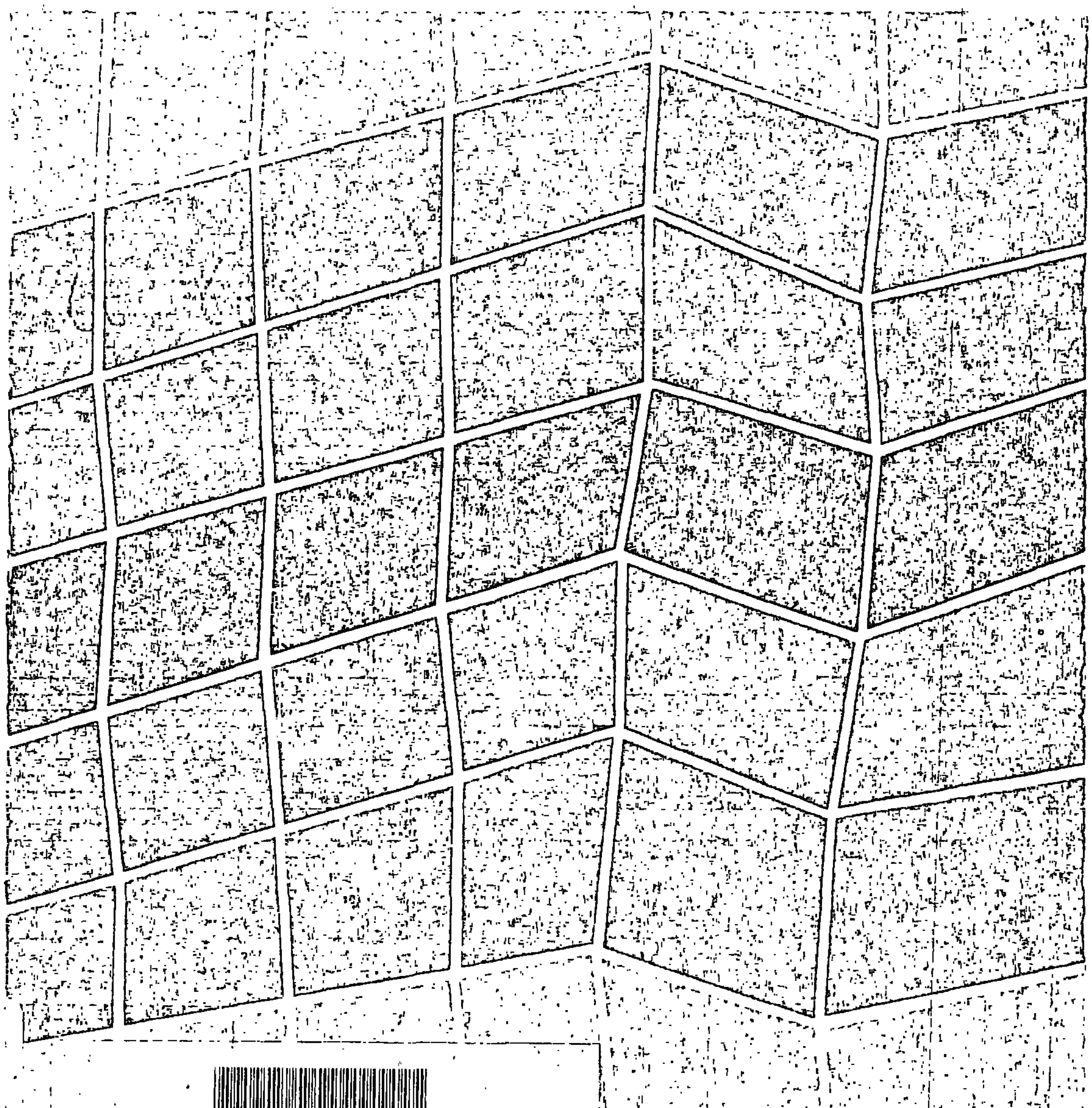
Rakesh Mohan

OCP-25

Urban Economic and Planning Models



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Rakesh Mohan

Urban Economic and Planning Models

Assessing the Potential for Cities in Developing Countries

WORLD BANK STAFF OCCASIONAL PAPERS □ NUMBER TWENTY-FIVE

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Foreword

I would like to explain why the World Bank does research work and why this research is published. We feel an obligation to look beyond the projects that we help finance toward the whole resource allocation of an economy and the effectiveness of the use of those resources. Our major concern, in dealings with member countries, is that all scarce resources—including capital, skilled labor, enterprise, and know-how—should be used to their best advantage. We want to see policies that encourage appropriate increases in the supply of savings, whether domestic or international. Finally, we are required by our Articles, as well as by inclination, to use objective economic criteria in all our judgments.

These are our preoccupations, and these, one way or another, are the subjects of most of our research work. Clearly, they are also the proper concern of anyone who is interested in promoting development, and so we seek to make our research papers widely available. In doing so, we have to take the risk of being misunderstood. Although these studies are published by the Bank, the views expressed and the methods explored should not necessarily be considered to represent the Bank's views or policies. Rather, they are offered as a modest contribution to the great discussion on how to advance the economic development of the underdeveloped world.

ROBERT S. MCNAMARA
President
The World Bank

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Preface

This book surveys the main varieties of urban models with a view toward finding approaches that can be useful for understanding cities in developing countries. Some comments are also offered on the complexity of urban phenomena that intrinsically cause difficulties in formulating urban economic models.

Notation is always a problem when comparing a large number of models. Here I have tried to maintain a consistent notation so that the same letter stands for similar variables in different models. One has to say "similar" because each model has its own variation in the definition of essentially the same variable. I have explained each notation as it is introduced and have made every effort to maintain reasonable comparability with the original articles.

I am indebted to Orville Grimes, Jr., Peter Watson, Edward Holland, J. Ben-Bouanah, Sven Sandstrom, Chris Turner, Raymond Struyk, James Ohls, and Edwin Mills, all of whom read the first draft with care and graciously provided useful comments. The book owes much to the diligence and care with which Bertrand Renaud and Douglas Keare have read and reread it, always providing helpful critical comments. This final version has benefited greatly from the comments painstakingly provided by Gregory Ingram and Alan Walters. Various drafts have been skillfully typed and retyped by Mary Ann Heraud, Ofelia Miranda, and Margot Clark. Elizabeth Hock edited the final manuscript. Christine Houle prepared the author index.

RAKESH MOHAN

Introduction

It is now a common observation that cities in poor countries have been growing at unprecedentedly high rates in the past two or three decades and that they continue to do so. Some cities have grown from a population of less than 1,000,000 people in 1950 to about 4,000,000 now. Such rapid growth has brought with it immense problems for public authorities who have to provide the public services that are essential to the functioning of these cities. Large public investments for the provision of water, sanitation, sewerage, roads, and electricity have been necessary. In addition, there has been both public and private investment in housing and transportation. The volume of this investment has been large even though it has not been able to keep up with the requirements. These responses to rapid urban growth have often had to be ad hoc reactions to problems that need urgent action. There has therefore been a growing feeling that these responses should be more systematic rather than ad hoc. As a result, the demand for methods of analysis to help in decisionmaking has also become apparent.

The World Bank now devotes about 30 to 35 percent of its total lending to projects that are directly urban related. As the urban lending program has expanded, it has become clear that an investment in one sector of the urban economy often affects the urban area in as many indirect ways as those directly intended. A water project, for example, is designed mainly to supply water to a certain section of a city; its indirect effects would include changes in land values that in turn affect the demand for housing and residential location, the demand for other infrastructure, and the pattern of transport. Similar statements may be made about any large, urban-related investment program. The project analysis of these programs should take account of the indirect effects along with the direct ones, neither of which are currently well understood. Therefore, an increasing need has been felt for models of urban areas that may help to cope with some of these problems. This book is in the nature of a partial response to this need.

Although it has often been alleged that investment in developing countries has been biased toward urban areas at the expense of the countryside, it appears that economic research has suffered from the opposite bias. There is a marked lack of research into the growth and structure of cities in developing countries, whereas there is a substantial body of work on aspects of agricultural and rural development. The literature on economic development has characteristically ignored the spatial concomitants of the development process even though considerable attention has been paid to the migration of people from rural to urban habitats. It is the paucity of relevant work related to urban areas in developing countries that has necessitated this review of work on models that have arisen mainly in Europe and North America. Such work is relatively recent and reflects some of the predominant and current concern about cities in these regions. How can these methods be transferred to cities in developing countries? What are the possibilities of applying current urban modeling techniques to these cities? These are the themes of this book.

Little descriptive information is currently available on the structure of cities in developing countries. Moreover, the available information crosses disciplines and is not easily accessible to individual researchers. It is difficult to describe the aspects in which cities in developing countries differ from, or are similar to, those in developed countries. Consequently, making judgments on the transferability of models developed in the West is somewhat hazardous. But on examining the available information, scanty though it is, I have come to believe that cities in developing countries do exhibit phenomena that are different from those observed in Western cities of today, as well as when these cities were at similarly low levels of income. These differences stem essentially from the unprecedentedly high rates of population growth in the non-Western cities, the coexistence of high and low levels of technology with predominantly low incomes, and the decline in relative prices of transport and communications as compared with a century ago. How these differences may affect the problems of modeling urban areas is discussed in some detail before the review of models.

Urban models are of various kinds, but they may be divided into two broad classes: analytic, or explanatory, models and operational, or policy-oriented, models. Naturally, policymakers at various levels are primarily concerned with the latter class. The former, based mainly on economic theory, are usually systematic attempts at explaining urban form. Why do households locate where they do? Why do firms locate where they do? Why are cities more densely populated in certain parts and not in others? The analytic or explanatory models offer basic insight at a general level into problems posed by such questions; they do not attempt to replicate or explain the configuration found in a particular city. Most of these models investigate the desired or optimal location of residences or employment; the search is either from the viewpoint of the household (or firm) or of society as a whole trying to maximize the welfare of all its citizens. Particular attention is paid in these models to (*a*) the tradeoff between the cost of the site itself and the cost incurred in traveling to and from the site; (*b*) the analytic problems caused by the unique quality of each location within a city; (*c*) the effects of transport congestion on city form; and (*d*) the consequences of a societal welfare concern that emphasize equity. Though these models are at a sufficiently theoretical plane, they should be regarded as conceptual building blocks toward less abstract models. To the extent that these models provide paradigms for understanding the basic structure of cities, they are useful for policy purposes and have been used as such. To the extent that they misrepresent reality because of gross simplification, they may have misguided some policymaking. There is therefore no hard line between a model that may be classified as analytical or explanatory and one that is termed operational or policy oriented. It is a matter of degree and the intent with which a model is developed.

Operational models have characteristically been large in the sense that they have usually required the use of a computer for their solution. There have been two basic strands in such modeling: the social physics variety now based on entropy maximization techniques and the behavioral variety that draws mainly from the analytic economic models introduced above. As its name suggests, the social physics variety of models is based on relations analogous to the physical laws of motion: it tries to replicate statistical regularities observed in the activities of people within a city. The concept of entropy, usually found in the field of thermodynamics where predictions are also based on the statistical regularities in the motion of particles, is now used in the justification of these models. The typical output of a model of this class would be the allocation of residential and employment location of people to zones in a city—often disaggregated by socioeconomic types of households, types of residential structures, types of employment, and so forth. Though such a model would be likely to replicate the current structure of the city being represented, it will not provide much of an understanding of why it is the way it is. It is therefore not a tool that is robust with respect to the rapid changes characteristic of cities in the developing countries. The economic models have many pleasing realistic features; they are easier to understand because their structure is drawn from behavioral relations derived from the analytic models. The versions thus far developed have, however, been quite unwieldy to operate because of their size; they have been highly data intensive and have not easily permitted evaluation of alternative courses of action within a time-bound realistic decisionmaking context. Indeed, they have been far more useful in illuminating new approaches for the explanation of the structure of urban areas than in policymaking. This characteristic once again illustrates that models cannot be clearly delineated between being analytical or explanatory, on the one hand, and policy oriented, on the other: that development of the two types has to be simultaneous and interdependent. Hence, many of the operational, or policy-oriented, models developed so far are judged to be of limited practical use in developing countries.

Lest it be concluded that there are no possibilities for fruitful attempts at modeling urban areas in developing countries, I provide four examples of approaches that may augur well for the future. Each of these is quite distinct in approach, though not without faults. The two operational models found to have potential for application are small in size and flexible in data requirements. They can be operated at various levels of aggregation and with varying degrees of quality of data. Their drawback is that they are mathematically complex. The two analytic, or explanatory, models suggested explore behavioral relationships which may subsequently be used in the development of policy-oriented models.

Because of the lack of systematic work on urban areas in developing countries, the development of tools for analyzing and understanding the operation of these areas will have to proceed simultaneously with the design of aids for policymaking. It is the major conclusion of this book that the search for models that can aid policymaking is likely to end with models that are small, comprehensible, and relatively easy to operationalize, though they are not necessarily simple; useful explanatory, or analytic, models are likely to require large sets of disaggregated data in order to provide the building blocks for the operational models even though their conceptual bases may remain simple.

Urban Economic and Planning Models

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Modeling Urban Phenomena: Why Do It and Why Is It Difficult?

> But as for those who posit the ideas as causes, firstly, in seeking to grasp the causes of the things around us, they introduce others equal in number to these, as if a man who wanted to count things thought he would not be able to do it while they were few, but tried to count them when he had added to their number.
>
> ARISTOTLE (*Metaphysics*, Book 1, Chapter 9)

Such was Aristotle's criticism of Plato's Theory of Forms. Plato sought to comprehend reality around him by defining general "forms" and interpreting things similar to them as their particular manifestations. His objective was to reduce the size of the problem by comprehending classes of things rather than the individual thing. The difficulty with this, however, is illustrated by the following question: What is the essence of a table that makes it a table? Thus, the number of forms equals the number of "things around us" ultimately and the quest for comprehension of the universe brings us back to the starting point.

Such is the predicament of the urban modelbuilder. The objective in building a model of an urban environment is to reduce the complexity of the observed world to the coherent and rigorous language of mathematical relationships.[1] When this language becomes complicated or the size of the simplified model becomes so large that it assumes a complexity of its own, the usefulness of such an exercise is questionable. Therefore, when a model of an urban environment is sought, it is essential to keep in mind the objectives of the exercise.

