IR_{mn} - SU_{mn} + 1)} - su_{mn} \dfrac{SU_{mn}}{(IR_{mn} - SU_{mn} + 1)} + cp_n \quad m \in COMG, n \in COM$

Factor wages:

28.b $w_{if} = cp_f - \dfrac{ST_{if}}{(1 - ST_{if})} st_{if} \quad i \in INST, f \in COMF$

Final consumption prices for commodities:

29.b $q_{im} = cp_m \qquad i \in INST, m \in COMG$

Consumer prices for leisure:

30.b $q_{hp} = w_{hp} - \dfrac{SUBST_{hp}}{(1 - SUBST_{hp})} subst_{hp} \quad h \in INSTH, p \in COMLAB$

Wholesale prices:

31.b
$$q_{wm} = \sum_{t \in TCOM} \frac{mt_{twm} MT_{twm}}{(1 + \sum_{t \in TCOM} MT_{twm} - \sum_{s \in SCOM} SUBMT_{swm})} - \sum_{s \in SCOM} \frac{submt_{swm} SUBMT_{swm}}{(1 + \sum_{t \in TCOM} MT_{twm} - \sum_{s \in SCOM} SUBMT_{swm})} + p_{wm} \qquad w \in INSTW, m \in COMG$$

Foreign prices in domestic currency:

32.b $p_{am} = exr_a + fp_{am} \quad a \in INSTF, m \in COMG$

II.10. Equilibrium Conditions for Factors

Two sets of factor markets must be in equilibrium: capital and labor. The model reflects a semi-"putty-clay" world, where land and capital are not perfectly mobile between competing uses. Adjustment to the long-run desired levels of capital may take place over a number of periods. Sectors are assumed to be able to disinvest when their demand for capital in any period is less than their depreciated stock of old capital. The disinvested capital is added to the supply of new capital. The capital market is in equilibrium when the value of the marginal product of capital is equalized between all sectors.

In contrast, the labor force is assumed to be fully mobile between sectors. Sectoral employment contracts when the value of the marginal product of labor is lower than the wage rate and expands otherwise. The equilibrium conditions for the factor markets are presented below.

Material balance for factors:

33.a $\sum_{i \in INST} S_{if} = \sum_{m \in COMG} V_{mf} \quad f \in COMF$

Material balance for factors (relative changes):

33. b $\sum_{i \in INST} \frac{S_{if}}{\sum_{e \in INST} S_{ef}} s_{if} = \sum_{m \in COMG} \frac{V_{mf}}{\sum_{j \in COMG} V_{jf}} v_{mf} \quad f \in COMF$

II.11. GDP and the Price Index

Real GDP

Real GDP is defined from the expenditure side of the economy as the sum of total private consumption, total government consumption, total investment and total exports minus total imports.

34.a $GDPR = \sum_{i \in INST} \sum_{m \in COMG} D_{im} + \sum_{k \in COMI} \sum_{m \in COMG} D_{km} + \sum_{a \in INSTF} \sum_{m \in COMG} X_{am} - \sum_{a \in INSTF} \sum_{m \in COMG} M_{am}$

Real GDP in relative changes:

34. b $gdpr = \left(\begin{array}{l} \sum_{i \in INST} \sum_{m \in COMG} \frac{D_{im}}{GDPR} d_{im} + \sum_{k \in COMI} \sum_{m \in COMG} \frac{D_{km}}{GDPR} d_{km} \\ + \sum_{a \in INSTF} \sum_{m \in COMG} \frac{X_{am}}{GDPR} x_{am} - \sum_{a \in INSTF} \sum_{m \in COMG} \frac{M_{am}}{GDPR} m_{am} \end{array} \right)$

Nominal GDP

Nominal GDP is generated from the value added side. Nominal GDP is the sum of value added, indirect taxes, and tariffs net of subsidies.

$$GDPN = \sum_{m \in COMG} \sum_{f \in COMF} CP_f V_{mf} + \sum_{m \in COMG} \sum_{n \in COM} CP_n V_{mn} (IR_{mn} - SU_{mn})$$

35.a $$+ \sum_{m \in COMG} \sum_{a \in INSTF} \left(\sum_{t \in TCOM} P_{am} M_{am} MT_{tam} - \sum_{t \in SCOM} P_{am} M_{am} SUBMT_{sam} \right)$$

$$- \sum_{b \in INSTB} \sum_{m \in COMG} CP_m D_{bm}$$

Nominal GDP in relative changes:

$$gdpn = \sum_{m \in COMG} \sum_{f \in COMF} \frac{CP_f V_{mf}}{GDPN} (cp_f + v_{mf})$$

$$+ \sum_{m \in COMG} \sum_{n \in COM} \frac{CP_n V_{mn}}{GDPN} ((IR_{mn} - SU_{mn})(cp_n + v_{mn}) + IR_{mn} ir_{mn} - SU_{mn} su_{mn})$$

35. b $$+ \sum_{m \in COMG} \sum_{a \in INSTF} \left(\begin{array}{l} \sum_{t \in TCOM} \frac{P_{am} M_{am} MT_{tam}}{GDPN} (p_{am} + m_{am} + mt_{tam}) \\ - \sum_{t \in SCOM} \frac{P_{am} M_{am} SUBMT_{sam}}{GDPN} (p_{am} + m_{am} + submt_{sam}) \end{array} \right)$$

$$- \sum_{b \in INSTB} \sum_{m \in COMG} \frac{CP_m D_{bm}}{GDPN} (cp_m + d_{bm})$$

The price index

The price index is defined as the GDP deflator - nominal GDP divided by real GDP. This index provides a numeraire price level against which all other prices can be measured. The GDP deflator provides a convenient choice since it is usually available from national accounts data.

36.a $$PINDEX = \frac{GDPN}{GDPR}$$

In relative changes:

36.b $$pindex = gdpn - gdpr$$

II.12. Walras' Law and Normalization of Prices

There are two aspects specific to general equilibrium models which must be discussed before the numerical specification is performed: *Walras' Law* and the *normalization of prices*.

Walras' Law states that for a given set of prices, the sum of excess demand over all markets must be equal to zero. In other words, if one market is in excess demand, another market must be in excess supply, and if all markets but one are in equilibrium, so is that last one. Alternatively, if all markets clear and all agents but one are on their budget constraint, the last economic agent will automatically be on his budget constraint. Hence one equation may be omitted from the list of equations. It is redundant in the sense that it can be derived from the remaining equations in the system and hence does not add any information to the model. In the model the investment - saving identity (equation (9b)) has thus been left unspecified.

The CGE framework embodies the real side of the economy, excluding a monetary sector. As a consequence, only the **relative prices** of goods and services enter into the determination of the decisions of agents and the resulting equilibrium allocation of resources. It will not be possible to determine the absolute price level. This proposition reflects the fact that if *all* prices increase in the same proportion but relative prices are unaltered, the real relationship in the economy remains unchanged. The income equation is seen to be homogeneous of first degree in

prices, i.e. an equal increase in all prices will increase the income by the same amount. The household demand equations are seen to be homogeneous of degree zero in prices and income. Hence, an equal increase in all prices does not change household demand. The intermediate and factor demand equations are seen to be homogenous of degree zero in prices. This indicates that there is an indefinite set of prices, which is consistent with the model. To enable the determination of the system of prices in the model, it is necessary to define a numeraire used to define relative prices, i.e. one price has to be arbitrarily fixed. The system is then solved for all the other (relative) prices. The good with the price set equal to unity is known as the *numeraire* commodity, and the prices of all other goods are determined in terms of the numeraire. Choosing one commodity to be numeraire is known as the *normalization* procedure.

Often the domestic price level is chosen to act as the numeraire. With the nominal exchange rate set exogenously, the model determines the real exchange rate, that is, the relative price of traded and non-traded goods, which is equivalent to the ratio of the domestic cost level relative to world prices. Alternative foreign exchange market closure choices are possible. With a **flexible exchange rate** closure, *foreign savings* are set exogenously, the equilibrating variable is the nominal exchange rate and the domestic price index serves as numeraire. Equilibrium will be achieved through movements in the nominal exchange rate that affect export and import prices relative to domestic prices, in other words, by changing the relative price of tradable and non-tradable goods. A macroeconomic model of this type can be used to determine only one of the following variables: the nominal exchange rate, the price level, or the balance of payments.

II.13. Model Closure

The single period part of the model outlined above can be characterized as an under-determined system of equations. The model has more variables than equations. This implies that the equations only describe a subset of the variables of the model. Thus, to solve this model the remaining variables must be declared exogenous. This is know as the closure of the model. The exogenous variables can be chosen in a number of different ways reflecting alternative assumptions concerning the working of the economy and the desired focus of the analysis. The closure rules that should be chosen in this context reflect assumptions on the working of the labor market, the rest of the world, and the policy environment. These assumptions are discussed briefly below.

The labor market

The labor endowment is specified exogenously at the demographic growth rate of each type of labor.

$$l_{ip} = \overline{l_{ip}} \qquad p \in COMLAB,\ i \in INST$$

Two alternative specifications for representing the working of a labor market can be analyzed in this model: the classical full employment assumption, and a Keynesian type unemployment assumption. The classical case assumes that the labor market can be described as an efficiently functioning market with a flexible wage rate determined as the market clearing price between household leisure and activity factor employment, on the one hand, and the endowment of labor, on the other. Changes in unemployment are in the classical case assumed to be voluntary (and exogenous).

The classical closure rule:

$$lu_{ip} = \overline{lu_{ip}} \qquad p \in COMLAB,\ i \in INST$$

Where $\overline{lu_{ip}}$ *is an exogenously fixed level of voluntary unemployment growth.*

The Keynesian assumption is that the real wage rates are exogenously fixed. Employment in this case is the corresponding demand for labor at this wage rate. Unemployment is endogenously determined as the residual between the available endowment of labor and the employment and leisure demand.

The Keynesian labor supply closure rule:

$$cp_p = \overline{cp_p} \qquad p \in COMLAB$$

Where $\overline{cp_p}$ *is an exogenously fixed change in the economy-wide wage rate for labor type p.*

When prices of each of the labor types are fixed, aggregate employment of each type is determined from the producer-side of the economy. However, the supply from each household type is not uniquely identified. For this purpose an additional equation is needed, the employment rationing scheme:

$$s_{hp} = s_{"non-poor",p} \qquad h \in INSTH,\ p \in COMLAB$$

This states that labor employment from all sources change in the same proportion – this assumption can be changed, for example poor farmers can have a higher propensity to unemployment than, say, non-poor employees.

Policy simulations - closure rules

Economic behaviors and conditions in the rest of the world have only been given rudimentary treatment in the core equations of the model described above. A number of variables are left to be specified exogenously. These include changes in world prices for each commodity group and changes in private and official transfer payments. In Table 2 the variables that need to be specified are summarized.

The government has a great number of policy tools at its disposal to influence the direction of the economy, all of which must be specified exogenously. The variables included in the model are changes in tax rates of intermediate goods and factors in each occupation, tax rates on factor incomes by supplier and tax rates on each commodity group from different origins, the composition and level of government commodity demand, and transfers between institutions. A primitive set of monetary instruments are included (exchange rates of currencies of the rest of the world).

Finally, the technological growth rates for each competitive productive sector as well as the substitution elasticities in production, consumption and trade must be specified exogenously.

Table 2. Exogenous Variables

INSTITUTIONS	
s_{ic} $i \in INST, c \in COMCAP$	Changes in institution *i*'s capital supplies (*this is exogenous to the static model discussed above, but determined in the recursive dynamic link below*)
l_{ip} $i \in INST, p \in COMLAB$	Changes in institution *i*'s supply of labor type *p*
sr_i $i \in INST$	Changes in institution *i*'s saving rate
ptr_{ie} $e, i \in INST$	Changes in transfers between institution *i* and institution *e*
d_{bm} $b \in INSTB, m \in COMG$	Changes in the statistical FISIM adjustment in the banking sector
GOVERNMENT	
d_{gm} $g \in INSTG, m \in COMG$	Changes in government consumption of commodity *m*
ir_{mn} $m \in COMG, n \in COM$	Changes in production-tax rates in sector *m* for intermediate or factor input *n*
su_{mn} $m \in COMG, n \in COM$	Changes in subsidy rate in sector *m* to intermediate or factor input *n*
it_i $i \in INST$	Changes in income tax rates for institution *i*
st_{if} $i \in INST, f \in COMF$	Changes in factor tax rate on factor *f* for institution *i*
$subst_{hp}$ $h \in INSTH, p \in COMLAB$	Changes in subsidy rate to household *h*'s consumption of leisure type *p*
mt_{twm} $t \in TCOM, w \in INSTW, m \in COMG$	Changes in commodity tax rate of type *t* on commodity *m* from source *w*
$submt_{swm}$ $s \in SCOM, w \in INSTW, m \in COMG$	Changes in commodity subsidy rate of type *s* on commodity *m* from source *w*
FOREIGN COUNTRIES	
fp_{am} $a \in INSTF, m \in COMG$	Changes in foreign currency prices for commodity *m* in country *a*
ts_a $a \in INSTF$	Changes in the current account deficit vis-à-vis country *a*
SECTORS	
a_{mn} $m \in COMG, n \in COM$	Technological progress parameter for sector *m* in use of input *n*

III. Calibration of Parameters

The parameters that need to be specified empirically range from the share parameters in the material balances and income equations to the own and cross-price elasticities in the demand and supply equations. To estimate this full set of parameters econometrically would be a daunting task, even if adequate data series were available. However, the required time-series or cross-sectional data rarely, if ever, exist. In consequence, the approach adopted here (and in

nearly all CGE applications) is to parameterize the model using information contained in the SAM, supplemented as needed by additional sources or, when possible, by econometric estimates.

Identifying the parameters of the model involves accounting for real flows, nominal flows, and the first order conditions of cost minimization or profit maximization. Incorporating these conditions in the model imposes constraints on possible parameter values analogous to identification conditions common in the simultaneous estimation of econometric models. The calibration procedure followed here uses these conditions coupled with exogenous estimates of certain parameters, to compute all other parameters so that all the production and trade equations in the model are satisfied using the price and quantity data taken from the base period SAM. Since there is only one observation for each parameter being estimated, this process should not be confused with statistical estimation. Model calibration is a *mathematical* procedure, not a *statistical* one.

The main benefit of using the Johansen method for CGE model specification lies in the ease of specifying the CES and CET functions. The elasticities needed by the flexible form model can be found using standard micro-economic theory. The elasticities in the production sectors and in import allocation are derived from cost shares and substitution elasticities. The households' compensated demand price elasticities are found using the definition of substitution elasticities. Given the households' compensated price elasticities and the households' income elasticities, Slutsky's equation provides the normal price elasticities in the structure of household demand.

The SAM provides a snapshot of the economy at a single point in time. The SAM documents the income and outflow (in value terms) in each and every market and account. Each row provides information on the income to an account, while the corresponding column portrays the outflow, and the row sum and column sum must balance. For the SAM, this balance implies: (1) costs (including distributed earnings) exhaust revenues for producers; (2) expenditure (plus taxes and savings) equal income for each actor in the model; and (3) demand equals supply of each commodity. Note that these conditions are the same as those associated with equilibrium in the CGE model. Calibration of the model involves determining a set of parameters and exogenous variables so that the CGE model solution exactly replicates the economy represented in the SAM.

The SAM is given in Appendix 2 with the individual field filled in with the corresponding notation from the model and the year to which the data of the numerical implementation refer. With this information we will in the following section demonstrate how the elasticity parameters of the model are calculated.

III.1. Activities' Price Elasticities

The activity contingent cost function was defined above as:

$$C_m(\underline{Q}^{mn}, Z_m) = \underset{\underline{V}^{mn}}{Min}\, C_m = \left[\sum_{n \in COM} V_{mn} Q_{mn} \middle| Z_m = A_m Z_m^{CES}(\underline{V}^{mn}) \right] \quad \text{m} \in \text{COMG}$$

The (Allen) elasticities of substitution are defined as:

$$\sigma_{nl}^{m} = \frac{C''_{mnl} C_m}{C'_{mn} C'_{ml}} = \frac{\dfrac{\partial \left(\dfrac{\partial C_m}{\partial Q_{mn}} \right)}{\partial Q_{ml}} C_m}{\left(\dfrac{\partial C_m}{\partial Q_{mn}} \right) \left(\dfrac{\partial C_m}{\partial Q_{ml}} \right)} = \frac{\left(\dfrac{\partial V_{mn}}{\partial Q_{ml}} \right) C_m}{V_{mn} V_{ml}} \qquad n \neq l \quad n,l \in COM, m \in COMG$$

Where
C'_{mn} *is the derivative of* C_m *with respect to* Q_{mn}
C'_{ml} *is the derivative of* C_m *with respect to* Q_{ml}
C''_{mnl} *is the derivative of* C'_{mn} *with respect to* Q_{ml}

The activity contingent input price elasticities are defined as:

$$\lambda^m_{nl} = \frac{\partial V_{mn}}{\partial Q_{ml}} \frac{Q_{ml}}{V_{mn}} \qquad n,l \in COM, m \in COMG$$

Combining the two definitions yields the relationship between the elasticity of substitution and the activity contingent input price elasticities as:

$$\lambda^m_{nl} = CS^m_l \, \sigma^m_{nl} \quad \forall \; n \neq l$$
$$= -\sum_{n \neq l} \lambda^m_{nl} \qquad \forall \; n = l \quad n,l \in COM, m \in COMG$$

Where
CS^m_l *is the share of commodity l in the costs of activity m :* $CS^m_l = \dfrac{Q_{ml} V_{ml}}{C_m}$

The activity contingent output price elasticities can be calculated in a similar manner.
The activity contingent revenue function was defined above as:

$$R(\underline{P^{mw}}, Z_m) = \underset{\underline{X}^{mw}}{Max} \; R_m = \left[\sum_{w \in INSTW} X_{mw} P_{mw} \middle| Z_m = Z^{CET}_m (\underline{X}^{mw}) \right] \quad \text{m} \in \text{COMG}$$

The elasticities of transformation are defined as:

$$\sigma^m_{ow} = -\frac{R''_{mow} R_m}{R'_{mo} C'_{mw}} = -\frac{\dfrac{\partial \left(\dfrac{\partial R_m}{\partial P_{mo}} \right)}{\partial P_{mw}} C_m}{\left(\dfrac{\partial R_m}{\partial P_{mo}} \right) \left(\dfrac{\partial R_m}{\partial P_{mw}} \right)} = -\frac{\dfrac{\partial X_{mo}}{\partial P_{mw}} R_m}{X_{mo} X_{mw}} \qquad o \neq w \quad o,w \in INSTW, m \in COMG$$

Where
R'_{mo} *is the derivative of* R_m *with respect to* P_{mo}
R'_{mw} *is the derivative of* R_m *with respect to* P_{mw}
R''_{mow} *is the derivative of* R'_{mo} *with respect to* P_{mw}

The activity contingent output price elasticities are defined as:

$$\lambda^m_{ow} = \frac{\partial X_{mo}}{\partial P_{mw}} \frac{P_{mlw}}{X_{mw}} \qquad o,w \in INSTW, m \in COMG$$

Again, combining the definitions yields the relationship between the elasticity of substitution and the activity contingent input price elasticities:

$$\lambda^m_{ow} = CS^m_w \, \sigma^m_{ol} \quad \forall \; o \neq w$$
$$= -\sum_{o \neq w} \lambda^m_{ow} \qquad \forall \; o = w \quad o,w \in INSTW, m \in COMG$$

Where
CS^m_w *is the share of destination w in the revenue from activity m :* $CS^m_w = \dfrac{P_{mw} X_{mw}}{R_m}$

III.2. Households' Price Elasticities

The consumer problem given above can also be specified as a cost minimization problem (the dual problem). The resulting expenditure function can be derived for households as:

$$E_h(\underline{Q^{hm}}, U_h) = (\min_{U_h} \sum_{m \in COMG} Q_{hm} D_{hm} \Big| U_h = U_h(\underline{D^{hm}})) \quad m \in COMG, h \in INSTH$$

Where

E_h *is the expenditure function for household h*

U_h *is the level of household h's utility obtained from consumption of the vector of commodities* $\underline{D^{hm}}$

The result of this optimization problem yields the *compensated* (Hicksian) demand equations:

Household *compensated* commodity demand equations

$$D_{hm} = D_{hm}(\underline{Q^{hm}}, U_h) \qquad \text{h} \in \text{INSTH}, \text{m} \in \text{COMG}$$

The (Allen) elasticities of substitution are then defined as:

$$\sigma^h_{mj} = \frac{E''_{hmj} E_h}{E'_{hm} E'_{hj}} = \frac{\dfrac{\partial\left(\dfrac{\partial E_h}{\partial Q_{hm}}\right)}{\partial Q_{hj}} E_h}{\left(\dfrac{\partial E_h}{\partial Q_{hm}}\right)\left(\dfrac{\partial E_h}{\partial Q_{hj}}\right)} = \frac{\left(\dfrac{\partial D_{hm}}{\partial Q_{hj}}\right) EY_h}{D_{hm} D_{hj}} \qquad m \neq j \quad m, j \in COMG, h \in INSTH$$

Where

E'_{hm} *is the derivative of* E_h *with respect to* Q_{hm}

E'_{hj} *is the derivative of* E_h *with respect to* Q_{hj}

E''_{hmj} *is the derivative of* $E^{©}_{hm}$ *with respect to* Q_{hj}

The *compensated* (Hicksian) price elasticities are defined as:

$$\gamma^i_{nl} = \frac{\partial D^c_{in}}{\partial P_l} \frac{P_l}{D_{in}} \qquad l, n \in COMG, i \in INST$$

The relationship between the elasticity of substitution and the compensated price elasticities is:

$$\gamma^h_{mj} = ES^h_j \sigma^h_{mj} \quad \forall \; m \neq j$$

$$\gamma^h_{mm} = \sum_{m \neq j} \gamma^h_{mj} \quad \forall \; m = j \quad m, j \in COMG, h \in INSTH$$

Where

ES^h_j *is the share of commodity j in the expenditure of institution h :* $ES^h_j = \dfrac{Q_j D_{hj}}{EY_h}$

The relationship between the ordinary (Mashallian) price elasticities, the compensated price elasticities and the income elasticity can be found from Slutsky's equation:

$$D^c_{hm}(\underline{Q}^{hj}, U_h) = D_{hm}(\underline{Q}^{hj}, E_h)$$

$$\Updownarrow$$

$$\frac{\partial D^c_{hm}}{\partial Q_{hj}} = \frac{\partial D_{hm}}{\partial Q_{hj}} + \frac{\partial D_{hm}}{\partial E_h}\frac{\partial E_h}{\partial Q_{hj}}$$

$$\Updownarrow$$

$$\frac{\partial D^c_{hm}}{\partial Q_{hj}}\frac{Q_{hm}}{D_{hm}} = \frac{\partial D_{hm}}{\partial Q_{hj}}\frac{Q_{hj}}{D_{hm}} + \frac{\partial D_{hm}}{\partial E_h}\frac{E_h}{D_{hm}}\frac{Q_{hj}D_{hj}}{E_h}$$

$$\Updownarrow$$

$$\varepsilon^h_{mj} = \gamma^h_{mj} - ES^i_j \eta^h_m$$

III.3. Import Allocation Elasticities

The import and domestic market split can be formulated as a cost minimization problem similar to the input demand problem in production.

$$MC_m(\underline{Q}^{mo}, TD_m) = \underset{\underline{M}^{mo}}{Min}\ MC_m = \left[\sum_{o \in INSTW} M_{mo}Q_{mo} \middle| TD_m = TD^{CES}_m(\underline{M}^{mw})\right] \quad \text{m} \in \text{COMG}$$

The (Allen) elasticities of substitution are defined as:

$$\sigma^m_{ow} = \frac{MC''_{mow}MC_m}{MC'_{mo}MC'_{mw}} = \frac{\dfrac{\partial\left(\dfrac{\partial MC_m}{\partial Q_{mo}}\right)}{\partial Q_{mw}}MC_m}{\left(\dfrac{\partial MC_m}{\partial Q_{mo}}\right)\left(\dfrac{\partial C_m}{\partial Q_{mw}}\right)} = \frac{\left(\dfrac{\partial V_{mo}}{\partial Q_{mw}}\right)MC_m}{V_{mo}V_{mw}} \qquad o \neq w \quad o,w \in INSTW, m \in COMG$$

Where
MC'_{mo} *is the derivative of* MC_m *with respect to* Q_{mo}
MC'_{mw} *is the derivative of* MC_m *with respect to* Q_{mw}
MC''_{mow} *is the derivative of* MC'_{mo} *with respect to* Q_{mw}

The total demand contingent origin price elasticities are defined as:

$$\varepsilon^m_{ow} = \frac{\partial M_{mw}}{\partial Q_{mo}}\frac{Q_{mo}}{M_{mw}} \qquad o,w \in INSTW, m \in COMG$$

As before combining the two definitions yields the relationship between the elasticity of substitution and the activity contingent input price elasticities as:

$$\varepsilon^m_{ow} = CS^m_w \sigma^m_{ow} \quad \forall\ o \neq w$$
$$= -\sum_{o \neq w} \varepsilon^m_{ow} \qquad \forall\ o = w \quad o,w \in INSTW, m \in COMG$$

Where

CS_w^m *is the share of origin w in the satisfaction of domestic demand for m* : $CS_w^m = \frac{Q_{mw} M_{mw}}{MC_m}$

CES and CET functions used in the model are characterized by:

1. A matrix of substitution elasticities for the household institutions.
2. A matrix of substitution elasticities for commodity origins.
3. A matrix of substitution elasticities for the inputs in production.
4. A matrix of transformation elasticities for commodity destinations.
5. A matrix of substitution elasticities for inputs in investment formation.
6. A vector of income elasticities for the institutions.
7. A number of cost and sales shares.

