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Office of the Vice President
Development Economics
The World Bank
August 1992
WPS 962

Background paper for World Development Report 1992

Do Markets Underprice Natural-Resource Commodities?

Margaret E. Slade

Not systematically, except for the environmental externalities associated with the production and use of natural-resource commodities — especially mineral commodities, which cause the most pollution.

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This paper — a product of the Office of the Vice President, Development Economics — is one in a series of background papers prepared for the *World Development Report 1992*. The *Report*, on development and the environment, discusses the possible effects of the expected dramatic growth in the world's population, industrial output, use of energy, and demand for food. Copies of this and other *World Development Report* background papers are available free from the World Bank, 1818 H Street, NW, Washington, DC 20433. Please contact the *World Development Report* office, room T7-101, extension 31393 (August 1992, 26 pages).

Slade examines the efficiency and equity of a market allocation of exhaustible resources and assesses the behavior of scarcity measures, such as relative price and rental rates. She finds little evidence of scarcity or impending shortage. Indeed, the evidence points to falling prices and rents for many commodities.

Do markets send the wrong signals? Are resource commodities systematically underpriced? Her conclusions are not completely optimistic.

Slade's analysis reveals many market failures, any of which would result in inappropriate resource commodity pricing. But, with one exception, she finds no systematic tendency to underprice. The exception concerns the environmental externalities associated with the production and use of natural-resource commodities. Similar externalities lead to underpricing and overuse of all commodities. Mineral commodities, however, are responsible for a large fraction of the pollution that is currently generated, so their underpricing is particularly significant.

The market failures associated with common-property and environmental resources can cause market prices to be lower than shadow prices or marginal values. They cannot, however, cause relative resource prices to fall, Slade argues. Falling prices would be associated with a relaxation of environmental standards and a move away from full-social-cost pricing. The tendency, however, is toward increased awareness of environmental damage and increased willingness to pay for its associated costs.

Nevertheless, the prices of many natural-resource commodities have fallen in real terms. Factors causing prices to decrease are not associated with market failure, and therefore do not support interference with the market mechanism. Indeed, says Slade, innovations that lower mining and processing costs, discoveries that increase resource stocks, and the provision of lower-cost substitutes are all features of efficiently operating markets.

Produced by the Policy Research Dissemination Center

Do Markets Underprice Natural-Resource Commodities?

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Prepared as a Background Paper for the
World Development Report 1992

* I would like to thank Nemat Shafik and B.J. Choe for thoughtful comments on an earlier draft.

The World Development Report 1992, "Development and the Environment," discusses the possible effects of the expected dramatic growth in the world's population, industrial output, use of energy, and demand for food. Under current practices, the result could be appalling environmental conditions in both urban and rural areas. The World Development Report presents an alternative, albeit more difficult, path - one that, if taken, would allow future generations to witness improved environmental conditions accompanied by rapid economic development and the virtual eradication of widespread poverty. Choosing this path will require that both industrial and developing countries seize the current moment of opportunity to reform policies, institutions, and aid programs. A two-fold strategy is required.

- First, take advantage of the positive links between economic efficiency, income growth, and protection of the environment. This calls for accelerating programs for reducing poverty, removing distortions that encourage the economically inefficient and environmentally damaging use of natural resources, clarifying property rights, expanding programs for education (especially for girls), family planning services, sanitation and clean water, and agricultural extension, credit and research.
- Second, break the negative links between economic activity and the environment. Certain targeted measures, described in the Report, can bring dramatic improvements in environmental quality at modest cost in investment and economic efficiency. To implement them will require overcoming the power of vested interests, building strong institutions, improving knowledge, encouraging participatory decisionmaking, and building a partnership of cooperation between industrial and developing countries.

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Other (unpublished) papers in the series are available direct from the World Development Report Office, room T7-101, extension 31393. For a complete list of titles, consult pages 182-3 of the World Development Report. The World Development Report was prepared by a team led by Andrew Steer; the background papers were edited by Will Wade-Gery.

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I. Introduction

Prices are the vehicle through which markets allocate goods and services. Under an idealized set of assumptions, the competitive-price mechanism results in a Pareto optimal allocation of resources. Economists, however, are concerned about the ability of the price system to allocate resources in a near-optimal fashion when the assumptions required for optimality are only approximately met. In what follows, some of the salient issues of this debate are examined with reference to exhaustible-resource markets. Can such markets be counted on to allocate non-renewable resources in an efficient and equitable fashion?

Non-renewable or mineral resources are unique because, unlike the potentially unlimited supply of labor and man-made capital, their stock is finite. They constitute the principal potential limit to economic growth. For centuries, economists and political philosophers have been concerned about resource availability and the capacity of the market mechanism to price scarcity. While some have been pessimistic about the market's ability to deal with the constraints imposed by finite resource stocks, others have taken more optimistic positions. The debate is by no means closed and many unsolved problems remain. For this reason, while theoretical models are discussed and empirical evidence bearing on the subject is marshalled, this paper provides no definitive answers. Many of its conclusions are far from universally accepted.

The outline of the paper is as follows. Section II develops a model of a competitive market for an exhaustible resource and demonstrates that, in this model, the price mechanism allocates the resource in a Pareto-optimal fashion. In other words, the market is shown to be *efficient*. Section III then asks whether this competitive-market allocation is *equitable*. The notion of equity clearly depends on a social-welfare function, and several well-known welfare criteria are discussed in this regard. Section IV examines *evidence*. Scarcity measures and natural resource price paths are addressed. Generally, the evidence is found to be negative: overall price behavior is inconsistent with the predictions of the simple model, while there is no evidence that we are "running out" of natural resources.

The simple model is then examined, the realism of its assumptions assessed, and possible *market failures* explored. Uncertainty is introduced. Section V questions the existence of markets for all commodities, for all states of the world and all future time periods. It also explores the implications of incomplete markets for Pareto optimality in the allocation of both commodities and risk.

In Section VI, possible *market failures* are assessed. The notion of shadow prices is introduced and the circumstances under which shadow and market prices can be expected to diverge are identified. These latter include the existence of monopoly power in resource markets (which causes a divergence between output prices and marginal output values), the presence of unemployment (which causes a divergence between wages and marginal input values), and an inappropriate choice of currency values (which causes a divergence between exchange rates and marginal-foreign-exchange values).

Finally, common-property and environmental problems are introduced. These problems and their associated market failures are not unique to mineral industries. Nevertheless, mining and refining, in the absence of controls, are heavily polluting activities. The situation is exacerbated by the transnational nature of the spillovers, which makes agreements to limit pollution difficult to enforce.