Urban planners and policymakers have become prisoners of the idea that everything affects everything else in the city.[2] If indeed everything is interrelated, every public decision should be an informed one if it is to achieve its aims. The urban policymakers must be aware not only of the direct effects of their decisions but of the indirect effects as well. For example, the provision of a new faster mode of travel has the direct effect of reducing travel time for its users. But its indirect effects on industry location and, consequently, on employment and residential location could be far greater in magnitude. The policymaker, aware of such relations, demands knowledge of their magnitudes: hence, the need for models. Therefore, models should help the policymaker understand the underlying determinants of spatial location within a city, analyze the causes of city growth and decay, and predict future land uses in specific parts of urban areas. For example, if a model shows that commercial development A will take up x hectares of city center land, cause y percent more traffic congestion, and generate z percent less tax income and jobs in ancillary industries than industrial development B, it will be of interest to the policymaker.

These, however, are an intimidating set of demands that social science can barely supply, although some of its practitioners pretend that they can. Recent developments in computer technology and in the use of mathematical techniques in the social sciences have raised expectations of what can be predicted about the future. Social scientists themselves have been instrumental in raising the level of demands made by policymakers. This results, more often than not, in disappointment.

1. I. S Lowry, "A Short Course in Model Design," *Journal of the American Institute of Planners*, vol. 31, no. 2 (May 1965), pp. 158–66.

2 Ibid.

The field in economics in which models have advanced most is that of macroeconomics. The 1960s saw the development of a number of large-scale macroeconomic models that attempted to model the whole economy. It was expected that the use of such models would turn economic policymaking into a science from an art: hence, the expression "fine tuning" gained wide use. After the initial enthusiasm, however, these models were found to be somewhat less useful than had been expected. They usually comprised a number of submodels, each of which was carefully developed and econometrically estimated. The problem often encountered was that when these submodels were put together, their interaction produced unforeseen results; they then had to be "tuned" for purposes of simulation. These models tracked the economy well in the short term but performed less well in the longer term. Discontinuous changes such as the rise in oil prices proved difficult to cope with in such models. The key problem, however, in these models was the difficulty in tracing chains of causation. When a variable is found to change significantly, in a simulation it is often difficult to find the cause in the inputs. Conversely, when inputs are changed, it is difficult to identify which change affects what. Thus, the relevance of these models was limited for policymakers. Large urban models tend to have similar problems.

In the context of urban systems, the demand for models is particularly great because of the large number of variables and available policy parameters, the complex relationships among them, and the long-term consequences of public decisions. The sheer size of the number of variables would not be such a problem if they were not thought to be correlated. If the variables were independent, problems could be solved partially or sequentially; there would be no need for models Planners, therefore, need models for the following purposes: prediction and projection, impact analyses of alternative strategies, plan design, educating planners, and controlling and directing urban change.

A comprehensive model would meet all of the above-mentioned needs, but such a model has yet to be formulated. Models have been and continue to be designed to meet different needs. The following kinds of urban models can be distinguished: theoretical, policymaking, data manipulating, educational, and measurement devices. Theoretical models are highly abstract and seek basic insights into urban structure. Such models can be useful in educating planners as well as in designing more operational models. Policymaking models can be merely predictive or can be optimizing or of the impact analysis type: they all seek to help planners and policymakers. Data-manipulating models (input-output models, for example) are good devices for checking the consistency of data and can also reveal the structure of some relationship. Educational models can be game-playing devices with which planners can be educated. Although these models may not be strictly operational, policymakers can use them to test possible effects of policies.

It is evident that this classification is not a mutually exclusive or an exhaustive one. It illustrates the point that different models do different things in attempting to satisfy the different needs mentioned earlier. Users of models must be as clear about what they cannot do as about what they can. If, for example, a model is predictive, it should not necessarily be regarded as prescriptive.

It is very important that policymakers understand why they want cities to be modeled before they begin financing modelbuilders' dreams. They should have definite ideas about the kind of information they seek from the model and must be able to communicate them to the modelbuilders. They will then have enhanced their chances of getting a useful model.

Some Reflections on City Phenomena

A need for models of the urban environment having been stated, it is appropriate to consider those intrinsic characteristics of cities which cause problems for modelbuilders. City phenomena are replete with analytical inconveniences such as increasing returns to scale, indivisibilities, interdependencies, and minimum-size thresholds. If these phenomena did not occur, there would be little reason for cities to arise. If there were no economies to be gained from agglomeration, there would be little reason for people to live in large clusters. Models, however, seldom capture these nonlinearities except in an elementary sense. For this reason, it becomes even more important to study city phenomena in order to understand their effects.

Many cities evolved as market or trading centers. This was essential for the expansion of product markets and, consequently, for economic growth. Roland Artle has suggested that the characteristics of the income elasticity of demand for goods provide clues as to why cities and economic growth invariably seem to accompany each other.[3] Basically, it is a simple idea: the income elasticity of the demand for primary goods such as food is low and *declines* with income, whereas that for urban goods and services is high and *increases* with income. As an economy grows from low levels of income, the proportion of income necessary to buy food declines; thus, demand is created for nonfood goods. Technical advances in agriculture allow part of the labor force to be released to work in urban pursuits. The question becomes: Can these products be manufactured and provided in rural areas? The answer appears to be negative. Even craft products need markets and capital. A craftsman can expect to sell his product only if he can afford to invest in capital. His risks are reduced if he has proximity to his customers, especially if the potential number of customers is large. The production process does not need economies of scale for the size of the expected market to be a factor. Such a consideration for production is difficult to capture in a mathematical model. What, then, is the relevance of these ideas to the modelbuilder? Because most models for policy purposes are concerned with projecting or predicting the future, the modeling of intraurban relationships will be improved if the nature of the city and its stage in the growth process are taken into account. Clearly, a city can be modeled more effectively if the modeling is done with a knowledge of the structural changes likely to occur within a decade.

As markets expand and production increases, the availability of inputs also becomes important. The likelihood of obtaining labor—more important, the proper kind of labor—at a given time is clearly greater in a concentrated population than in one which is dispersed. Other inputs, such as raw materials and manufactured goods are also easier to obtain: hence, the notion of interdependencies among products. There are, however, two kinds of interdependence. One is represented by an input-output matrix. A dense matrix, that is, one with many nonzero entries, represents a high level of interdependence. The process of production becomes an intricate mesh; most products require many other products as inputs. This can be termed technological interdependence. Such interdependence does not necessarily produce a city if transport costs are not high. According to Moses and William-

3. Roland Artle, "Urbanization and Economic Growth in Venezuela," *Papers and Proceedings of the Regional Science Association*, vol. 27 (1972), pp. 63–93.

son, cities that grew in the nineteenth century have the structure they do because the cost of moving goods within cities was high relative to moving people within cities, and to moving goods between cities (by train or water transport).[4] Thus, industries were located in clusters near major transport nodes. Trucking made intracity transportation of goods cheaper, and this is posited as one cause of decentralization. The second kind of interdependence also arises from technological interdependence but has to do with uncertainty. For example, the physical proximity of establishments is of some economic benefit. Vernon found this to be a major characteristic of the existence of certain types of industries in New York City.[5] Those activities which depend on changing output demand must respond by changing their own input demands. The necessity of doing this in a short time makes proximity necessary. Furthermore, risk is spread: a supplier who loses one client can easily switch to another before deciding to change his product line. In summary, technological interdependence is important for spatial reasons because of transport costs; the other, because of information costs and uncertainties.

In the broader context of modern economic activity, the interdependence of production with such services as banking, insurance, and marketing has become more important in the agglomeration economies of modern cities. Some of these economies can be dealt with by considering indivisibilities (or minimum threshold size) within the size of such activities, and can be represented by constraints to a production function. If, however, such activities are regarded as inputs into the production process, their representation is not easily accomplished. Agglomeration is necessary for these activities because of the need for face-to-face contact. The telephone and other communication advances were expected to make face-to-face contact unnecessary, but this does not appear to have happened. It is, however, difficult to place economic values on the benefits of such contact in economic analysis.

The existence of economies of scale in certain industries is the second most important reason for the existence of cities. Even if a

4. Leon N. Moses and H. F. Williamson, Jr., "The Location of Economic Activity in Cities," *American Economic Review*, vol. 57, no 2 (May 1967), pp. 211–22.

5. R. Vernon, *The Changing Economic Function of the Central City* (New York: Committee for Economic Development, 1959).

few industries exhibit economies of scale, the effect of these economies through backward and forward linkages will be much greater. Entire cities, such as Detroit, are based primarily on a single industry that has economies of scale. Of course, this can be represented easily by production functions: for example, by a Cobb-Douglas production function whose sum of exponents is greater than one. The problem created by such functions is the loss of many of the nice properties of constant returns or diminishing returns to scale production functions.

It is also important to recognize that there are many activities that are characteristically performed jointly. Learning and research are two such activities. Moreover, they are tightly correlated and are instrumental in producing technical change. They thrive on agglomeration. The provision of health services, transport, and recreation also tends to be collective although their use is personal or individual. Certain kinds of modern medical services are so expensive that they can be provided only if a hospital serves a large number of people. Similarly, theaters, operas, concerts, and other forms of entertainment require a certain minimum size of audience in order to be viable. The more densely an area is settled (within limits), the greater the potential for the use of such services and entertainments. Because these services require minimum threshold levels, the private market often fails to manage them well. Thus, some kind of public authority is frequently required to take on their management. Consequently, policymakers come into play and ask for guidelines to help in making decisions.

Finally, there are necessary evils associated with agglomeration. On the production side, negative externalities include pollution and monopoly resulting from the operation of economies of scale. On the consumption side, overcrowding can make the provision of collective services difficult.

In addition to the problems mentioned above, and in part because of them, cities change in rather unpredictable ways. The growth process is of two kinds: the multiplication of existing facilities and the beginning of new activities. The stability of a city depends on its ability to cope with change. Most industries have long-term cycles: creation, boom period, and decline. If a city is based on only one industry, the city itself goes through the same cycle. If, however, the city is based on a variety of industries, it is unlikely that all of their cycles will be in phase; consequently, the city is more flexible and able to cope with change. At any one time, then, there will be both efficient and inefficient industries. This is perhaps what Jane Jacobs (1970) means when she says that cities need to have inefficiencies and impracticalities built into them to cope with the uncertainties of changing technology and the economic environment.[6] This is not unimportant, for urban infrastructure usually lasts about fifty years. Apart from single-industry bases, cities built with single-transport modes are more vulnerable than others. European cities that were built when no motorized transport was available have adapted well to the many changes in the modes of transport during the past century. In contrast, North American cities built in the last seventy to a hundred years are so dependent on automobile transport that they might find the effects of rising oil prices more difficult and costly to handle. Models can hardly capture such characteristics of cities. If a model is an optimizing one and is calibrated on contemporary data, its prescriptions may well produce an efficient city for the present but a disastrous one for the future.

Uncertainty about the present, changes in the future, indivisibilities, and economies of scale have been posited as the reasons for the existence of cities. These are the phenomena most difficult to codify and to include in models. At best, approximations can be attempted. Nonlinearities can be built in, but they make solution and handling of the model more difficult. Complex relationships between variables and the durability of structures also make the impact of decisions more difficult to analyze. Thus, modelbuilders should be modest about what they are trying to accomplish and policymakers less sanguine about what they can expect from models.

Cities in Developing Countries

For a number of reasons, cities in developing countries are in many ways more difficult to model than those in the Western world.

Comparisons with Western cities

Unlike most Western cities, those in the developing countries have grown suddenly and explosively in this century and have attained sizes comparable to cities elsewhere but are at much lower levels of

6. Jane Jacobs, The Economy of Cities (London: Jonathan Cape, 1970).

income. Some cities in the United States have also grown rapidly, but this has been at high levels of income and technology. Analysis of cities in poor countries is more complex because of the coexistence of different kinds and levels of technology. Analysis of transport systems for example, in Western cities, needs to consider a limited number of modes that should be modeled. In an Asian city, however, the electric train coexists with hand-pulled rickshaws, bicycles, scooters, automobiles, and walking.

Even during the period of rapid urbanization in Europe, annual rates of national population growth were typically on the order of 0.5 percent,[7] whereas the annual population growth rate in developing countries is at present 2.5 to 3 percent. With an annual growth rate of 0.5 percent, European cities had time to adapt and evolve as they grew in size. Diffusion of innovation, the pace of technical change, and the rate of income growth were all in balance. Economic, social, and political institutions emerged to regulate patterns of growth and to govern cities as they grew. This can be viewed as a process that was basically in equilibrium at every stage.

The current experience in developing countries is quite different. A primary factor is that the rate of population growth is explosive as compared with historical experience. Even with the best of policies, controlling this growth rate is not easy, especially in the short run. One consequence of a high population growth rate is that cities can become larger without significant increases in the level of urbanization for the country as a whole, as is the case in India. Such a phenomenon causes a misunderstanding of the process of urbanization and associated policies that affect urban structure. Because this phenomenon is usually seen in a short-term context, myopic policies often emerge from such a view. If population growth were to be curtailed, however, cities might grow rapidly in any case. The nature of their growth would be different, of course, and urbanization levels of the country would rise because, in the absence of overall rapid population growth, the population would be released rapidly from agricultural pursuits to move to urban pursuits, which would cause high rates of migration and urbanization. In either case, cities in poor countries are growing rapidly and are likely to continue so doing.

7 George Beier and others, "The Task ahead for the Cities of the Developing World," World Bank Staff Working Paper, no. 209 (Washington, D.C., July 1975).

My point is that for the purposes of understanding and modeling urban structure, it is necessary to have an appreciation of the nature and causes of this growth. Institutions are not able to adapt as rapidly to such growth; consequently, the structural growth of the city becomes unpredictable and difficult to manage. The need for urban planners arises because market institutions often are not able to develop rapidly enough. Without knowledge and understanding of the underlying phenomena, urban planners themselves add to the unpredictability inherent in the situation.

The second major difference from the European experience is the coexistence of different levels of technology in cities of developing countries. Technological inequality is probably even more pronounced than income inequality in these cities as compared with those existing in Western cities in their early stages. This makes the demand structure of the rich qualitatively different from that of the poor in a manner more pronounced than that in European cities at similar levels of incomes. Currently, the demand pattern of the rich in developing countries corresponds roughly with that of the rich in Western countries. They demand and receive services and products that use twentieth-century technology, while the poor still live in much as they did a century ago. Thus, it is possible to find both a modern shopping center (comparable to American ones) and a traditional bazaar in the city of a developing country. Technological inequality is particularly noticeable in housing. There is, therefore, a wider variety of urban structural patterns in these cities, which makes modeling more difficult. I emphasize the point about technological inequality because income inequality in these cities is often given more attention, even though it was probably quite similar to that in Western cities in their early stages.