Standard practice is for the modelers to specify the elasticities (of substitution or transformation) outside the model, based (when possible) on econometric estimates. For the CES functions, the elasticity of substitution measures the degree to which commodities in the household demand, input in production, or imported and domestic versions of the same commodity can be substituted for one another in demand. For CET functions, the elasticity of transformation measures the degree to which commodities destined for different destinations can be transformed to other destinations in supply. A substitution/transformation elasticity of less than one is generally assumed to be low. An elasticity between one and two is normal. Elasticities higher than two are high. It is highly recommended in CGE models to run sensitivity tests on the values specified.

The remaining parameters in the model are all cost or revenue shares.

IV. Solving the Static Model

The model presented above in its linearized form can be represented algebraically as:

$A \bullet z = 0$

Where

z is the [qx1] vector of variables
A is the [rxq] matrix of coefficients
q is the number of variables
r is the number of equations

The number of variables (q) exceeds the number of equations (r), hence it is necessary that a number (q-r) of variables be specified exogenously as shown above. The closed system can thus be rearranged as follows:

$A_1 \bullet z_1 + A_2 \bullet z_2 = 0$

Where

z_1 *is the [rx1] vector of endogenous variables*
z_2 *is the [(q - r)x1] vector of exogenous variables*
A_1 *is the [rxr] submatrix of A*
A_2 *is the [rx(q - r)] submatrix of A*

The solution to the endogenous variables can then be found by a couple of simple matrix operations:

$z_1 = - A_1^{-1} \bullet A_2 \bullet z_2$

An equivalent way of solving the model is through the use of linear programming techniques. In this case a variable is optimized subject to the constraints formed by the equations of the model. Since the system only has one solution, which is compatible with all the equation constraint, the model will generate the same solution no matter which of the variables is optimized. This method is employed by the computer implementation of the model in GAMS. When the GAMS program has found a solution to the variables in the relative change form it updates the levels, and a new set of coefficients corresponding to the updated data set can be calculated.

V. The Dynamic Structure

The intertemporal component of the model is captured by a series of equations that describe how exogenous characteristics such as the policy environment, international prices, and population, as well as model-driven conditions such as capital stocks, evolve over time. The sequence of static equilibria that results from the framework described above is related through these equations. For example, the current decisions of agents affect future economic developments through the accumulation or contraction of stocks, but are not affected by expectations regarding future economic outcomes (assumptions regarding the nature of expectations could in principle be accommodated within this framework.)

V.1. The Initial Capital Stock

The SAM discussed above is formulated in flows and no information about the initial stocks of primary factors in general and capital factors in particular is included. We seek to overcome this problem by making a number of very restrictive assumptions on the workings of the capital market and the distribution of wealth in the economy. First, we assume that the rate of return on any type of assets is the same in all occupations. Second, this return is assumed equal to an average nominal long term interest rate in the economy in 2002 - our base year.

V.2. Capital Accumulation and Supply

It is assumed in the model that the government does not hold any capital. This is a standard assumption in economics. The government is assumed to be an agent for the private sector. Ultimately, the revenue or loss from any government-operated commercial activities will lower or raise the revenue needs from taxation. In the model, the banking sector's and the government's savings are combined with any net-capital transfers from the households to determine the magnitude of banking sector investments.

Updating the banks' capital stocks:

$$CAP_b^{t+1} = CAP_b\,(1 - DPR) + \frac{TS_b}{PI} + \sum_{i \in INST} \frac{CTR_{ib}}{PI} \quad b \in INSTB$$

Endogenous capital transfers to firms:

$$CTR_{gb} = TS_g \quad g \in INSTG, b \in INSTB$$

Where

CAP_b *is capital stock of bank b*

CTR_{ib} *is capital transfer from institution i to bank b*

DPR *is the economy-wide capital depreciation rate*

The foreign sectors are assumed not to own capital, but own shares in firms. Foreign savings are combined with firm savings and any capital transfers from the households. Firm investment is subsequently determined as this combined savings deflated by the investment price.

Updating firms' capital stocks:

$$CAP_r^{t+1} = CAP_r\,(1 - DPR) + \frac{TS_r}{PI} + \sum_{i \in INST} \frac{CTR_{ir}}{PI} \quad r \in INSTC$$

Endogenous capital transfers to firms:

$$CTR_{ar} = TS_a \quad a \in INSTF$$

Where

CAP_r^t *is the capital stock of firm r*

CTR_{ir} *is capital transfers from institution i to firm r*

Household investment is determined as household savings net of capital transfers to firms and banks deflated by the aggregate investment price.

$$CAP_h^{t+1} = CAP_h\,(1 - DPR) + \frac{TS_h}{PI} - \sum_{i \in INST} \frac{CTR_{hi}}{PI} \quad h \in INSTH$$

Where

CAP_h^t *is the capital stock of household h*

CTR_{hi} *is capital transfers from household h to institution i*

The final element of the model is the capital accumulation functions, which equate the current capital stock to the depreciated stock inherited from the previous period plus gross investments. The equations for the intertemporal link are given below.

Next period's bank capital supply closure:

$$s_{bk} = \frac{CAP_b^t}{CAP_b^{t-1}} - 1 \quad b \in INSTB,\ k \in COMCAP$$

Next period's firm capital supply closure:

$$s_{rk} = \frac{CAP_r^{t+1}}{CAP_r^t} - 1 \quad r \in INSTC,\ k \in COMCAP$$

Next period's household capital supply closure:

$$s_{hk} = \frac{CAP_h^{t+1}}{CAP_h^t} - 1 \quad h \in INSTH,\ k \in COMCAP$$

V.3. Updating the SAM

The model is solved for the percentage changes in the level variables described in the SAM. After each solution, these individual data blocks are updated by these changes in order for a new set of parameters to be calibrated for use in the next year's (or step's) model run. Table 3 below provides the formulas used after each run.

Table 3. SAM Updating Procedure

Data set provided to the model	
$V_{mn}^{t+1} = V_{mn}(1+v_{mn}) \quad m \in COMG, n \in COM$	Sector m's intermediate and factor demands n
$M_{wm}^{t+1} = M_{wm}(1+m_{wm}) \quad w \in INSTW, m \in COMG$	Imports - domestic demand
$X_{wm}^{t+1} = X_{wm}(1+x_{wm}) \quad w \in INSTW, m \in COMG$	Exports - domestic supply
$D_{in}^{t+1} = D_{in}(1+d_{in}) \quad i \in INST, n \in COM$	All institutions' final consumption demand (incl. household leisure demand)
$D_{km}^{t+1} = D_{km}(1+d_{km}) \quad k \in COMI, m \in COMG$	Final demand for investment goods formation
$L_{ip}^{t+1} = L_{ip}(1+l_{ip}) \quad i \in INST, p \in COMLAB$	Labor endowments
$S_{if}^{t+1} = S_{if}(1+s_{if}) \quad i \in INST, f \in COMG$	Factor supplies
$LU_{if}^{t+1} = LU_{if}(1+lu_{if}) \quad i \in INST, f \in COMG$	Unemployment
$TS_{i}^{t+1} = TS_{i}(1+ts_{i}) \quad i \in INST$	Institution savings
$CAP_{ik}^{t+1} = CAP_{ik}(1+s_{ik}) \quad i \in INST, k \in COMCAP$	Capital stock by owner
$PTR_{ie}^{t+1} = PTR_{ie}(1+ptr_{ie}) \quad i, e \in INST$	Transfers
$EXR_{a}^{t+1} = EXR_{a}(1+exr_{a}) \quad a \in INSTF$	Exchange rates
$FP_{am}^{t+1} = FP_{am}(1+fp_{am}) \quad a \in INSTF, m \in COMG$	Foreign prices (foreign currency)
$P_{wm}^{t+1} = P_{wm}(1+p_{wm}) \quad w \in INSTW, m \in COMG$	Domestic and foreign producer prices (local currency)
$CP_{n}^{t+1} = CP_{n}(1+cp_{n}) \quad n \in COM$	All pre-tax consumer prices
$TI_{if}^{t+1} = (1+TI_{if})\cdot(1+it_{if})-1 \quad i \in INST, f \in COMF$	Income tax rates
$IR_{mn}^{t+1} = (1+IR_{mn})\cdot(1+ir_{mn})-1 \quad m \in COMG, n \in COM$	Production tax rates
$SU_{mn}^{t+1} = (1+SU_{mn})(1+su_{mn})-1 \quad m \in COMG, n \in COM$	Production subsidy rates
$IT_{i}^{t+1} = (1+IT_{i})(1+it_{i})-1 \quad i \in INST$	Income tax rates
$ST_{if}^{t+1} = (1+ST_{if})(1+st_{if})-1 \quad i \in INST, f \in COMF$	Factor tax rates
$SUBST_{hp}^{t+1} = (1+SUBST_{hp})(1+subst_{hp})-1 \quad h \in INSTH, p \in COMLAB$	Subsidy rates to households' consumption of leisure
$MT_{twm}^{t+1} = (1+MT_{twm})(1+mt_{twm})-1 \quad t \in TCOM, w \in INS, m \in COMG$	Commodity tax rates
$SUBMT_{swm}^{t+1} = (1+SUBMT_{swm})(1+submt_{swm})-1 \quad s \in SCOM, w \in INSTW, m \in COMG$	Commodity subsidy rates

Appendix I: Model Summary Tables

Table 4. Variables in the Model

1.	a_{mn}	$m \in COMG, n \in COM$	Technological progress parameter for intermediate or factor input *n* in activity *m*
2.	cp_n	$n \in COM$	Before-tax prices of commodity or factor *n*
3.	d_{in}	$i \in INST, n \in COM$	Demand for commodity and leisure type *n* by institution *i*
4.	d_{km}	$k \in COMI, m \in COMG$	Demand for commodity *m* in transformation to investment goods *k*
5.	eny_h	$h \in INSTH$	Disposable income inclusive of value of own consumption of leisure of household *h*
6.	exr_a	$a \in INSTF$	Exchange rate of country *a*
7.	ey_h	$h \in INSTH$	Consumption expenditure of household *h*
8.	fp_{am}	$a \in INSTF, m \in COMG$	Country *a*'s prices of commodity *m*
9.	$fptr_{ai}$	$a \in INSTF, i \in INST$	Transfers from country *a* to institution *i* in currency of country *a*
10.	$Gdpn$		Gross Domestic Product
11.	$Gdpr$		Gross Domestic Product in constant prices
12.	ir_{mn}	$m \in COMG, n \in COM$	Tax rate on intermediate or factor input *n* in activity *m*
13.	it_i	$i \in INST$	Income tax rate of institution *i*
14.	l_{hp}	$h \in INSTH, p \in COMLAB$	Household *h*'s endowment of labor type *p*
15.	lu_{hp}	$h \in INSTH, p \in COMLAB$	Household *h*'s unemployment of labor type *p*
16.	mt_{twm}	$t \in TCOM, w \in INSTW, m \in COMG$	Commodity tax rate of type *t* on commodity *m* from source *w*
17.	m_{wm}	$w \in INSTW, m \in COMG$	Demand for commodity *m* from source *w*
18.	pi_k	$k \in COMI$	Composite investment price of investment good *k*
19.	$pindex$		GDP deflator
20.	ptr_{ie}	$i, e \in INST$	Transfers from institution *e* to institution *i*

21.	p_{wm}	$w \in INSTW, m \in COMG$	Before-tax producer prices for commodity *m* from source *w*
22.	q_{hn}	$h \in INSTH, n \in COM$	After-tax prices of commodity and leisure type *n* facing household *h*
23.	q_{mn}	$m \in COMG, n \in COM$	After-tax prices for intermediate inputs and factors *n* in activity *m*
24.	q_{wm}	$w \in INSTW, m \in COMG$	After-tax prices for commodities *m* from source *w*
25.	s_{if}	$i \in INST, f \in COMF$	Employment of factor type *f* supplied by institution *i*
26.	ss_h	$h \in INSTH$	Saving rate of institution *h*
27.	st_{if}	$i \in INST, f \in COMF$	Tax on factor *f* from supplier institution *i*
28.	$submt_{swm}$	$s \in SCOM, w \in INSTW, m \in COMG$	Subsidy rate of type *s* to commodity *m* from source *w*
29.	$subst_{hp}$	$h \in INSTH, p \in COMLAB$	Subsidy rate to household *h*'s consumption of labor type *p*
30.	su_{mn}	$m \in COMG, n \in COM$	Subsidy rate to intermediate or factor input *n* in activity *m*
31.	td_m	$m \in COMG$	Total demand for commodity *m*
32.	ti_k	$k \in COMI$	Total investment in investment good *k*
33.	ts_i	$i \in INST$	Saving of institution *i*
34.	v_{mn}	$m \in COMG, n \in COM$	Demand for intermediate or factor input *n* in activity *m*
35.	w_{if}	$i \in INST, f \in COMF$	After-tax wage rate for factor *f* supplied by institution *i*
36.	x_{mw}	$m \in COMG, w \in INSTW$	Supply of commodity *m* to destination *w*
37.	yd_i	$i \in INST$	Disposable income of institution *i*
38.	yt_i	$i \in INST$	Taxable income of institution *i*
39.	z_m	$m \in COMG$	Gross output of activity *m*

Table 5. Model Equations

	Behavior of activities	
1.b	$v_{mn} = \sum_{l \in COM} \lambda_{nl}^{m} q_{ml} + z_m - a_{mn} \qquad m \in COMG,\ n,l \in COM$	Factor and intermediate demand equations
2.b	$x_{mw} = \sum_{o \in INSTW} \lambda_{wo}^{m} p_{om} + z_m \qquad m \in COMG, w \in INSTW$	Domestic and export supply equations
3.b	$\sum_{w \in INSTW} \frac{X_{mw} P_{mw}}{R_m} p_{wm} = \sum_{n \in COM} \frac{V_{mn} Q_{mn}}{C_m} q_{mn} \qquad m \in COMG$	Zero-profit condition equations
	Behavior of households	
4.b	$d_{hn} = \sum_{l \in COM} \varepsilon_{nl}^{h} q_{hl} + \gamma_{hn} ey_h \qquad h \in INSTH, n \in COM$	Household demand equations
5.b	$yt_h = \sum_{f \in COMF} \frac{W_{hf} S_{hf}}{YT_h} (s_{hf} + w_{hf}) + \sum_{i \in INST} \frac{PTR_{ih}}{YT_h} ptr_{ih} \qquad h \in INSTH$	Household taxable income equations
6.b	$yd_h = \frac{(1 - IT_h) YT_h}{YD_h} (yt_h - \frac{IT_h}{(1 - IT_h)} it_h) - \sum_{i \in INST} \frac{PTR_{hi}}{YD_h} ptr_{hi}$	Household monetary disposable income equations
7.b	$ts_h = \frac{SS_h}{TS_h} ss_h + \frac{YD_h}{TS_h} yd_h$	Household savings equations
8.b	$eny_h = \sum_{p \in COMLAB} \frac{W_{hp} D_{hp}}{ENY_h} (w_{hp} + d_{hp}) + \frac{YD_h}{ENY_h} yd_h$	Household disposable income (incl. leisure value) equations
9.b	$ey_h = \frac{ENY_h}{EY_h} eny_h - \frac{TS_h}{EY_h} ts_h$	Total household consumption equations
10.b	$l_{hp} = \frac{S_{hp}}{L_{hp}} s_{hp} + \frac{D_{hp}}{L_{hp}} d_{hp} + \frac{LU_{hp}}{L_{hp}} lu_{hp}$	Household labor supply equations
	Behavior of firms	
11.b	$yt_r = \sum_{f \in COMF} \frac{W_{rf} S_{rf}}{YT_r} (w_{rf} + s_{rf}) \qquad r \in INSTC$	Firm taxable income equations
12.b	$yd_r = \sum_{f \in COMF} \frac{YT_r (1 - IT_r)}{YD_r} (yt_r - \frac{IT_r}{(1 - IT_r)} it_r) + \sum_{i \in INST} \frac{PTR_{ir}}{YD_r} ptr_{ir} \qquad r \in INSTC$	Firms disposable income equations

13.b	$ts_r = \frac{YD_r}{TS} yd_r - \sum_{i \in INST} \frac{PTR_{ri}}{TS} ptr_{ri}$	Firm retained earnings equations
	Behavior of banks	
14.b	$yt_b = \sum_{f \in COMF} \frac{W_{bf} S_{bf}}{YT_b} (w_{bf} + s_{bf}) + \sum_{i \in INSTHBC} \left(\frac{PTR_{ib}}{YT_b} ptr_{ib} - \frac{PTR_{bi}}{YT_b} ptr_{bi} \right) \quad b \in INSTB$	Bank taxable income equations
15.b	$yd_b = \frac{YT_b (1 - IT_b)}{YD_b} (yt_b - \frac{IT_b}{(1 - IT_b)} it_b) + \sum_{a \in INSTF} \frac{PTR_{ab}}{YD_b} ptr_{ab} + \sum_{g \in INSTG} \frac{PTR_{gb}}{YD_b} ptr_{gb}$	Bank disposable income equations
16.b	$ts_b = \frac{YD_b}{TS_b} yd_b - \sum_{i \in INST} \frac{PTR_{bi}}{TS_b} ptr_{bi} - \sum_m \frac{CP_m D_{bm}}{TS_b} (cp_m + d_{bm})$	Bank retained earnings equations
	Government balance	
17.b	$TS_g = \frac{1}{TS_g} \left[\begin{array}{l} \sum_{m \in COMG} \sum_{n \in COM} CP_n V_{mn} ((IR_{mn} - SU_{mn})(cp_n + v_{mn}) + IR_{mn} ir_{mn} - SU_{mn} su_{mn}) \\ + \sum_{w \in INSTW} \sum_{j \in COMG} \sum_{t \in TCOM} P_{wj} M_{wj} MT_{twj} (p_{wj} + m_{wj} + mt_{twj}) \\ - \sum_{w \in INSTF} \sum_{j \in COMG} \sum_{s \in SCOM} P_{wj} M_{wj} SUBMT_{swj} (p_{wj} + m_{wj} + submt_{swj}) \\ + \sum_{i \in INST} \sum_{f \in COMF} ST_{if} S_{if} CP_f (st_{if} + s_{if} + cp_f) \\ - \sum_{h \in INSTH} \sum_{p \in COMLAB} SUBST_{hp} W_{hp} ((D_{hp} + LU_{hp})(subst_{hp} + w_{hp}) + D_{hp} d_{hp} + LU_{hp} lu_{hp}) \\ + \sum_{r \in INSTC} IT_r YT_r (it_r + yt_r) + \sum_{b \in INSTB} IT_b YT_b (it_b + yt_b) + \sum_{h \in INSTH} IT_h YT_h (it_h + yt_h) \\ - \sum_{g \in INSTG} \sum_{j \in COMG} CP_j D_{gj} (cp_j + d_{gj}) + \sum_{i \in INST} (PTR_{ig} ptr_{ig} - PTR_{gi} ptr_{gi}) \end{array} \right] \quad g \in INSTG$	Government budget equations
	External sector behavior	
18.b	$td_j = \sum_{m \in COMG} \frac{V_{mj}}{TD_j} v_{mj} + \sum_{i \in INST} \frac{D_{ij}}{TD_j} d_{ij} + \sum_{k \in COMI} \frac{D_{kj}}{TD_j} d_{kj} \quad j \in COMG$	Total demand equations
19.b	$m_{wj} = \sum_{o \in INSTW} \varepsilon_{wo}^{j} q_{oj} + td_j \quad w \in INSTW, j \in COMG$	Armington split of demand equations