Clearly, in assessing the price system, it is inadequate merely to conclude that markets are not ideal and market failures exist. What is required is a comparison of the market allocation to its feasible alternatives. If governments are to put forward policies that constrain resource production, consumption, or pricing, they should confront the problems associated with alternative institutions and enforcement mechanisms. In this paper, these issues are only mentioned in passing. They are, however, considered in greater detail in a companion paper (Slade 1991a).

II. Intergenerational Efficiency

In this section, a model is constructed in order to address the question of efficient resource allocation formally.[1] The notion of Pareto optimality is introduced and conditions for an efficient program are derived. The competitive-market outcome is then compared to the efficient program.

The standard analysis of the efficiency of competitive markets is static. Static analysis is adequate because there is nothing that links consumption in different periods. However, when an exhaustible resource is introduced, the problem becomes dynamic and the standard analysis no longer applies. Consequently, this study develops a dynamic model of intertemporal resource allocation. We begin with an economy with a single consumption good, C. Consumers derive utility from its consumption, and the marginal utility of consumption is assumed to be strictly positive. An allocation is thus assumed to be intergenerationally efficient (equivalently, Pareto optimal), if it is impossible to increase consumption in one period, t, without causing less to be consumed in some other period, t'.

We now introduce a homogeneous exhaustible resource. Resource flows and stocks in period t are denoted R_t and S_t, respectively, and the initial stock is assumed to be finite. In the simplest case, where $C_t=R_t$ and the entire stock is eventually consumed, *all allocations are efficient*. An increase in consumption in period t must be exactly offset by a decrease in some other period t'. This simple example serves to illustrate two points: first, the notion of Pareto efficiency is fairly weak; and second, radically different programs can be similarly efficient. To make the notion of efficiency non-trivial, we must recognize that most minerals are not consumed directly. Instead, they are used as inputs to the production of final goods. Furthermore, when production relies on more than one input, intergenerational efficiency constrains relative input use. In what follows, this more realistic problem is analysed.

The economy is now modified to include aggregate output, Q, which is produced by two inputs, man-made capital, K, and an exhaustible resource, R. This relationship is governed by the production function

$$Q_t = f(K_t,R_t,t) \tag{1}$$

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[1] A similar model can be found in Dasgupta and Heal (1979).

where f is assumed to be increasing, concave, and twice continuously differentiable in K and R, and the marginal products of K and R are assumed to be strictly positive. The presence of t in the production function reflects the possibility of technical progress.

Output can be consumed (C) or invested (I) so that

$$Q_t = C_t + I_t \tag{2}$$

and

$$I_t = dK_t/dt =: \dot{K}_t \tag{3}$$

At any time t, therefore, the stock remaining is

$$S_t = S_0 - \int_0^t R_t d_t \tag{4}$$

so that $\dot{S}_t = -R_t$ and $S_t \geq 0$.

An evolution of this economy $\{K_t, S_t, R_t, C_t\}^\infty_0$ is intertemporally inefficient if there exists another feasible program $\{\tilde{K}_t, \tilde{S}_t, \tilde{R}_t, \tilde{C}_t\}^\infty_0$ with $\tilde{C}_t \geq C_t$ for all t and $\tilde{C}_t > C_t$ for some t. If such a program does not exist, then the original program is *intertemporally efficient*. With an efficient program, therefore, it is impossible to increase consumption in one period without decreasing it in another. In general, there can be an infinite number of efficient programs.

The assumption that the marginal product of R is strictly positive implies that, with an efficient program, the entire stock will eventually be used up. In other words

$$\lim_{t \to \infty} S_t = 0 \tag{5}$$

This condition, however, is no longer sufficient.

To compare competitive and efficient market allocations, we now derive conditions that each must satisfy in order to determine whether they are the same. The attached annex (pp 24-26) explores the implications of intergenerational efficiency formally. The necessary condition derived there is:[2]

$$f_{Kt} = \dot{f}_{Rt}/f_{Rt} \tag{6}$$

To interpret equation (6), suppose that input and output markets are competitive and let Q be the numeraire good. Then f_{Kt} is the rental rate of capital which, since it does not depreciate, is also the rate of return to holding K, which we denote r. Moreover, f_{Kt} is the spot price of the resource, which we denote P_t, and $\dot{f}_{Rt}/f_{Rt}$ is the rate of resource-price appreciation.

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[2] A subscripted function denotes the partial derivative of that function with respect to the argument that corresponds to the subscript.

Equation (6) is thus seen to be the familiar Hotelling (1931) rule that, in a competitive market, the resource price appreciates at the rate of interest:[3]

$$\dot{P}_t/P_t = r \tag{7}$$

Equation (7) is a local condition and, by itself, does not guarantee efficiency. This should be obvious since it only determines the rate of change in price, not the price level itself. With many technologies (production functions f), however, the combination of (7) and (5) is sufficient for efficiency; competitive markets allocate an exhaustible resource in an efficient manner and no government intervention is required. The intergenerational or intertemporal efficiency of competitive-resource markets is a very strong result and one that calls for more careful analysis. First, however, we turn to the question of equity.

III. Intergenerational equity

As noted above, competitive markets, under idealized circumstances, will allocate a scarce resource in an efficient manner. However, this does not imply the *optimality* of competitive markets. Intergenerational, like cross-sectional optimality requires some balance between intertemporal efficiency and equity. Again. standard (static) analyses of equity do not apply to exhaustible-resource markets; consequently, equity must be examined within a dynamic setting.

Since equity is defined with respect to some social-welfare function, it cannot be discussed without making a value judgement of some kind. A cross-sectional social-welfare function, for instance, weights the utilities of different consumers or households. However, since the focus here is on aggregate consumption, cross-sectional problems are ignored.[4] An intertemporal social-welfare function, by contrast, weights the utilities of different generations. The choice of generational weights is, of course, significant. There are many welfare criteria common in the literature, each involving different welfare weights. For example, the sum of generational utilities can be maximized (the utilitarian criterion) or the minimum utility can be maximized (the maximin criterion). With the former, each generation is given equal weight, and with the latter, the highest sustainable consumption level is sought.[5] For either criterion, there is the further choice of whether or not to discount the utility of future generations. At first glance, discounting may seem like an unfair practice. Why should earlier generations be given

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[3] More generally, with positive extraction costs, it is the net marginal product of R, P - MC, that appreciates at the rate of interest.

[4] The mere fact that only aggregate consumption, C_t, is analyzed, however, means that we are implicitly assuming a cross-sectional social-welfare function of a particularly simple sort. Indeed, we are maximizing the sum of individual consumption in each period, which is equivalent to using a utilitarian criterion with the utility of consumption equal to consumption itself.

[5] It is initially assumed that the utility of consumption is consumption itself.

higher welfare weights than later? On closer consideration, however, there may be valid welfare grounds for discounting.