Unlike the European experience, then, technological growth, income growth, and population growth are unbalanced in cities in developing countries today. High rates of population growth and technological growth make it possible to have cities with a population numbering in the millions at low levels of income. This was not possible in the nineteenth century. The solution of problems arising from sewage disposal, water supply, and transport in a city of 8,000,000 people requires modern technology. These solutions are qualitatively different from those required by a city of 500,000, which was the size of major Western cities at similar income levels.

The third major difference is the decline in relative cost of transport and communication. This makes for less centralized cities, as seen in some of the urban densities in Latin America. Workers can live some distance away from their place of work. Information and innovation travel faster. Other imbalances render this diffusion uneven and urban centered and make primary cities more important. The international diffusion of information and innovation reinforces this tendency of concentration in primary cities and further exacerbates technology and income inequalities. The elite are internationally mobile: they characteristically have more contact with Western cities than with their own hinterlands. Their demand structure is therefore more responsive to Western factor proportions. The urban effects of such polarized income levels are especially evident in housing, where differences between rich and poor are extreme.

These major differences combine to produce effects that suggest the idea that such cities have a different spatial structure from those for which models and plans have been developed. Until now, cities in developing countries have been lumped together and discussed as if they were all similar, yet different as a group from Western cities. It is worthwhile, however, to make some distinctions among them. Clearly, there are systematic historical and geographic differences among cities in developing countries. The classification by George Beier and others, based on the urbanization experience, is interesting,[8] but I am more concerned with urban form. The classification of cities, suggested below, into preindustrial, industrial, and postindustrial is a hybrid one[9]; consequently, it is less well ordered than the Beier one.

Preindustrial cities existed in developing countries before colonization or before the modern era. They have grown naturally and have highly congested central areas reflecting their founding before the motorized age. Having grown within and along with the traditional economy, they have tight links with their immediate hinterland. Their patterns of production, commercial relationships, and transport routes reflect this. These cities often have modern influences superimposed on them; this is usually characterized by a modern business center coexisting with the old market area though spatially separated. Ex-

8. Ibid.

9. This hybrid classification was suggested originally in Rakesh Mohan, "Urban Land Policy, Income Distribution, and the Urban Poor," in *Income Distribution and Growth in Less Developed Countries,* ed. Charles Frank and Richard C. Webb (Washington, D.C.: Brookings Institution, 1977).

amples of these are cities such as Ibadan in Nigeria, Hyderabad and Lucknow in India, and Mombasa in Kenya.

Industrial cities are usually large port cities that were the main colonial importing and exporting centers. As such, they have been heavily influenced by the Western world. They have links with a wider hinterland than preindustrial cities, but the links are mostly commercial. Their structure is more like Western cities and is more geared to motorized transport. This category can be further divided to reflect varieties of colonial experience. Singapore, Hong Kong, Calcutta, Bombay, Rio de Janeiro, and Buenos Aires are examples of such cities.

Postindustrial cities should really be called administrative cities; they are termed postindustrial for logical neatness! They are postindustrial in the sense that they have only become "cities" in the last two or three decades. They are, for the most part, planned cities and are usually quite sparsely populated. They have grown primarily as capital cities with somewhat questionable economic bases. Some are in the process of attracting more industry and diversified economic activity in addition to governmental activity. Islamabad, Brasilia, and many African capitals are examples of postindustrial cities.

My point in differentiating among kinds of cities in developing countries is to illustrate why they can be expected to require different kinds of analyses. In addition, they have large differences in income levels that must be taken into account.

Effect of the differences on modelbuilding

How do the three major differences between cities in developing countries and Western cities combine to make the life of modelbuilders difficult?

RAPID POPULATION GROWTH. A rapid growth in population at low levels of income in cities having high and low levels of technology produces segmented markets. Here the focus is on the distinctions between the formal and informal labor markets. The modelbuilder is accustomed to looking almost exclusively at the formal markets. In such circumstances, the journey to work is regarded as one of the most important components of an urban economic model and as the major transport activity. Whether this is so in cities in developing countries is not clear. Where there is less organized economic activity, employment appears to be more diverse. The large informal sector provides moving employment, such as hawkers and service-oriented people who have no fixed place of work but who do have areas of operation. Such employment may well account for about 10 percent of all urban employment, as in Peru.[10] The journey to work for these people cannot be defined accurately. Further, their work location covers a large area.

Also included in the informal sector are the small manufacturers, artisans, and shopkeepers who often live and work in the same location. Most are self-employed and probably account for another 10 percent of urban employment. Some people in this sector commute from the neighboring countryside, stay in the city for part of the week, and return to their homes for the remainder. It would be surprising, therefore, if journey-to-work–oriented models of urban location were appropriate for cities in developing countries. (There is, however, a paucity of statistics on these transport patterns, so these remarks should be considered speculative.) At best, the informal market is heterogeneous and the range of incomes within it is large. Analysis of it is further complicated by the fact that a majority of its workers are secondary breadwinners in their households. This makes the distinction between residential and work locations much more complicated. If poorer households characteristically have more than one breadwinner, this problem becomes a serious one for models generating residential location distributions. The existence of informal markets also causes difficulties for models that are based on the basic or export-oriented employment concept that is then used to generate all other kinds of employment. The relationships between markets in cities in developing countries requires more investigation in order to find such multipliers.

TRANSPORT MODES. Analysis and modelbuilding are simplified considerably when there is a single or, at the most, two modes of transport. The coexistence of low incomes and high technology produces great problems for the modelbuilder in this area. In developing countries, the poorest people walk, those in slightly better circumstances use a bicycle, the next group use a bus or a train, and the richest own automobiles. There are also other modes such as the jeepney, scooter, rickshaw, and cycle-rickshaw.

Some evidence from Delhi is worth relating here. Excluding walk-

10. Richard C. Webb, *Government Policy and the Distribution of Income in Peru, 1963-73* (Cambridge, Mass.: Harvard University Press, 1977).

ing, 42 percent of the commuters used bicycles and about 37 percent used public transport. At least 35 percent of work trips were walking trips for people with low incomes as against 25 percent for middle-income people. The average length of the work trip was remarkably short: twelve minutes for fast vehicles, sixteen minutes for mass transit, and eleven minutes for bicycle users. (There are no data for walkers.)[11] This evidence, coupled with the spatial origin of vehicular work trips, indicates dispersed employment. Those who commute to work by fast vehicles live closest to the city center; those using slow vehicles, the farthest from it. The calculation of urban transport costs and their effect on residential location is not a simple task. It is one that requires ingenuity if useful models of cities in developing countries are to be built.

HOUSING. High migration rates, low incomes, and scarcity of capital produce squatter settlements, shanty towns, and slums. Such housing accounts for about half of the total housing in many cities in developing countries. This produces two problems for modelbuilders. First, data are intrinsically difficult to gather from such neighborhoods because many structures are illegal. Second, the variety in housing is as great or greater than in modes of transport. The production technology for straw huts is clearly different from that for tall buildings. Even if the form of the production function is not different, the factor proportions are clearly so. In Western cities this is not the case; it is possible to make useful distinctions between low-rise and high-rise buildings. The production technology for both is not much different. In the cities of developing countries, different kinds of housing require different kinds of inputs from different markets. Further, the housing market itself is more segmented.

FACTOR PROPORTIONS. The existence of a city implies higher ratios between capital and land and capital and labor (although the latter is not obvious). Because developing countries have a scarcity of capital but have cities similar in size of population to the West, their structures can be expected to be different. Because cities in developing countries do have some skyscrapers, it would appear that the mere existence of such structures makes them more capital intensive than

11. A. C. Sarna, "A Disaggregated Land Use Transport Model for Delhi, India" (Ph.D. dissertation, University of Waterloo, Ontario, 1975).

might have been expected. Large populations that are densely situated drive up the price of land in the centers of cities, resulting in the use of skyscrapers as a form of capital to substitute for land. In the context of the whole economy, this may not be the best allocation of resources. The provision of urban services such as sewerage, water supply, electricity, roads, and mass transport, is necessary as well as capital intensive. Generally, the factor proportions in these activities are probably nearer to those in developed countries than to those in the developing country generally. Here the problem is undoubtedly one of limited technological choice. Nonetheless, it creates allocational and financial problems for cities in developing countries.

CHOICE. Low levels of income limit the alternatives available to the poor. This fact may be of some help to the modelbuilder. (The limiting case is one in which the poor are so constrained that their location decision involves no element of choice.) It may be expected, though, that however poor the nature of choices available is, poor people do have to choose between a range of alternatives.

ROLE OF PUBLIC AUTHORITIES. Finally, there is one aspect of cities in developing countries that is difficult to take account of in economic models although it is one of the main reasons for building them. This is the role public authorities play in making spatial decisions for the city. Households can be regarded as utility maximizing, and firms, as profit maximizing. Modeling the behavior of public authorities can follow no such rules. In the first place, centralized political authority has long been a reason for the existence of some cities, and it has played an especially strong role since World War II. Many governments have become more active since then, particularly those in developing countries. A further complication is that, although there is much evidence of the effects of governmental decisions in the formal sector, these effects are less visible in the informal sector. Consequently, little is known about them. Theories that adequately capture the interaction between governmental decisions and economic activity have yet to be developed, although governmental-reaction functions and the like are now being posited. Obviously, some role must be assigned to governmental activity in urban models of cities in developing countries. This role cannot be merely exogenous, where the effects of such decisions are measured by sensitivity analysis, but endogenous as well.

Summary

The major characteristics of cities in developing countries may be summarized as follows:

- unprecedentedly high population growth rates;
- coexistence of high and low levels of technology and predominantly low incomes; and
- declining relative prices of transport and communication compared with those of the last century.

These characteristics give rise to segmented markets, many transport modes, squatter settlements, shanty towns, and slums. Further, the high intensity of capital in urban areas is not commensurate with factor proportions existing elsewhere in the country. These effects make the development of formal models of cities in developing countries difficult. Because information on urban phenomena in developing countries is scanty, these ideas remain somewhat speculative.

Criteria for Model Evaluation

The first question to be posed when reviewing urban models should be: What are the model's objectives? Ultimately, models should be judged on economic criteria; that is, their benefits should be seen in relation to their costs, their results in relation to their objectives. This will not be attempted in any rigorous manner in this review, although costs of models are mentioned whenever possible. This section lists criteria for judging models according to what I regard as desirable qualities in different types of models. Specifically, one set of criteria is presented for explanatory or analytic models and another set for the policy-oriented ones, because the objectives of the models differ.

Explanatory or analytic models

MODEL STRUCTURE. Explanatory models operate at high levels of abstraction that require many compromises with reality to be made. It is essential, therefore, that their structure not be obtuse. Of course, if a complex reality is being modeled, the structure cannot always be simple. But a complex model structure can be presented so that relationships within the model are clearly articulated. This is important because larger policy-oriented models are often based on these analytic models.

CORRESPONDENCE TO REALITY. As I have mentioned, it is understood that explanatory, or analytic, models have to make certain compromises with reality in order to remain a reasonable size. They are explanatory, however, only to the extent that they explain some urban phenomena. Thus, a good model should remain close to reality in its structure and output.

OUTPUT. The results of a good analytic model should provide insight into some aspect of urban phenomena; if it does not, the model is of little value. For example, a model can focus on the location decision and illuminate its behavioral determinants. It is at this level of modeling that the rationale behind urban structure emerges.

NORMATIVE AND POSITIVE RESULTS. Models should not create confusion between what is and what ought to be. The assumptions behind a good model should be made explicit. This is important, for the results can then be judged on the basis of the assumptions. Moreover, the role of the assumptions in the results can be identified. If a model predicts something, it should be made clear whether the prediction is a projection or a prescription for a plan of action. If it is a prescription, the criteria for it should be spelled out. When such distinctions are not made, projections of current trends, for example, are often mistaken for targets.

Before discussing the criteria for policy-oriented models, I should point out some connections between the two kinds of models. Explanatory models are not necessarily small. Essentially, they seek to improve basic understanding of the urban process. Usually they cannot be used directly in policy planning. Policy-oriented models should be reasonably close in their simulation of reality to aid decisionmaking. They must take into account the relationships that bear on the problem under consideration. This usually necessitates the presence of submodels within the model. The relationships in these submodels usually manifest the behavioral relationships found in explanatory models. To use the analogy of macroeconomic models once again, all the research on the consumption function is used to form the consumption equations in large econometric models. This set of equations can be disaggregated for different product markets and can comprise a set of between thirty and forty equations. The form of all these equations comes from basic consumption function studies. Similarly, if a policy-oriented model accounts for the fact that rental prices for land decline with distance from a city's center, the area would probably be divided into segmented markets for different types of demands, such as office space, residential, and manufacturing.

Even highly theoretical models are useful as long as they exhibit the qualities specified above. Ultimately, they are useful for purposes of policy design through their use in policy models.

Policy-oriented models

Models that are policy oriented are more difficult to evaluate because they are usually larger and more complex. They are difficult to understand because of the many inputs that must be fed into them, the many equations that constitute their structure, and the many outputs they provide. They can be judged according to the following criteria.

OBJECTIVES. Complex as they are, policy-oriented models must have clearly stated objectives so policymakers can use them and can judge whether the model is achieving its objectives. A predictive model is quite different in structure from a prescriptive one. A spatially disaggregated one is again different from an aggregated one, and so forth.

DATA REQUIREMENTS. In evaluation of a model for application to developing countries, its data requirements have to be examined closely. As described later, half of the budget for a policy-oriented model is characteristically devoted to data collection. Since data collection in developing countries is a problem, the data requirements of models for developing countries have to be scrutinized even more carefully. Several of the issues involved are measurability, accessibility, and cost.

It often happens that some kinds of data requirements are simply not measurable, although they might well fit into neat conceptual slots. Other kinds of data are measurable but are often not easily accessible. The length of time involved in their collection is a particularly important consideration, given the high rate of growth of cities in developing countries. If, for example, a set of data requires two years to collect, it might well become obsolete before the collection is completed. Resources for such data collection are scarce, as are the facilities for processing it. Thus, the more a model uses readily available data, the cheaper it will be to obtain and the more quickly the model can be put into operation. The cost of data collection must also be viewed in the light of the benefits expected from the exercise.