20.b	$cp_j + td_j = \sum_{w \in INSTW} \frac{Q_{wj} M_{wj}}{CP_j TD_j}(q_{wj} + m_{wj}) \quad j \in COMG$	Market price equations
21. b	$m_{md} = x_{md} \quad m \in COMG, d \in INSTD$	Domestic goods equilibrium equations
22. b	$ts_a = \frac{1}{TS_a}\left[\begin{array}{l} \sum_{j \in COMG} P_{aj} M_{aj}(p_{aj} + m_{aj}) \\ + \sum_{f \in COMF} Q_{af} S_{af}(q_{af} + s_{af}) \\ - \sum_{j \in COMG} P_{aj} X_{aj}(p_{aj} + x_{aj}) \\ + \sum_{i \in INST}(PTR_{ia}\, ptr_{ia} - PTR_{ai}\, ptr_{ai}) \end{array}\right] \quad a \in INSTF$	Foreign savings equations
23.b	$ptr_{ai} = fptr_{ai} + exr_a$	External transfers payments equations
	Capital formation	
24.b	$ti_k = \sum_{i \in INST} \frac{TS_i}{\sum_{e \in INST} TS_e} ts_i - pi_k \quad k \in COMI$	Investment equations
25.b	$d_{kj} = \sum_{m \in COMG} \lambda^k_{jm} cp_m + ti_k \quad j,m \in COMG, k \in COMI$	Intermediate investment demand equations
26.b	$pi_k = \sum_{j \in COMG} \frac{CP_j D_{kj}}{PI_k TI_k} cp_j \quad k \in COMI, j \in COMG$	Investment price aggregation equations
	Price definitions	
27.b	$q_{mn} = ir_{mn} \frac{R_{mn}}{(IR_{mn} - SU_{mn} + 1)} - su_{mn} \frac{SU_{mn}}{(IR_{mn} - SU_{mn} + 1)} + cp_n \quad m \in COMG, n \in COM$	Intermediate and factor prices equations
28.b	$w_{if} = cp_f - \frac{ST_{if}}{(1 - ST_{if})} st_{if} \quad i \in INST, f \in COMF$	Factor wage equations
29.b	$q_{im} = cp_m \quad i \in INST, m \in COMG$	Final consumption prices for commodities equations
30.b	$q_{hp} = w_{hp} - \frac{SUBST_{hp}}{(1 - SUBST_{hp})} subst_{hp} \quad h \in INSTH, p \in COMLAB$	Consumer prices for leisure equations

31.b	$q_{wm} = \sum_{t \in TCOM} \frac{mt_{twm} MT_{twm}}{(1 + \sum_{t \in TCOM} MT_{twm} - \sum_{s \in SCOM} SUBMT_{swm})} - \sum_{s \in SCOM} \frac{submt_{swm} SUBMT_{swm}}{(1 + \sum_{t \in TCOM} MT_{twm} - \sum_{s \in SCOM} SUBMT_{swm})} + p_{wm} \quad w \in INSTW, m \in COMG$	Wholesale price equations
32.b	$p_{am} = exr_a + fp_{am} \quad a \in INSTF, m \in COMG$	Foreign prices in domestic currency equations
	Material balances for factors	
33. b	$\sum_{i \in INST} \frac{S_{if}}{\sum_{e \in INST} S_{ef}} s_{if} = \sum_{m \in COMG} \frac{V_{mf}}{\sum_{j \in COMG} V_{jf}} v_{mf} \quad f \in COMF$	Material balance for factors equations
	GDP aggregation	
34. b	$gdpr = \left(\sum_{i \in INST} \sum_{m \in COMG} \frac{D_{im}}{GDPR} d_{im} + \sum_{k \in COMI} \sum_{m \in COMG} \frac{D_{km}}{GDPR} d_{km} + \sum_{a \in INSTF} \sum_{m \in COMG} \frac{X_{am}}{GDPR} x_{am} - \sum_{a \in INSTF} \sum_{m \in COMG} \frac{M_{am}}{GDPR} m_{am} \right)$	Real GDP equation
35. b	$gdpn = \sum_{m \in COMG} \sum_{f \in COMF} \frac{CP_f V_{mf}}{GDPN} (cp_f + v_{mf}) + \sum_{m \in COMG} \sum_{n \in COM} \frac{CP_n V_{mn}}{GDPN} ((IR_{mn} - SU_{mn})(cp_n + v_{mn}) + IR_{mn} ir_{mn} - SU_{mn} su_{mn}) + \sum_{m \in COMG} \sum_{a \in INSTF} \left(\sum_{t \in TCOM} \frac{P_{am} M_{am} MT_{tam}}{GDPN} (p_{am} + m_{am} + mt_{tam}) - \sum_{t \in SCOM} \frac{P_{am} M_{am} SUBMT_{sam}}{GDPN} (p_{am} + m_{am} + submt_{sam}) \right) - \sum_{b \in INSTB} \sum_{m \in COMG} \frac{CP_m D_{bm}}{GDPN} (cp_m + d_{bm})$	Nominal GDP equation
36.b	$pindex = gdpn - gdpr$	GDP deflator equation
	Closure assumption	
37.b	$(s_{hp} = s_{"non-poor", p} \quad h \in INSTH, p \in COMLAB)$	Keynesian closure: labor market rationing scheme equations

Table 6. Parameters in the Model

$\lambda_{nl}^{m} \quad m \in COMG, n, l \in COM$	Factor and intermediate good demand own and cross price elasticities (used in factor and intermediate demand equations)
$\lambda_{wo}^{m} \quad m \in COMG, w, o \in INSTW$	Destination commodity supply own and cross price elasticities (used in supply equations)
$\lambda_{mj}^{k} \quad k \in COMI, m, j \in COMG$	Intermediate investment demand price elasticities (used in investment aggregation equation)
$\varepsilon_{mj}^{h} \quad h \in INSTH, m, j \in COMG$	Household commodity demand own and cross price elasticities (used in household commodity demand equations)
$\varepsilon_{wo}^{m} \quad m \in COMG, w, o \in INSTW$	Origin commodity demand own and cross price elasticities (used in import demand equations)
$\eta_{m}^{h} \quad h \in INSTH, m \in COMG$	Household income elasticities (used in household commodity demand equations)

Table 7. Substitution and Transformation Elasticities

	Elasticity	Value
	Substitution elasticities in input demand	
σ_{ff}^{m}	Substitution elasticities in activities in factor nest	high – basic 0.1, high – medium 0.3, medium – basic 0.6, see Table 8 for elasticities of labor factors and capital
σ_{jj}^{m}	Substitution elasticities in activities in intermediate input nest	0.02
	Transformation elasticities in output supply	
σ_{ow}^{m}	Transformation elasticity between Poland and EU and Non-EU countries	see Table 8.
	Substitution elasticities in household demand	
σ_{mj}^{h}	All household substitution elasticities	between goods 0.7, between leisure types 1.3, between leisure and goods – for employees, self-employed and farmers 0.8, for pensioners and others 0.4
	Substitution elasticity in Armington aggregation	
σ_{wo}^{m}	Substitution elasticities between Poland and EU and Non-EU countries	see Table 8.
	Substitution elasticities in investment formation	
$\sigma_{\mathrm{mj}}^{\mathrm{k}}$	All substitution elasticities in investment	0.02
	Household income elasticities	
η_{m}^{h}	All household income elasticities	see Table 9.

Table 8. Substitution/Transformation Elasticities Used in the Model

	Input demand elasticities			Transformation elasticities		Armington elasticities	
	basic-capital	medium-capital	high-capital	Poland-EU	Poland - Non-EU	Poland-EU	Poland - Non-EU
Agriculture, hunting, forestry, logging and related serivice activities	1.21	0.61	0.64	-2.40	-2.00	2.40	1.80
Fishing, operation of fish hatcheries and fish farms	1.21	0.61	0.64	-2.40	-2.00	2.40	1.80
Mining of coal and lignite, extraction of peat	2.01	1.00	1.05	-0.40	-0.40	1.40	1.40
Mining nec	2.01	1.00	1.05	-0.40	-0.40	1.40	1.40
Manufacture of food products and beverages	1.78	0.89	0.94	-1.60	-1.60	2.60	2.40
Manufacture of tobacco products	1.78	0.89	0.94	-0.80	-0.80	1.40	1.40
Manufacture of textiles	1.73	0.86	0.91	-0.01	-0.01	0.80	0.60
Manufacture of wearing apparel; dressing and dyeing of fur	1.73	0.86	0.91	-0.04	-0.02	0.80	0.60
Manufacture of leather and leather products	1.73	0.86	0.91	-0.04	-0.04	0.80	0.60
Manufacture of wood and wood products	1.73	0.86	0.91	-0.40	-0.40	1.40	1.40
Manufacture of pulp, paper and paper products	1.49	0.75	0.78	-0.40	-0.40	1.40	1.40
Publishing, printing and reproduction of recorded media	1.78	0.89	0.94	-1.60	-1.60	2.00	2.00
Manufacture of coke, refined petroleum products and nuclear fuel	1.86	0.93	0.98	-1.60	-1.60	1.40	1.40
Manufacture of chemicals and chemical products	1.86	0.93	0.98	-1.20	-1.20	1.40	1.40
Manufacture of rubber and plastic products	1.63	0.82	0.86	-1.20	-1.20	1.40	1.40
Manufacture of other non-metalic mineral products	1.86	0.93	0.98	-1.60	-1.60	1.40	1.40
Manufacture of basic metals	1.86	0.93	0.98	-0.20	-0.20	1.60	1.60
Manufacture of fabricated metal products,except machinery and equipment	1.63	0.82	0.86	-0.60	-0.60	1.60	1.60
Manufacture of machinery and equipment n.E.C.	1.93	0.97	1.01	-1.60	-1.60	1.20	1.20
Manufacture of office machinery and computers	1.40	0.70	0.74	-1.60	-1.60	1.20	1.20
Manufacture of electrical machinery and apparatus	1.40	0.70	0.74	-0.60	-0.40	1.20	1.20
Manufacture of radio, television and communication equipment and apparatus	1.40	0.70	0.74	-1.00	-1.00	1.40	1.40
Manufacture of medical, precision and optical instruments, watches and clocks	1.40	0.70	0.74	-1.60	-1.60	2.40	2.40
Manufacture of motor vehicles, trailers and semi-trailers	1.55	0.77	0.81	-0.20	-0.10	2.40	2.40
Manufacture of other transport equipment	1.55	0.77	0.81	-0.40	-0.20	2.40	2.40
Manufacture of furniture, manufacturing n.e.c.	1.73	0.86	0.91	-1.00	-1.00	1.00	1.00
Electricity, gas steam and hot water supply	1.77	0.88	0.93	-0.40	-0.40	1.40	1.40
Collection, purification and distribution of water	1.77	0.88	0.93	-0.20	-0.20	0.40	0.40
Construction	1.35	0.68	0.71	-0.40	-0.40	0.40	0.40
Wholesale and retail trade, repair of motor vehicles and motocycles; retail sale of automotive fuel	1.20	0.60	0.63	-1.20	-1.20	1.20	1.20
Hotels and restaurants	1.20	0.60	0.63	-1.20	-1.20	1.20	1.20
Transport and storage	1.20	0.60	0.63	-0.80	-0.80	1.20	1.20
Post and telecommunications	1.20	0.60	0.63	-0.80	-0.80	1.20	1.20
Financial intermediation	1.20	0.60	0.63	-1.00	-1.00	1.20	1.20
Real estate activities, renting of equipment, computer activities, R&D	1.20	0.60	0.63	-0.80	-0.80	1.00	1.00
Public administration and defence; compulsory social scurity	1.05	0.53	0.55	-0.80	-0.80	1.00	1.00
Education	1.05	0.53	0.55	-0.80	-0.80	1.00	1.00
Health and social work	1.05	0.53	0.55	-0.80	-0.80	1.00	1.00
Other community, social and personal service activities, recycling	1.05	0.53	0.55	-0.80	-0.80	1.00	1.00

Table 9. Income Elasticities of Households

	Non-poor					Poor				
	Employees	Farmers	Self-employed	Pensioners	Other	Employees	Farmers	Self-employed	Pensioners	Other
Agriculture, hunting, forestry, logging and related serivice activities	0.50	0.48	0.47	0.64	0.63	0.55	0.59	0.66	0.79	0.69
Fishing, operation of fish hatcheries and fish farms	0.70	0.84	0.70	0.68	0.70	0.89	1.11	0.79	0.80	0.80
Mining of coal and lignite, extraction of peat	0.15	0.30	0.15	0.30	0.30	1.70	2.30	1.70	2.30	2.30
Mining nec	0.35	0.40	0.40	0.35	0.35	0.20	0.30	0.30	0.30	0.30
Manufacture of food products and beverages	0.51	0.49	0.51	0.56	0.57	0.56	0.60	0.62	0.63	0.69
Manufacture of tobacco products	0.32	0.38	0.33	0.45	0.42	0.63	0.56	0.31	0.66	0.59
Manufacture of textiles	1.37	1.45	1.41	1.46	1.47	1.35	1.50	1.50	1.36	1.43
Manufacture of wearing apparel; dressing and dyeing of fur	1.33	1.34	1.32	1.19	1.28	1.48	1.62	1.48	1.61	1.29
Manufacture of leather and leather products	1.14	1.05	1.21	1.10	1.18	1.34	1.70	1.42	1.60	1.41
Manufacture of wood and wood products	1.85	2.15	1.70	1.60	1.60	1.72	1.80	1.00	1.40	1.40
Manufacture of pulp, paper and paper products	1.45	1.80	1.20	1.75	1.70	1.15	1.30	1.30	1.35	1.20
Publishing, printing and reproduction of recorded media	1.10	1.00	1.10	1.10	1.10	1.15	1.10	1.20	1.15	1.15
Manufacture of coke, refined petroleum products and nuclear fuel	0.90	0.95	0.90	1.00	1.00	0.90	0.95	0.90	1.00	1.00
Manufacture of chemicals and chemical products	1.15	1.14	1.15	1.01	1.08	1.17	1.38	0.99	1.20	1.16
Manufacture of rubber and plastic products	1.61	1.77	1.58	1.67	1.70	1.65	1.81	1.50	1.54	1.49
Manufacture of other non-metalic mineral products	1.85	2.20	1.74	1.70	1.72	1.81	1.82	1.28	1.57	1.23
Manufacture of basic metals	1.04	0.85	0.94	1.05	1.09	0.67	0.61	1.01	0.56	0.73
Manufacture of fabricated metal products,except machinery and equipment	1.62	1.79	1.49	1.67	1.70	1.43	1.64	1.32	1.50	1.29
Manufacture of machinery and equipment n.E.C.	1.89	2.52	1.53	1.85	2.08	3.05	2.62	2.21	1.77	2.33
Manufacture of office machinery and computers	2.70	3.11	2.54	2.75	3.01	2.65	2.87	2.61	1.70	1.87
Manufacture of electrical machinery and apparatus	1.36	1.80	1.33	1.35	1.38	1.42	1.30	1.24	1.30	1.41
Manufacture of radio, television and communication equipment and apparatus	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Manufacture of medical, precision and optical instruments, watches and clocks	1.50	1.50	1.50	1.50	1.50	1.30	1.30	1.30	1.30	1.30
Manufacture of motor vehicles, trailers and semi-trailers	2.64	4.22	2.47	2.24	3.56	2.98	3.36	3.71	2.51	3.04
Manufacture of other transport equipment	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Manufacture of furniture, manufacturing n.e.c.	1.92	2.17	1.78	2.07	2.05	1.75	2.48	2.46	2.22	2.55
Electricity, gas steam and hot water supply	0.54	0.78	0.55	0.67	0.60	0.75	0.52	0.54	0.80	0.79
Collection, purification and distribution of water	0.32	0.50	0.32	0.38	0.38	0.28	0.40	0.28	0.32	0.32
Construction	1.80	2.00	1.80	1.60	1.50	0.90	0.90	0.90	0.90	0.90
Wholesale and retail trade, repair of motor vehicles and motocycles; retail sale of automotive fuel	1.40	1.50	1.50	1.20	1.20	0.75	0.75	0.75	0.75	0.75
Hotels and restaurants	0.55	0.96	0.78	0.35	0.10	0.34	0.42	0.55	0.10	0.10
Transport and storage	1.04	0.88	1.20	1.10	1.12	0.86	0.72	1.20	1.23	0.90
Post and telecommunications	0.76	0.77	0.82	0.78	0.86	0.72	0.76	0.85	0.97	0.87
Financial intermediation	1.36	1.17	1.28	1.43	1.35	1.20	0.97	1.39	0.94	1.22
Real estate activities, renting of equipment, computer activities, R&D	0.70	0.80	0.70	0.70	0.70	0.80	0.90	0.80	0.80	0.80
Public administration and defence; compulsory social scurity	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Education	1.16	1.72	1.12	0.91	1.34	1.22	1.79	1.76	0.87	0.70
Health and social work	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Other community, social and personal service activities, recycling	1.57	1.22	1.34	1.61	0.99	0.95	0.83	0.52	0.54	0.51

PART II

Data Compilation

I. Introduction

The computable general equilibrium model, being a structural economy-wide model, requires a lot of data, originating from many sources, like production, foreign trade, household incomes and expenditures, and balance of payments statistics. In case of the CGE model for Poland, the following data sources have been utilized to create 2002 database for the model:

- input-output table;
- national accounts by institutional sectors and sub-sectors;
- aggregated social accounting matrix (SAM);
- conversion matrix for consumption (conversion of COICOP to NACE classification);
- compilation of household data (incomes, transfers, consumption, savings, taxes) by 10 group classification; and
- foreign trade and balance of payments.

These data-sources have been used to compile a disaggregated social accounting matrix which was the main data input into the model.

The methods of compilation of the data and the problems encountered are described below.

II. Input-Output Table

The estimation of the input-output (IO) table for 2002 is based on the IO table for 2000 published by the Central Statistical Office (CSO) in 2004 and the data for 2002 from the Statistical Yearbook. The original classification of 55 divisions (NACE) of production activity was aggregated into 39 industries due to the lack of disaggregated data for 2002 in case of some sectors. The list of industries included in the IO table for 2002 is given below.

1. Agriculture, hunting, forestry, logging and related service activities
2. Fishing, operation of fish hatcheries and fish farms
3. Mining of coal and lignite, extraction of peat
4. Mining not elsewhere classified
5. Manufacture of food products and beverages
6. Manufacture of tobacco products
7. Manufacture of textiles
8. Manufacture of wearing apparel, dressing and dyeing of fur
9. Manufacture of leather and leather products
10. Manufacture of wood and wood products
11. Manufacture of pulp
12. Publishing, printing and reproduction of recorded media
13. Manufacture of coke, refined petroleum products and nuclear fuel

14. Manufacture of chemicals and chemical products
15. Manufacture of rubber and plastic products
16. Manufacture of other non-metallic mineral products
17. Manufacture of basic metals
18. Manufacture of fabricated metal products, except machinery and equipment
19. Manufacture of machinery and equipment not elsewhere classified
20. Manufacture of office machinery and computers
21. Manufacture of electrical machinery and apparatus
22. Manufacture of radio, television and communication equipment and apparatus
23. Manufacture of medical optical instruments, watches and clocks
24. Manufacture of motor vehicles, trailers and semi-trailers
25. Manufacture of other transport equipment
26. Manufacture of furniture, manufacturing not elsewhere classified
27. Electricity, gas steam and hot water supply
28. Collection, purification and distribution of water
29. Construction
30. Trade
31. Hotels and restaurants
32. Transport
33. Post and telecommunications
34. Financial intermediation
35. Real estate activities, renting of equipment, computer activities
36. Public administration and defense, compulsory social security
37. Education
38. Health and social work
39. Other community, social and personal service activities, recycling.

The aggregated IO table for 2000 has been used in order to estimate the table for 2002. The base IO table has been transformed to include redistribution of trade and transport margins into respective sectors under the assumption of constant margins (imposed on intermediate and final demand of a given industry) from 2000 table. The overall trade and transport margins were calculated for the whole IO table, based on detailed information on basic and purchaser prices as well as product taxes and subsidies. The redistribution of trade & transport margins between those two components (two matrices) was estimated by using the relevant proportions on a more aggregated level (two vectors).

Using the data on gross output, intermediate consumption and the components of value added (compensation of employees, taxes and subsidies on production and gross operating

surplus) as published in industrial and National Accounts statistics,[2] the 39x39 matrix of intermediate consumption for 2002 was estimated under the assumption of constant intermediate cost structure in the production process of the given industry.[3]

The components of final demand were treated separately, according to the availability of disaggregated data for 2002. In estimating the export and import vectors, multiple sources of information have been used: Statistical Yearbook for 2002, Statistical Yearbook on Industry and Balance of Payments statistics (in compiling the data on the trade of services). Moreover, in the case of exports, some additional estimations have been imposed to take into consideration cross-border trade, as it is included in the National Accounts statistics. Some missing data were estimated on the basis of an assumption of constant proportions (in a small set of industries) from the year 2000. The split of the data into trade with the EU-25 countries and the rest of the world have been made using information from the database on Polish trade (POLHAZ) and the proportions from IO 2000 in case of services.

The disaggregation of private and public consumption, due to the lack of adequate data, assumes a constant structure of consumption between 2000 and 2002. The same assumption is imposed on the purchase of commodities used in the investment activity – the structure of Gross Fixed Capital Formation and Changes in Inventories from 2000 are imposed on the IO table for 2002.

Given the aggregate data from the National Accounts, the VAT and excise duties for 2002 were calculated under the assumption of constant sectoral structure of tax payments. The split between tax levied on domestic, EU-25 and the rest of the world products was based on the sectoral structure of supply to the domestic market.

Aggregate import tariffs were calculated using the tariff rates given from the database on Polish trade POLHAZ in case of agriculture and industry products. When considering the import tariff rates on services, these were calculated from the IO data from 2000. The data from POLHAZ were also used to split the tariff rates into those levied on imports from EU and the rest of the world.

The resulting demand side of the IO table was not balanced with the supply side (because of the relative unavailability of data concerning the demand side of the economy). The Generalized Friedlander Balancing Procedure was applied in order to balance the Input-Output table (see Appendix II).

III. Aggregated Social Accounting Matrix

Aggregated SAM matrix has been based on national accounts by institutional sector for 2002 as published by GUS.[4] For the following groups of units: households, private non-profit institutions, non-financial corporations, financial corporations, general government and the rest of the world the full set of current accounts is covered, i.e.:

1. Goods and services account (C0)
2. Production account (C1)
3. Income generation account (C2)
4. Allocation of primary income account (C3)
5. Secondary distribution of income account (C4)

[2] Source of data: "Statistical Yearbook 2004", CSO, Warsaw, 2004; and "Statistical Yearbook of Industry 2004", CSO, Warsaw, 2004

[3] The assumption is consistent with the Leontief model of economic activity.

[4] Mainly: "Rachunki narodowe według sektorów i podsektorów instytucjonalnych 2001 – 2003" *('National accounts by institutional sector and subsector,* 2001 - 2003'), GUS, Warsaw, 2004.