Suppose that there is some positive probability ρ_t, that the world as we know it will end in period t. Collapse might be due to a sudden climatic change -- a new ice age, for example -- or to a man-made disaster, such as war. In either case, the aim is to maximize the expected value of the relevant welfare criterion. With a utilitarian criterion, for example, the expected value is:

$$E[\sum_{t=0}^{\infty} C_t] = \sum_{t=0}^{\infty} [\prod_{\tau=0}^{t}(1-\rho_\tau)]\, C_t \tag{8}$$

In the special case where $\rho_t = \rho$ for all t, (8) reduces to

$$E[\sum_{t=0}^{\infty} C_t] = \sum_{t=0}^{\infty}(1-\rho)^t C_t. \tag{9}$$

Equation (9) looks very much like a discounted consumption stream. The discount factor $1-\rho$, however, has a different interpretation. Instead of valuing the utilities of future generations less, their utilities are weighted by the probability that they will be around to enjoy their consumption.

In addition, when the utility of consumption in period t is not equated with C_t, it is reasonable to assume that the marginal utility of C diminishes with C. Under these circumstances, there is a second reason for discounting the consumption of future generations. If per-capita income rises over time, as has been the case historically, then future generations will derive less and less utility from the same level of consumption. This gives rise to a utility discount factor in addition to an uncertainty discount factor.

Different welfare criteria clearly result in different patterns of resource extraction. For example, it should be intuitively obvious that the maximin criterion results in a constant level of consumption for every period.[6] Constant consumption can be contrasted with the standard rising-price falling-consumption pattern produced by competitive markets. A utilitarian criterion gives rise to still another consumption path.

The choice of social-welfare function is a complex issue that is beyond the scope of this paper. However, it is to be hoped that the discussion above makes clear the basic mistake involved in equating efficiency with optimality. Moreover, even though the idealized

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[6] For a formal demonstration of this claim, see Solow (1974). This program can be implemented using the "Hartwick Rule" (1977), which states that a constant level of consumption can be achieved by investing all of the rents from the extraction of exhaustible resources under competitive conditions.

competitive market allocates resources efficiently, there is no evidence that it does so equitably.

IV. Evidence from Natural-Resource Markets

We now turn from theory to evidence, in order to assess whether real-world markets give appropriate signals of resource scarcity and whether the necessary conditions for efficient-resource allocation are met in practice. Three common scarcity indicators are defined, their strengths and weaknesses discussed, and their historic behavior analyzed. Evidence from econometric testing of the Hotelling model is then examined.

1. Measuring Scarcity

As a resource becomes scarce, it is hoped that the market will signal this fact and that consumers will adapt their usage patterns accordingly. Several indices that might signal scarcity have been proposed in the literature. The most popular of these are relative price (the ratio of an extractive-industry price index to an overall price index), unit cost (the value of factor inputs per unit of extractive-industry output), and rental rate (the marginal value of the unextracted resource).

Unit cost is the least appealing of the three measures. For example, in Hotelling's classic article (1931) the resource is assumed to be extracted costlessly. Under these circumstances, unit cost can provide no signal of increasing scarcity. Nevertheless, price rises at the rate of interest and causes consumers to conserve on use. Whether relative price or rental rate is a better measure of scarcity is much debated (see Brown and Field, 1978; Smith, 1978; and Fisher, 1979). With the simplest competitive model, rental rate (price net of marginal-extraction cost) is predicted to rise at the rate of interest. The measure therefore has a certain theoretical appeal.

On the other hand, the rate of price increase also has a simple intuitive decomposition. If we add extraction cost to the competitive model, where cost can depend on both current and cumulative extraction, it can be shown that the rate of price change is equal to a weighted average of the return on capital and of the rate of change in marginal-extraction cost, MC:[7]

$$\frac{\dot{P}}{P} = \frac{P - MC}{P}\, r + \frac{MC}{P}\,\frac{\dot{MC}}{MC} \tag{10}$$

In other words, it is an average of a pure scarcity rent and a Ricardian or differential quality rent.

The choice between the two measures, relative price and rental rate, ultimately rests on the nature of the scarcity; we can be concerned with the scarcity of "ore" (the unextracted resource) or of "metal" (the refined commodity). If we think of ore as an input to processed metal, it becomes clear that changes in underlying economic conditions can affect factor and

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[7] For a formal demonstration, see Smith (1979).

product prices differently. Moreover, the two indices do not always move in the same direction. In other words, qualitative *and* quantitative conclusions drawn from the two measures can differ. This would be of considerable concern if each index were to give a clear, but conflicting, signal. However, as the next section demonstrates, this is not the case.

2. The Behavior of Scarcity Measures

The following subsection looks at the behavior of unit costs, relative prices, and rental rates for selected non-renewable resources. The models assessed can be thought of as reduced forms; the focus of analysis is on the time-series properties of either gross-scarcity measures or the residuals obtained after conditioning these measures on exogenous variables. (More formal structural tests of the Hotelling model are considered in the next subsection.) Here, the classic study is by Barnett and Morse (1963); in examining both relative-price and unit-cost trends, they conclude that, because both fall over time, scarcity is not a problem. In an update, Barnett (1979) reaches the same conclusion -- that there is no sign of an upturn in either the unit cost or relative price of major mineral commodities.

Barnett and Morse's findings are not universally accepted. For example, Smith (1979) looks at the stability of coefficients of estimated price-trend relationships and argues that the data are too volatile to support definitive conclusions. Slade (1982) finds that, after substantial initial declines, the 1970s show some evidence of an upturn in the price paths of many mineral commodities. However, since then prices have been increasingly volatile; large run ups have been followed by equally large declines (Slade 1991b), but there is little evidence of a sustained trend.

Rent is more difficult to measure directly. However, Fisher (1979), and Devarajan and Fisher (1982), argue that rent can be measured indirectly. They advocate the use of unit-exploration cost as a proxy for scarcity rent and find evidence that this cost has been rising, at least for petroleum.

To summarize, if scarcity is measured by unit-extraction cost, there is no evidence of an increase. If on the other hand, relative price or unit-exploration cost is used as a scarcity index, there is weak evidence of increased scarcity for some commodities. Nevertheless, when we consider a century of data, the most striking feature is the decline in the relative price of the majority of mineral commodities. It is therefore of interest to examine the simple competitive-market model to see how it might be modified so as to produce prices that fall over substantial periods of time. There are several such modifications.