INPUTS. In the description of a model there is often much confusion about which variables are endogenous and which are exogenous or provided externally. Among these exogenous variables, the distinction between control variables and other variables should be made clear. Control variables are those which policymakers can play with and should therefore be made explicit. The operational meanings of these control variables should also be clear. The clear specification of inputs is also important because it facilitates the understanding of what a model is—and is not—simulating.

MODEL STRUCTURE. The structure of a model is the core on which its usefulness really depends. A model is expected to simulate reality to the extent that its empirical or behavioral foundations are believed to be sound. But there are other aspects of model structure that need to be considered: simplicity, flexibility, and the role model structure plays in decisionmaking.

The structure of a good model is characterized by its simplicity. It need not be so facile that it avoids modeling complex relationships within it, however. Rather, it should be easy to explain. Opaque structures render models difficult to comprehend. An explanation of such a structure is facilitated if it has clear connections to theory and is consistent with it. If it is not consistent, it is useful to explain why it is not. Thus, the theoretical bases of a good model are made explicit. This is particularly important if a model is to be used as a policy device. Policymakers are more inclined to believe results if they can appreciate the reasons that produce these results; hence the need for logical structures.

Models are usually built with one city in mind; once completed, attempts are made to apply them to other cities. On one hand, the more flexible a model, the better it is. On the other hand, flexibility implies that the model is not that tailored to the original city; this may create grounds for suspicion about its usefulness. If a model is pronounced flexible, justifications should be given. These depend on the kind of relationships embedded in the model. Is the flexibility due to changeable parameters or because the relationships themselves can be changed easily? If a model is capable of being moved from one country to another or from one culture to another, explanations about the feasibility of such a move should be provided.

Another kind of flexibility exists in the changeability of the parts of a model. When a model has a number of objectives and a number of submodels, it is useful to know how tightly related these parts are. Are they modules that can be removed or whose sequence can be changed to fit different circumstances and objectives? In this sense, the more flexible the model, the better it is.

The structure of the model affects the outputs. When the output is used for decisionmaking, it should be possible to determine whether the results are due to technical quirks of the model or are believed to be inherent in the city as modeled. Although this is often a difficult and sometimes impossible task, the robustness of a model can be tested by intensive sensitivity analysis. In other words, a model's role in decisionmaking should be clear.

OUTPUT. Because the policymaker receives the output from the model, it is most important that the output be intelligible. It should also correspond to reality. If a projection seems rash, it must be explained. Calibration of the model serves to make the outputs reasonable. Since all outputs are actually point estimates of some distribution, an indication of their level of accuracy should be given. In view of this, some tolerance measures should be included. Probability estimates are useful when extrapolation is involved.

Policymakers are also interested in the robustness of outputs. They want to know how sensitive they are to policy changes. Thus, a good model should provide sensitivity tests. In addition, predictive outputs should be clearly distinguished from prescriptive ones.

COST OF OPERATION. There are various kinds of costs: (a) those of developing the model and making it operational; (b) time costs, which depend on the length of time required to put the model into operation; (c) costs for the development of skill; and (d) costs of running the model on a computer. Clearly, minimizing costs is one of the objectives of the modelbuilder. Skills are in particularly short supply in developing countries and are therefore a crucial variable. Furthermore, intimate knowledge of a city is essential to the building of a good model. It is therefore important to find models that can be built with the available skills.

In the reviews that follow, I have attempted to separate description from evaluation. Evaluative comments are based on the above-mentioned qualities, but the evaluations do not appear as checklists. As I have stated earlier in this chapter, there is yet no general agreement about what constitutes a good model; until there is, evaluations that are essentially subjective must suffice.

2

Analytic, or Explanatory Models

Economic models of the spatial structure of cities are relatively recent in origin and are still in the process of development. Most of the work has been conducted in North America and Great Britain and is therefore derived from the urban problems prominent in those areas. The phenomenon that has occupied analysts most in North America has been the continuous suburbanization of both jobs and residences over the last half century. The problems addressed in Britain concern the design of new towns. Thus, most of the work conducted in the United States has been analytical and explanatory, whereas that in Great Britain has been more practical.

This chapter reviews the theoretical bases of a large class of models that attempt to explain urban spatial structure at a rather high level of abstraction. Their objective is to explain general trends over time or spatial arrangements at a fairly coarse level of detail. On the one hand, there are theories of residential location that focus largely on the trade-off between housing (that is, consumption of space) and transport costs; on the other hand, there are theories of employment location.

The theories of residential location usually assume explicitly or implicitly that employment is concentrated in the center of the city. The traditional model is one in which the household is in equilibrium, but the notion of disequilibrium has received more emphasis lately. As will be evident in the following sections, the modeling of the household location decision is more advanced than that of industrial or employment location. The latter is still an assortment of conjectures in search of a theory—or a theory in search of empirical verification.

The Classical Economic Models

The classical economic models are examined here as being either equilibrium or disequilibrium models.

Equilibrium models

A review of urban economic models must begin with the seminal contributions of Muth, Wingo, Kain, Alonso, and Mills.[1] The models are of the same family: utility-maximizing households constrained by their budgets in their attempt to find optimum residential locations. The city is located in a featureless plain and possesses a single central business district (CBD) in which all employment is located.

Muth's households have a utility function:

$$U = U(x,q), \tag{1}$$

where x represents consumption of goods other than housing and q is consumption of housing. Their budget constraint is:

$$M = x + p(u)q + T(u,M) \qquad T_u > 0, \tag{2}$$

where M is household income; u is the distance from the CBD; the price of x is unity; $p(u)$ is the price per unit of housing, a function of distance from the CBD; T is the cost per trip and is a function of location and income; and the subscripts denote partial derivatives

1. Richard F. Muth, *Cities and Housing* (Chicago: University of Chicago Press, 1969); Lowden Wingo, Jr., *Transportation and Urban Land* (Washington, D.C.. Resources for the Future, 1961); John F. Kain, "The Journey to Work as a Determinant of Residential Location," *Papers and Proceedings of the Regional Science Association*, vol. 9 (1962); W. Alonso, *Location and Land Use: Toward a General Theory of Land Rent* (Cambridge, Mass.: Harvard University Press, 1964); and Edwin S. Mills, "An Aggregative Model of Resource Allocation in Metropolitan Areas," *American Economic Review*, vol. 57, no. 2 (May 1967), pp. 197–210.

(that is, T_u is $\partial T/\partial u$). Income includes money value of travel and leisure time. Prices of goods other than housing and transport are the same throughout the city. Housing is regarded as a bundle of services yielded both by structures and by the land they occupy. These services are a flow, not an asset; thus, the price is also of the flow, not of the asset. No distinction is made between owners and renters because they are both seen as consuming a bundle of services. In the basic model, Muth assumes that households make a fixed number of trips to the CBD, which implies that it is not a decision variable. These costs comprise money costs that vary with distance ($T_u > 0$) from the CBD and time costs that are assumed to vary with income ($T_M > 0$).

Maximizing equation (1), constrained by equation (2), yields the standard first-order Lagrangian conditions:

$$\frac{\partial L}{\partial x} = U_x - \lambda = 0, \tag{3a}$$

$$\frac{\partial L}{\partial q} = U_q - \lambda p = 0, \tag{3b}$$

$$\frac{\partial L}{\partial u} = -\lambda(qp_u + T_u) = 0, \quad \text{and} \tag{3c}$$

$$\frac{\partial L}{\partial \lambda} = M - \{x + p(u)q + T(u,M)\}. \tag{3d}$$

The marginal utilities are in the same proportion as the prices (from equations 3a and 3b):

$$\frac{U_x}{U_q} = \frac{1}{p(u)}, \tag{4}$$

and equation (3c) yields:

$$-qp_u = T_u \text{ or } p_u = -\frac{1}{q} \cdot T_u \qquad T_u > 0. \tag{5}$$

Equation (5) shows that, in equilibrium, a small move will not result in any savings. T_u is the marginal change in transport costs and qp_u is the change in housing expenditure occasioned by such a move. Thus, a move outwards (that is, $\delta u > 0$) will increase transport costs by T_u, but this will be balanced exactly by a saving, qp_u, in housing expenditure. A small move inwards balances the saving in transport costs by an equivalent increase in housing expenditure, $T_u > 0$ hence $p_u < 0$ (that is, housing costs per unit decline with distance).

Muth then investigated the effect of small changes in each variable and obtained the following results by totally differentiating equation (5) with respect to u to get:

$$-qp_{uu} - p_u \frac{dq}{du} - T_{uu} \leq 0. \tag{6}$$

He concluded that:

$$p_{uu} > 0, \tag{6a}$$

the price of housing decreases with distance at a numerically decreasing rate, and:

$$\frac{\partial u^*}{\partial M} \geq 0, \tag{6b}$$

where u^* is the optimum location of the household.

The optimum location becomes further from the CBD as the income gets higher. This is an ambiguous result and depends on various assumptions concerning the income elasticity of the demand for housing and the elasticity of T_u with respect to income—assumptions regarded as plausible by Muth. Put more simply, this result states that the benefits of increased housing consumption outweigh the increase in transport costs.

This is a very important implication for city structure and for change in structure over time. If Muth's assumptions are correct, a general increase in incomes leads to an increase in housing consumption on the part of all households and the city spreads out. Where transport is slow, as in developing countries, the time costs for people with higher incomes may well be high enough to contradict this assumption. The location of people with high incomes near the CBDs rather than in the suburbs could then be regarded as an optimal location within that framework. This could be tested econometrically if good data on housing expenditures and trip times for different income groups and location were available for cities in developing countries.

$$\text{(6c)} \qquad \frac{\partial u}{\partial p_0} \leq 0,$$

where p_0 is the price of housing services located at the center. Therefore, optimum location will be nearer the CBD if the price of housing services increases (keeping the gradient of price and distance unchanged).

$$\text{(6d)} \qquad \frac{\partial u^*}{\partial T_0} \leq 0 \quad \text{and} \quad \frac{\partial u^*}{\partial T_u} \leq 0.$$

An increase in either the fixed or marginal cost of transport decreases the optimal distance from the CBD.

Muth assumed the income elasticity of demand for housing to be greater than 1 in the derivation of all these results. This assumption is supported by his own empirical work. He then modified his basic model by relaxing some initial assumptions: (a) the number of CBD trips is made a decision variable by introducing it in the utility function; (b) similarly, preferences for location are introduced in the utility function; and (c) uniformly distributed local employment is introduced in addition to the CBD employment. Assumption (a) does not affect the results of the model in any substantive sense; (b) makes the derivation of qualitative results almost impossible; and (c) makes possible the derivation of a wage gradient with distance from the CBD. Obviously, this appears because the local workers incur no transport cost and therefore can accept lower wages to remain at the same utility level at the same location.

Muth has a production side to his model that describes the behavior of profit-maximizing firms producing housing services:

$$\text{(7)} \qquad \Pi = pq(L, NL) - rL - \rho(NL),$$

where Π is profits; L is quantity of land inputs; NL is quantity of nonland inputs; r and ρ are their respective prices; and $q(L, NL)$ is the production function of housing services. All the firms are identical and have the same production functions. Thus, they all have the same profits regardless of their location. The prices of the inputs vary with locations, so differing combinations are used according to location. Equilibrium conditions yield:

$$dr^* = \frac{1}{S_L} dp^* - \frac{S_{NL}}{S_L} d\rho^*, \tag{8}$$

where indicates natural logarithm of the number; and S_L and S_{NL} are the shares of land and nonland in the firms revenue, that is:

$$S_L = \frac{rL}{pq} \text{ and } S_{NL} = \rho \frac{(NL)}{pq}$$

Equation (8) shows that land price is high where the price of housing (due to location, for example) is high and where the price of other inputs, such as raw materials, is low. Equation (8) can be rewritten as:

$$\frac{r_u}{r} = \frac{1}{S_L}\left(\frac{p_u}{p}\right) - \frac{S_{NL}}{S_L}\left(\frac{\rho_u}{\rho}\right). \tag{9}$$

This shows that the land rent gradient, r_u/r, is a multiple of the housing price gradient because $S_L < 1$, assuming that the gradient of other input prices, such as wages, is negligible. For a land share of 5 to 20 percent, the land rent gradient can be anything from five to twenty times the housing price gradient. By assuming a Cobb-Douglas production function for housing, Muth derived the housing price, land rental, and population density functions as declining exponentially with distance from the CBD.

This model has been presented in some detail because it is an example of simple economic reasoning stretched to its limit. The assumptions underlying the model are highly unrealistic, but that is the price of a simple, manageable model. Muth did extensive empirical testing of his propositions; indeed, his work is an example of the close relationship of good economic theorizing and empirical work. Many qualitative results are impossible to obtain without the assumption of robust parameter estimates, such as the income elasticity of the demand for housing.

Wingo concerned himself with the cost and programming of transport in a city, although these are embedded in a larger model of land use and transport. Indeed, he set out to develop a transport model but soon discovered that land use and transport were too related to be separated. Wingo was much more guided by policy considerations than Muth and tried to articulate a model that would be of operational use for policy planners. It proved, however, to be of greater interest as an analytical model. His approach was guided by three considerations: (a) a model should have explicit differentiation between policy and structural effects; (b) it should enhance its analytical value by bringing the main elements of the problems within the framework of economic theory; and (c) it should have conditions for treating the problem of intraurban distribution of population as a part of the urban economy as a whole.

His model concentrates on the manner in which labor services are organized in space, given the characteristics of the transport system, the spatial arrangements of production, the nature of the labor force, and the institutions by which the labor force is articulated with the processes of production. The model provides the following: (a) the concept of transport demand based on characteristics of the labor force and of the journey to work; (b) a systematic general description of the transport function; (c) a general transport cost function; (d) a system of location rents which result from the transport cost function; and (e) the manner in which a household demands space and the way in which supply is equated to demand.

Here I will describe in some detail the derivation of the transport cost function which really forms the core of the model, and will neglect the rest of the model. Moreover, it is especially interesting because it is based on fairly simple notions but takes into account many of the complexities of transport. Wingo treated the problems of congestion in the context of an urban location model at that early stage although Walters and Beckman, McGuire, and Winsten,[2] among others, had analyzed the congestion problem, earlier; their findings were rediscovered later. The development of this cost function illustrates the way in which a particular part of an urban model requires care and thought as well as empirical observation for a proper specification.