6. Use of disposable income account (C5)
7. Capital account (C6)

In the aggregated SAM, the following accounts and groups of accounts have been distinguished:

PRODUCTION
- commodities
- activities

PRODUCTION FACTORS
- labor
- capital

DOMESTIC INSTITUTIONS
- households
- firms
- banks
- government

TAXES
- households
- firms
- banks
- government

SAVINGS/INVESTMENT
- fixed investment
- change in stocks

REST OF THE WORLD

Entries in the aggregated SAM are explicitly linked to the national accounts figures. As an intermediate step, the following sub-matrices have been compiled:

PROPERTY INCOME FLOW SUB-MATRIX
- Interest income
- Dividends income
- Income from investment

TRANSFERS SUB-MATRIX
- Direct taxes and similar
- Social transfers
- Net insurance premiums and claims

	Activities	Commodities	Production factors	Households	Firms, banks	Government	Capital / savings	Rest of the world
Activities		Domestic supply						Export
Commodities	Intermediate consumption			Private consumption		Public consumption	Investments	
Production factors	Value added							
Households			Factors income		Dividends	Transfers		Transfers
Firms, banks			Factors income	Transfers	Transfers	Transfers		Transfers
Government	Producer taxes	VAT, excise duties, import tariffs	Social contributions	PIT, other income taxes	CIT			Official transfers
Capital / savings				Savings	Savings	Savings		Savings
Rest of the world		Import	Factors income	Transfers	Dividends	Official transfers		

The Social Accounting Matrix is the algebraic format to record transactions in the economy in given year but may be also treated as the scheme of model relationship. For instance, Table 10 presents a so-called "symbolic SAM," the entries of which represent variables and parameters of the model.

Summing of the entries of the SAM along the rows provides the formulas for the revenues of the respective group of agents (e.g. households or firms) while expenditures are recorded along the columns of the SAM. By setting revenues equal to expenditures, with savings being an equilibrating category, we obtain equations of the model.[5] For instance, for the households we have the following entries in the SAM:

Revenues (row):

- Factor income ($(1-ST_{f,h}^{2002})\cdot CP_f^{2002}\cdot S_{f,h}^{2002}$ $\begin{matrix} c\in INSTC \\ ,\, f\in COMF \end{matrix}$)
- Dividend ($TR_{c,h}^{2002}$ $\begin{matrix} c\in INSTB \\ ,\, h\in INSTH \end{matrix}$)
- Transfers from financial institutions ($TR_{c,b}^{2002}$ $\begin{matrix} b\in INSTB \\ ,\, h\in INSTH \end{matrix}$)
- Transfers from the government ($TR_{g,b}^{2002}$ $\begin{matrix} g\in INSTG \\ ,\, b\in INSTB \end{matrix}$)

Expenditures (column):

- Private consumption ($D_{h,m}^{2002}\cdot CP_m^{2002}$ $\begin{matrix} h\in INSTH \\ m\in COMG \end{matrix}$)
- Transfers to financial institutions ($TR_{h,b}^{2002}$ $\begin{matrix} h\in INSTH \\ ,\, b\in INSTB \end{matrix}$)

[5] These are balances of revenues and expenditures for the groups of agents. Behavioral equations, which are the core of the model, cannot be obtained by summing symbolic SAM entries. These equations have the functional forms derived from first order conditions of the optimization problems of the agents, as explained in the chapter on the description of the model.

- Personal income taxes ($IT_h^{2002} \cdot \left(\begin{array}{l} \sum_{f \in COMF} (1 - ST_{f,h}^{2002}) \cdot CP_f^{2002} \cdot S_{f,h}^{2002} \\ + \sum_{i \in INST} TR_{i,h}^{2002} \end{array} \right), h \in INSTH$)
- Private transfers to the rest of the world ($TR_{h,a}^{2002}$ $\begin{array}{l} h \in INSTH \\ , a \in INSTF \end{array}$)
- Household saving (TS_i^{2002} , $i \in INSTH$)

The resulting revenue – expenditure balance for households looks as follows:

$$TS_h^{2002} = \left((1 - IT_h^{2002}) \left(\sum_{f \in COMF} (1 - ST_{f,h}^{2002}) \cdot CP_f^{2002} \cdot S_{f,h}^{2002} + \sum_{i \in INST} TR_{i,h}^{2002} \right) \right) \quad h \in INSTH$$
$$- \left(\sum_{m \in COMG} D_{h,m}^{2002} \cdot CP_m^{2002} + \sum_{i \in INST} TR_{h,i}^{2002} \right)$$

Table 10. Symbolic SAM

	Activities $j \in COMG$	**Commodities** $m \in COMG$	**Factors** $f \in COMF$	**Firms** $c \in INSTC$
Activities $j \in COMG$		***Domestic supplies*** $X_{m,d}^{2002} \cdot P_{m,d}^{2002}$ $m \in COMG$ $d \in INSTD$		
Commodities $m \in COMG$	***Intermediate inputs*** $V_{j,m}^{2002} \cdot CP_{m}^{2002}$ $j, m \in COMG$			
Factors $f \in COMF$	***Factor wages*** $V_{j,f}^{2002} \cdot CP_{f}^{2002}$ $j \in COMG$ $f \in COMF$			
Firms $c \in INSTC$			***Factor income*** $(1 - ST_{f,c}^{2002}) \cdot CP_{f}^{2002} \cdot S_{f,c}^{2002}$ $c \in INSTC$ $f \in COMF$	
Banks $b \in INSTB$			***Factor income*** $(1 - ST_{f,i}^{2002}) \cdot CP_{f}^{2002} \cdot S_{f,i}^{2002}$ $b \in INSTB$ $f \in COMF$	***Transfers*** $TR_{c,b}^{2002}$ $c \in INSTC$ $b \in INSTB$
Households $h \in INSTH$			***Factor income*** $(1 - ST_{f,h}^{2002}) \cdot CP_{f}^{2002} \cdot S_{f,h}^{2002}$ $h \in INSTH$ $f \in COMF$	***Dividend*** $TR_{c,h}^{2002}$ $c \in INSTB$ $h \in INSTH$
Government $g \in INSTG$	***Misc. producer taxes*** $\sum_{n \in COM} IR_{j,n}^{2002} \cdot CP_{n}^{2002} \cdot V_{j,n}^{2002}$ $j \in COMG$	***Import tariffs+excises+VAT*** $IM_{m}^{2002} \cdot \left(\sum_{a \in INSTF} P_{m,a}^{2002} \cdot M_{m,a}^{2002} + \sum_{d \in INSTD} P_{m,d}^{2002} \cdot X_{m,d}^{2002} \right)$ $m \in COMG$	***Social contributions*** $\sum_{i \in INST} ST_{f,i}^{2002} \cdot CP_{f}^{2002} \cdot S_{f,i}^{2002}$ $f \in COMF$ ***Factor income*** $\sum_{f \in COMF} CP_{f}^{2002} \cdot S_{fg}^{2002}$ $g \in INSTG$	***Corporate income taxes*** $IT_{c}(\sum_{f \in COMF} (1 - ST_{f,c}^{2002}) CP_{f}^{2002} \cdot S_{f,c}^{2002})$ $c \in INSTC$
Rest of world $a \in INSTF$		***Imports*** $M_{m,a}^{2002} \cdot P_{m,a}^{2002}$ $a \in INSTF$ $m \in COMG$	***Factor income*** $(1 - ST_{f,a}^{2002}) \cdot CP_{f}^{2002} \cdot S_{f,a}^{2002}$ $a \in INSTF$ $f \in COMF$	***Dividend*** $TR_{c,a}^{2002}$ $c \in INSTC$ $a \in INSTF$
Capital accounts				***Firm savings*** TS_{c}^{2002} $c \in INSTC$

Banks $b \in INSTB$	Households $h \in INSTH$	Government $g \in INSTG$	Rest of world $a \in INSTF$	Capital accounts
			Exports $X_{j,a}^{2002} \cdot P_{j,a}^{2002}$ $j \in COMG$ $a \in INSTF$	
	Private consumption $D_{h,m}^{2002} \cdot CP_m^{2002}$ $h \in INSTH$ $m \in COMG$	***Government consumption*** $D_{g,m}^{2002} \cdot CP_m^{2002}$ $g \in INSTG$ $m \in COMG$		***Investment goods*** $D_{k,m}^{2002} \cdot CP_m^{2002}$ $k \in COMI$ $m \in COMG$
Transfers $TR_{b,c}^{2002}$ $b \in INSTB$ $c \in INSTC$				
	Transfers $TR_{h,b}^{2002}$ $h \in INSTH$ $b \in INSTB$	***Transfers*** $TR_{g,b}^{2002}$ $g \in INSTG$ $b \in INSTB$		
Transfers $TR_{b,h}^{2002}$ $b \in INSTB$ $h \in INSTH$		***Government transfers*** $TR_{g,h}^{2002}$ $g \in INSTG$ $h \in INSTH$		
Corporate income taxes $IT_b \left[\begin{array}{l} \sum_{f \in COMF} (1 - ST_{f,b}^{2002}) CP_f^{2002} S_{f,b} \\ + \sum_{h \in INSTH} (TR_{h,b} - TR_{b,h}) + \sum_{c \in INSTC} (TR_{c,b} - TR_{b,c}) \end{array} \right]$ $b \in INSTB$	***Personal income taxes*** $IT_h^{2002} \cdot \left(\begin{array}{l} \sum_{f \in COMF} (1 - ST_{f,h}^{2002}) \cdot CP_f^{2002} \cdot S_{f,h}^{2002} \\ + \sum_{i \in INST} TR_{i,h}^{2002} \end{array} \right)$ $h \in INSTH$		***Official transfers*** $TR_{a,g}^{2002}$ $a \in INSTF$ $g \in INSTG$	
Transfers $TR_{b,a}^{2002}$ $b \in INSTB$ $a \in INSTF$	***Private transfers*** $TR_{h,a}^{2002}$ $h \in INSTH$ $a \in INSTF$	***Official transfers*** $TR_{g,a}^{2002}$ $a \in INSTF$ $g \in INSTG$		
Bank savings TS_b^{2002} $b \in INSTB$	***Household savings*** TS_i^{2002} $i \in INSTH$	***Government savings*** TS_g^{2002} $g \in INSTG$	***Current Acc.*** TS_a^{2002} $a \in INSTF$	

IV. Other Disaggregated Data

IV.1. Estimation of Labor Costs (wages and salaries, social contributions) by Type of Labor and Industry (NACE classification)

Factor income structure was estimated based on individual net income data from the 2002 Household Budget Survey.

In estimating the individual social security contributions and labor costs, the following assumptions were made:

- the individual is employed on a full time basis by only one employer
- the workplace is located in the same locality in which the individual lives
- the individual calculates the personal income tax separately
- Based on current regulations, the following composition of the social security part of tax wedge has been assumed:
- paid by the employee:
  - 9,76 % – retirement
  - 6,5% – pension
  - 2,45% – illness
- paid by the employer:
  - 9,76 % – retirement
  - 6,5% – pension
  - 1,62% – accident
  - 2,45% – Labor Fund (LF)
  - 0,15% – State Fund for Rehabilitation of Disabled People

In the final SAM and the model, all social security contributions have been recorded in production (activities) and then factor accounts, i.e. as if paid by employers. Therefore they differentiate with respect to:

- the NACE sections and the type of labor (activities accounts)
- the type of labor and household (factor accounts).

As for the personal income tax, there are the following rates: 19%, 30% and 40% for the following income brackets: up to 37.024 PLN, from 37.024 PLN up to 74.048 PLN and 74.048 PLN or more.

- the tax exemption amounts to 2790 PLN annually
- the cost of earning income amounts to 1227 PLN annually
- the calculations do not include PFRON.

In the SAM, actual income tax payments for all household groups are recorded, i.e. the resulting PIT is paid according to effective tax rates.

IV.2. Estimation of Social Transfers.

Social transfer structure by household type was estimated based on the social transfer data from the 2002 Household Budget Survey. These were aggregated according to the type of household. This structure has then been applied to aggregates from the national accounts.

IV.3. Classification of Households

Households have been classified into five socio-economic groups, similar to the classification used by the Central Statistical Office. These groups are: employees, farmers, self-employed, pensioners, and other households.

Additionally, household types were classified along a poor vs. non-poor dimension. The distinction has been made using the concept of living standards per consumption unit. The approximation of living standards is LS=(I+C)/2,[6] where C is consumption as calculated by GUS and I is disposable income, $I=I_w+I_a$, where I_w is non-agricultural income and I_a is imputed agricultural income.[7] For the calculation of consumption unit, the Polish social assistance scale - which is similar to the standard scale recommended by the OECD - was used.[8]

The poor are defined as the lowest quintile of individuals (i.e. about 15% of households) with respect to living standard per measure of consumption unit.

Eventually, the following ten groups of households have been distinguished:

Employees - non-poor
- poor

Farmers - non-poor
- poor

Self-employed - non-poor
- poor

Pensioners - non-poor
- poor

Other - non-poor
- poor

IV.4. Consumption Conversion Matrix

For the purpose of estimating the conversion matrix (linking consumption by COICOP categories with NACE groups), the following data sources have been used:

- Input-Output Table 2002
- Household Budget Survey 2002 (individual data)

The initial data for the consumption matrix were taken from the Household Budget Survey 2002.

In the model, the household sector was divided into 10 accounts according to the head of the family and poor/non-poor criterion. The data concerning the consumption of households in the Household Budget Survey are presented in the COICOP classification.

The initial step therefore was the transformation of data into the NACE classification as the commodities and activities accounts are presented in such classification.

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[6] This is a kind of half-way solution to the well known problem of whether to use consumption or income as a proxy for living standards.

[7] The problems with agricultural income are that (i) it is highly seasonal; and (ii) the Central Statistical Office allows it to become negative (with outlays for agriculture production greater than revenues in some months). To deal with these problems we proxy household agricultural income by AO , where AO is outlays for agriculture production and is a group-specific scaling factor (ensuring that total agricultural income remains unchanged).

[8] The equivalence scale is: 1.1 units for a single adult household, 1 unit for the first adult and 0.7 for each additional adult (for other households), and 0.5 for children.

The transformation of data from one classification to the other usually is not a very demanding task. The tool that is needed is the straight key that presents which commodities are produced by which branch. However the transformation of data from COICOP to NACE is not that simple as no straight key is available.

The key that is available requires the following chain of transformations: from COICOP to CPC VER. 1.0., then to ISIC Rev. 3, then to ISIC Rev. 3.1, and finally to NACE Rev. 1.1.

While the last three transformations are straightforward (one aggregate from former classification is to be mapped to only one aggregate from the latter), the first is quite problematic because aggregates from COICOP are mapped to many different aggregates in the CPC ver. 1.0. classification.

The transformation was therefore prepared on the basis of available information combined with expert knowledge.

The COICOP classification includes 12 main groups of household expenditures:

1. *Food and non-alcoholic beverages*

   All products from this category were mapped to one of the three NACE categories (agriculture, hunting and related services, fishing, logging and related services as well as manufacture of food products and beverages). Salt was mapped to mining, other food products were mapped to manufacture of chemicals and chemical products, and some categories were partly mapped to forestry, logging and related services.

2. *Alcoholic beverages, tobacco products and drugs*

   Expenditures on alcohol were treated as the income of producers of food products and beverages as well as hotels and restaurants. Expenditures on tobacco products were all mapped to tobacco producers.

3. *Clothes, shoes*

   Products from this category were mostly mapped to manufacture of wearing apparel, dressing, dyeing or furs as well as production of textiles. A part was also mapped to manufacture of leather and leather products. A small part was mapped to manufacture of rubber and plastic products and fabricated metal products.

4. *Housing and energy*

   Part of the expenditures was mapped to NACE 70 (real estate activities) as well as to NACE 55 (hotels and restaurants), NACE 63 (supporting and auxiliary transport activities, activities of travel agency) and NACE 74 (other business activities). The most problematic issue was the mapping of expenditures for materials to conservation - that part of expenditures was classified to NACE 20, 21 24, 25, 26 and 28, with the percentage shares based mainly on expert knowledge. Part of expenses was also mapped to NACE 40 (electricity, gas and water supply), NACE 41 (collection, purification and distribution of water), NACE 45 (construction) and NACE 90 (sewage and refuse disposal, sanitation and similar activities). Payments for energy were mapped to NACE 2, NACE 10 and NACE 23. A small part of expenditures was also mapped to NACE 29 (maintenance of rooms and equipment of common use).

5. *House equipment and housekeeping*

   The mapping of expenditures from this category was problematic as there is a big diversity of goods included (household equipment made of almost all materials). Thus, the mapping was based to a high degree on expert knowledge.

6. *Health*

   The goods included in this category are different medical equipment, drugs as well medical services. Therefore, the products from this category are were mapped to manufacture of textiles, rubber and plastic products, medical precision and optical instruments, watches and

clocks, other transport equipment (as for the first group), food products and beverages, chemicals and chemical products, agriculture and hunting services (for the second group) as well as health and social work (for the third one).

7. Transport
The category was classified to NACE 34 (manufacture of vehicles, trailers and semi-trailers), NACE 35 (manufacture of other transport equipment), NACE 23 (manufacture of coke, refined petroleum products and nuclear fuel), NACE 24 (manufacture of chemicals and chemical products), NACE 52 (retail trade), as well as to NACE 60-63 (aggregated in the model which simplifies the problem). The most problematic was the mapping of expenditures for parts of vehicles (based to a great extent on expert knowledge).

8. Communication
All expenditures for this category were mapped to NACE 32 (manufacture of radio, television and communication equipment and apparatus) and NACE 64 (post and telecommunication).

9. Leisure and culture
Expenditures for audiovisual, photographic and computer science equipment was mapped to NACE 32 (manufacture of radio, television, and communication equipment and apparatus), for tourism to NACE 22, and for newspapers, books, etc to NACE 22 (publishing, printing and reproduction of recorded media). Other expenditures were highly problematic to map directly - each category was mapped to many different NACE categories based to great extents on expert knowledge.

10. Education
All expenditures were mapped to NACE 80 (education).

11. Restaurants and hotels
All the expenditures were mapped to NACE 55 (hotels and restaurants).

12. Other expenditures on goods and services
This category is very broad and diversified and the transformation therefore complicated and based in great part on expert knowledge.

It should be noted that before transformation of expenditures from COICOP to NACE, expenditures for alcohol and tobacco were artificially raised as households are likely to under-report them in the poll survey.

Finally, expenditures for rents were added to the final consumption of goods from NACE 70.

The RAS method was then used to balance the matrix.

V. Disaggregated Social Accounting Matrix

A disaggregated social accounting matrix (SAM) for Poland, 2002 was compiled in the CGE model format, using partial data described above (mainly the IO table and the aggregated SAM based on national accounts data). Additionally, Household Budget Survey data have been used to disaggregate the household account into 10 groups and the labor account into three education level categories. For the latter purpose, labor force survey data have also been used.

The disaggregated SAM contains information about the flows in the overall economy. The accounts of the SAM include information about the 39 activities and commodities, 4 factors of production, 10 types of households, firms, banks, government and 2 accounts of the rest of the world (EU-25 countries and other countries) with additional savings/investment accounts.

The following discussion will focus on the content, source of data and assumptions underlying estimation of each of the accounts of the disaggregated SAM.

The 39x39 matrix of intermediate inputs in the production process is taken from the estimated IO table at basic prices.

Estimation of the matrix of factor wages (with dimensions of 4x39) is based on the data from HBS and National Accounts. Compensation of the capital factor is assumed to be reflected by the gross operating surplus in a given activity. Estimation of the compensation of the 3 types of labor is based on the individual net income data from the HBS.

The taxes levied on the production process in a given activity are reflected by the taxes on production. Additionally, the subsidies to producers are separated from the taxes and treated as a flow of funds from the government to the activities accounts. These data are taken from the National Accounts statistics.

The 39x39 diagonal matrix of domestic supplies to the domestic market contains information about the gross output net of exports from the IO table. Government receipts from taxation of the transactions recorded in the commodities accounts (VAT, import tariffs and excise duties) as well as the (2x39) matrix of imports were assumed to be reflected by the corresponding elements of the IO table. Additionally, government subsidies to products are separated from the tax payments.

The 10x4 matrix of household factor income was estimated using information from the HBS and National Accounts by Institutions. Household data on compensation of employees from the National Accounts (including the following subsectors: farmers, self-employed, employees, pensioners and disabled, and others) was updated through a proportional distribution of compensation of employees from non-commercial institutions to the structure of household data. The National Accounts classification of households was transformed to the classification used in the SAM according to the following mapping:

- data on farmers, self-employed and employees reflect the exact data from the national accounts (one-to-one mapping)
- pensioners constitute part of the national accounts data on pensioners and disabled (according to the proportion observed in the HBS (income from work of pensioners and disabled)
- other household primary income is the sum of remaining figures on pensioners and disabled and the data on income from non-earned sources and others.

The resulting data on labor factor income of 5 types of households was transformed to the classification of 10 household types (including the distinction of poor and non-poor of each type of household) using appropriate proportions of the data on income from work calculated from the HBS. The disaggregation of each household income from owning a particular kind of labor factor was conducted according to the information given in the HBS.

Household income from owning capital was estimated on the basis of the same procedure, using the information on gross operating surplus by subsections of the household sector from the National Accounts by Institutions and the data from the HBS on income from ownership to estimate the disaggregation among poor and non-poor households.

Factor income of firms and banks is, under the assumption used in the NA, only the income from ownership of capital and is reflected by the corresponding gross operating surplus, given in the aggregated SAM.

Labor income of the aggregated foreign sector was distributed among labor factor types using the aggregate proportions calculated for the distribution of compensations for the domestic agents. The distribution of these among residents of the EU-25 countries and the Rest of the World (RoW) was estimated using appropriate proportions from the balance of payments statistics.

The distribution of government receipts from labor factors of production (assumed to be social contributions paid by both the employee and the employer) was estimated utilizing information from the HBS. Government receipt from capital ownership is the corresponding account of the aggregated SAM.

Data on transfers between firms, banks and selected foreign accounts are given by the appropriate accounts from the Aggregated SAM. In case of transfers from/to foreign accounts, these were calculated using additional information on current transfers from the balance of payments statistics.

Firm payments to households (dividends) were distributed among various household types according to the structure of household income from the ownership of capital, described above.

Bank transfers to households and household transfers to banks were distributed among household types using the relevant shares in total household income from owning all productive factors.

Foreign net transfers to households were distributed among household types using the information on net foreign transfers to the household sector from the National Accounts by Institutions and the mapping (between household classification by NA and household classification used in SAM) described in the section of household labor income. The relevant information on distribution among pensioners and disabled and distinction of poor and non-poor was taken from the net foreign transfers from the HBS. The split of foreign transfers to households between EU-25 and other countries was based on the information from the balance of payments (inflow of current transfers structure). The same procedure was applied to redistribute the transfers from households to foreign sectors.

Government net transfers to banks and firms were given by the corresponding accounts of the aggregated SAM. Official transfers from the foreign accounts to government and *vice versa* were redistributed among EU-25 and other countries using the information on current transfers from the balance of payments statistics.