First, in the simple model, the initial stock of the resource is known with certainty as of time zero. In practice, however, extraction of known deposits proceeds simultaneously with exploration for previously unknown ore bodies. Indeed, large discoveries increase the size of the stock and can cause prices to fall. This issue is examined by Pindyck (1978). Second, although the model outlined in Section II allows for technical change, the idea that new methods of extraction can lower costs is not formally developed. The issue is explored by Slade (19[illegible]), who shows that prices can fall when cost-reducing mining techniques are introduced. Third, substitute materials can cause the demand for mineral commodities to shift inwards. Several authors (Heal, 1976; Hanson, 1980) have assessed the behavior of resource prices and rental rates when a backstop technology is introduced and have shown that both can fall in the presence of obsolescence.

The three modifications outlined above can produce falling prices without introducing market failures of any sort, a subject that is dealt with subsequently. In practice, the first two -- discoveries of previously unknown deposits and new cost-lowering extraction techniques -- are more likely than obsolescence to contribute to falling prices. Finally, it should be noted that for all of the above explanations of price decline, price-falling phases are apt to be temporary.[8] Ultimately, the exhaustibility underlying Hotelling's model should reassert itself.

3. Structural Tests of the Hotelling Model

More formal tests of the Hotelling model are of two sorts. The first relies on estimates of the extractive firm's productive technology combined with Euler equations for its dynamic-profit maximization. Examples include Stollery (1983), Farrow (1985), Halvorsen and Smith (1984 and 1991), and Young (1991). Once the firm's technology is known, rental rates can be approximated either by: (1) the difference between price and marginal cost (Stollery, 1983; Farrow, 1985); or by (2) the shadow price of the unpriced ore to the vertically integrated metal producer (Halvorsen and Smith 1984 and 1991). Most such tests have been conducted using data for metal-mining firms. Although some studies support the Hotelling model, its overall performance has been poor. In particular, rental rates for some commodities decline even faster than product prices.

Notice that this structural approach is able to deal with the shortcomings of the reduced-form models. In particular, technical change and demand shifts can be accounted for. And, as the data pertain to firms whose reserves are known at the beginning of the estimation period, new discoveries cannot explain falling rental rates.

The second structural approach was developed by Miller and Upton (1985), who base their tests on a less widely known implication of Hotelling's analysis of the optimal time pattern for the exploitation of an exhaustible resource. They show that, for the competitive-market model, the value of the reserves in any currently operating, optimally managed mineral deposit will depend solely on the current spot price, net of marginal-extraction cost and regardless of when the reserves are extracted. They refer to this proposition as the Hotelling Valuation Principle. Miller and Upton test their model using stock-market valuations of oil and gas reserves from a sample of U.S. companies and find the data to be consistent with their Principle. Unfortunately, to the author's knowledge, there have been no subsequent tests of the Hotelling Valuation Principle.

We can thus see that, with a few exceptions, their is little empirical support for the predictions of the Hotelling model. This can mean either that firms do not maximize profits or that the model is too simple to explain their observed behavior. The second explanation seems more fruitful, and is explored in the next section.

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[8] The exception is the discovery of a perfect substitute that is cheaper to produce than the mineral commodity.

V. Trading under Uncertainty

The model of optimal extraction of an exhaustible resource developed thus far has several undesirable features. One such feature is the assumption that the institutions required for trading exist and that all relevant information is known to market participants as of time zero. In other words, there are markets for each commodity in every time period, and there is no uncertainty. These assumptions are now relaxed and the consequent implications for prices and rental rates are explored.

1. Introducing Uncertainty

While the Hotelling model is completely deterministic, the real world is inherently uncertain. Uncertainty in natural-resource markets takes many forms including unknown initial stocks of reserves (Gilbert 1979), potential expropriation of deposits (Long 1975), stochastic discoveries of new supplies (Loury 1978), and uncertain timing of backstop technology availability (Kamien and Schwartz 1978). Each source of uncertainty can affect the behavior of prices and rental rates for natural-resource commodities.

When uncertainty is introduced, prices become random variables; a deterministic pattern, such as the r-percent rule, cannot be expected to hold. At best, price can be expected to rise at the rate of interest, which is the stochastic analog of Hotelling's rule.[9] This notion can be made more precise.

Suppose that the rate of price appreciation is not deterministic, but instead a random variable whose expectation is r. Thus:

$$\dot{P}_t/P_t = r + \epsilon_t \qquad (11)$$

where ϵ is a random variable that is identically and independently distributed. When (11) holds, the discounted price (i.e., the price that is constant in the Hotelling model)[10] is said to be a martingale. A characteristic feature of a martingale is that the optimal forecast for any future value is the current value.

Deshmukh and Pliska (1985) develop a stochastic model of optimal-resource depletion under competitive-market conditions that includes all of the above-mentioned sources of uncertainty as special cases. They then ask when the discounted price will be a martingale. The answer is that discounted prices are martingales if, and only if, the conditional distribution of the timing of the uncertain event (the discovery of a new deposit or technology, for example) is independent of the current stock of reserves S_t or if the occurrence of the event does not affect the profit function of the extractive firm. Despite the rather special conditions, this answer may very well hold for many uncertain events of

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[9] For ease of exposition, in what follows the word "price" is used instead of "price net of marginal-extraction cost."

[10] If $P_t = P_0 \epsilon^{rt}$, then discounted price, $\epsilon^{-rt} P_t = P_0$, is constant.

interest. For example, the timing of the discovery of additional stocks, the development of a backstop technology, and political expropriation of reserves are apt to be unaffected by the current level of reserves. In addition, given that the Hotelling model performs poorly over a wide range of discount rates, uncertainty (which essentially modifies the discount rate) is unlikely to be the principal source of the problem.

2. Incomplete Markets

With the simplest Hotelling model, markets must exist for every commodity in every time period. Although this is in itself a formidable requirement, the situation worsens still further when uncertainty is introduced. Markets have to exist for every time period and every state of the world. Since the number of potential states in a given time period is unbounded, the dimensionality of the problem also grows without bound. We know from Arrow (1964) and Debreu (1959) that a market for contingent claims can be treated just like a market with no uncertainty. Identical goods in different states of the world are assumed to be different goods. Therefore, if competitive markets allocate resources in an efficient manner under conditions of certainty, then, so long as markets are complete, the competitive-market allocation of risk will also be Pareto optimal.

The efficiency of competitive markets for contingent claims is a powerful result. Unfortunately, it rests on the very strong assumption of complete markets. Spot and futures markets exist for many mineral commodities such as copper and petroleum. But it is not possible to purchase one futures contract for delivery of a commodity three months forward if the world is at war, and another contract for the same commodity with the same due date that differs only by the world being at peace. Clearly, there are many more possible states of the world than futures contracts.