Wingo assumed the journey to work to be the most important transportation function and proceeded to analyze its cost. He observed that the journey to work is both spatially and temporally concentrated; consequently, these peaks in demand result in saturation of a given capacity. Other demands on the transportation

2. Alan Walters, "The Theory and Measurement of Private and Social Cost of Highway Congestion," *Econometrica*, vol. 29 (October 1961), pp. 676–99; and Martin J. Beckman, C. B. McGuire, and Christopher B. Winsten, *Studies in the Economics of Transportation* (New Haven: Yale University Press, 1956).

system can then be excluded and the journey to work analyzed. The costs can be broken into two components: the time costs incurred and the actual transportation costs. Both involve a prior characterization of technology of the urban transportation system. A key assumption made here is that of the homogeneity of travelers and carriers (in his case, the automobile). This obviously simplifies the analysis; indeed, it makes it manageable. The calculation of time in the journey to work is done by the function:

$$T = T(u, v, n, c), \tag{10}$$

where u is distance traveled; v is velocity; n is number of workers; and c is a measure of capacity of the transport system. The specification of the function depends on the mode of transport used. Wingo's contribution lies in the inclusion of n and c in this function; they demonstrate the interdependence between the users of a transport system when demand (n) exceeds capacity (c). The time lost because of this excess demand is a result of ingression, which is the irreducible minimum of time loss because of aggregation of demand. This is a technological function analagous to changes in water pressure in a pipe according to the volume moving through it. It is also a result of congestion, which is caused by the reduction of free flow (bad driving, for example). This condition arises through human errors, which are proportional to the pressure of traffic. If t_n is defined as the time loss to the nth unit entering the transport system at peak time, then:

$$t_n = \frac{n-1}{c} \text{ and} \tag{11}$$

$$t = \frac{n(n-1)}{2c} \tag{12}$$

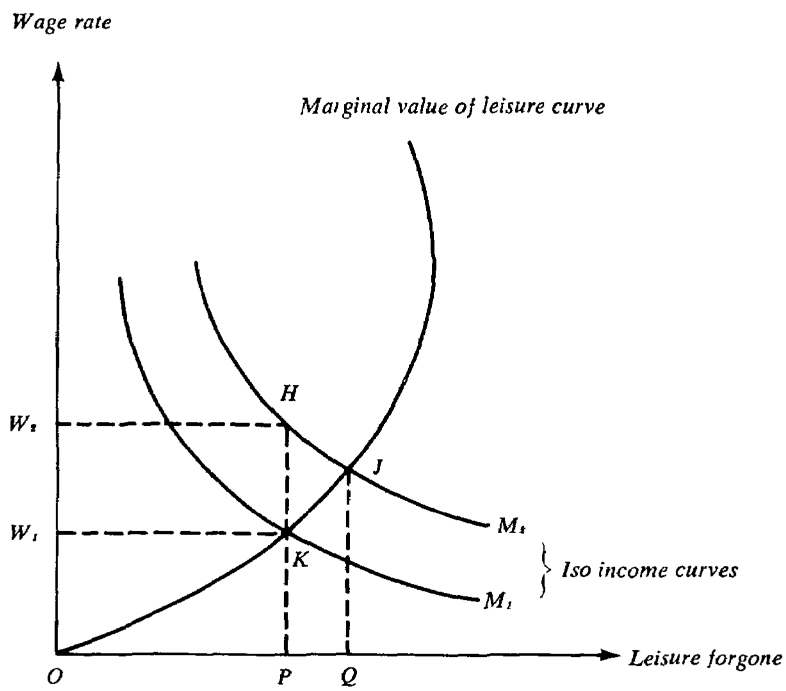
is time lost to all units in queue. This is proportional to the square of demand. Capacity of a system is also determined technologically:

$$c = c\ (v,\ v^{\beta},\ \xi), \tag{13}$$

where ξ is the length of the carrier.[3] The exact specification of the

3. Wingo's expression is $c = \dfrac{v}{\rho(\alpha' v^{\beta} + r' v) + \xi}$ where ρ is a risk coefficient, β is approximately 2-2.5 for automobiles and r is reaction time.

Figure 1. Valuing Time Spent on the Journey to Work



function clearly depends on the mode of transport used, the condition of the roads, and so forth.

The time spent on the journey to work having been derived, its value is derived from the marginal value of leisure function: the supply function of labor. Although a worker is paid according to time spent at work, Wingo argued that the wage rate subsumes the time costs of the journey to work. In figure 1, *OP* is the time spent at work and *PQ* is the journey to work. The worker needs to be compensated for *OQ* hours at hourly wage *QJ*. He gets paid, however, for *OP* hours only, hence his actual wage rate may be *PH*—that is W_2—to give him the same income. His pure wage rate is then W, and W_1 W_2 is compensation for time spent in traveling. W_1KH W_2 provides a measure of the value of the time spent on the journey to work.

The actual transport cost (money costs), have two components:

those which vary according to distance traveled, and those which vary according to number of trips made. The sum of these plus the time cost yields the total cost of transport for an individual.

For the sake of completeness, I will now describe the rest of Wingo's model. All workers are paid the same wages, so the differences in transport cost incurred by each account for the differences in land rental. In other words, land near the center of the city commands a higher rent, the difference being equivalent to the excess transport cost incurred in living at a distance from the center. A household's demand for land depends on the rental value of land; the elasticity of demand is constant. The supply of land is proportional to the distance from the city center and is given exogenously. The model is closed by balancing the supply and demand of residential land. Wingo's condition for locational equilibrium is that the saving in transport cost is equal to the increase in outlay on residential land. This is not strictly correct: the condition should be that no one can increase his utility by moving. The two are equivalent if the time costs of commuting include the disutility of commuting, for which Wingo appears to account.

The importance of Wingo's work lies in his demonstration of the complexity involved in specifying a single component of an urban economic model. The valuation of transport costs involves knowledge of the technological relations of the particular transport system and of the workings of the labor market. While the assumptions about homogeneity of carriers and travelers are justified at this level of abstraction for developed countries, they are not so for the developing countries. The modes of transport in the latter are far more mixed, from walking to bicycles to electric trains, as are the workings of the informal labor market. Wingo himself made the point that valuing time with money implies fungibility, and this is only valid for markets that are operating well. If time and money are not exchangeable, a person may behave as though the scarcity of his time or his money were governing his behavior. In either case the other is ineffectual in allocation. Money and time constraints are probably distributed among the population in accordance with income levels. Money is more likely to be the binding constraint for low-income groups; time, for upper-income groups. Such considerations can go a long way toward the explanation of location patterns in cities of developing countries. Modeling this can be done with a Wingo's kind of approach employing the modifications I have suggested.

Kain focused on the journey to work as a determinant of residential location. His assumptions are largely similar to those of Muth and Wingo—with two major exceptions. He allowed for multiple workplaces, although it is assumed that each household knows the workplaces of its members before making the decision about residential location. Second, households are allowed to be heterogeneous, with different space preferences or requirements according to family size and the like. Kain viewed transport costs as consisting of three components: those which vary according to workplace location, T_w; those which depend on the distance of the residence from other points of interest to the household, T_0; and those incurred for services obtainable within the residential area, T_r. He asserted that, for the majority of urban households, the sum of T_r and T_0 is small compared with T_w. Consequently, in the context of the tradeoff between housing and transport costs, residential location can be explained by a consideration of the variation in journey-to-work costs.

Kain's model was not articulated mathematically; instead, it was primarily diagrammatic. He assumed that residential space varies across locations because households compete for more accessible sites and economize on transportation expenditures. He further assumed, unlike Muth, who derived this result, that unit costs of housing (or rents) decline with distance from the workplace. The justification of this assumption, given multiple workplaces, is not immediately obvious.[4] The household decision model posited that the household first decides on the quantity and quality of residential space desired and then finds the optimal location. With a declining rent curve, the optimal location is one at which the additional transport costs incurred due to a marginal move outwards are just balanced by the savings in total rent. Thus, those households desiring more space live farther away from those desiring less. Kain's results are therefore consistent with those of Muth. His simple model is of interest, however, because it provides ideas on how to deal with multiple workplaces and different household preferences and requirements. Evidence for the validity of his conclusions comes from his data on Detroit.

Alonso provided a rather complete and general model of urban location and urban land markets. He started with utility-maximizing

4. In his empirical work in Detroit, Kain merely assumed that rents decline with distance from the CBD and hence in one direction outward from every workplace.

households constrained by their budgets:

$$U = U(x, q, u). \tag{14}$$

This is similar to Muth's formulation except that q is quantity of land rather than housing and u—the distance from the CBD—is introduced explicitly in the utility function with $U_u < 0$. Alonso derived a bid-price function for each household from the equilibrium conditions. Each household has a bid-price curve for a given level of utility. The result of adding u to the utility function is that one of the equilibrium conditions becomes:

$$-\frac{\partial r}{\partial u} = \frac{1}{q}\left(\frac{\partial T}{\partial u} - \frac{1}{\lambda}\frac{\partial U}{\partial u}\right), \tag{15}$$

where λ is the Lagrange multiplier denoting the marginal value of money. This condition says that residential rent compensates for different travel times. Because $T_u > 0$ and $U_u < 0$, the righthand side of equation (15) is positive and the rent (of land) distance function has a negative slope. The reason Alonso did not use demand curves to analyze the land market is because each location has a different demand curve. The uniqueness of locations makes the derivation of an aggregate demand curve invalid. This problem is circumvented by the use of bid price curves. A bid price curve represents the prices a household is willing to pay for land in each location in order to maintain a constant level of utility. A bid price curve is therefore derived by fixing utility and varying distance to obtain a function:

$$b = b(u). \tag{16}$$

People with the steeper curves locate nearer the center. The market also yields a price structure curve showing the market price (rent) of land at each location, $r(u)$. Tangency between these determines a household's location, that is:

$$p(u^*) = b(u^*) \quad \text{and} \quad p'(u^*) = b'(u^*) \tag{17}$$

for equilibrium.

On the production side, firms are profit maximizing. Profit is defined as revenue minus the sum of land and nonland production costs. Alonso used curious revenue and cost functions:

$$\begin{aligned} &\text{Revenue (volume of business)} &&= R(u,q), \\ &\text{Operating (nonland) costs} &&= NL\ (R,u,q), \text{ and} \\ &\text{Land costs} &&= p(u)q. \end{aligned} \tag{18}$$

The firm's bid-price function is derived for each level of profits: that is, the rent a firm is willing to pay for each location in order to make the same profits. Their location is then determined by the equilibrium tangency condition mentioned above for households. At each location, of course, profit is maximized.

Market equilibrium is achieved when each user's land bid price is tangent to the price structure. The price structure is the envelope of all bid-price functions. Finally, supply of and demand for land should be equal. Alonso analyzed the case where each user's bid-price function is a family of parallel straight lines. He concluded that users will be ranked from the city center according to the ranking of the slope of their bid price lines, the steeper ones being nearer the center.

Alonso's approach was largely diagrammatic, though he provided some mathematical analysis. A rigorous mathematical formulation of his model would be quite complex because he allowed for different tastes among households; indeed, that is what produces different bid-price functions. Mills showed that Alonso's assumptions are not capable of producing a solution to his model. The specification of an equilibrium utility level is necessary for a solution. Particular specification of the form of the utility functions is also necessary if stronger implications from the model are to be derived.

Alonso's work is essentially an extension of von Thünen's theory of the values of agricultural land.[5] He adapted it to an urban area but encountered difficulties because urban land does not have the intrinsic productivity differences that agricultural land has because of variations in fertility. He thus lost one determining variable and ended up with an *n*-person, *n*-firm game. A solution to such a game requires assumptions concerning strategy, permitted coalitions, and so forth.

Although Alonso's model is not entirely satisfactory, it is useful because it again illustrates the difficulty of modeling urban areas, even at a highly simplified level. The difficulties arise from the nature of urban areas: the uniqueness of each location is created by a complex set of interdependencies.

Mills, drawing on the work of Wingo, Alonso, and Muth, at-

5. See Johann Heinrich von Thunen, *Von Thunen's Isolated State,* trans. Ceila M. Wartenburg (London· Pergamon Press, 1966; originally published 1826).

tempted to build simple general equilibrium models for urban structure. He provided a family of models with similar bases, though each has a different wrinkle. I will begin by reviewing the most complex and earliest of his models and then comment on the others.[6]

Mills began by speculating about the primary reason for the existence of cities and posited that nonhomogeneity of land and nonconstant returns to scale in production functions are sufficient to justify the existence of cities. If land is heterogeneous (some land is more productive than other land), it will pay to concentrate production on the better land and thus produce a city. This can be represented two ways in models. One is to introduce variables such as natural resources, topography, and climate into the production function; the other is to have just one land input while associating different efficiency parameters with different sites. Mills chose the latter for the purposes of this model. Agglomeration economies of different kinds are broadly interpreted to be scale economies and are represented as such in an aggregative model.

The city is assumed to be a homogeneous plain and has three activities. The first is the production of goods. The goods production function, having nonconstant returns to scale, is represented by:

$$(19) \qquad X_{1s} = A_1 L_1^{\alpha_1} N_1^{\beta_1} K_1^{\gamma_1}, \ (\alpha_1 + \beta_1 + \gamma_1 = H_1 \gtrless 1),$$

where X_{1s} is total output of goods produced; subscript s denotes supply; L_1, N_1, K_1 are the land, labor, and capital inputs; and $H_1 \lesseqgtr 1$ represents nonconstant returns. All goods production takes place in the CBD; hence:

$$(20) \qquad X_1 = \int_{CBD} X_1(u)\,du,$$

where $X_1(u)$ refers to the amount of goods produced in a ring of width du, u miles from the center. The other two activities are transport and the production of housing. Transport has only one input, land, with a fixed coefficient:

$$(21) \qquad L_2(u) = bX_{2s}(u),$$

where X_{2s} is transport produced and $L_2(u)$ is land used at distance u.

"Housing" subsumes all goods but those produced in the CBD.

6. Mills, "An Aggregative Model."

The assumption is that all goods with nonconstant returns to scale will be produced in the CBD, whereas the others will be forced by competition to locate near customers in order to avoid transport cost. The production function for this definition of housing is:

$$X_{3s}(u) = A_3 L_3(u)^{\alpha_3} N_3(u)^{\beta_3} K_3(u)^{\gamma_3} \qquad (\alpha_3 + \beta_3 + \gamma_3 = 1). \tag{22}$$

On the demand side, X_1 is considered an export good with a price elasticity, λ_1, given exogenously. Thus:

$$X_{1D} = a p_1^{-\lambda_1}. \tag{23}$$

A fixed proportion, δ, of the workers resident at each u work (presumably in housing and transport) near their residences in the suburbs. The demand for transportation is, then:

$$X_{2D}(u) = (1 - \delta) \int_{k_0}^{k_1} N(u')du', \qquad k_0 \leq u \leq k_1 \tag{24}$$

where k_1 is the radius of the city; k_0 is the radius of the CBD; and $N(u')du'$ is the number of people living in a ring of width du' and radius u'.