Government transfers to households were distributed among household types using the information on government transfers to household sector from the National Accounts by Institutions and the mapping (between household classification by NA and household classification used in SAM) described in the section on household labor income. The relevant information on distribution among pensioners and disabled and distinction of poor and non-poor was taken from the government transfer data in the HBS.

The 39x10 matrix of household consumption was estimated using jointly the information from the National Accounts by Institutions and the HBS. The primary structure of the pattern of consumption of different commodities by each household type was indicated by the data from the HBS. This structure was adjusted by the Friedlander Balancing Procedure to make the consumption of commodities (aggregated across household types) and the consumption of households (aggregated across commodities) reflect the aggregates from the National Accounts by Institutions and the IO table. The semi-aggregated numbers correspond to the following calculations:

- consumption of commodities is given by private consumption from the IO table
- consumption of households was calculated using the information on consumption of the household sector from the National Accounts by Institutions and the mapping (between NA household classification and household classification used in SAM) described in the section on household labor income. The relevant information on distribution among pensioners and disabled and distinction of poor and non-poor was taken from the consumption data from the HBS

The applied procedure assures the consistency of household consumption data with National Accounts by Institutions and the estimated IO table.

Firm and bank savings accounts reflect the corresponding data from the aggregated SAM. Household savings were distributed among household types using the information on gross savings of the household sector from the National Accounts by Institutions and the mapping (between household classification by NA and household classification used in SAM) described in the section on household labor income. The relevant information on distribution among pensioners and disabled and distinction of poor and non-poor was taken from the data on the remaining cash available to an individual at the end of the period from the HBS.

The government savings account reflects the corresponding data from the aggregated SAM. Foreign sectors savings are split between EU-25 and other countries using the appropriate information from the aggregated SAM and the balance of payments statistics.

Taxes levied by the government on banks and firms (Corporate Income Taxes -CIT) reflect the data given in the aggregated SAM. Taxes paid by the households, Personal Income Taxes (PIT) and other income taxes (OIT – other liabilities of the households sector to the government, mainly property taxes), are reflected in the aggregated SAM. These were disaggregated across household types using the information from the National Accounts by Institutions and the relevant information on distribution among pensioners and disabled and distinction of poor and non-poor from the HBS (described above).

The corresponding information from the IO table was used to estimate the remaining accounts of the SAM: government consumption, investment goods consumption and exports.

The resulting Social Accounts Matrix for the year 2002 was not balanced (although the imbalances were not huge), mainly due to disaggregation of factors of production and households (the imbalances in the production accounts were eliminated by balancing the IO table) and different data sources used in estimation. The balancing of the SAM was performed using the Generalized Friedlander Procedure with the relative confidence of initial accounts set to reflect the uncertainty connected with data sources, availability of data and assumptions made in estimation.

VI. Additional Data Problems

Disaggregated SAM entries provide statistical description of the economy in a given year and this is the core of the data-base of the CGE model. The other important part of the data is parameters of the model, mostly shares and substitution elasticities. While share parameters (e.g., cost, consumption, investment, foreign trade etc.) are computed based on the data from the SAM, this is not the case with elasticities of substitutions. As explained above, in practical CGE modeling their values are set at levels consistent with the empirical literature, result from the calibration procedure (i.e., specific values of elasticities allows the model to produce results consistent with the historical growth path) or - preferably - are econometrically estimated. In this study, most of the elasticities were set initially to the levels consistent with empirical literature and were then fine-tuned during the calibration process. The key elasticities are listed below and numerical values displayed in Table 8 above.

- substitution elasticities for the households,
- substitution elasticities for commodity origins,
- substitution elasticities for the inputs in production,
- transformation elasticities for commodity destinations.
- substitution elasticities for inputs in investment formation,

Regarding income elasticities of demand for households, they were econometrically estimated by the procedure described below.

VI.1. Household Income Elasticities of Demand

The household income elasticity matrix was estimated using individual data from the Polish Household Budget Survey for 2002. Since the classification of individual consumption by purpose (COICOP) is used to classify individual consumption expenditures in the budget survey and the other data in the SAM are based on the NACE classification, it was necessary to convert data from COICOP to product classification. The pass-through matrix was used for this purpose.

The basis for estimation of the income elasticities was the Engel curve (the relationship between real income and consumption of a good). Demand for a given type of production activity can be defined as physical values (money) or as percentage shares of income or total expenditure. The last two definitions are frequently used as they allow one to estimate also elasticities for aggregates of expenditures (as in our model). Single equation models were estimated, where the share of total expenditure for a given sector was the explained variable and the sum of household expenditures was an explanatory variable. Other explanatory variables like the number of persons in the household or the logarithm of the age of the household head were also included in the model.[9] The Working-Leser model was selected from among several functional forms of the Engel curve based on theoretical and empirical considerations.

$$w_i = a_i + b_i \ln y + d_i (\ln y)^2 + \sum_j g_{ij} z_j$$

Where
w_i – share of expenditure for given type of production activity in total expenditure
y – total household expenditure for consumption
z_j – vector of household characteristics

The least square method was applied to obtain estimates of the parameters a_i, b_i and d_i. In case of correlation between residuals from the model and explanatory variables, the instrumental variable (IV) method was applied provided that two additional tests of the usefulness of IV application were satisfied.

Endogeneity of total expenditure was tested using Durbin-Wu-Hausman (DWH) tests. The test statistics reveals whether ordinary least squares estimates deviate significantly from instrumental variable estimates in the regressions when the total expenditure regressors are treated as endogenous variables.

Successful application of the instrumental variable method also requires: that the instruments are relevant, in the sense that they are correlated with the endogenous variable; and that the instruments are uncorrelated with the disturbance terms in the structural regression. In our case, they have to satisfy the over-identification test proposed by Davidson-MacKinnon on the joint hypothesis that the instruments are uncorrelated with the errors and that the second stage regression is correctly specified.[10]

The model assumes that the weighted sum of elasticities for each group of households equals 1.

In some cases it was infeasible to calculate income elasticities due to a high level of disaggregation of the social accounts matrix and the lack of degrees of freedom in extreme

[9] Other variables: number of children under 16, dummy for country localization, the square of the logarithm of the age of the household head, dummy for households questioned in June, July or August.

[10] Davidson, R. and J. MacKinnon, 1993, *Estimation and Inference in Econometrics*, New York: Oxford University Press.

situations. Under such circumstances income elasticities were estimated for households without breakdown into groups by socio-economic characteristic and/or by financial status. These results were later generalized across all type of households. Due to these generalizations and because all other elasticities were calculated independently, the weighted sum of elasticities for each group of households did not equal 1. The differences were proportionally adjusted to meet the model conditions.

The results of the estimations are shown in Table 9 above.

VI.2. Time Endowment and its Utilization Rate

The other data problem that had to be solved, in addition to the standard calibration procedure, was estimation of the time endowment of the households and rate of its utilization for work. It was assumed that potential time of work and leisure for each person is equal to 12 hours a day and that working week is 6 days long. Accordingly, 12 hours per day is assumed to be the minimal time to spend on physiological needs, socially determined necessities (e.g., social life, religious practice), and family life. Household budget survey (HBS) data have been used to compute the time endowment and utilization rate. In the first step, time endowment was computed for each household group as a sum of employed, unemployed and economically inactive persons in each household group multiplied by the annual time endowment. In the second step, information from the HBS on average working hours was used to compute the utilization rate of the time endowment, according to the formula:[11]

$$UR_{ij} = \frac{ha_{ij} e_{ij}}{h_p (e_{ij} + u_{ij} + p_{ij})}$$

Where
ha_{ij} – average number of hours worked by respondents
h_p – potential number of hours
e_{ij} – number of employed for i-th type of household and j-th type of labor
u_{ij} – number of unemployed for i-th type of household and j-th type of labor
p_{ij} – number of economically passive for i-th type of household and j-th type of labor

[11] This data was taken from HBS 2000 since only till then there was a question about the number of hours worked during the last week. Other data: number of employed, unemployed and economically inactive persons relate to 2002.

Time endowment and its utilization were computed for working age persons (15 – 64). The results of the computations are presented below:

Table 11. Utilization of Time Endowment in 2002 (%)

		EDUCATION LEVEL			
		HIGH	MED	BASIC	Average
HOUSEHOLD TYPE	EMPR	41,7%	39,5%	13,5%	34,5%
	EMPP	34,1%	31,3%	16,3%	25,6%
	FARMR	52,3%	52,7%	33,1%	45,2%
	FARMP	51,4%	46,4%	27,2%	37,8%
	SELFR	48,3%	42,4%	9,7%	37,8%
	SELFP	26,4%	32,6%	13,3%	25,3%
	PENR	14,3%	14,2%	5,8%	11,9%
	PENP	3,5%	16,1%	10,5%	13,4%
	OTHERR	14,4%	11,9%	6,8%	10,3%
	OTHERP	5,5%	7,9%	5,9%	6,9%
	Average	37,8%	31,9%	12,3%	27,1%

Appendix II: Balancing the SAM Accounts

1. Summary

Estimates of Social Accounting Matrices are usually derived from survey information and may often conflict with basic accounting identities because of random sample errors and differences in classifications. A new, consistent and more reliable set of data can be computed on the basis of the original data and the information represented by the identities. The ideal balancing process lets the least reliable of the original estimates bear the burden of the necessary adjustment, and there is agreement in the literature that the most appropriate framework for achieving this goal is the Generalized Least Squares (GLS) approach.

There are, however, few practical applications of this method as procedures are cumbersome and can tax even large computer systems. This paper shows how a new, conceptually simple, algorithm can be implemented. Because this approach is much easier to use and can be applied with even relatively small computers using standard software, it should provide for the practical use of GLS-balancing in Poland in the future.

2. Introduction

In most countries, each item of the Social Accounting Matrix (SAM) is estimated on the basis of several different sources. Usually such estimates are derived from surveys and it is highly unlikely that independent estimates of the same item have identical values. However, the differing estimates can be used as a basis for the calculation of a new reconciled estimate, which is more reliable than each of the original ones.

This process of reconciling conflicting estimates is called "balancing the accounts." In general, the need to balance the SAM data arises whenever some of the original estimates conflict with the relevant set of accounting identities. The balancing process eliminates such conflicts and leads to a consistent set of estimates for all items.

Since the second World War, the systems of national accounts in industrialized countries have become increasingly sophisticated, and the search for ways of systematizing and automating the balancing process have led to numerous suggestions of methodological approaches. The ones most frequently cited are the RAS method, the Friedlander method, and the Generalized Least Squares (GLS) method. The GLS estimator is the best (that is, the most reliable) estimator.[12] It makes optimal use of available information about the reliability of the original, inconsistent estimates and is therefore theoretically superior to the RAS and the Friedlander method. Stone, Champernowne, and Meade first presented the case for GLS balancing in 1942. For a long time technical problems hindered practical application of the method but these obstacles have been reduced by recent developments in numerical analysis and progress in computer technology (Byron, 1978; van der Ploeg, 1982; Barker et. al., 1984; and Stone, 1984).

This paper implements an algorithm originally proposed by Kasper Bartholdy (Bartholdy, 1982), which provides for yet another step in the direction of facilitating practical use of GLS balancing. The following sections show how a slight generalization of the Friedlander algorithm renders it possible to use that algorithm for GLS balancing. The "Generalized Friedlander/Bartholdy" algorithm is conceptually simple and more amenable to programming than previously used algorithms.

[12] Here, reliability is defined as the reciprocal value of the sum of the variances of the balanced estimates for the items in the Social Accounting Matrix.

3. The General Linear Balancing Problem

The framework for the presentation below is a specific balancing problem, to which the RAS method, the Friedlander method as well as the Generalized Friedlander method apply. Before outlining this specific case, it is appropriate to define what is generally meant by "a balancing problem."

Let the "true" values of the items in the SAM comprise the column vector y, and assume that a first estimate, y*, has been computed on the basis of available primary data. The two vectors, y and y*, may differ due to stochastic errors, contained in the vector:

(1) $y^* = y + e$

The "true" values comply with a set of SAM identities:

(2) $f_i(y) = h_i$, $i = 1, \ldots, z$

Where h_i is a scalar.
When f is linear for all values of i, the balancing problem is said to be linear.

In this case (2) can be rewritten as follows:

(2a) $C\,y = h$

Where C is a matrix of constant scalars.

If $f_i(y^*) \neq h_i$ for some value of i, the elements of y* are said to be "inconsistent." The thrust of the balancing process is to calculate a revised estimate, y**, for which:

(3) $f_i(y^{**}) = h_i$, $i = 1, \ldots, z$

y** is called "a balanced estimate".

4. A Concrete Balancing Problem for Poland

The general balancing problem outlined above can be given a more concrete interpretation on the basis of the following SAM:

	Activities	Commodities	Factors	Households	Government	Capital accounts	Rest of world	Total
Activities		Domestic supplies (D)			Production subsidies (GTR_a)	Stock changes (SI)	Export (E)	Total revenues
Commodities	Intermediate inputs (A)			Private consumption (C)	Government consumption (G)	Investment goods (I)		Commodity absorption
Factors	Factor wages (Y)							Wages and profit
Households			Factor income ($Y-T_i$)		Government transfers (GTR_h)		Remittances (FTR_h)	Household income
Government	Indirect taxes (T_a)	Import tariffs (T_m)	Income taxes (T_i)	Consumption taxes (T_h)			Foreign aid (FTR_g)	Government revenue
Capital accounts	Retained earnings (S_a)			Household saving (S_h)	Government saving (S_g)		Current account (S_w)	Total savings
Rest of world		Imports (M)		Transfers to ROW (PTR_h)	Foreign debt interest (GTR_w)			Foreign expenditure
Total	Total cost (TC)	Commodity supply (TS)	Wages and profit (TFI)	Household expenditure (THE)	Government expenditure (TGE)	Total investment (TI)	Foreign payment (TFR)	

The SAM generalizes the input-output idea that one sector's purchase is another sector's sale to include all transactions in the economy, not just inter-industry flows. Any flow of money from, say, a household to a productive sector (representing the purchase of that sector's output by the household), or from a household to the government (representing tax payments) is recorded in the SAM as a delivery by some actor (the column) to some other actor (the row).

The SAM requires a balance in the accounts of every actor in the economy. For example, the income from sales in the agricultural sector must equal its total expenditure on intermediate inputs, labor, imports, and capital services. Traditionally this is captured in double-entry bookkeeping by the requirement that the two sides of the ledger must equal. In the SAM, incomes appear along the rows, and expenditure appears down the columns; thus the budget constraints require that the row sum (income) must equal the column sum (expenditure).

The entries in the SAM can be summaries as follows:

	Activities	Commodities	Factors Households Government Capital Accounts	Rest of world
Activities Commodities Factors Households Government Capital Accounts Rest of world	V	M	U	X
Total	I'V	I'M	f'	I'X

Here, V shows the distribution of cost in domestic production by industrial branch while M contains the domestic supply and import figures for each commodity. U specifies the allocation of incomes, the use of commodities in production and for purposes of consumption and investment, and sectoral savings and transfers, while X records the exports of each commodity and net foreign transfers. The vector f shows the column totals for each category of intermediate and final domestic demand and I is a vector of appropriate dimension in which all elements take on the value 1.

Assume now (realistically) that highly reliable estimates of V, M and X are available while initial estimates of U and f have been computed on the basis of relatively weak information (U and f are assumed to have been estimated independently). A proper approach to the resulting balancing problem would be to adapt the initial estimates of U and f to the much more reliable estimates of V, M and X; that is to find U^{**} and f^{**}, for which:

(4) $I'U^{**} - (f^{**})' = 0'$

(5) $U^{**}I - (V^{**}I + M^{**} - X^{**}) = 0$

(6) $V^{**} = V^{*}, M^{**} = M^{*}, X^{**} = X^{*}.$

In order to simplify the specification of the balancing problem, U* and f* can be concatenated into a matrix A*:

(7) $$A^{*} = \begin{bmatrix} U^{*} \\ -(f^{*})' \end{bmatrix}$$

Equations (4), (5), and (6) imply that the balancing process should adapt A* to the known sums of the rows and columns of the "true" matrix, A:

(8) $$A^{**}I = \begin{bmatrix} V^{**}I + M^{**} - X^{**} \\ -I'(V^{**}I + M^{**} - X^{**}) \end{bmatrix}$$

(9) $I'A^{**} = 0'$

Any A** which satisfies (8) and (9) is a balanced estimate of A. The RAS method, the Friedlander method and the Generalized Friedlander method can all be used to compute a balanced estimate.

Equations (8) and (9) are derived solely on the basis of the Social Accounts. This may appear to conflict with basic ideas in the guidelines laid down in the UN System of National Accounts, which provide for a consistent set of tables covering also financial accounts. Ideally, the whole set of tables should be balanced simultaneously so as to apply all available information to the adjustment of each item.

5. The RAS Method

The RAS method can be used whenever an initial estimate A^* of a matrix A has been generated, and the "true" row and column totals are known.[13] The method is based on a model for the elements of the matrix A. The model contains a descriptive and a definition part. First, it is assumed that the "true" value of each individual element, a_{ij}, is a product of the initial estimate, a^*_{ij}, a row factor, r_i, and a column factor, s_j:

(10) $$a_{ij} = r_i a^*_{ij} s_j + e_{ij} \quad (i = 1, \ldots, m; j = 1, \ldots, n)$$

Here e_{ij} denotes a stochastic variable whose mean is zero. According to (10), a set of unbiased balanced estimates can be calculated as follows:

(11) $$a^{**}_{ij} = r_i a^*_{ij} s_j \quad (i = 1, ..., m; j = 1, ..., n)$$

Equation (11) constitutes the descriptive part of the RAS model. The identities of the RAS model express that the row and column totals of the balanced matrix-estimate must equal the exogenously given values. Let the vectors u and v contain these values. Then:

(12) $$\sum_{j=1}^{n} a^*_{ij} = u_i \quad (i = 1, ..., m; j = l, ..., n)$$

(13) $$\sum_{i=1}^{m} a^*_{ij} = v_j \quad (i = l, ..., m; j = 1, .., n)$$

The standard method for solving the system (11), (12) and (13) with respect to A^{**} is iterative. During each iteration two estimates of the matrix A and one estimate of r and s, respectively, are computed. In the p'th iteration, the formulas applied are the following:

(14) $$a^{2p-1}_{ij} = \left(1 + \frac{u_i - \sum_j a^{2p-2}_{ij}}{\sum_j a^{2p-2}_{ij}}\right) \cdot a^{2p-2}_{ij} = r^p_i \cdot a^{2p-2}_{ij}$$

(15) $$a^{2p}_{ij} = a^{2p-1}_{ij} \cdot \left(1 + \frac{v_j - \sum_i a^{2p-1}_{ij}}{\sum_i a^{2p-1}_{ij}}\right) = a^{2p-1}_{ij} \cdot s^p_j$$

a^{2p}_{ij} is the 2p'th approximation to a^{**}_{ij}. In the first iteration, p equals 1 and a^0_{ij} equals a^*_{ij}.

In (14) the difference between the i'th row sum (in the most recently calculated A-estimate) and the value u_i is distributed proportionately to the elements of that row. Similarly, (15) distributes the difference between the current j'th column sum (in the most recently computed

[13] The method was originally proposed in Deming and Stephan (1940), but the mathematical characteristics of the method were only explored much later (Gorman, 1963; and Bacharach, 1970).

A-estimate) and the value v_j proportionately to the elements of that column. Under normal circumstances the iterative process described by (14) and (15) will converge toward a unique balanced estimate of A.[14] This balanced estimate is the solution A^{**} to (11), (12) and (13).

By comparing (12) and (13) it is seen that the specific balancing problem given by (8) and (9) can be solved by means of the RAS method. The primary advantage of choosing the RAS approach is its great computational and conceptual simplicity. The price for this simplicity is non-optimal use of available information on the relative reliability of the individual estimates in A^*. The j'th column error is distributed proportionately on the elements of the j'th column even if the variance of the estimator for one element is significantly larger than the variance of the estimator for another. Thereby, the RAS method violates the intuitively reasonable requirement of any balancing process that the most unreliable estimates should bear the main share of the balancing burden.

6. The Friedlander Method

The Friedlander-method is very similar to the RAS-method.[15] The precondition for using the Friedlander method is the existence of an initial estimate A^* of some "true", non-negative matrix A and perfect knowledge of the "true" row and column totals. The model on which the Friedlander method is based can be formulated as follows:

(16) $$a_{ij}^{**} = a_{ij}^{*}(1 + r_i + s_j) \quad (i = 1, ..., m; j = 1,..., n)$$

(17) $$\sum_j a_{ij}^{**} = u_i \quad (i = 1, \ldots, m; j = 1, \ldots, n)$$

(18) $$\sum_i a_{ij}^{**} = v_i \quad (i = 1, \ldots, m; j = 1, \ldots, n)$$

As in the formulation of the RAS model above, r_i and s_j are row and column factors, respectively, while u_i and v_j are the "known" values of i'th row sum and the j'th column sum, respectively.

The Friedlander model can be solved by using the following iterative procedure (corresponding to equation (14) and (15) of the RAS process):

(19) $$a_{ij}^{2p-1} = a_{ij}^{2p-2} + \left(\frac{u_i - \sum_j a_{ij}^{2p-2}}{\sum_j a_{ij}^{*}} \right) \cdot a_{ij}^{*} = a_{ij}^{2p-2} + r_i^p \cdot a_{ij}^{*}$$

(20) $$a_{ij}^{2p} = a_{ij}^{2p-1} + \left(\frac{v_j - \sum_i a_{ij}^{2p-1}}{\sum_i a_{ij}^{*}} \right) \cdot a_{ij}^{*} = a_{ij}^{2p-1} + a_{ij}^{*} \cdot s_j^p$$

This process keeps the coefficients used for distribution of the discrepancies constant. The discrepancy in each row is distributed on the individual element a_{ij}^{2p-1} in proportion to the size of the initial estimate a_{ij}^{*}. The same principle applies to the distribution of column differences.

[14] In the SAM, the sum of the known row totals equals the sum of the known column totals ($\sum u_i = \sum v_j$) and a unique solution to the RAS model, (1l)-(13) exists and the process described by (14) and (15) will converge toward that solution (see Gorman, 1963; and Bacharach, 1970).

[15] Like the RAS method, the Friedlander method was originally presented in Deming and Stephan (1940) but the mathematical properties of the method were first analyzed in Friedlander (1961).

If the process converges (that is, if all the row and column discrepancies approach zero for $p \rightarrow \infty$) then the estimate A^p for $p \rightarrow \infty$ is a balanced estimate.

It can be proven that the Friedlander balanced estimate solves the following minimization problem:

$$(21) \quad \text{Min} \sum_{(i,j)\in\phi} \frac{(a^*_{ij} - a^{**}_{ij})^2}{a^*_{ij}} \text{ subject to } \sum_j a^{**}_{ij} = u_i \ , \ \sum_i a^{**}_{ij} = v_j$$

Where $\phi = \{(i,j) \mid a^*_{ij} > 0\}$

The Friedlander process will always converge toward the solution to this problem if such a solution exists.