The problem can be partly overcome by the introduction of derivative financial assets such as options. An option gives the buyer the right, but not the obligation, to buy or sell (or both) an agreed upon quantity of a commodity at a fixed price on a specified date in the future. Financial economists have expended considerable effort in determining the number of securities such as options that are needed to complete a market. The option-pricing model of Black and Scholes (1973) is a major contribution to this literature. They show that the ability to trade securities *frequently* can enable a few multiperiod securities to span many states of nature. In fact, their model contains only two securities, but an uncountable number of states of the world. Because trading is continuous and uncertainty resolved smoothly, markets are effectively complete.

Mineral-commodity markets such as the London Metal Exchange have existed for more than a century. More recently, new markets have opened and new derivative securities have been introduced.[11] Today, trading in commodity futures and options is virtually continuous and takes place at all hours of the day (albeit in different locations). Incomplete

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[11] For a description of the new markets and contracts, see Slade, Kolstad, and Wiener (1991).

markets may not therefore pose as large a problem as originally indicated.[12]

However, there remains another difficulty. The futures contracts currently traded are at best for periods of thirty-six months forward. The Hotelling model, in contrast, assumes that one can trade forward into the indefinite future. There are two possible solutions to this problem. The first is to increase the maximum length of the contracts traded, and initially this may seem a good idea. On closer inspection, however, it has less appeal, given our existence in a second-best world. With respect to financial markets, the general theory of second best translates into the principle that opening a new market can reduce welfare, as long as the overall market remains incomplete.[13]

The second possible way out is to substitute a sequence of short-term contracts for one long-term contract; in other words, the short-term contracts can be continuously rolled over. Unfortunately, this solution is also not ideal. The theory of contracts tells us that if there is a need for consumption smoothing, short and long-term contracts are not equivalent. Mineral commodities are storable and therefore inventories can help to smooth consumption. Nevertheless, disruptions in mineral markets, due to embargoes or political disruptions for example, can be lengthy. This means that short-term contracts might not lead to an optimal allocation of consumption or of risk.

To summarize, the introduction of uncertainty leads to several difficulties not encompassed by the simple Hotelling model. Expected prices may not increase at the rate of interest, markets may be incomplete, and contract terms may be too short to allocate goods and risk efficiently. In spite of all of these difficulties, however, it is unlikely that uncertainty leads to a systematic underpricing of resource commodities. Moreover, the resulting complications do not seem to be large in magnitude.

3. Hotelling vs. Efficient-Market Hypothesis

Many non-mineral commodities are also traded in futures markets. Naturally, there are many economic theories that attempt to explain the price behavior of such commodities. One of these, the efficient-market hypothesis, is examined in this subsection.

When the market for a commodity is efficient, "new" information is instantly incorporated into the price level; if it were not, there would exist opportunities for profitable arbitrage. By definition, "new" information cannot be forecast and is therefore uncorrelated with anything known in previous periods. This line of reasoning leads to the efficient-market hypothesis, which can be expressed formally as

$$\dot{P}_t/P_t = \epsilon_t \tag{12}$$

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[12] The Black-Scholes model relies on rather special assumptions concerning the nature of uncertainty, which may not be met in practice. For this reason, many financial economists are skeptical that markets are in fact complete. Nevertheless, my feeling is that incompleteness is not central to the problem of concern here.

[13] This idea is developed formally by Hart (1975).

where ϵ is defined as for equation (11). The EMF, as expressed in (12), states that percent changes in prices are stochastic and that undiscounted prices are martingales.[14] This can be contrasted with (11), where discounted prices are martingales, and with (7), where undiscounted prices increase at a deterministic rate and discounted prices are constant.

Equation (11) clearly nests all three possibilities. Moreover, it lends itself to empirical testing. In other words, it is possible to discriminate between the following three models: (1) a simple Hotelling model with the deterministic rate of price appreciation equal to the rate of interest; (2) a stochastic Hotelling model with the expected value of price appreciation equal to the rate of interest; and (3) an efficient-market model with the expected value of price appreciation equal to zero.

If we assume that ϵ - $N(0,\sigma^2)$, then tests of the three possibilities are as follows: (1) $r > 0$, $\sigma^2 = 0$, 2) $r > 0$, $\sigma^2 > 0$ and 3) $r = 0$, $\sigma^2 > 0$. Slade (1988) investigates these possibilities using data on prices of seven major-mineral commodities over the period 1906-1973, and finds that, although none of the three models captures price behavior exactly, the third receives most support. This means that the prices of natural-resource commodities behave very much like the prices of non-exhaustible commodities that are traded on futures markets.

Figure 1 shows auto-correlation functions for price changes, ΔP_t, of the seven commodities. In the figure, the solid lines represent 95 percent confidence intervals. Estimated auto-correlation coefficients a_j that lie outside the confidence intervals (those circled in the diagram) indicate violations of the predictions of model three.[15] The figure shows that the efficient-market hypothesis is only strongly rejected for petroleum, and it is interesting that a petroleum-futures contract did not exist during the period of the data. It is also striking that the commodities that have been traded on the London Metal Exchange since the turn of the century, copper, lead, and silver, show no auto-correlations significantly different from zero.

To conclude, even though others have found evidence that mineral forward and futures markets are not fully efficient (Goss, 1981; Gilbert, 1986; and Jones and Uri, 1990),[16] the behavior of prices for mineral commodities traded on exchanges is not significantly different from the behavior of renewable commodity or financial asset prices.

VI. Market Failures

The theoretical model developed in Section II pertains to an ideal competitive market.

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[14] Strictly speaking, it is the logarithm of price that is a martingale in (12).

[15] a_j is the correlation coefficient between ΔP_t and ΔP_{t-j}

[16] Market inefficiency is not unique to mineral commodities. It is also often rejected for renewable commodities and financial assets (see for example, Hodrick and Srivastava, 1987).