Within the CBD:

$$X_{2D}(u) = \int_{0}^{k_0} N_1(u')du', \qquad (0 \leq u \leq k_0) \tag{25}$$

where $N_1(u')du'$ is the number of workers working in a ring of width du' and radius u'.

The demand for housing is constant per worker:

$$X_{3D}(u) = N(u)x_3. \tag{26}$$

MARKET CONDITIONS. All factor markets are competitive. The wage rate w, and the rental rate, ρ, for capital are given exogenously. Rental rate for land, $r(u)$, is exogenous.

In industry 1:

$$w = \frac{\partial(p_1 X_1)}{\partial N_1}, \; \rho = \frac{\partial(p_1 X_1)}{\partial K_1} \quad \text{and} \quad r(k_0) = \frac{\partial(p_1 X_1)}{\partial L_1}, \tag{27}$$

according to normal marginal productivity conditions. The competition for land is between the CBD industry and suburban uses; thus, land use is determined by the rent at the edge of the CBD distant k_0 from the center.

Land is the only transportation input; thus:

$$p_2(u) = a_1 r(u), \tag{28}$$

that is, the cost per passenger mile depends only on the rent, $r(u)$, at that u, for a_1 is a constant.

Housing is produced with competitive input *and* output markets; thus:

$$w = \beta_3 \frac{p_3(u)X_3(u)}{N_3(u)},\ \rho = \gamma_3 \frac{p_3(u)X_3(u)}{K_3(u)},\ r(u) = \frac{\alpha_3 p_3(u)X_3(u)}{L_3(u)},\ \text{and} \tag{29}$$

$$p_3(u) = \bar{A}_3 r(u)^{\alpha_3}, \tag{30}$$

where

$$\bar{A}_3 = \{A_3 {\alpha_3}^{\alpha_3} \beta_3^{\beta_3} {\gamma_3}^{\gamma_3}\}^{-1} w^3 \rho^{\gamma_3}.$$

OTHER CONDITIONS. The main equilibrium condition is that a worker at u cannot decrease his location cost by moving toward the city center. The decrease in his transport cost would be balanced by an increase in his housing cost:

$$p_2(u) + p_3'(u) X_3 = 0, \quad \text{where} \tag{31}$$

$$p_3'(u) = \frac{dp_3(u)}{du}.$$

This is a crucial condition and should really be derived from some maximization conditions. Embedded in it are notions concerning the disutility of transportation and relative prices of housing and transport.

The rent at the edge of the city is given exogenously. Assume it to be agricultural rent; that is:

$$r(k_1) = r_A. \tag{32}$$

In equilibrium all land must be used up, thus:

$$L_1(u) + L_2(u) = 2\Pi u \tag{33}$$

$$(0 \leq u \leq k_0)$$

in the CBD; and:

$$L_2(u) + L_3(u) = 2\Pi u \qquad (k_0 \leq u \leq k_1) \tag{34}$$

in the suburbs. Finally:

$$N_1 \equiv \int_0^{k_0} N_1(u)du = (1 - \delta) \int_{k_0}^{k_1} N(u)du, \tag{35}$$

which merely ensures that all workers live somewhere.

SOLUTION. Although this model is based on highly simplified notions of the structure of the city, it does not have a straightforward solution. The solution should provide: all output quantities and prices, all the input quantities and prices, and parameters k_0 and k_1 for the size of the city.

It is useful to note that a large amount of information is given exogenously: (1) all the parameters in production functions; (2) demand function parameters for goods; (3) rental rates of labor and capital; (4) fraction of labor force employed in the suburbs; (5) demand per worker for housing; and (6) value of agricultural land.

Inspection of the model shows that the rent-distance function, $r(u)$, for land is the critical function to be determined; many of the other functions can then be derived from it. Mills provided some interesting insights but did not solve the whole model. Within the CBD:

$$L_1(u) = \frac{2\Pi}{\lambda r(k_0)} (1 - e^{-\lambda r(k_0)u}), \tag{36}$$

that is, the amount of land used in production increases at a decreasing rate with distance from the city center even though the amount of total land grows with u. As a consequence:

$$L_2(u) = 2\Pi u - L_1(u); \tag{37}$$

hence the land required for transport increases at an increasing rate up to the edge of the CBD. Mills noted that $r(k_0)$ and k_0 are both large and in the limit for a very large city: $L_1(u) \rightarrow 0$, and $L_2(u) \rightarrow 2\Pi u$, that is, all land at the edge of the CBD is required for transport. This result can be regarded as the requirement for a ring road around the CBD.

The implications of the growth of the city can be found by varying k_0 and analyzing the results. Optimal reallocation of land, based on the assumptions of the model, will be provided.

For the suburbs (the city outside the CBD) Mills derived:

$$r(u) = (A_0 + B_0u) - \frac{1}{1 - \alpha_3}, \tag{38}$$

where $A_0 = A_0(r_A, \alpha_3, X_3, k_1)$ and $B_0 = B_0(\alpha_3, X_3)$, which shows that rent declines, though not exponentially, as one moves towards the edge of the city. An exponential decline results only when $\alpha_3 = 1$: that is, when land is the only input in housing. The implication is that factor substitution makes the land rent profile flatter. It should be noted here that this is more likely to occur in earlier stages of development when land is the major input in housing construction.

Finally, Mills derived the population density function:

$$\frac{N(u)}{L_3(u)} = (C + Du)^{-1}, \tag{39}$$

where C and D are functions of r_A, α_3 and k_1: that is, rent of agricultural land, size of the city, and techniques of house construction. The density therefore declines with distance but not exponentially as has often been suggested, by Clark, for example.[7]

It is interesting to compare this model with some of Mills' later work. His modifications are simpler in some ways and more refined in others. This early model is curious in a number of ways. First, the transport production function has only land as an input. Second, the city is artificially divided into the CBD and suburbs. Workers in the suburbs are imagined as working on housing and transport even though transport has only land in the production function. Third, housing has a constant per worker demand. This robs the model of part of the flexibility given it by the possibility of utility optimization between consumption of space and transport. In a later model (1969), Mills simplified it into two sectors—goods and transport—where goods are interpreted to include housing.[8] He made the production

7. Colin Clark, "Urban Population Densities," *Journal of the Royal Statistical Society*, series A, 114 (1951), pp. 490–96.

8. Edwin S. Mills, "The Value of Urban Land," in *The Quality of Urban Environment*, ed. H. S. Perloff (Washington, D.C.: Resources for the Future, 1969).

function a constant returns-to-scale function. All employment is not in the CBD, and the center is seen only as a major transport node through which all exports pass; this is considered the justification for the existence of the city. Transport is produced by a Cobb-Douglas production function with constant returns to scale. The demand for transport is linked to the production of $X_1(u)$, with the assumption that each unit of X_1 generates a fixed demand for transport. As a result, the land rent, wages, and the rental rate of capital are all linked with the cost of transport, which itself is a function of land rental; w and ρ are exogenously given. This model is more internally consistent than the previous one and the economy is more integrated. It is mainly used to derive the rent-distance function. It is, however, of a form similar to the earlier model. Land rents decline exponentially if the two production functions (for goods and transport) have similar exponents, that is, similar shares of land are used in the production of each:

$$r(u) = r_0 e^{-Au}, \tag{40}$$

where A is a function of all the other parameters of the two production functions. All other variables—for example, land use intensity (amount of capital per acre) or land used for transport or output per acre—can be derived as functions of land rent. Therefore, if land rent is a negative exponential function, so too are all the land-use functions.

Finally, Mills suggested a complex model including congestion.[9] Here he ignored the CBD and concentrated on the suburbs. All employment is now in the CBD; consequently, both the housing and transport functions do not use any labor. Transport is again produced with land only. Housing is produced with a Cobb-Douglas production function using land and capital. The demand side is richer: housing demand per worker is made to be price and income elastic. In this way the effects of general income changes on the structure of a city can be investigated. The cost of transport is now affected by congestion. Mills took the congestion function from the earlier work of others (for example, Walters), it is really not very

9. Edwin S. Mills, *Studies in the Structure of the Urban Economy* (Baltimore: Johns Hopkins University Press, 1972). See also, Walters, "The Theory and Measurement."

different from that of Wingo. It is:

$$(41) \qquad p_2(u) = \bar{p}_2 + C\left(\frac{X_{2D}(u)}{X_{2S}(u)}\right)^D,$$

where $p_2(u)$ is cost of transport at u per mile (as before); $\bar{p}_2$ is some constant (free of congestion) cost; and C and D are parameters determined technologically by the transport system. Much of Wingo's work was interpreted as the specification of these parameters. D for automobiles is believed to be about 2. The point to note is that congestion cost is seen to be a power function of excess demand. The equilibrium conditions of this model are essentially the same as in Mills' earlier model.[10]

Mills found that the introduction of congestion and of elastic housing demand makes the model impossible to solve analytically. The rest of his book is devoted to a numerical solution and a demonstration of the way in which sensitivity analysis can be performed on such a model. Some of the interesting results are:

- Technical progress in transport induces workers to use more transportation by moving farther out and expanding the city. Land rents fall in the city center.
- An increase in income elasticity of housing demand increases the size of the total area and, not suprisingly, reduces population density. The magnitude of the effect, however, is suprising: a 9 to 10 percent increase in elasticity causes a 90 percent increase in city area. The disutility of travel is probably not taken into full account, as suggested earlier for the 1967 model. An increase in income has somewhat similar effects.
- An increase in D in equation (41)—the elasticity of congestion cost with respect to amount of congestion—has a somewhat paradoxical result. Travel cost increases near the CBD but decreases with distance from the CBD. This decrease amounts to "decongestion" farther out; thus, people move farther away. Rents in the CBD rise while they decrease with distance: that is, the rent-distance function increases in curvature.

Much of this model is geared to the explanation of decentrali-

10. Mills, "An Aggregative Model."

zation of urban areas in the United States over time, as observed empirically. Increases in incomes and population and technical change in transport are viewed as the main causes of decentralization.

Mills' work is interesting because it attempts to see the urban area as a whole in general equilibrium models. It demonstrates that even extremely simple models tend to be mathematically cumbersome. Each addition of complexity necessitates a simplification somewhere else in the model. The more realistic the assumption, the more unlikely the possibility of an analytical solution. It does demonstrate, however, that even simple models provide us with insight into prevailing urban structures. Mills' general equilibrium model also incorporates some of the notions of the housing market from Muth, the land market from Alonso, and transport characteristics from Wingo.

Disequilibrium models

As shown by the above, one of the principal preoccupations of urban economists has been the explanation of the population density function that relates population per unit area within a city to distance from the CBD. The variables being explained, whether housing density, housing prices, or land prices, are essentially variations on the same theme. The models reviewed above are all equilibrium models; they assume, therefore, that cities are instantly built, rebuilt, or altered according to the current values of the independent variables. Residential location and housing consumption are adjusted according to the levels of employment in the CBD, transport costs, prices, and incomes. Changes in any of these variables produce instantaneous adjustments to new equilibrium levels.

Recent work has questioned these assumptions and emphasized the durability of residential and nonresidential capital and the disequilibrium nature of urban growth. Harrison and Kain (1974) sought to explain urban spatial structure through an alternative model that depicts urban growth as a layering process cumulating over time.[11] Their main idea is that current levels of population, commuting cost, transport cost, and other factor prices determine

11. David Harrison and John F. Kain, "Cumulative Urban Growth and Urban Density Functions," *Journal of Urban Economics*, vol. 1, no. 1 (January 1974), pp. 61–96.

the density of development during the current period, but that the density of development in other periods depends on the levels of these variables during those periods. More formally:

$$(42) \qquad D_t = \frac{\sum_{i=0}^{t} d_i n_i}{\sum_{i=0}^{t} n_i},$$

where D_t is average net residential density; d_i is net residential density of dwelling units added to the area in time period i; and n_i is the number of dwelling units added to the area in time period i.

In each time period, the level of d_i is explained by the standard variables (transport cost, factor prices, incomes, and the like) as used in the equilibrium models. As pointed out by Michelle White, the implications of the new model are quite different.[12] Consider two cities, one originally built in the nineteenth century (such as Boston) and the other in the twentieth century (such as Los Angeles). The new model implies that the two cities would have vastly different density functions even if their current income, population, and transport costs were identical. Under an equilibrium model, current identical values of exogenous parameters would yield identical density functions for the two cities. In accordance with this view, the empirical work of Harrison and Kain shows that the change in the slopes of density functions over time has been much less than had been supposed (by Mills, for example).[13] The data and estimation methods used in various studies have been quite different, so conclusive comparisons between them are still not possible.

That physical capital and structures are durable is scarcely debatable. The question to be answered is how malleable these structures are in response to changes in factor prices. That only 4.3 percent of total U.S. housing stock in 1959 could be attributed to modified structures supports Harrison and Kain's emphasis on the nonmalleability of existing structures and their exclusive focus on new ones.[14]

12 Michelle White, "On Cumulative Urban Growth and Urban Density Functions," *Journal of Urban Economics*, vol. 4 (1977), pp. 104–12.

13. Mills, *Studies*.

14. Harrison and Kain, p. 64.

What is ignored, however, are changes in use without structure modification. As Kain himself recognized: "A more realistic theory and more accurate predictions could clearly be obtained by introducing depreciation and by allowing succession and physical transformations of the stock."[15] Similarly, the results of the instant equilibrium assumption are probably not as far from the truth as the assumption itself because of gradual adjustments that take place continuously. A Boston neighborhood built in 1850, for example, may now reflect factor prices in say, 1930 rather than in 1850 or 1977. The explanation of urban spatial structure, then, lies somewhere between the two extremes of equilibrium and disequilibrium models.

The stock adjustment approach is also useful in the modeling of micro household decisions that seek to explain housing demand, as distinguished from the derivation of macro results such as the density function. In recent papers, Hanushek and Quigley focused on the disequilibrium of households resulting from departures from their optimal level of demand according to household incomes, preferences, and relative prices.[16] As in Harrison and Kain, this is considered a result of the nonmalleability of dwelling units and also of substantial transaction costs that are incurred with changing the set of housing services consumed.