Like the RAS method, the Friedlander method yields conceptual and computational simplicity. But it also contains the same inherent weakness; namely that the column and row discrepancies are distributed automatically without regard to the relative reliability of the initial estimates.

The GLS method, which is presented below, overcomes this weakness and can be combined with the Friedlander iteration scheme to avoid loss of computational simplicity.

7. Balancing by Means of the Generalized Least Squares Method

The GLS solution to the general linear balancing problem is based on the following equation:

$$(22) \quad y^* = y + e; \ E(ee') = W$$

Here y^* is the first estimate of the "true" Social Accounting Matrix vector y, and W is the covariance matrix for the residuals, contained in the vector e. Since W is positive and definite (as are all non-singular covariance matrices), it can be expressed in the form PP', where P is a non-singular matrix:

$$(23) \quad W = PP'$$

This leads to:

(24) (P-1)' P-1 = W-1 and

(25) P-1W(P-1)'= I

Where I is a unit matrix.

Pre-multiplying (22) by P^{-1} gives:

$$(26) \quad P^{-1}y^* = P^{-1}y + P^{-1}e$$

These steps convert (22) into an ordinary least squares model, since the covariance matrix of the vector $P^{-1}e$ is equal to the unit matrix:

(27) E(P-1e (P-1e)') = P-1W(P-1) = I

The principle of GLS is that the estimate, y**, should minimize the sum of the squared residuals in (26). Thus, the balanced estimate solves the following problem:

$$(28) \quad \text{Min } (P^{-1}y^* - P^{-1}y^{**})' (P^{-1}y^* - P^{-1}y^{**}) \text{ s.t. } Cy^{**} = h$$

Where the constraints express the Social Accounting Matrix identities (which are assumed to be linear). Using (24), (28) can be simplified:

$$(29) \quad \text{Min } (y^* - y^{**})' W^{-1}(y^* - y^{**}) \text{ s.t. } Cy^{**} = h$$

The first order conditions for a minimum are the following:

(30) $W^{-1}(y^{*} - y^{**}) - C'\alpha = 0$ and

(31) $Cy^{**} = h$

Here α is a vector of Lagrange multipliers. The solution to the equation system, (30) and (31), is:

(32) $\alpha = -(CWC')^{-1}(Cy^{*} - h)$ and

(33) $y^{**} = y^{*} + WC'\alpha$

The vector y^{**} in (33) is the balanced BLUE estimate, if the initial estimate, y^{*}, is unbiased.

In practice the values of the elements in the covariance matrix W are rarely known. Moreover, it is usually difficult to ensure that all estimates in y^{*} are unbiased. However, given the knowledge about biases and relative reliabilities, an "assumed" BLUE estimate (i.e., an estimate which uses the statisticians' knowledge optimally) can be calculated by:

(i) eliminating "assumed" biases from y^{*} before inserting it into (32) and (33) and

(ii) using the statisticians' "guessed" covariance matrix for W in these formulas.[16]

The computation of y^{**} can be divided into two phases: one in which only α is calculated (via equation (32)); and one in which the value of α is inserted into (33) to give the balanced estimate of y.

However, the inversion of (CWC') may often not be feasible in practice because of the dimensions of that matrix.

Therefore it remains worthwhile to look for alternative algorithms which may simplify the GLS computations. The following section shows how the Friedlander method can be used for that purpose.

8. The Generalized Friedlander/Bartholdy Method

As mentioned above, the Friedlander process solves balancing problems which are characterized by perfect knowledge of the row and column totals for a matrix A, given that (i) a first estimate, A*, has been computed; and (ii) a solution to the balancing problem exists. The minimization problem solved by the Friedlander process may be written as follows:

(34) $$\text{Min} \sum_{(i,j)\in\phi} \frac{(a_{ij}^{*} - a_{ij}^{**})^2}{a_{ij}^{*}} \text{ subject to } \sum_j a_{ij}^{**} = u_i \;,\; \sum_i a_{ij}^{**} = v_j$$

Where $\phi = \{(i,j) \mid a_{ij}^{*} > 0\}$

By comparing (34) to (29), it is easily seen that the solution, A^{**}, to (34) is actually the GLS estimate in the special case for which:

$\text{var}(a_{ij}^{*}) = \zeta a_{ij}^{*}$ and $\text{cov}(a_{ij}^{*}, a_{kl}^{*}) = 0$ for all i, j, k and l (ζ is a scalar constant).

[16] Note that only relative values - not absolute values - of the elements in W matter for the solution to (33). This property can be utilized in the case in which no initial estimate for a particular element is available. An arbitrary value for the initial estimate can be chosen, and the variance given a very large value.

With this in mind, it seems straightforward to try to use the computationally simple Friedlander algorithm for GLS balancing in more general cases. In most practical applications of GLS balancing it is natural to assume that cov(a^*_{ij}, a^*_{kl}) = 0 for all values of i,j, k, and l (this assumption is often unavoidable because of lack of knowledge about covariances). But it is rarely in accordance with the knowledge of the statisticians involved to assume, as in the Friedlander case above, that the variance of the stochastic error of the individual estimate, a^*_{ij}, is proportional to the initial estimate, (var (a^*_{ij}) = ζa^*_{ij}).

Thus, it is desirable to find an algorithm which is computationally similar to the Friedlander method, but makes it possible to solve the following GLS minimization problem for any value of the variances, w^*_{ij} :

$$\text{(35)} \qquad \text{Min} \sum_{(i,j)\in\phi} \frac{(a^*_{ij} - a^{**}_{ij})}{w^*_{ij}} \text{ subject to } \sum_j a^{**}_{ij} = u_i \ , \ \sum_i a^{**}_{ij} = v_j$$

Where $\phi = \{(i, j) \mid w^*_{ij} > 0\}$

It turns out that only a slight generalization of the Friedlander process is needed. The necessary generalization is reflected in the following formulas for the calculations of the p'th iteration:

$$\text{(36)} \qquad a^{2p-1}_{ij} = a^{2p-2}_{ij} + (u_i - \sum_j a^{2p-2}_{ij}) \frac{w_{ij}}{\sum_j w_{ij}}$$

$$\text{(37)} \qquad a^{2p}_{ij} = a^{2p-1}_{ij} + (v_j - \sum_i a^{2p-1}_{ij}) \frac{w_{ij}}{\sum_i w_{ij}}$$

This "generalized" Friedlander process is started by setting p equal to 1 and $A^0 = A^*$. Thereafter, (36) and (37) are solved in turn during the p'th iteration. It is obvious that if the process converges (that is if the row and column differences tend to zero for p→∞, then $\lim_{p\to\infty} A^{2p}$ is a balanced estimate of A. The resulting matrix is the GLS balanced estimate for the case in which there is no covariance between errors on initial estimates. Whenever a solution to the minimization problem (35) exists, the generalized Friedlander process will tend toward that solution.

In sum, if (i) the problem of balancing a Social Accounting Matrix does not contain other restrictions on the balanced estimate than those dictated by predefined values of row and column sums; and (ii) the covariance matrix of stochastic estimation errors is diagonal, then the GLS estimate can be computed in a simple and efficient manner by means of the generalized Friedlander method, defined by equations (36) and (37) above.

PART III

Illustrative Policy Simulations

I. Introduction

This part of the paper applies the CGE model in order to analyze the implications of key structural fiscal policy actions for the labor market in Poland. CGE models have been widely used to simulate different types of fiscal reforms. Early-generation CGE tax models were static and did not incorporate intertemporal effects. Shoven and Whalley (1984) provide a review of this kind of CGE studies. The more recent applications of CGE modelling to simulate tax reforms involve dynamic effects (e.g. Knudsen et al. (1998), Bovenberg et al. (2000)). General discussion of applicability of CGE models to analyses of tax reforms versus other approaches may be found, for instance, in Kesselman and Cheung (2004).

In Section II, we present the economic background with a special focus on the labor market and public finances. This chapter identifies high labor taxation and the discouraging effects of social transfers to be among the main reasons for the difficult labor market situation in Poland.

According to these conclusions we design policy packages (two scenarios described below) aimed at improving the labor market situation through a reduction of the tax wedge on labor combined with consolidation of social transfers or increase in other taxes.

In Section III we present the results of these simulations.

In the first scenario we analyze a budget neutral 10% reduction of the tax wedge compensated by an appropriate increase in the VAT rate. This represents a reform where taxation is shifted away from labor to general taxes with the size of the government unchanged.

In the second scenario we analyze a 10% decrease of the tax wedge compensated by an appropriate reduction in social transfers. This represents a reform where labor taxes are reduced and social transfers consolidated leading to a smaller government.

In both simulations we assume that the labor market is fully flexible (the so-called neoclassical closure), which means that wages are flexible and equilibrate the enterprises' demand for labor with the households' supply of time for market activities, while unemployment is fully voluntary and exogenous to the model.

Next, we introduce rigidities in the labor market (the so-called Keynesian closure). This means that wages are exogenously fixed and employment is determined by the corresponding demand for labor at this wage rate, while unemployment varies according to the changes in household endowment and supply of labor. Keynesian closure is analyzed for both scenarios and compared to the effect obtained using the neo-classical closure.

In order to get closer to reality, we then introduce the so-called mixed closure, in which the labor market is segmented: for high skill labor, the market is assumed to be flexible (neo-classical closure); while for low-medium skill labor, the market is rigid (Keynesian closure).

Further, we simulate a three-year (2006-08) government reform program consisting of sequenced changes to the tax and social contribution systems. This simulation is thus "dynamic" in the sense that it represents a path of static equilibria for every simulated year. Results are compared to the baseline scenario described in Poland's Convergence Program[17].

Finally, we present summary and conclusions.

The main findings of this part are as follows:

(i) the CGE model is an excellent tool for modeling policy reforms. It allows analyzing the economy at any available level of disaggregation and for tracking channels by which the reform package influences the economy.

[17] Program Konwergencji (przyj ty przez Rad Ministrów w dniu 30.04.2004 r.), Warszawa, kwiecień 2004 r.

(ii) Simulations show that it is indeed possible to improve the situation in the labor market in Poland using the available policy tools, such as reduction/reorientation of taxes and/or improvement in labor market incentives embedded in social transfers.

(iii) The key ingredient of such reform should be a decisive reduction in labor taxation (social contributions). Complementing reforms with a consolidation of social transfers and related labor market incentives greatly improves the outcome of the reform.

(iv) Out of two analyzed policy packages, the one envisaging a reduction in social transfers (adding to a cut in a tax wedge) yields visibly superior results, in terms of higher growth, higher employment, higher investment, higher activity rate, lower taxes and so on. Thus, stimulating the labor market on both the supply and demand sides is highly conducive to long-run growth.

(v) Most importantly, poor households are not hurt in such scenario. The increased disposable income (from higher wages) makes up for the loss of a part of government transfers, on average and for most household types although income inequality increases. On the other hand, poorer households gain in terms of employment and participation rate.

(vi) When wages are rigid (fixed), a reduction in the tax wedge increases employment and GDP in both scenarios, which become somewhat similar. However, the second scenario retains better characteristics. The important feature in a Keynesian closure is a much lower level of economic activity (due to unresponsive wages). The mixed closure produces results similar to the Keynesian one.

II. Economic Background

Introduction

The adjustment in the labor market in Poland during transition generally happened in two waves. There were large declines in employment in the initial period of transition, paralleling a rapid liberalization of markets and an initial temporary recession during 1989-1992. Most of this job destruction reflected widespread labor shedding in state-owned enterprises, which during the central planning era, under a deliberate artificial policy of full employment, had accumulated a large surplus of redundant labor. Fortunately, free market conditions facilitated the emergence of a thriving private sector, particularly small and medium-size enterprises, which have absorbed a large part of those who lost jobs in state- owned companies.

During 1993-1998 and particularly after 1995, the corporate sector concentrated its effort on modernization of its capital stock, which resulted in an investment boom and related economic growth acceleration. However, during the same period, reform efforts lost momentum, state bureaucracy grew, the investment climate gradually deteriorated, already high taxes increased further and fiscal slippages become notorious, which was overcompensated by a relatively tight monetary policy.

The 1998 Russian crisis triggered a second round of enterprise restructuring. This time productivity increases came only through labor shedding, as firms with a modern capital stock could afford a large reduction of labor. Massive job destruction continued for five years until 2002 and totaled almost 15% of the labor force. Lay-offs were concentrated predominantly in the low-skill segment of the labor market as firms simply substituted capital for low-skill labor. At the same time, policymakers failed to recognize the problem in due time. Economic and employment policies were not labor friendly; on the contrary, they provided incentives to withdraw from the labor force (on the supply side) such as early retirement schemes, while punitive rates of labor taxes (both demand and supply side) priced out many workers from the labor market. As a result, Poland has the lowest employment rate in the European Union – only 51% of the population are employed.

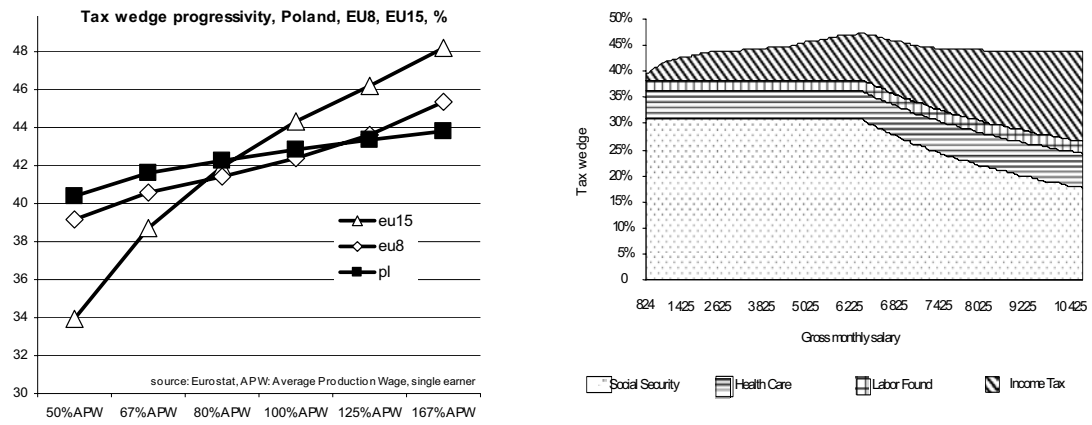
The introduction above has identified three major medium-term causes of the labor market depression in Poland: deterioration in investment climate (demand side), excessive labor taxes (demand and supply) and disincentives provided by the system of social transfers (supply side). Two of them, i.e. tax wedge and the role of benefits, can be analyzed within the framework of this model.

The role of the tax wedge

The structure of labor taxes in Poland (as % of gross wage) is as follows: (i) paid by the employee: 9,76 % – retirement; 6,5% - pension; 2,45% - sickness insurance; and (ii) paid by the employer: 9,76 % – retirement; 6,5% - disability insurance; 1,62% - accident insurance; 2,45% - Labor Fund; 0,15% - State Fund for Rehabilitation of the Disabled.

Among EU-8 countries, Poland has the second highest tax wedge for a minimum wage earner (after the Czech Republic), equal to 40.4% of total labor costs (Figure 5). Similarly, it has the second highest tax wedge for an average wage earner (after Hungary), equal to 43.9% of total labor costs (2003 Eurostat data). Furthermore, the tax wedge in Poland is the least progressive among its European peers at low- to mid-income ranges. This puts a relatively higher burden on low-wage, low-skilled workers. Moreover, when gross income exceeds three times average salary, the tax wedge becomes regressive.

Figure 5. Tax Wedge in Poland



Sources: Eurostat (left) and Ministry of Labor and Economy (right). Note: average gross wage is 2289 PLN per month.

Many studies find a negative relationship between the tax wedge and employment. Although the magnitude of the disemployment effects of labor taxes differs from study to study (elasticities generally between -0.1 to -0.5), this effect is consistently found to be significant. In theory (and in reality), the relationship between the tax wedge and employment has (among others) the following characteristics: (i) a high tax wedge affects employment much more for low-wage earners than for high-wage earners; (ii) the disemployment effect is magnified by the presence of any kind of "wage floor", such as a minimum wage; (iii) existence of any alternative non-employment benefits works in a similar way by producing a reservation wage which acts as a wage floor; and (iv) an increase in payroll taxes (labor taxes paid by employers) is expected to have a bigger negative impact on employment than an increase in income taxes (labor taxes paid by employees).

All the above characteristics prevail in Poland: (i) the share of low skills in population is relatively high; (ii) minimum wage is likely to be binding for a relatively large part of the labor force, particularly in disadvantaged regions; (iii) social benefits are relatively generous, particularly in regions were cost-of-living is low; and (iv) payroll taxes (paid by the employer)

constitute the largest part of the tax wedge. Therefore, there are grounds to believe that in Poland a large tax wedge is particularly harmful to employment.

The role of non-employment benefits

Provision of social transfers in the initial period of transition provided a necessary cushion for those left behind by the rapid change. Transfers were used as an instrument to maintain social cohesion and win support for further economic reform. However, despite the tightening of eligibility criteria, this generous safety net was not effectively withdrawn in due time leading to serious fiscal and labor market problems. Moreover, social transfers are badly targeted. As much as 50% of them either go directly to non-poor households or are in excess of what is needed to bring households to the national poverty line. Finally, social transfers are not conditional on labor market participation.

The decision to be economically active is ultimately a financial decision. Easy access to relatively generous, unconditional transfers (early retirement, pre-retirement allowances, disability benefits) provides a disincentive towards job search, increases the reservation wage and effectively decreases labor supply.

Labor market and public finances

The analysis above suggests that a deep reform of the combined tax-benefit system should be pursued in order to stimulate labor demand and supply and improve labor market conditions.

Given the already very strained fiscal situation in Poland and the need to further reduce the deficit in order to meet Maastricht euro adoption criteria, the question is whether the fiscal-labor market reform be done in a budget neutral way. In principle, the answer could be positive. First, lowering of the tax wedge may partly finance itself through higher employment and output already in the short term. Second, it is not a given that current pensions must be financed from social security contributions levied on labor only – they could be partially financed from general taxes, as is the case in several countries in Europe. Finally, streamlining and rationalization of social benefits systems is still an untapped source of fiscal savings.

Current policy discussions on tax wedge and social benefits in Poland

In recognition of the above mentioned problems, the Ministry of Economy and Labor has for some time been working on a strategy to decrease the disincentives for work provided by the combined tax-benefit system. Currently two ideas are on the table.

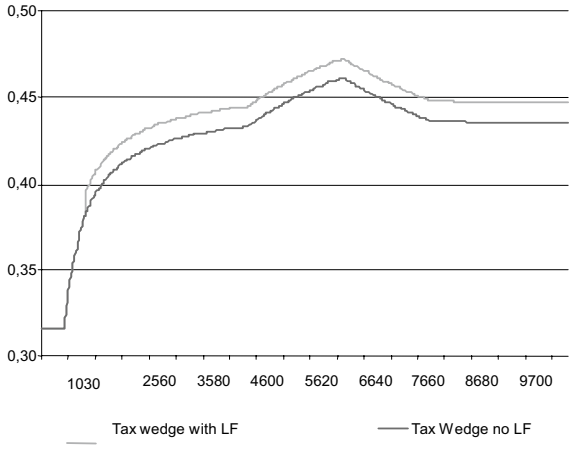
The first is to abolish contributions to the Labor Fund (2.45% of gross wage), which would be compensated by an increase in indirect taxes. The feature of this solution is that it reduces labor cost, but does not affect net wages directly. Therefore, it should increase demand for labor and reduce the counter-cyclical pattern of the Labor Fund's revenues and its pro-cyclical spending.

The second policy option would be to introduce a so-called activation allowance that reduces the employee's part of the personal income tax and health care contribution for low-wage earners (i.e. people who earn less than 1,200 PLN per month). This solution is expected to increase net wages, but would not affect labor costs directly. The advantage is that it should motivate people earning (close to) the minimum wage to increase their labor supply.The expected impact of the two proposals is presented in Figure 6 below.

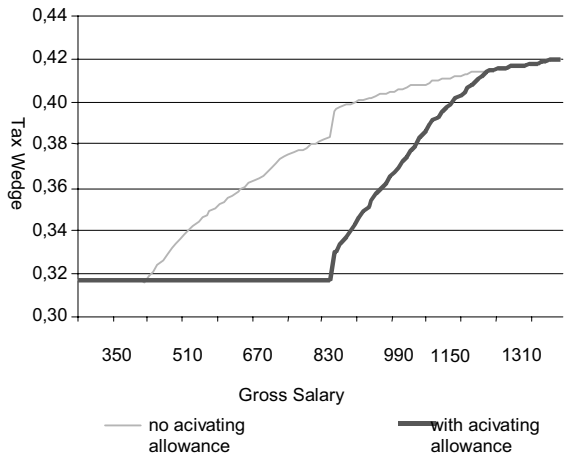
Moreover, one of the objectives of the National Plan for Employment for 2005 is reducing the de-activating role of social transfers. The reform agenda envisages: limiting access to pre-retirement and disability benefits, rationalization of many social programs and eliminating inactivity traps that exist on the border of labor market and social protection system.

Figure 6. The Expected Impact of Alternative Proposals

Option 1: Abolishing Labor Fund Contribution



Option 2: Introducing Activation Allowance



Source: Ministry of Labor and Economy

III. Description of Simulations

This section presents the result of the simulations. The two scenarios (namely, the reduction in the tax wedge offset by (i) a VAT rate increase; and (ii) a reduction in social transfers) are analyzed using the neo-classical closure. This is later expanded to Keynesian and mixed closures. Finally, we simulate the effects of a three-year, sequenced reform package.

The reaction of GDP, output, balance of payments, capital and labor market and households are all described.[18] All results presented are expressed as percentage point (pp) deviation from the baseline scenario. The baseline is the path of growth assumed in the 2005 government Convergence Program.

III.1. Scenario 1: 10% Decrease of Tax Wedge Compensated by an Appropriate Change in the VAT Rate

The design of the reform

In this scenario, the government reduces the tax wedge (social taxes, assumed to be paid by the employer) by 10%. Simultaneously, it increases the general VAT rate proportionally across all goods, in order to fully offset the loss of revenues and arrive at the initial deficit level. Thus, the scenario assumes a budget neutral shift from direct taxes (on labor) to indirect taxes (on consumption), which leaves the overall "size of the government" unchanged. The 10% reduction of the tax wedge results in a 1.4% of GDP decline in revenues. To compensate for that the government increases the general VAT rate by as much as 20% (revenues from VAT are about 60% of those from social contributions).

[18] Although the unemployment rate is important from the political economy perspective, changes in employment (and in the activity rate) are more relevant from the economic perspective.

Channels at work

The general channels in which those changes are transmitted through the economy are as follows: the initial reduction in the tax wedge implies a decline of the cost of labor, which raises demand for labor, and also drives wages up. It leads to an increase in production and GDP. On the other hand, the increase in the VAT rate creates a negative demand effect on the commodities market, dampening the response of overall activity in the economy.