Figure 1 **Auto-correlation Functions for Price Changes**

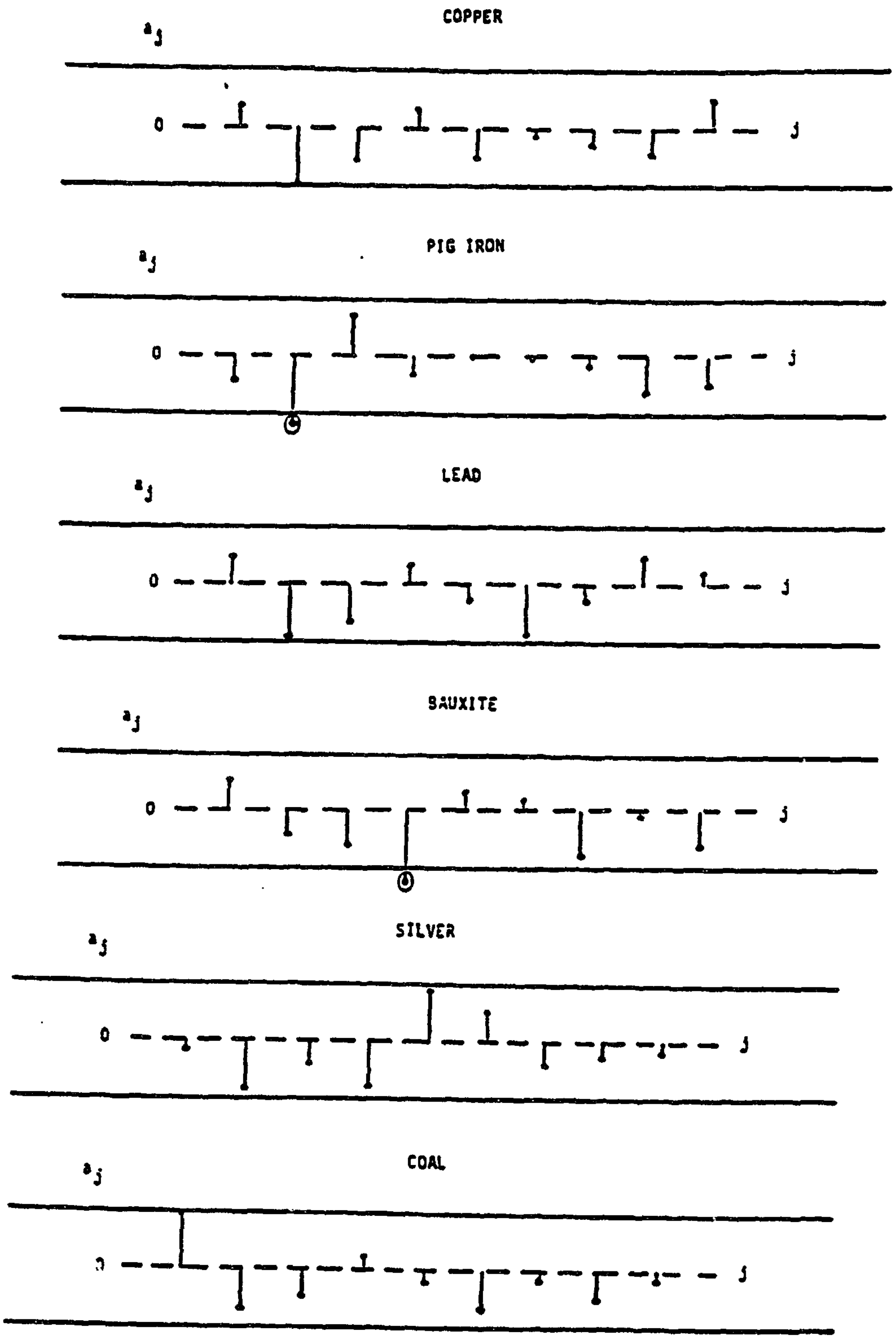
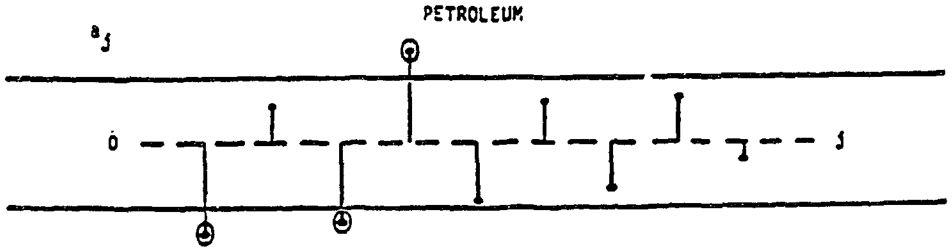


Figure 1 (cont.)



In particular, firms are price takers in input and output markets, all factors are fully employed, there are no constraints on profit-maximizing behavior, property rights are well defined, and extraction does not produce unwanted byproducts such as pollution. In the absence of market failure, market and shadow prices (marginal values) coincide. Market failure, however, can cause a divergence between market and marginal values. In this section, each of the above assumptions is relaxed in turn, and the implications for prices and rental rates are explored.

1. Output Market Distortions

Imperfect competition in output markets occurs when firms face downward-sloping demand schedules for their products. This is the case for both the monopolist and oligopolist; when a firm possesses price power in output markets, the shadow price of output is marginal revenue, not market price. It is therefore marginal revenue that appreciates at the rate of interest, and equation (7) becomes

$$\dot{MR}_t/MR_t = r \tag{13}$$

As price is above marginal revenue, monopoly prices might be higher than competitive prices; but this need not be the case. To understand why, consider a demand function of constant-elasticity μ (a positive number). Marginal revenue and price are then related thus:[17]

$$MR_t = (1 - 1/\mu)P_t \tag{14}$$

In other words, marginal revenue is a constant markdown under price, which implies that price and marginal revenue increase at the same rate.

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[17] Equation (14) holds for both monopolists and oligopolists as long as μ is the elasticity of *firm* demand.

Relationship (14) holds for any firm with market power. What differentiates exhaustible-resource markets, however, is the finite stock S_0.[18] Since price must clear the market, and the entire stock eventually be exhausted, *monopoly and competitive price paths are identical in the constant-elasticity case.*

This result, which is very strong, is derived using a range of simplifying (and perhaps unreasonable) assumptions. When some of these assumed conditions fail to hold, monopoly and competitive prices need not coincide.[19] Nevertheless, if a monopolist charges higher prices in early time periods, they will charge lower prices in later periods and vice versa. The important point is that the finiteness of the stock limits the scope for monopolists to exercise market power.

Moreover, when the conditions that lead monopoly and competitive price paths to diverge *are* fulfilled, exhaustible-resource markets are probably not characterized by significant monopoly power. Most mineral commodities are traded in world rather than national markets, and in many of these markets levels of horizontal concentration have fallen as a result of the entry of new extractive firms or the nationalization of privately owned reserves. Monopoly power is therefore not an overriding consideration.[20]

2. Factor-Market Distortions

In the standard analysis of exhaustible resource markets, marginal cost plus scarcity rent is taken to be a good measure of social value. Due to factor-market distortions, this is less apt to be true in developing countries. In the presence of distortions, shadow and market-input prices differ and cause a divergence between social and marginal cost. Consider unemployment. In a fully employed competitive economy, the social opportunity cost of a worker is their market-wage rate. With unemployment, however, the two diverge. Particularly in less developed countries, there are often systematic forces that cause the shadow price of labor to be less than the market wage.[21] When this is true, marginal-extraction cost exceeds social-extraction cost.