Their basic model is conceptually simple. As before, households derive utility from housing consumption, q, and other goods X. The utility function for a household with characteristic A is described by:

$$U_A(q, X) = V(A, q, X). \tag{43}$$

This utility is maximized subject to the standard budget constraint:

$$M = pq + X, \tag{44}$$

where p is the price of housing services, and X is the numéraire. A conventional demand curve is the result:

$$q = f(A, M, p). \tag{45}$$

15. John F. Kain, *Essays on Urban Spatial Structure* (Cambridge, Mass.: Ballinger, 1975), p. 22.

16. Eric A. Hanushek and John M. Quigley, "An Explicit Model of Intra-Metropolitan Mobility" (New Haven: Institute for Social and Policy Studies, Yale University, June 1977; processed); and "The Dynamics of the Housing Market: A Stock Adjustment Model of Housing Consumption," *Journal of Urban Economics* (forthcoming).

This is the desired or optimal level of housing services and q_t^d denotes the optimal level at time t. With an instantly adjusting equilibrium model:

$$(46) \qquad q_t^d = f(A_t, M_t, p_t) = q_t,$$

but rather:

$$(47) \qquad q_t = B(q_{t-1}^d - q_{t-1}) + (q_t^d - q_{t-1}) + \phi\, q_{t-1},$$

where B measures the adjustment speed from the initial disequilibrium position; γ measures that from the initial desired position to the current desired position; and $\phi = 1 + \dfrac{dp_t/dt}{p_t}$, the relative rate of housing price inflation.

Hanushek and Quigley tested a variant of the last equation and estimated it from data obtained from a Housing Allowance Demand Experiment. The main problem with such models is that q_t^d—the level of optimal housing consumption—is an unobserved variable. Hanushek and Quigley assumed that recent movers are in equilibrium; thus, q_t^d can be observed directly. The deviation from equilibrium of nonmover households with similar characteristics, A_t, is defined as the difference between their level of housing consumption and that of recent movers. This assumption is suspect, for it is reasonable to suppose that recent movers plan for the future; hence q_t observed for them is really q_{t+h}^d, where h is the relevant time horizon. Notwithstanding this problem, this new approach is clearly more realistic than the one employing instantaneous adjustment.

These are called the classical urban economic models because they represent pioneering attempts at modeling cities from the economist's vantage point, even though the disequilibrium models are newer than the "new urban economics" reviewed below. Moreover, they are still the most influential in the building of urban economic models. As I will discuss in the next chapter, many of the policy-oriented models derive much of their methodological base from them. These models bear the same relationship to large, economically oriented policy models as do the basic Keynesian macroeconomic models to the large macroeconomic models.

The recent and more theoretical models, which have not been quite so influential, are reviewed in the next section.

The New Urban Economics

This section reviews a spate of urban modeling work that has appeared in recent years and that has already been dubbed "the new urban economics."[17] It is distinguished from earlier work by being more rigorously theoretical, with a higher disregard for reality. The attempt is to explore the extent of possible conclusions from simple formulations. While the work of Wingo, Muth, Kain, Alonso, and Mills is also theoretical, it is rooted in their own extensive empirical work. Little operational relevance can be derived from these new urban economic models, but it is important to review them because of the following:

- They do offer some counterintuitive results. For example, it is optimal to have an unequal distribution of utility even where all households have the same tastes and income.
- They demonstrate again the intrinsic complexity of urban areas. Even highly simplified assumptions often lead to models that do not have analytical solutions.
- The analytical innovation occurs in the use of control theory or the calculus of variations that is in many ways similar to the growth theory literature. Here, space is the crucial variable over which optimization of one kind or another is taking place—such as time in growth theory.

Almost all the models are monocentric. The city is in a flat plain, travel is equally costly in all directions, and all travel occurs between home and work. These assumptions make it possible to use one-dimensional analysis, where distance from the CBD acts as the main spatial variable. The issues explored are the distribution of rent, residential density and space, consequences of travel congestion on city structure and rents, implications of different incomes on people's location pattern, and the ability of competitive markets to sustain optimum city structure. All of these models are static;

17. Edwin S. Mills and J. Mackinnon, "Notes on the New Urban Economics," *Bell Journal of Economics and Management Science,* vol. 4, no. 2 (Autumn 1973), pp. 593–601.

thus, they have no implications for urban growth. The reason for this is probably technical. It is difficult enough to optimize over space; adding time would make the exercise impossible.

Distribution of land rent, population density, and income

Beckmann, in an article that can probably be regarded as the first in the new urban economics, attempted to derive rent, population density, and distribution of income groups as functions of distance.[18] Delson and Montesano have pointed out various errors in Beckmann's analysis.[19] The altered results are reviewed here.

The model assumes a Pareto income distribution:

$$N(m) = Am^{-a}, \tag{48}$$

where m is household income; N is number of households with incomes greater than m; and A, a are positive constants (empirically, a has been found to be in the region of 2). The utility function is:

$$U = C_0 \log q + C_1 \log u + \sum_{i=2}^{n} C_i \log Z_i, \tag{49}$$

where q is amount of land occupied by a household; u is distance from the CBD; and Z_2 to Z_n are all other goods. The problem is to maximize equation (43) subject to the budget constraint:

$$r(u)q + tu + \sum_{i=2}^{n} P_i Z_i = m, \tag{50}$$

where t is daily transport cost per unit distance, and P_i is the price of the ith consumption good.

The general problem is similar to that investigated by Alonso,[20] with the crucial difference that everyone does not have the same income although they do have the same utility function. In his solu-

18. Martin J. Beckmann, "On the Distribution of Urban Rent and Residential Density," *Journal of Economic Theory*, vol. 1, no. 1 (June 1969), pp. 60–67.

19. J. K. Delson, "Correction on the Boundary Conditions in Beckmann's Model on Urban Rent and Residential Density," *Journal of Economic Theory*, vol. 2, no. 3 (September 1970), pp. 314–18; and A. Montesano, "A Restatement of Beckmann's Model on the Distribution of Urban Rent and Residential Density," *Journal of Economic Theory*, vol. 4 (April 1972), pp. 329–54.

20. Alonso, *Location and Land Use*.

tion, Beckmann assumed the CBD to have a radius of 1. Delson's correction asserted that this was an unnecessary assumption. It really does not make much difference if it is regarded as a normalization procedure which makes the radius of the CBD the measure of distance. Montesano provided the correct and complete solution to the problem. First, Beckmann asserted that all households with the same income locate at the same distance from the CBD. The first-order conditions can be used, however, to show only that all households at the same distance have the same income. We have:

$$q = -\frac{tu}{\dfrac{r(u)}{C_0} + u\dfrac{dr}{du}}, \tag{51}$$

where $r(u)$ is only a function of u so q is also only a function of u; hence, a household located at u pays the same rent, occupies the same space, has the same transport cost, and must therefore have the same income. The converse, however, does not follow.

Second, Montesano asserted that the explicit assumption $dm/du > 0$ is needed—that is, that income increases monotonically with distance—in order to solve the model. Montesano called this an assumption and then proceeded to show that second-order conditions for maximization of utility require that this be so. My interpretation is that this is a result of the model and not an assumption. The second-order conditions depend on the form of the utility function; the result, therefore, probably depends on the particular form used.

Montesano showed that the assumption of $t = 0$ (money cost of transport equal to zero) leads to multiple solutions. This can be seen intuitively, as Montesano did at the end of his article after having derived multiple solutions. If $t = 0$, it is possible to derive an expression for the utility function that is dependent only on income; households are then indifferent to location. It is not surprising, therefore, that this assumption leads to multiple solutions. My point here is that it is sometimes useful to look at second-order conditions before tediously solving a model. To the extent that the model tells us something about urban patterns, it can be observed that in developing countries where the money cost of transport (walking to work) is zero for the poor, there may be greater difficulty in "optimum" urban design.

With the assumption of $t > 0$, Montesano obtained a unique solution that showed that (a) $r(u)$ is convex but decreasing less than proportionally; (b) $q(u)$ is increasing (that is, residential density declines with distance); and (c) $y(u)$ is increasing. All of these are expressed in rather complicated functions with no simple interpretations. None of these results are, of course, surprising; what is surprising is the complexity of the analysis, given a straightforward utility function, budget constraint, and description of the income distribution. This does not augur well for the inclusion of income distribution in models of cities in developing countries.

Congestion and transportation

The next group of models is concerned with the optimal allocation of the urban area to transport; the costs of congestion are usually given particular attention. Solow used the standard model: he maximized utility with consumption and housing space as arguments in a logarithmically additive utility function subject to a budget constraint.[21] He derived the declining rent-distance function and concluded that "the rent profile must fall fast enough that those living farther from the center and spending on space a fixed fraction of income after transportation cost occupy more space than those living closer to the center." [22] Solow then introduced congestion in the following way. The aggregate width of the road network at distance u is:

$$2\Pi u(1 - b(u)),$$

where $b(u)$ is proportion of space devoted to housing. Then annual cost of round-trip travel per person-mile at distance u is:

$$C\left(\frac{N(u)}{2\Pi u(1 - b(u))}\right)^D,$$

where $N(u)$ is the number of people living beyond u Total cost per person is then:

$$t(u) = C\,(2\Pi)^{-m} \int_1^u \left(\frac{N(u)}{u(1 - b(u))}\right)^D du, \tag{52}$$

21. R. M. Solow, "Congestion, Density and the Use of Land in Transportation," *Swedish Journal of Economics*, vol. 74, no. 1 (March 1972), pp. 161–73; and "Congestion Cost and the Use of Land for Streets," *Bell Journal of Economics and Management Science*, vol. 4, no. 2 (Autumn 1973), pp 602–18.

22. Solow, "Congestion, Density . . . Transportation."

where the radius of the CBD is normalized as 1. This merely states that congestion costs are proportional to traffic density. This formulation is no different from Mills,[23] (see equation (41)) which itself was taken from Walters.[24] The point here is that this explicitly shows some of the interdependence characteristics of urban areas. $N(u)$, the number of people living beyond u, are dependent on $t(u)$, which itself depends on $N(u)$. Solow solved for the unknown functions $t(u)$ and $N(u)$, but he had to make the following assumptions:

- The same fraction of land area is devoted to housing at every distance: that is, $b(u) = b$.
- The typical person spends half of his total income on housing.

Both are assumptions that deprive the model of its interesting components—in particular the first assumption. Solow and Vickrey addressed this problem in a more simplified context of the long, narrow city.[25] Their conclusion was that a higher proportion of land would be needed for transport near the center of the city in a case in which city size was limited.

Solow solved his model numerically and found one interesting result. The introduction of congestion makes the rent profile more convex: the rent falls sharply as one leaves the CBD and less sharply near the city limits. Mills' model as noted earlier, found a similar result. Solow investigated this result more thoroughly in his later article.[26] Both of the assumptions described above are relaxed and numerical solutions are obtained for different parameter values. The main result is that the addition of roads near the CBD flattens the rent gradient strikingly. Because congestion is greatest near the CBD, the addition of roads there reduces congestion costs, hence transport cost. It therefore flattens the rent gradient because the rent differences are transport cost differences.

Solow finally did some cost-benefit calculations on the allocation of land to roads. In the absence of congestion tolls, market land values reflect differences in private transport cost, not total social

23. Mills, *Studies*.
24. Walters, "The Theory and Measurement."
25. R. M. Solow and W. S. Vickrey, "Land Use in a Long, Narrow City," *Journal of Economic Theory*, vol. 3 (December 1971), pp. 430–47.
26. Solow, "Congestion Cost."

cost. Land values do not fall as fast as they should, and the market rent function lies below the "correct" rent function everywhere. Land is therefore undervalued; if these values were used in cost benefit calculations, too much land would be used for roads.

Although Solow's 1973 article does provide some interesting numerical results, these models do not provide any new insight into urban structure. Their use, perhaps, lies in Solow's pedagogical style, which is a good example of a gradual, step-by-step approach to modelbuilding. Solow himself suggested that the model could be improved by explicit inclusion of housing, in addition to land, as a residential cost; the addition of time cost in transport cost; and the existence of two or more income classes. The inclusion of production functions for transport and housing—that is, a richer specification of the supply side—can be added to these.

More recent analyses of the congestion cost problem have emanated from the Berkeley group, which is heavily influenced by the control engineering approach. Optimizing is done from the social point of view: total costs of some kind are minimized. These are in the form of a citywide integral, so the problem is a fairly straightforward one in the calculus of variations. The solutions are not straightforward, however, and economic implications are often difficult to interpret.

Mills and de Ferranti can be said to have posed this problem first.[27] They concerned themselves with finding the optimum allocation of land to transportation in the suburbs in the presence of congestion. I use the more general formulation of Livesey to illustrate this class of models.[28] The usual circular city assumptions are made, with N people given as working in the CBD. The model itself is rather simple:

$$L_1(u) + L_2(u) = \theta u, \tag{53}$$

where $L_2(u)$ denotes land used for transportation at radius u; $L_1(u)$ is land used for residence in the suburbs and for business in the CBD; and θu is land available. The density of workers in business is constant at a_c, and residential density in the suburbs, a_s, is also constant. Thus, the number of people working at radius u is:

$$N_c(u) = a_c L_1(u) = a_c(\theta u - L_2(u)), \tag{54}$$

27. Edwin S. Mills and D. M. de Ferranti, "Market Choices and Optimum City Size," *American Economic Review*, vol. 61, no. 2 (May 1971), pp. 340–45.

28. D. A. Livesey, "Optimum City Size: A Minimum Congestion Cost Approach," *Journal of Economic Theory*, vol. 6, no. 2 (April 1973), pp. 144–61.

the number residing at u is:

$$N_s(u) = a_s L_1(u) = a_s(\theta u - L_2(u)), \tag{55}$$

and congestion costs are:

$$t(u) = \bar{t} + C\left(\frac{T(u)}{L_2(u)}\right)^D, \tag{56}$$

where $\bar{t}$ is some constant cost, here assumed zero, and $T(u)$ is the number of travellers at u. This is now a familiar formulation.