Table 12. Simulation with VAT Adjustment – Macro Variables and Labor Market

	Quantity	Price
GDP	0.18%	0.00%
Private consumption	0.44%	0.31%
Investments	-0.51%	0.41%
Exports	-0.38%	-0.49%
Imports	-0.37%	0.53%
Gross output	0.13%	-0.84%
Labor	0.62%	-3.00%
Capital	0.00%	-0.75%
Unemployment	17.9%	
Change of unemp.	-0.1%	
Labor participation	0.51%	
Inactivity	-0.29%	
G. net lending (% GDP)	-3.97%	
Change in VAT	19.9%	

	Education		
	high	medium	basic
Demand	0.67%	0.58%	0.87%
Wages	2.35%	2.23%	1.96%
Unemplyment	5.0%	21.5%	21.4%
Change in unemp.	0.0%	-0.1%	-0.2%
Participation	0.64%	0.45%	0.68%

GDP and components

The net effect on GDP growth is positive – it increases by 0.18 pp (Table 12).

Private consumption also rises (by 0.44 pp). The positive impact of the growth in employment and wages and, hence, in disposable income (due to cuts in the tax wedge) outweighs the negative impact of higher relative prices of consumption (due to the VAT increase).

Investments decline by 0.51 pp as the supply of savings decreases by 0.1 pp and the price of the investment good (relative to the GDP deflator) increases by 0.4 pp.

Balance of payments

Due to the increase of VAT, the prices of imports rise (on aggregate by 0.5 pp), so demand shifts towards domestic production (the prices of domestic production rise slower than in the baseline case due to the decreased costs of labor), which leads to a decline in imports by 0.37 pp. For the balance of payments in foreign currency to remain unchanged, the domestic currency appreciates by 0.5 pp., meaning a slight decline in the prices of exports. As exports become less profitable, their volume declines (by 0.38 pp). Altogether, it leads to a slightly positive contribution of net exports to GDP growth.

Capital market

As the change in final demand is relatively moderate, the additional demand for capital is limited, which - given the unchanged supply of capital - leads to a decline of capital prices by 0.75 pp.

Labor market

Total employment increases by 0.62 pp as a result of two opposite effects. First, as labor costs fall, the demand for labor rises, along with wages (+2.2 pp). Higher wages make people more willing to work, so we observe a flow from inactivity to activity - the participation rate

increases by 0.51 pp. Second, the rise in VAT dampens the demand for goods and this constrains producers in employing additional labor.

As both activity and the employment rate rise in parallel and at a similar rate, unemployment remains broadly unchanged.

Looking at the disaggregated labor market, the growth of the demand for labor with basic education (by 0.87 pp) is somewhat higher than for medium- and higher education (0.58 pp and 0.67 pp, respectively).

This reflects the different situation in various segments of the labor market. Unemployment among the higher educated labor is low (about 5%), which makes this type of labor relatively scarce and results in relatively higher wage growth (+2.4 pp). This leads to a relatively higher inflow of inactive people into the market, but relatively lower additional demand. The unemployment rate, thus, remains unchanged.

The opposite effect occurs in the market for labor force with basic education; unemployment is high (about 21%), which helps to keep wage growth moderate (+2.0 pp) and in turn supports demand for labor. Higher demand and wages attract people to the labor market (activity rate rises by 0.68 pp). The net effect on the unemployment rate is slightly positive (it drops by 0.15 pp).

Table 13. Simulation with VAT Adjustment – Household Sector

	Disposable income	Consumption	Prices
Employees - non-poor	1.6%	1.4%	0.3%
Employees - poor	1.7%	1.0%	0.7%
Farmers - non-poor	0.5%	0.1%	0.4%
Farmers - poor	0.1%	-0.8%	0.9%
Self-employed - non-poo	-0.6%	-0.8%	0.3%
Self-employed - poor	-0.5%	-1.1%	0.6%
Pensioners - non-poor	0.8%	0.6%	0.3%
Pensioners - poor	0.9%	0.2%	0.7%
Others - non-poor	0.8%	0.5%	0.4%
Others - poor	0.9%	0.1%	0.8%
Non-poor	0.7%	0.4%	0.3%
Poor	1.0%	0.3%	0.7%

Households

The disposable income of employees grows by about 1.6 pp, reflecting the fact that they supply the bulk of labor in the economy (Table 13). The disposable income of farmers, pensioners and other households grows moderately by 0.1-0.9 pp. The disposable income of self-employed falls by 0.5 – 0.6 pp, despite the fact that their supply of labor rises the most. This is because the price of the services of capital falls, with revenues from renting capital to the production sector the main source of income for the self employed (they own the majority, i.e. two-thirds, of the total capital stock).

The consumption of employees rises relatively faster than for other types of households, paralleling the movements of disposable income. Similarly, the consumption of self-employed falls due to the drop in disposable income.

The disposable income of poor households (+1.0 pp) rises faster than that of non-poor households (+0.7 pp). The main reason is, again, the falling income from capital ownership - an important income source for non-poor households. However, at the same time, the prices of consumption for poor households rises relatively faster. The main reason is rising prices of foodstuff, which constitute a considerable part of their consumption basket. Higher growth of food prices originates in the production pattern of the economy, where a decline in labor costs

for the food processing sector is the smallest among all sectors, resulting in a relatively limited supply response. Higher income and higher prices net out and the consumption of non-poor and poor households rise at a similar pace (0.4 pp and 0.3 pp, respectively).

Production sector

As regards the production sector, gross output rises by 0.13 pp on average (Table 14). This growth is mainly driven by patterns of consumption demand and by a drop in labor costs, which positively affects such industries as production of energy, trade, transport and financial intermediation (output growth 0.4-0.6 pp above baseline).

For industries with a high share of exports (such as mining and manufacturing of investment goods), the decline in labor costs means a significant improvement in competitiveness and an expansion of output and exports.

Table 14. Simulation with VAT Adjustment – Production Sector

Industry	Gross output	Producer costs	Export	Import	Employ-ment
Agriculture	0.2%	-0.5%	0.1%	0.3%	0.7%
Mining	0.9%	-1.5%	1.5%	0.0%	1.3%
Food	-0.5%	-0.3%	-1.0%	0.5%	-0.4%
Light	-0.7%	-0.6%	-1.3%	-1.4%	-0.7%
Intermediate light	-0.6%	-0.5%	-1.3%	-0.6%	0.0%
Intermediate heavy	0.3%	-0.7%	0.4%	-0.2%	0.5%
Investments	1.2%	-0.9%	1.7%	-0.4%	1.5%
Motor vehicles	-2.9%	-0.4%	-2.9%	-0.7%	-0.2%
Energy	0.4%	-1.1%	0.7%	-0.2%	1.0%
Construction	-0.2%	-0.6%	-0.1%	-0.2%	0.2%
Trade	0.4%	-0.7%	0.7%	0.2%	0.9%
Transport	0.4%	-0.8%	0.5%	0.1%	0.8%
Financial interm.	0.6%	-0.9%	1.2%	0.2%	1.2%
Non-market serv.	0.4%	-1.7%	1.2%	0.3%	0.5%

Although consumption demand rises faster than GDP, the production in the industry of food processing declines by 0.5 pp. As noted above, this is mainly due to a relatively small decrease of overall production costs in this industry (-0.3 pp). This means higher domestic prices for food, and lower exports and higher imports.

Limited supply and relatively higher prices of food products crowds out consumption demand for light manufacturing, where output of domestic producers declines by 0.7 pp (along with imports).

Growth of investment demand below the baseline results in lower domestic production of manufacturing of light intermediate goods, motor vehicles and construction. The decline is observed despite lower costs of production in these industries. Also, due to a weak domestic (investment) demand, imports in these industries (as well as in mining, heavy intermediate goods and investments goods) decline.

Since government demand for goods is fixed, gross output of non-market services (which is mainly consumed by the government) rises by 0.4 pp.

As regards employment, the highest growth rates are recorded in sectors where the labor costs decline the most. This includes mining, manufacturing of investment goods, energy, trade, transport and financial intermediation.

III.2. Scenario 2: 10% Decrease of Tax Wedge Compensated by an Appropriate Reduction in Social Transfers

The design of the reform

In this scenario, the government reduces the tax wedge (social taxes) by 10% and simultaneously reduces social transfers in order to fully offset the fiscal impact and arrive at the initial deficit. Compared to the previous scenario, the government decreases both revenues and expenditures, which leads to a decrease in the overall level of fiscalism in the economy. The simulations results reveal that in order to match the declining revenues from taxes, the government needs to reduce social transfers by 14%.

Channels at work

The reform package directly affects both the demand and supply of labor. Broadly speaking, in addition to stimulating labor demand through the decline of the tax wedge and the resulting decline of labor cost, it also affects positively the supply of labor through the decline in social transfers. This is because social transfers act as a "subsidy to leisure", discouraging people from labor market participation. Cutting them, apart from putting direct downward pressure on disposable income, changes the relative price of work and leisure and makes people more willing to search for a job.

Compared to the previous scenario, we would expect that the reaction of wages should be more moderate while the reaction of employment much higher. This is indeed the case.

GDP and components

The overall effect of a change in social contributions and social transfers is an increase of GDP by 1 pp (Table 15).

Private consumption rises by 0.84 pp in response to an increase of employment and wages. This is possible, because growth of the wage bill more than offsets the adverse effect of a cut in social transfers. Additionally, the price of renting capital rises which increases disposable income of households.

Investment rises by 2.2 pp, reflecting the increase of total savings by 2.4 pp. These additional savings originate mainly in corporate sector (3.1 pp), reflecting both a higher level of economic activity and higher prices of capital services, and the household sector (2.8 pp), due to higher disposable income.

Balance of payments

The growth of domestic demand (by 0.97 pp), driven in large part by more import-intensive investment demand, results in an increase in demand for imports by 1.6 pp. Since the balance of payments position is fixed, the domestic currency depreciates slightly, while the expansion of output provides for an offsetting increase in exports (+1.8 pp).

Table 15. Simulation with Social Transfers Adjustment – Macro Variables and Labor Market

	Quantity	Price
GDP	1.04%	0.00%
Private consumption	0.84%	0.37%
Investments	2.19%	0.23%
Exports	1.81%	0.01%
Imports	1.55%	0.01%
Gross output	1.15%	0.15%
Labor	2 26%	-4.20%
Capital	0.00%	3.41%
Unemployment	17.6%	
Change of unemp.	-0.3%	
Labor participation	1.85%	
Inactivity	-1.06%	
G. net lending [% GDP]	-3.94%	
Change in transfers	-14.0%	

	Education		
	high	medium	basic
Demand	1.95%	2.26%	3.66%
Wages	1.10%	1.17%	0.32%
Unemplyment	5.0%	21.2%	20.9%
Change in unemp.	-0.1%	-0.4%	-0.6%
Participation	1.85%	1.77%	2.87%

Capital market

As the capital supply is the same as in the baseline scenario, it becomes relatively more scarce in the presence of growing employment and overall demand in the economy and its price increases by 3.4 pp.

Labor market

Total employment rises by 2.3 pp. The reaction of employment is, as expected, stronger than in the previous scenario (0.6 pp). The direct cause is a steeper fall in labor costs (-4.2 pp compared with -3 pp in the previous case). This happens through lower growth in wages (1.1 pp vis-à-vis 2.2 pp in scenario 1) induced by increased supply of labor. Note that wages are still higher than in the baseline scenario.

Wage pressure are subdued by a considerable inflow of people to the labor market. This is possible because, as transfers (subsidy to inactivity) are cut, the relative price of leisure to work declines and people are more willing to search for a job. As a result, the activity rate increases by 1.9 pp.

The unemployment rate declines slightly (-0.3 pp), reflecting the opposite forces of an increase of employment and an increase of activity rate.

Although for all labor types the demand and employment is higher than in the baseline, labor market adjustments differ between submarkets.

Labor with basic education is relatively more substitutable with capital, which becomes a more scarce factor of production. Therefore, the demand for basic skills rises the most (by 3.7 pp). On the other hand, the decline in social transfers, which constitutes a considerable part of income of the households supplying this kind of labor, affects their labor market decisions significantly and triggers a relatively high inflow into the labor force (participation rate grows by 2.9 pp). As a result, wages increase moderately (+0.3 pp), but the decline in the unemployment rate is still relatively high (-0.6 pp).

At the other end of the labor market, the low unemployment rate of highly educated labor translates into inflows to the market only moderately higher than in the baseline (the activity rate rises by 1.9 pp) so wage growth is higher (1.1 pp) and the demand increase (+2 pp) more muted. The two offsetting effects of rising employment and activity leave the unemployment rate almost unchanged (-0.1 pp).

For the medium skills labor, social transfers induce a relatively moderate inflow into the labor market (activity rate up by 1.8 pp), while the labor demand effect (+2.3 pp) is relatively

high (because this type of labor is important in the production process and constitutes about 70% of total employment). Thus, the resulting wage growth is higher (+1.2 pp) and the fall in unemployment more pronounced (-0.4 pp).

Households

The distribution of disposable income differs somewhat across types of households (Table 16). The highest increase occurs among the self-employed households, mainly due to the increased income from capital renting, as prices of capital services rise and self-employed own the majority of capital among households. The farmers' and employees' disposable income also grows, mainly due to the increased income from labor. Disposable income of those households who rely on social transfers (pensioners and "other" households) falls in general by 0-2.7 pp, despite being supported by higher labor income.

The distribution of prices (cost of living) changes among households is rather homogenous (ranging from 0.2 to 0.4 pp).

As a result, the change in consumption reflects changes in disposable income and the strength of the consumption-leisure substitution effect, which depends on the importance of social transfers as a source of income. Disposable income of households who depend the most on social transfers (e.g. pensioners, farmers, and others) are adversly afected by the decline in transfers. However, they are also more willing to work (to compensate the decrease of income) and thus the decline of their disposable income is rather moderate. Households who depend less on transfers (e.g. employees) are less affected in their work/leisure and consumption decisions.

Table 16. Simulation with Social Transfers Adjustment – Household Sector

	Disposable income	Consumption	Prices
Employees - non-poor	1.1%	0.6%	0.4%
Employees - poor	0.9%	0.4%	0.4%
Farmers - non-poor	1.8%	1.0%	0.4%
Farmers - poor	1.8%	0.7%	0.3%
Self-employed - non-poor	3.1%	2.7%	0.3%
Self-employed - poor	3.0%	2.8%	0.2%
Pensioners - non-poor	-0.3%	-1.1%	0.4%
Pensioners - poor	-1.7%	-2.5%	0.3%
Others - non-poor	0.0%	-0.6%	0.4%
Others - poor	-2.7%	-3.3%	0.4%
Non-poor	1.5%	0.9%	0.4%
Poor	0.4%	0.1%	0.3%

The disposable income of the non-poor households rises faster than that of the poor. This happens because: (i) the decline in social transfers mostly affects poor households; and (ii) non-poor households own more capital for which the price is going up. However, it is important to note that despite cuts in social transfers, poor households are not hurt on average. Increased income from higher employment and wages more than offsets the fall in transfers. With the higher consumer prices (stemming from the increase of overall demand in the economy), their consumption remains broadly unchanged in real terms.

Production sector

The strong increase in demand and falling price of labor result in a rise of gross output by 1.2 pp.

The output reaction of individual industries depends on their capital intensity and the source of demand (Table 17). This is because relative prices of capital rise (and of labor fall) while

investment demand drives total demand to a large extent. On the other hand, the reaction of employment depends on how much demand increases and how substitutable cheaper labor is for more expensive capital. Finally, export performance depends on the cost development in individual sectors.

Accordingly, the highest growth rates of output are recorded in mining, light manufacturing, heavy intermediate and investment good manufacturing, and manufacturing of motor vehicles (ranging from 2 to 4 pp). In this group, demand comes from the investment side while capital intensity is relatively low. In construction, where capital intensity is somewhat higher (but demand is investment driven), the rise in demand for labor is mainly due to the substitution effect because of the increasing prices of capital. In this group, on average, export growth is sizable, as costs of production decline and the price of exports rises.

In the other group, i.e. manufacturing of light intermediate goods, energy, transport and financial intermediation, where capital intensity is higher, the overall cost of production rises somewhat (by 0 to 1 pp) so the increase in demand for labor is modest (from 2 to 3 pp) and so is the reaction of gross output (0.8 to 1.2 pp) and value added. Export growth is positive, but moderate.

In agriculture and food processing costs of production grow and capital-labor substitution takes place. Gross output and value added are almost unchanged. The rise of costs in these industries makes their production less competitive on foreign markets and, as a result, exports decline.

Table 17. Simulation with Social Transfers Adjustment – Production Sector

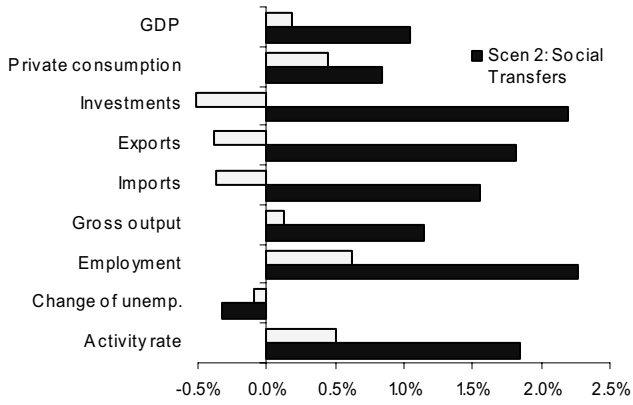
Industry	Gross output	Producer costs	Export	Import	Employ-ment
Agriculture	-0.2%	1.0%	-2.5%	2.2%	1.3%
Mining	1.9%	-1.2%	2.6%	0.7%	3.0%
Food	0.2%	0.4%	-0.3%	1.3%	0.8%
Light	2.4%	-0.2%	3.3%	1.8%	3.3%
Intermediate light	1.2%	0.1%	1.0%	1.5%	2.5%
Intermediate hea	1.9%	-0.2%	2.2%	1.3%	2.9%
Investments	3.3%	-0.5%	3.9%	1.6%	4.1%
Motor vehicles	3.9%	-0.2%	4.0%	2.2%	6.4%
Energy	1.1%	0.0%	1.1%	1.1%	3.0%
Construction	1.6%	0.4%	1.5%	1.8%	3.0%
Trade	0.8%	1.0%	-0.5%	1.4%	2.6%
Transport	0.8%	0.3%	0.5%	1.6%	2.1%
Financial interm.	1.0%	0.6%	1.1%	1.5%	2.7%
Non-market serv.	0.5%	-1.4%	1.0%	1.1%	1.2%

Finally, in the case of non-market services, where labor intensity is one of the highest in the economy, the decline of the tax wedge means a considerable reduction in overall costs (-1.4 pp), but since the consumption of government, which constitutes the bulk of demand for these services, is the same as in the baseline scenario, growth of gross output and value added (by 0.5 pp) is relatively low. This growth is mainly based on increased employment (+1.2 pp).

Comparison of scenario 1 (VAT adjustment) and scenario 2 (social transfer adjustment)

Figure 7 presents a rough comparison of behavior of the main economic variables under the two scenarios.

Figure 7. Economic Developments under the Two Scenarios (pp deviation from baseline)



III.3. Introducing Labor Market Rigidities

Keynesian (fixed wages) closure

In both analyzed simulations we assumed that the labor market was fully flexible (the so-called neo-classical closure), which means that wages are flexible and equilibrate the enterprise labor demand with household labor supply.

In this section, we introduce rigidities in the labor market (the so-called Keynesian closure). This means that wages are exogenously fixed and employment is determined by the corresponding demand for labor at this wage rate, while unemployment varies according to the changes in households' endowment and supply of labor.

We analyze the implications of Keynesian closure for both fiscal reform scenarios.

Channels at work

With wages fixed, the decrease of the tax wedge leads to the maximum decrease of labor costs (as there is no reaction of wages). The resulting demand for labor is much higher, and so is the level of economic activity and GDP (Table 18).

The higher level of economic activity increases the government's receipts from various taxes and reduces the magnitude of the adjustment necessary to restore the initial deficit. Specifically, in scenario 1, an increase in the VAT rate of 13% is enough, instead of the 20% in the case of neo-classical closure. In scenario 2, social transfers need to be reduced only by 12%, instead of 14%. In the first case, the lower increase of VAT needed to restore the initial fiscal deficit leads to a higher level of economic activity as the dampening effect of VAT on overall demand is reduced. However, in the case of social transfers adjustment, the lower decrease of transfers results in a smaller, but still positive, shift of labor supply.

We can expect that the implication of assuming labor market rigidities will be more pronounced in the case of the first scenario (VAT rate adjustment) than the second (adjustment through transfers). This is because Keynesian closure implies fixed wages and in the second scenario the reaction of wages to the reform package under classical closure was already relatively moderate.

GDP and components

Due to higher economic activity, the impact of the reform is positive. GDP increases by 0.9 pp and 1.3 pp for scenario 1 and 2, respectively.

Table 18. Main Economic Variables (Comparison of Classical and Keynesian Closure)

	Scen. 1: VAT rate adjusting				Scen. 2: social transfers adjusting			
	Classical closure		Keynesian closure		Classical closure		Keynesian closure	
	Volume	Price	Volume	Price	Volume	Price	Volume	Price
GDP	0.18%	0.00%	0.93%	0.00%	1.04%	0.00%	1.30%	0.00%
Private consumption	0.44%	0.31%	1.03%	0.49%	0.84%	0.37%	1.09%	0.46%
Investments	-0.51%	0.41%	1.10%	0.45%	2.19%	0.23%	2.63%	0.30%
Exports	-0.38%	-0.49%	1.20%	-0.32%	1.81%	0.01%	2.31%	0.00%
Imports	-0.37%	0.53%	1.01%	0.35%	1.55%	0.01%	1.98%	0.00%
Gross output	0.13%	-0.84%	0.99%	-0.43%	1.15%	0.15%	1.45%	0.19%
Labor	0.62%	-3.00%	2.19%	-5.30%	2.26%	-4.20%	2.85%	-5.30%
Capital	0.00%	-0.75%	0.00%	2.20%	0.00%	3.41%	0.00%	4.31%
Unemployment	17.9%		14.4%		17.6%		15.8%	
Change of unemp.	-0.1%		-3.5%		-0.3%		-2.2%	
Labor participation	0.51%		-2.04%		1.85%		0.17%	
Inactivity	-0.29%		1.16%		-1.06%		-0.10%	
G. net lending (% GDP)	-3.97%		-3.94%		-3.94%		-3.93%	
Change in VAT	19.9%		13.0%		-14.0%		-11.9%	

Private consumption also rises (by about 1 pp in both cases). Similar to the classical closure, this means that the positive impact of the growth in the wage bill (through higher employment) outweighs the negative impact of higher relative prices of consumption (scenario 1) or lower social transfers (scenario 2). On top of that, under Keynesian closure, the lower necessary adjustment of VAT (scenario 1) or social transfers (scenario 2) adds to the increased private consumption.