Distortions also occur in capital markets. Less developed economies, in particular, often operate under capital-market constraints. Moreover, if a domestic currency is overvalued, the social opportunity cost of imported capital will exceed its official financial cost. Given these distortions, marginal-extraction cost underestimates social-extraction cost.

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[18] For expositional purposes, we assume that the stock is homogeneous and that extraction costs are zero.

[19] For a discussion of the conditions under which a monopolist will extract more or less slowly than firms in a competitive industry, see Stiglitz (1976).

[20] This conclusion may seem strange, in light of OPEC price increases in the 1970s. However, we are concerned here with systematic long-run tendencies; today the crude-oil market *is* workably competitive.

[21] For a discussion of these issues, see Little and Mirrlees (1974).

Capital and labor-market distortions tend to work in opposite directions, and their net effect is therefore difficult to predict. What is important, however, is that there is no presumption of a systematic tendency for factor-market distortions to lead to the underpricing of natural resources.

3. Foreign Debt

In recent years, many resource-based developing countries have become heavily indebted. Overly optimistic estimates of future petroleum revenues have been one cause for excessive borrowing, but there are others. It has been suggested that heavily indebted countries are forced to over-expand their resource exports in an effort to obtain foreign exchange.

There are several explanations for this alleged behavior. The most important is the constraint imposed by debt-servicing when borrowing capacity has been exhausted. When resource prices fall, a higher level of exports is needed to keep foreign-exchange earnings constant. Under these circumstances, the supply schedule for natural-resource commodities will be downward-sloping or at least backward-bending. This behavior will in turn depress commodity prices, thus continuing the cycle. The situation can be exacerbated if a combination of high debt levels and the desire to make exports competitive leads developing countries to depreciate their currencies. Under these circumstances, shadow prices of foreign exchange differ from official exchange rates and shadow-commodity prices differ from commodity-market prices.

The evidence for such a hypothesis is mixed. Gilbert (1986) finds a negative correlation between developing-country debt levels and non-fuel primary-commodity prices. His work, however, is criticized by Chang (1987) for relating changes in commodity prices to levels of debt service; high debt-servicing should imply low, not falling commodity prices. Chang reformulates the specification and finds no evidence of either an outward shift in the supply schedule for primary commodities or a change in its slope. He concludes that there is no reason to believe that developing country debt can account for either low or falling commodity prices. Thus, although the debt-servicing hypothesis for underpricing natural-resources has a certain theoretical appeal, the evidence in its favor is not strong. Until further investigation yields more consistent results, the issue must remain unresolved.

A second possible link works indirectly, through the interest rate. Suppose that interest rates rise, and that their increase is expected to be permanent. Debt payments will clearly rise by the same fraction, while mineral commodity prices will rise at the higher rate (equation 7). However, higher commodity-price appreciation -- implying lower consumption in all future periods -- is incompatible with equation (5) unless a downward jump in price occurs first. In other words, the initial response to higher interest rates is deflationary.

VII. Common Property and Environmental Externalities

Common-property resources present special problems for the definition and enforcement of property rights. Their key feature is the large number of users, each of whom ignore the effect that their use has on others. Consider one common property resource bad, namely the pollution associated with the extraction of an exhaustible resource.[22] Suppose that there are N identical producers -- countries or firms -- who manufacture a homogeneous output, q, using the services of capital, k, and a common-property resource, P.[23] This can be summarized by the production function

$$q_i = h(k_i,P) \qquad \Sigma i\ q_i = Q \qquad i = 1 \ldots N \tag{15}$$

where capital letters stand for aggregate quantities. Moreover, pollution P is produced as a byproduct of the production of q, a relationship assumed to be linear. Thus:

$$p_i = \beta\ q_i \qquad \Sigma_i\ p_i = P \tag{16}$$

Note that it is aggregate pollution P that enters into the production relationship (15).

Under well known regularity conditions, there exists a total-cost function C that is dual to the production function h. C has as arguments the rental price of capital, r, the level of pollution (which is unpriced), and the quantity of output produced, q_i.

Each producer seeks to maximize their private profit, π_i:

$$\max_{q_i}\ \pi_i = q_i - C(r,P,q_i) \tag{17}$$

subject to the constraint (16). Without loss of generality, output price is chosen to equal 1. In what follows, the subscript i is suppressed; this is justified because all producers are assumed identical. For a non-cooperative solution or Nash equilibrium, the first-order condition for the maximization of (17) is:

$$\delta\pi/\delta q = 1 - C_p\ \beta - C_q = 0 \tag{18}$$

or

$$C_p = (1 - C_q)/\beta \tag{19}$$

Now suppose that a cooperative solution is sought. In other words, a single agent or planner acts to maximize joint profit, $\Pi = \Sigma_i\ \pi_i$. Thus:

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[22] In fact, mineral commodities are responsible for a large fraction of the pollution that is generated today. Jorgenson and Wilcoxen (1990) find that three sectors - primary metals, petroleum refining, and chemicals - account for 55% of US spending on pollution abatement.

[23] This model is taken from Slade (1987). That paper, in turn, makes use of Baumol and Oates (1975) and Dasgupta (1982).

$$\max_{\{q_i\}_{i=1}^{N}} \pi = \Sigma_i \, [q_i - C(r,P,q_i)], \tag{20}$$

subject to the N constraints (16). Since each producer is identical, (20) is equivalent to

$$\max_{Q} \pi = Q - NC(r,P,q) \qquad q = Q/N \tag{21}$$

The first-order condition for this maximization is:

$$1 - N[C_p \, \beta + C_q/N] = 0 \tag{22}$$

or

$$C_p = (1 - C_q)/N\beta \tag{23}$$

The difference between (19) and (23) is obvious. The planner takes into account the fall in every producer's output due to an increase in producer i's polluting activities, and i's output is therefore increased only up to the point at which the private marginal cost of polluting is equal to the private-marginal value of pollution divided by N -- in other words, when social-marginal costs and benefits are equated.[24]

In the case of a noncooperative or market solution, Pareto optimality is not achieved. Too much pollution is produced and everyone could be made better off through cooperation. When N is large, this difference can be substantial. Market failure is caused by the lack of a market for pollution: pollution is both unpriced and commonly owned. The latter feature is crucial. To see this, consider the case where P is a private good, so that only p_i enters π_i. In this circumstance, even though pollution has no market price, it does have a shadow price, C_p, that gives the correct signals. In contrast, where P is a common-property resource, the private-shadow value, $(1 - C_q)/\beta$, does not equal the social-shadow value, $(1 - C_q)/N\beta$, and market failure ensues. Here, there is a definite case for public intervention. The marginal social damage of pollution is NC_p. If the government charged each firm a tax, T, per unit of pollution generated, equal to the difference between the social and private damage, $T = NC_p - C_p = (N - 1)C_p$, then optimality would be achieved. In other words, each firm would set private-marginal benefit, $(1 - C_q)/\beta$, equal to private marginal cost, $C_p + (1 - N)C_p$, which yields the cooperative condition (23).

To summarize, we have seen that common-property environmental resources are sources of potentially serious market failure. Avoiding the latter requires intervention or some form of cooperative behavior. When the common-property resource is a bad (pollution), it is underpriced (at zero) and overproduced. Moreover, the output produced in this polluting manner is also underpriced and overproduced.

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[24] Equations (19) and (23) are special cases of a result due to Samuelson (1954).

Our concern here is with exhaustible resources, whose production and use is closely associated with various pollutant byproducts. The combustion of fossil fuels, for example, generates carbon dioxide and contributes to global warming; metal-smelting is associated with acid rain; and strip mining results in unsightly surface damage. Often the market price of an exhaustible resource does not reflect its true social production cost. The obvious remedy is to tax either the producer or the consumer.[25] However, given that environmental spillovers are transnational, setting appropriate taxes or standards requires international cooperation, and unfortunately, international agencies have no legal power to enforce cooperative agreements. For this reason, if an agreement is to be effective, it must be self-enforcing. The design of environmental taxes and their incidence is a broad subject that cannot be covered here. Nevertheless, since environmental damage can be irreversible, this is a problem that deserves immediate attention.

VIII. Summary and Conclusions

The efficiency and equity of a market allocation of exhaustible resources has been examined and the behavior of scarcity measures, such as relative price and rental rates, has been assessed. Little evidence of scarcity or impending shortage has been uncovered. Indeed, the evidence points to falling prices and rents for many commodities. This raises the important question of whether markets are providing the wrong signals. Are resource commodities systematically underpriced? The conclusions of this study are not completely optimistic. Many market failures have been revealed in the analysis, any of which could result in inappropriate resource commodity pricing. But, with one exception, no systematic tendency to underprice has been found. The exception concerns the environmental externalities associated with the production and use of natural-resource commodities. Similar externalities lead to underpricing and overuse of all commodities. Mineral commodities, however, are responsible for a large fraction of the pollution that is currently generated, and therefore their underpricing is particularly significant.

The market failures associated with common-property and environmental resources can cause market prices to be lower than shadow prices or marginal values. They cannot, however, cause relative-resource prices to fall. Falling prices would be associated with a relaxation of environmental standards and a move away from full-social-cost pricing. The tendency, however, is towards increased awareness of environmental damage and increased willingness to pay for its associated costs. Nevertheless, the prices of many natural-resource commodities have fallen in real terms. Factors causing prices to decrease are not associated with market failure, and therefore do not argue for interference with the market mechanism. Indeed, innovations that lower mining and processing costs, discoveries that increase resource stocks, and the provision of lower-cost substitutes are all features of efficiently operating markets.

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[25] The magnitude and incidence of such taxes are examined in Slade (1991a). On average, they are found to be progressive. If this finding is robust, then it helps to mitigate the problems associated with environmental preservation.

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Annex: Derivation of Equation (6)*

Consider an economy where aggregate output, Q, is produced by two inputs, man-made capital, K, and an exhaustible resource, R. This relationship is governed by the production function

$$Q_t = f(K_t, R_t, t). \tag{A1}$$

f is assumed to be increasing, concave, and twice continuously differentiable in K and R, and the marginal products of K and R are assumed to be strictly positive. Finally, it is assumed that

$$\lim_{K \to 0} f_K = \lim_{R \to 0} f_R = \infty. \tag{A2}$$

Output can be consumed (C) or invested (I) so that

$$Q_t = C_t + I_t \tag{A3}$$

and

$$I_t = dK_t/dt =: \dot{K}_t. \tag{A4}$$

The economy is endowed with an initial stock of the exhaustible resource, S_0. At any time t, therefore, the stock remaining is

$$S_t = S_0 - \int_0^t R_\tau d_\tau, \tag{A5}$$

so that $\dot{S}_t = - R_t$ and $S_t \geq 0$.

We restrict attention to programs where K_t, R_t, and C_t are strictly positive for all t and partition time into discrete intervals of length D. Let C_tD, R_tD, and I_tD represent consumption, resource utilization, and investment during the interval (t, t+D) and consider two adjacent intervals (t, t+D) and (t+D, t+2D). Fixing the program everywhere except on these two

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* A similar model can be found in Dasgupta and Heal (1979).

intervals, we would like to see if we can have a higher level of consumption on the first without reducing consumption on the second.

On the two intervals, we have

$$C_tD + I_tD = f(K_t, R_t, t)D$$

and (A6)

$$C_{t+D}D + I_{t+D}D = f(K_{t+D}, R_{t+D}, t+D)D.$$

Conducting a variation on the first equation in (A6) yields *

$$\Delta C_t + \Delta I_t = f_{K_t} \Delta K_t + f_{R_t} \Delta R_t = 0 + f_{R_t} \Delta R_t, \qquad \text{(A7)}$$

and conducting a variation on the second equation in (A6) yields

$$\Delta C_{t+D} + \Delta I_{t+D} = 0 + \Delta I_{t+D} = f_{K_{t+D}} \Delta K_{t+D} + f_{R_{t+D}} \Delta R_{t+D}.^{**} \qquad \text{(A8)}$$

Since both K and S are held fixed at t and at t+2D, it must be true that $\Delta I_t + \Delta I_{t+D} = \Delta R_t + \Delta R_{t+D} = 0$. This fact, together with (A7) and (A8) yields

$$\Delta C_t = (f_{R_t} - f_{R_{t+D}})\Delta R_t + f_{K_{t+D}} \Delta K_{t+D}. \qquad \text{(A9)}$$

Along an efficient program, variation can yield no extra consumption in the interval (t, t+D), which implies that $\Delta C_t = 0$. And, as K_t is fixed, $D\Delta I_t = \Delta K_{t+D}$. Substituting these facts and equation (A7) into (A9), we obtain

$$f_{K_{t+D}} = (f_{R_{t+D}} - f_{R_t})/(Df_{R_t}). \qquad \text{(A10)}$$

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* Readers unfamiliar with the calculus of variations are referred to Kamien and Schwartz (1981).

** $\Delta K_t = 0$ because there was no change in investment in the previous period, and $\Delta C_{t+D} = 0$ by assumption.

Equation (A10) must be satisfied for every adjacent pair of intervals along an efficient program. If we take the limit as D → 0, (A10) becomes

$$f_{K_t} = \dot{f}_{R_t}/f_{R_t}. \tag{A11}$$

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