Investment rises in both scenarios (by 1.1 pp and 2.6 pp for scenarios 1 and 2, respectively) reflecting the increase in total savings (1.6 pp and 2.9 pp, respectively). These additional savings come from the corporate sector (+2 pp and +4 pp, respectively) and from households (+1.8 pp and 3.2 pp, respectively) reflecting both the higher level of economic activity, higher prices of capital services and higher disposable income (in the presence of constant propensity to save).

Compared to neoclassical closure, Keynesian closure means higher economic activity (GDP, private consumption, investment) for both scenarios, but considerably more so for scenario 1.

Balance of payments

The pattern of foreign trade prices is not much different under Keynesian closure. In the first scenario, the increase in import prices due to the VAT rate hike are offset by a slight depreciation of the exchange rate. In the second scenario prices are virtually unchanged.

Due to higher demand, not least from investment, import rise (by 1 pp and 2 pp for scenario 1 and 2, respectively) – more than under classical closure.

Exports, driven by the supply effect (increase in output) and according to a fixed balance of payments requirement, rise as well (by 1.2 pp and 2.3 pp, respectively).

Capital market

With the flow of capital services being the same as in the baseline scenario and with higher total demand on the goods market, capital becomes a relatively more scarce commodity and its price rises relative to the baseline (by 2.2 pp and 4.3 pp for scenarios 1 and 2, respectively). The difference from the classical closure is more pronounced in scenario 1, where capital prices actually rise instead of fall (notice the considerable difference in the level of economic activity induced by the fiscal reform considered in scenario 1 depending on the labor market assumption). This triggers capital-labor substitution and adds to the increase in the demand for labor.

Labor market

When wages are fixed, the decline of the tax wedge is fully passed into lower labor costs (-5.3 pp in both scenarios) and the positive employment effect is maximal. The resulting growth in total employment above baseline reaches 2.2 pp and 2.9 pp for scenarios 1 and 2, respectively. In scenario 1, the difference in employment growth from the classical closure is as high as 1.6 pp.

Under Keynesian closure, flat wages (combined with the relatively smaller - compared to the neo-classical case - decrease of social transfers in the second scenario) significantly affect the leisure-consumption decisions of households, discouraging them from being active in the labor market. The participation rate is much lower (about 2 pp) than under classical closure. Specifically, in the second scenario, despite a drop in social transfers, the activity rate is almost unchanged (+ 0.2 pp) compared to the baseline. In the first scenario, it even reduces labor force participation (activity rate is 2 pp lower than in the baseline).

Higher employment and lower activity mean (by construction) a lower unemployment rate, which drops by 3.5 pp and 2.2 pp (in scenarios 1 and 2, respectively).

Table 19. Labor Market Dynamics (Comparison of Classical and Keynesian Closure)

		Classical closure			Keynesian closure		
		Education					
		high	medium	basic	high	medium	basic
VAT	Demand	0.67%	0.58%	0.87%	1.98%	2.18%	3.23%
	Wages	2.35%	2.23%	1.96%	0.00%	0.00%	0.00%
	Unemplyment	5.0%	21.5%	21.4%	1.1%	18.5%	13.5%
	Change in unemp.	0.0%	-0.1%	-0.2%	-3.9%	-3.1%	-8.0%
	Activity	0.64%	0.45%	0.68%	-2.08%	-1.67%	-6.31%
		Classical closure			Keynesian closure		
		Education					
		high	medium	basic	high	medium	basic
Social Transfers	Demand	1.95%	2.26%	3.66%	2.45%	2.88%	4.31%
	Wages	1.10%	1.17%	0.32%	0.00%	0.00%	0.00%
	Unemplyment	5.0%	21.2%	20.9%	2.9%	19.4%	18.8%
	Change in unemp.	-0.1%	-0.4%	-0.6%	-2.2%	-2.2%	-2.8%
	Activity	1.85%	1.77%	2.87%	0.17%	0.12%	0.76%

As regards labor market segments, due to the relatively higher growth of the price of capital services and the fact that the labor force with basic education is the most substitutable with capital, the demand for basic labor increases the fastest in both scenarios (Table 19). The participation rate in this market performs the worst, both relative to the baseline (in scenario 1 by 6.1 pp) and relative to the classical closure (both scenarios). In scenario 1, fixed wages and rising VAT mean a higher price of consumption and more appealing leisure. In scenario 2, fixed wages and the lower reduction in social transfers reduce the incentive for labor market participation. Accordingly, unemployment declines considerably.

The reaction of higher- and medium-educated labor is similar, but more moderate, mainly due to lower substitutability with the capital factor of production.

III.4. Mixed Closure

In this section, we introduce the so-called mixed closure, in which the labor market is segmented: for high-skill labor, the market is assumed to be flexible (neo-classical closure); while for medium- and low-skill labor, the market is rigid (Keynesian closure). This is done in order to better approximate the labor market reality.

As expected, the reaction of the economy is somewhere between the results obtained for the Keynesian and neo-classical assumptions (Table 20). However, because the share of workers with medium or basic education constitutes about 75% of total employment, the reaction of the economy is more "Keynesian" (Figure 8). Moreover, the difference between mixed and Keynesian closures is visibly smaller for the second scenario. The main differences appear, of course, in the reaction of disaggregated labor markets.

The composition of economic growth resembles that obtained with wages fixed in all labor markets. In scenario 1, GDP growth is balanced between expenditure components, while in scenario 2, investment demand is stronger.

As under the Keynesian closure, rising demand for goods triggers higher prices of the fixed factor of production (capital), leading to higher demand for basic labor (substitution effect), higher disposable income of households (mainly self-employed), higher profits of firms, and, thus, higher savings. This in turn stimulates investment demand. These effects are somewhat weaker than under the Keynesian closure.

Table 20. Mixed Closure (Comparison between Simulation with VAT and Social Transfers Adjustment)

	VAT		Social transfers	
	Volume	Price	Volume	Price
GDP	0.74%	0.00%	1.24%	0.00%
Private consumption	0.88%	0.42%	1.02%	0.42%
Investments	0.71%	0.39%	2.54%	0.25%
Exports	0.82%	-0.44%	2.20%	-0.03%
Imports	0.67%	0.34%	1.88%	-0.03%
Gross output	0.78%	-0.57%	1.38%	0.17%
Labor	1.80%	-4.70%	2.70%	-5.00%
Capital	0.00%	1.43%	0.00%	4.08%
Unemployment	15.2%		16.3%	
Change of unemp.	-2.7%		-1.7%	
Labor participation	-1.46%		0.66%	
Inactivity	0.83%		-0.38%	
G. net lending [% GDP]	-3.95%		-3.93%	
Change in VAT	15.1%		-12.7%	

As under other closures, investment growth is higher in scenario 2 than in scenario 1.

The direction and magnitude of movements in foreign trade, the balance of payments and exchange rate is similar as under Keynesian closure and the underlying economic mechanism is the same.

Although employment grows for all types of labor, the demand for labor with basic education increases the most (Table 21). This is due to increasing prices of capital services and higher substitutability of this type of labor with capital. As the wages of labor with medium and basic education are held fixed the costs of employing them declines significantly. In scenario 1 fixed wages and higher consumption costs discourage basic and medium-skill workers from labor market participation. In scenario 2, the reduction in social transfers (mainly for households supplying low-skill labor) keeps this type of labor in the market.

Table 21. Mixed Closure (Comparison of Labor Market Behavior Between Simulation with VAT and Social Transfers Adjustment)

	Education					
	VAT			Social transfers		
	high	medium	basic	high	medium	basic
Demand	1.28%	1.91%	2.74%	2.13%	2.79%	4.11%
Wages	2.19%	0.00%	0.00%	1.16%	0.00%	0.00%
Unemplyment	5.0%	18.5%	14.1%	4.9%	19.5%	19.7%
Change in unemp.	-0.1%	-3.0%	-7.4%	-0.1%	-2.1%	-1.8%
Participation	1.22%	-1.87%	-6.14%	2.02%	0.16%	1.78%

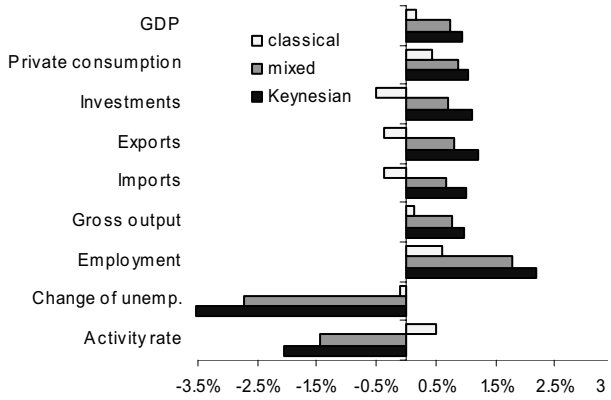
The growth of demand for highly-educated labor is constrained by higher wages which flexibly adjust to equilibrate demand with supply. Higher wages also induce an inflow of workers into activity, reflecting the fact that working becomes relatively more appealing than consuming leisure. As a result of these offsetting tendencies, the unemployment rate for high-skill labor remains almost unchanged.

Comparison of various closures

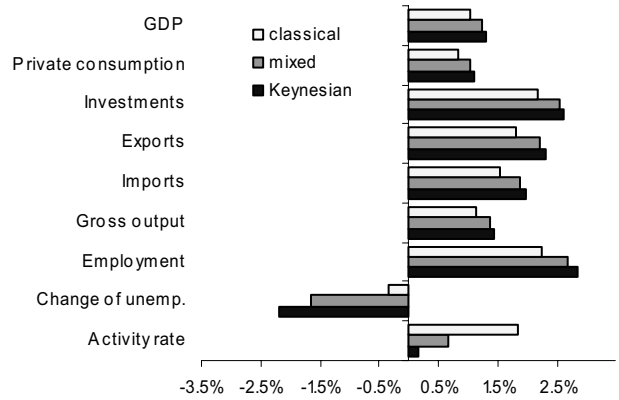
Figure 8 presents a graphical comparison of the behavior of main economic variables under the various closures.

Figure 8. Economic Developments Under Various Closures (pp deviation from baseline)

Scenario 1: VAT rate adjustment



Scenario 2: Social transfers adjustment



III.5. Dynamic Simulation

The purpose of this section is to model a reform that is implemented in two steps. The effects of each step of reform materialize during the year when it has been implemented and the final outcome of the "dynamic" simulation is a path of static equilibria for every simulated year.

Reform design

As in the case of static simulations, the reform focuses on the reduction of labor taxes offset by an adjustment of other taxes or social transfers to secure the exogenous path of government saving (budget deficits). In the first step, personal income taxes (PIT) are reduced, with a proper adjustment of the VAT tax. In the second step, the social security contribution rate is reduced and social transfers are adjusted accordingly. The reforms are assumed to occur in years 2006 and 2007.

We assume the environment of fully fixed wages in the labor market (Keynesian closure). The response of the economy is showed relative to the baseline scenario, consistent with the government Convergence Program.

More specifically, the reform package analyzed envisages: (i) in the first year (2006), the introduction of a uniform PIT rate at a level of about 17%, with tax free income and the deduction of costs of obtaining income at current levels. Simultaneously, the government introduces a VAT rate uniform across all products (currently the VAT rate is differentiated). The VAT rate is set at the level consistent with the path of budget deficit assumed in the baseline; (ii) in the second year (2007), the social contribution rate is reduced by 5% and social transfers adjusted to keep government savings at the baseline level; and (iii) in the third year (2008), no new reforms are introduced, but we record the impact on the economy of the previous reform packages.

The sequencing of reforms reflects the political economy constraints. Reforms that are easier to implement (tax reform) are introduced first. Reforms that are more difficult (consolidation of social transfers) are postponed to the second year.

2006 - the first year of the simulation

The reform. A decrease of the PIT rate results in a decline of government revenues from this source of income by 17.7%, i.e. 0.73% of GDP. In order to balance the budget, the government needs to increase the effective average VAT rate by 8%.

Channels at work. The following main channels are at work: (i) the effective decrease of PIT increases disposable income and thus endowment of households which now demand more goods and more leisure; (ii) the higher VAT makes consumption relatively less appealing than leisure; (iii) fixed wages mean that more people are discouraged from working; (iv) as the supply of capital is predetermined, given the weaker overall demand and lower employment, the price of capital falls; and (v) most importantly, the reduction of PIT in the environment of fixed wages means no reduction of labor costs, so employment is not growing (it *de facto* falls).

GDP and components. The net effect of these reforms on GDP is positive (it is higher by 0.23 pp than in the baseline, see Table 22). However, gross output declines slightly (by 0.1 pp). GDP growth is mainly driven by private consumption (+0.6 pp) due to higher disposable income via lower PIT.

Investment declines by 0.5 pp, reflecting slightly lower saving (-0.1 pp) and an increase in the price of the investment good (+0.5 pp). Although household savings rise by 0.3 pp and foreign savings increase by 1.4 pp, firm savings decline considerably (by -1.2 pp) because of lower activity and declining cost of capital services.

Balance of payments. The depreciation of the currency necessary to maintain external equilibrium is 1.3 pp. Higher prices make it possible to expand exports by 0.5 pp. This increase is relatively small, reflecting the supply constraint of domestic firms. Imports increase by 0.6 pp, filling the shortage of domestic supply in the domestic market.

Capital market. As mentioned above, the price of capital falls (by 1.3 pp).

Table 22. Results of Dynamic Simulation (pp changes from the baseline)

		2006	2007	2008	Cumulative
GDP and components	GDP	0.23%	0.63%	0.11%	0.97%
	Private consumption	0.59%	0.57%	0.14%	1.30%
	Public consumption	0.00%	0.00%	0.00%	0.00%
	Total consumption	0.47%	0.45%	0.12%	1.04%
	Investments	-0.52%	1.16%	0.05%	0.69%
	Domestic demand	0.27%	0.59%	0.10%	0.96%
	Exports	0.52%	1.12%	0.09%	1.73%
	Imports	0.64%	0.94%	0.11%	1.69%
	Net export contribution	-0.05%	0.02%	0.00%	-0.03%
Factors of production	Gross output	-0.05%	0.72%	0.10%	0.77%
	Labor	-0.18%	1.39%	0.07%	1.28%
	Capital	0.00%	0.00%	0.10%	0.10%
Prices of work	Cost of labor	0.00%	-2.60%	0.00%	-2.60%
	Wages	0.00%	0.00%	0.00%	0.00%
Labor market	Change in unemployment	-1.36%	-2.51%	-2.64%	-6.51%
	Participation rate	-1.77%	-0.02%	-0.52%	-2.31%
	Inactivity	1.01%	-0.01%	0.21%	1.21%
Savings	Total	-0.06%	1.31%	0.05%	1.30%
	Firms	-1.15%	1.93%	0.06%	0.84%
	Households	0.28%	1.59%	0.06%	1.93%
	Foreign	1.41%	0.03%	0.01%	1.45%
	Government	0.00%	0.00%	0.43%	0.43%
Exch. rate	Exchange rate	1.26%	0.03%	0.01%	1.30%
Government	Gov. debt to GDP	-0.01%	-0.02%	-0.03%	
	VAT revenues (change)	8.0%	9.1%	9.4%	
	PIT revenues (change)	-17.7%	-16.7%	-16.6%	
	Social transfers (change)	0.1%	-7.2%	-7.2%	

Labor market. Total employment falls by 0.2 pp, mainly because there is no labor cost reduction involved and because domestic demand is weaker. As a result of fixed wages and higher prices of consumption (VAT hike), households prefer to consume more leisure, so the labor force activity rate drops significantly (by 1.8 pp). Accordingly, the unemployment rate is about 1.4 pp lower than in the baseline scenario.

Households. The decrease of primary income of households, due to the declining prices of capital good services and the fact that wages are flat and employment is falling, partly offsets the increase of disposable income due to the reduction in PIT tax rates. Eventually, the disposable income of households rises by only 0.28 pp. Private consumption increases by 0.59 pp and becomes the main source of GDP growth (it rises by about 2.5 times more than GDP).

Summary of the first year. The first package yields mixed results. On the positive side, the reduction of PIT affects positively the incomes of households, thus raising consumption and helping to maintain GDP at least at the baseline level. On the negative side, investment, gross output, employment and activity are lower than in the baseline scenario, while higher VAT makes consumption relatively more expensive (particularly the consumption of food, where the initial VAT rate was much lower). As a result of lower participation rate, the unemployment rate falls.

This scenario may be interpreted as a short run, somewhat artificial, improvement in the labor market (drop in the unemployment rate) but at the cost of worsening long-run growth prospects (falling labor market activity and lower investment dynamics).

2007 – the second year of the simulation

The reform. In this year, the government reduces the social contribution rate (tax wedge) by 5% and simultaneously cuts social transfers proportionally across all households in an amount necessary to keep government savings consistent with the baseline scenario. The necessary adjustment is a cut of 7.2%.

Channels at work. (i) most importantly, costs of labor decline (by 2.6 pp), which raises the demand for labor; (ii) the lower capital stock inherited from the previous period of low investment makes capital a more demanded good, so its price rises with a (slightly) negative impact on the economy; (iii) capital prices become even higher as, with rising employment, capital becomes a more scarce good; (iv) this triggers substitution between capital and labor adding to the demand for labor; and (v) fixed wages work towards lower economic activity.

GDP and components. GDP grows by 0.63 pp more than in the baseline (Table 22). Gross output is higher by about 0.8 pp. Main drivers are private consumption and investment.

Increased savings of households (reflecting the higher wage bill and a constant prospensity to save) adds to the higher savings of the corporate sector (+1.9 pp) due to higher capital prices, resulting in a rise of overall savings by 1.3 pp. The higher level of savings leads to an increase of investment demand by 1.2 pp, almost twice as high as the growth of GDP. The share of investment demand in GDP rises accordingly.

Balance of payments. The expansion of the supply of domestically produced goods (the increase of gross output is higher than the increase in domestic demand) triggers higher exports (+1.1 pp). On the other hand, the rise of domestic demand, driven mainly by relatively import-intensive investment demand, affects the inflow of imported goods (+0.94 pp). As a result the exchange rate remains virtually unchanged compared with the baseline.

Capital market. As mentioned above, the price of capital rises (by 2.1 pp).

Labor market. The situation in the labor market is visibly better than during the previous year. Total employment increases by 1.4 pp (due to higher demand, lower labor costs and higher capital prices). As wages remain fixed, the willingness of people to participate in the labor market is constrained, but this is offset by the decline in social transfers (a subsidy to leisure). As a result the activity rate remains unchanged. The unemployment rate declines by 2.5 pp, reflecting the increasing employment.

Households. Although the decline of social transfers affects negatively the disposable income of households, the rise in employment (and hence the wage bill) and higher income from capital services lead to an increase of income of households by 1.6 pp. This makes it possible to expand private consumption (by 0.57 pp) and savings.

Summary of the second year.

The second part of economic reforms produces results that seem to be of much better "quality." GDP rises faster, sustained by an increase in investment demand. Higher investment contributes to additional growth during subsequent periods. The demand for labor rises, facilitating a decline of the unemployment rate. Finally, the size of the government decreases, as the second part of the reform affects negatively both revenues and expenditures.

2008 – the last year of the simulation

During this year, there are no new reforms and we observe only the second round effects of the previous changes. These are changes that result mainly from additional capital accumulation (induced by increased investment activity) as well as the results of the different composition of taxes. Obviously, these changes are much smaller than in the years of the reforms.

As there is more supply of capital, its price declines slightly (by 0.02 pp). The demand for labor rises by 0.07 pp as the demand for goods is higher than in the baseline.

The GDP growth is higher by 0.11 pp compared to the baseline.

The savings of all institutions except the government increase slightly (by 0.01 – 0.09 pp). As the accumulated growth of the economy is higher, the additional tax revenues are also higher than in the baseline, leading to an increase of government savings by 0.43 pp. As a consequence of increased overall savings, investment activity increases (by 0.05 pp).

Gross output of the production sector grows faster than domestic demand. A small depreciation of the currency adds to an increased supply to foreign markets. This leads to an increase of exports by 0.1 pp. The increase of imports needed to restore the balance of payments is of a similar magnitude.

Fixed wages contribute to a decrease in the activity rate (-0.52 pp). As a result of those two processes, the unemployment rate declines by 2.6 pp.

Increased primary income from labor leads to higher disposable income of households and increased private consumption (by 0.14 pp).

Overall assessment of the reform package (3 year perspective)

The combined reform package affects positively the overall level of economic activity. GDP increases by about 1% above the baseline, due to both demand and supply shocks. The strongest reaction is recorded in the case of private consumption (accumulated growth over the baseline is 1.3 pp). Investment also reacts positively, but only in the two last years of the simulation. Its accumulated growth is about 0.7 pp above the baseline, contributing to higher potential growth.

As a result of the assumption of rigid wages, the reforms have both positive and negative effects on the labor market. Initially, wage discipline is conducive to higher employment (+1.3 pp higher than in the baseline over 2005 – 2008). However, it limits the motivations of inactive population to enter the labor market, particularly when the system of social transfers is not reformed enough to provide appropriate incentives. Therefore, the structural problem of low economic activity of the population is not adequately addressed. Finally, as a result of higher employment and lower activity the unemployment rate declines visibly: the unemployment rate falls from 17.9% in 2005 to about 10% in 2008, i.e. by 6.5 pp more than in the baseline.

III.6. Conclusions

The results of simulations show that it is indeed possible to improve the situation in the labor market in Poland using the available policy tools, such as reduction/reorientation of taxes and/or improvement in labor market incentives embedded in social transfers.

The key ingredient of such reform should be a decisive reduction in labor taxation (social contributions). Complementing reforms with a consolidation of social transfers and related labor market incentives greatly improves the outcome of the reform.

Out of the two analyzed policy packages, the one envisaging a reduction in social transfers (in addition to a cut in the tax wedge) yields visibly superior results, in terms of higher growth, higher employment, investment, and activity, lower taxes and so on. The outcome is also beneficial for longer run growth because of higher investment demand and participation rates. Thus, it pays off to stimulate the labor market on both the supply and demand sides.

As regards the labor market, total employment increases, as long as there is a decline in labor costs. In the neo-classical environment, the employment reaction is higher in the markets where the unemployment rate is relatively high (the markets for labor with medium and basic education). This is because high unemployment imposes better wage discipline, leading to lower labor costs.

Household income rises in all scenarios. Most importantly, poor households are not hurt. When social transfers are reduced, the increased disposable income (from the higher wage bill) makes up for this loss of income. These households, trying to sustain their living standards, become active on the labor markets.

When wages are rigid (fixed), the reduction in the tax wedge increases employment and GDP in both scenarios which become similar. However, the second scenario still has better characteristics. The important feature in a Keynesian closure is a much lower level of economic activity because unresponsive wages discourage people from working.

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