Land value is given as a constant, R_a, which is the cost of agriculture use regarded as the relevant opportunity cost. Hence total social cost is:

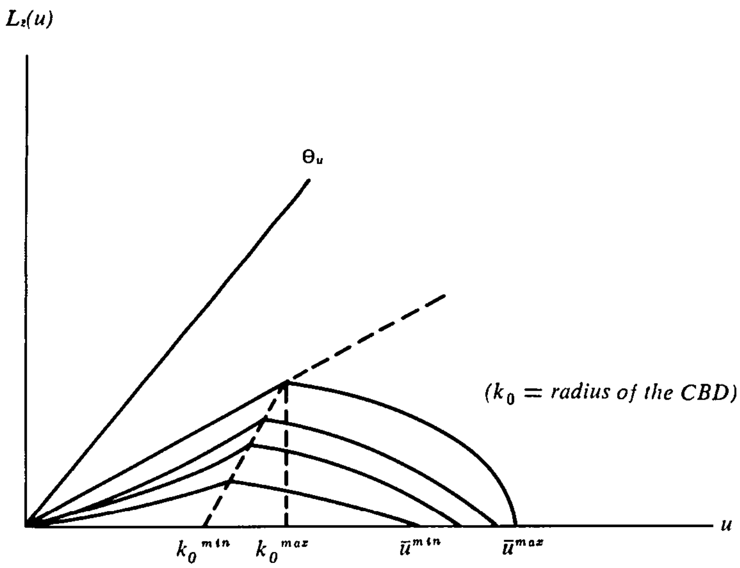
$$\int \{CT(u)C(u)^D + R_a\theta u\}\, du, \tag{57}$$

when congestion cost $C(u) = T(u)/L_2(u)$. This is the integral to be minimized, subject to the given constraints. The problem is first tackled separately for the CBD and suburbs and then in a unified way for both parts of the city. The form of the solution is best seen in a diagram, for the analytical expressions are quite cumbersome and uninformative. Figure 2 shows that the optimal allocation of land for transport in moving from the center to the edge of the CBD is a monotonically increasing concave function and then a monotonically decreasing convex function until the boundary of the city is reached. The maximum is at the edge of the CBD.

This is really quite an uninteresting model, for it has little economic and behavioral content. The opportunity cost of land is regarded as constant, so the model is deprived of any pretense as a serious (though abstract) model of an urban area. It would be consistent if all business activity were spread equally over the city and all employment were local—but there would then be no transport. The assumptions of constant business and residential densities are equally restrictive. The model would clearly be too complex, perhaps unmanageable, if more reasonable assumptions were used. As it is, it should only be called a problem in the calculus of variations, for it does not shed any light on the structure of a city.

A subsequent model from the Berkeley group by Legey, Ripper, and Varaiya extended the analysis by allowing for substitution

Figure 2. Optimal Allocation of Land for Transport in Both the CBD and the Suburbs for a Fixed Working Population (Livesey Model)



between land and capital.[29] Housing and transport are produced by Cobb-Douglas production functions using land and capital. Demand for housing and transport is perfectly inelastic: that is, everyone demands the same amount of each commodity. The total social cost includes capital cost. The interest rate on capital is taken as given. Land value is again constant, reflecting the alternative agricultural value. The sum of transport cost, which have the usual formulation (see equation 56), capital cost, and land cost is minimized for optimality. The solution gives the magnitudes of land and capital devoted to housing and transport and the optimum size of city, given the population. Two solutions are obtained: the optimal solution, corresponding to what a central planning board would do, and a market solution. The central result is that

29. L. Legey, M. Ripper, and P. Varaiya, "Effects of Congestion on the Shape of a City," *Journal of Economic Theory*, vol. 6, no. 2 (April 1973), pp. 162–79.

a market city would be more spread out than an optimal city. If, however, congestion tolls are charged, the market city could be the same size as the optimal one. This model is of somewhat greater interest than the other Berkeley one, even though the demand side is primitive. The criticism of using constant agricultural value holds true again.

Another model inspired by Solow and Vickrey's "long, narrow city" is Kraus' "circular city."[30] It is concerned with questions of optimal land use in an urban environment, with particular emphasis on transport rights of way and the pricing procedures necessary to use them efficiently. This model has no residential sector, and business valuation of sites reflects only travel on the city's roads. Demand for trips between any two units of business area is inelastic. The only costs of roads are the value of the land they cover, and tolls can be levied on all roads without cost. All intersections have signals; these are costless, too. The object is to minimize total transport costs incurred per unit of time in the city. The total area to be allocated to business is given, but the city's radius is to be determined as is the distribution of business area with the circumference. Each trip uses a route which minimizes the cost of the trip taken, which is assumed to include money as well as time costs. Optimization of the signal and toll system minimizes the total cost because trip demands are inelastic.

Radial as well as circumferential traffic is allowed in this model. Each unit area generates a demand of g trips per hour, which are uniformly distributed over all units of business area. All land within a central disc, a circle of radius u_0 concentric with the city's circumference (radius u_1), is allocated to a circumferential inner road.

Let $y(u)$ be area of land devoted to radial roads and $s(u)$ be the area devoted to business within the ring bounded by circles of radius u_0 and u. Now:

$$f_y(u) = \frac{y'(u)}{2\Pi u} \; (y'(u) \geq 0) \text{ and} \tag{58}$$

$$f_s(u) = \frac{s'(u)}{2\Pi u} \; (s'(u) \geq 0). \tag{59}$$

30. Marvin Kraus, "Land Use in a Circular City," *Journal of Economic Theory*, vol. 8, no. 4 (August 1974), pp. 440–57.

These functions characterize the intensity with which land is allocated to alternative uses. There is also:

$$y(u) + z'(u) \leq 0. \tag{60}$$

Let $V_1(u)$ be volume of radial traffic through an arc of radius r and length dr and let $C_1(u)du$ be the capacity of such an arc. Similarly, $V_2(u)$ is the volume of circumferential traffic through a radial segment of infinitesimal length dr at radius r, and $C_2(u)du$ is the capacity of such a segment. The cost per trip mile in direction i, u miles from the center, is:

$$AC_i(u) = f\left(\frac{V_i(u)}{C_i(u)}\right), \qquad i = 1,2 \tag{61}$$

which is the familiar volume divided by capacity type function. Here:

$$\begin{aligned} &f'(\cdot) > 0, \\ &f''(\cdot) > 0, \text{ and} \\ &f(0) > 0; \end{aligned} \tag{62}$$

that is, $f(\cdot)$ is an increasing strictly convex function and is nonzero at zero density, V_0 is the hourly flow of circumferential traffic crossing any radial line segment of the inner ring road, and ring-road hourly travel costs are given by the function $2\Pi G(V_0, U_0)$. Thus, total travel costs for the city per unit time are:

$$2\Pi \int_{u_0}^{u_1} \sum_{i=1}^{2} uv_i(u) AC_i(u) dr + 2\Pi b(V_0, U_0). \tag{63}$$

This is the expression to be minimized.

Further specification of road capacity relates it to land and green time (when signals are green) allocation patterns. Further:

$$P_i(u) = AC_i(u) + T_i(u), \qquad i = 1,2 \tag{64}$$

where $P_i(u)$ is the price of a trip mile and $T_i(u)$ is the toll per trip mile.

Before the problem can be solved, rules for trip patterns are provided. In the solution, analytical expressions are first obtained for $V_1(u)$, $V_2(u)$ and V_0; that is, the traffic follows in each of the directions. The final results for an optimal solution can be summarized as follows:

- In the absence of an inner ring road, any configuration of trip prices inducing travel through the city center leads to explosive travel costs.
- Not surprisingly, toll charges should be the difference between average and marginal costs.
- On every circle, the marginal rates of substitution of land for variable trip costs are equal in the production of radial and circumferential travel. This ensures the optimal allocation of land between radial and circumferential roads.
- A similar condition holds for the allocation of green time to radial and circumferential roads. The marginal rates of substitution of the value of green time for variable trip costs should be equal for travel in both directions.

This model is not noted for its realism either. It is, however, a noteworthy attempt to relax the general assumption that all travel is radial. The treatment of the two directions in travel can be extended to the modeling of different models of travel. This would be particularly important for cities in developing countries where the modes are heterogeneous.

Optimal towns

The last group of models considered in this section are those concerned with deriving the conditions and consequences of "optimal towns." They are somewhat difficult to compare because optimality clearly depends on the welfare function used and the type and extent of constraints in the model. The welfare function reflects the moral or other preferences of the modeler, whereas the constraints constitute his conception of the city.

Mirrlees originated this group of models and his approach was somewhat different from that of others.[31] He posed the problem in almost the simplest form possible. The welfare function is the sum of all individual utilities. Individual utilities depend on consumption of

31. J. A. Mirrlees, "The Optimum Town," *Swedish Journal of Economics*, vol. 74, no. 1 (March 1972), pp. 114–35.

goods, space of residence, and location:

$$U = U(x,q,u), \tag{65}$$

where x is consumption of goods other than housing; q is amount of space used in housing; and u, the distance from the center, is the location variable. This is the same formulation used by Alonso.[32] Further:

$$q = q(u), \quad \text{and} \quad x = x(u) \tag{66}$$

are assumed but are also required for optimality. The main conclusion of the paper is that optimal allocation requires utility to be a function of distance. There is a seemingly surprising result: welfare maximization for identical individuals requires unequal treatment except in special cases. In fact, this is actually not surprising, for u is included explicitly in the utility function. As Mirrlees' model shows, when u is not explicitly included, it is possible to achieve equal utility. The problem arises because identical individuals have to be placed in different locations. They can all have the same utility if some tradeoffs are possible. If, for example, more space is traded off against higher transport costs, equal utility becomes possible; explicit preferences for location, however, have to be dismissed. A richer specification of locational preferences that can be traded would also allow equal utilities. When location is dependent on u alone, only a special case (in which the rent gradient and the utility function are such that the changing consumption of goods and housing are always peculiarly balanced against distance changes) allows equal utilities.

This is an interesting theoretical exercise even though the nature of an urban area is in a very rudimentary form: distance from the center is the all-important variable. Otherwise the problem is straightforward utility maximization (summed to form the social welfare function) with budget constraints. It is of interest because it shows forcefully that:

- Cities imply inequality even if all individuals are identical and if the city is uniform.
- With identical individuals it is possible to mitigate this inequality

32. Alonso, *Location and Land Use.*

only if locational preferences are more complicated than distance from the city center. This implies that the city would then be less homogeneous.

- As a corollary, there can also be an equality of sorts if people have different utility functions; equality is then difficult to define operationally, except by income.

Riley created a similar model but included the number of leisure hours explicitly in the individual's utility function.[33] These vary with transport time, so distance is now a "pure" location preference variable in the utility function. It does not include the disutility of commuting. Riley made the social welfare function more egalitarian by making it logarithmically additive:

$$W = \prod_{i=1}^{N} U^i, \tag{67}$$

where W is total social welfare; U^i is the utility of individual i; and N is the population of the city.

Riley found that utility increases exponentially with distance: that is, optimality requires unequal treatment of identical individuals even with an egalitarian social welfare function. The reason here is not quite the same as in Mirrlees' model because distance is now a "pure" location variable in the utility function, depending on the parameters:

$$\frac{\partial U}{\partial u} > 0 \quad \text{or} \quad \frac{\partial U}{\partial u} < 0 \tag{68}$$

in this problem. Riley explained his result as follows: The fact that an individual can live at only one location (and not at two) causes a nonconvexity that is not present in the usual case of utility and welfare maximization. Here, everyone does have the same marginal utility of income; because everyone cannot have an identical consumption bundle (nor can he have an identical total utility level), however, the degree of inequality is primarily governed by the elasticity of utility with respect to distance. Riley uses:

$$U = x^{\alpha} q^{\beta} h_3^{\gamma} u^{\delta}, \qquad \alpha,\beta,\gamma > 0 \tag{69}$$

33. J. G. Riley, "'Gammaville': An Optimal Town," *Journal of Economic Theory*, vol. 6, no. 5 (1973), pp. 471–82.

where h_3 is number of leisure hours and the other symbols are the usual ones. Inequality is an increasing function of δ. If this is positive, the degree of inequality decreases with β—the elasticity of utility with respect to residence area. If δ is negative, the degree of utility increases with β. Congestion costs are not considered and transport costs are linearly proportional to u in this model.

Dixit and Oron and others had the same concern as Mirrlees and Riley but created more developed models of the urban area. Both use substantively similar models, Dixit following Oron and others.[34]

Dixit's main theme is that optimum city size is determined by the balance between economies of scale in production and diseconomies in transport, congestion being an important part of the latter. The two models can be summarized as follows. Goods are produced with increasing returns to scale:

$$X = AN^{\alpha}L^{\beta}, \tag{70}$$

where $\alpha + \beta > 1$, $0 \leq \alpha$, $\beta \leq 1$ and N is number of man-hours worked. (Oron and others have only labor as input because they fixed CBD size.) Housing and transportation are produced with land only: $L_1(u)$ and $L_2(u)$, respectively, at distance u.

Dixit's two major additions are, first, the inclusion of congestion in the conventional way: that is, a power of traffic density plus some constant as in equation (41). Oron and others assumed that these costs were directly proportional to traffic density. Second, the individual's utility function is:

$$U(u) = x^{\sigma/(1+\sigma)}q^{1/(1+\sigma)}, \tag{71}$$

where $x = x(u)$ and $q = q(u)$ follow the usual symbols. Oron and others assume $\sigma = 1$.

This form of the utility function means that expenditure on goods and housing occurs in the proportion σ:1, with a given income and prices. Dixit's point, which is well taken, is that σ is at least 3.

Workers supply a fixed number of hours devoted to work and commuting. It would be more realistic to assume that work hours are fixed.

34. A. Dixit, "The Optimum Factory Town," *Bell Journal of Economics and Management Science*, vol. 4, no. 2 (Autumn 1973), pp. 637-51; and Y. Oron, D. Pines, and E. Sheshihski, "Optimum U.S. Equilibrium Land Use Pattern and Congestion Toll," ibid., pp. 619–36.

Dixit's social welfare function is:

$$\int_{k_0}^{k_1} - U(u)^{-m} \{-n'(u)\}, \tag{72}$$

where $m > 0$; $n(u)du$ is the number of people between u and $u + du$; k_0 is the radius of the CBD; k_1 is the radius of the city; and m is a parameter which controls the level of inequality in the optimum allocation. A higher m means less inequality and in the limit, $m \rightarrow \infty$ means full equality. Oron and others constrain their model to this case.

Dixit derived analytical functions for traffic density, residential density, and utility level, all in terms of consumption $x(u)$. $x(u)$ is then expressed in terms of u, for which analytical expressions are obtained for the case of pure congestion costs in travel.

$$\text{Residential density} = K(C_1 + C_2u)^{-\left\{1 + \frac{a}{D(a-1)}\right\}}, \tag{73}$$

where K, C_1, C_2 are constants; D is the exponent in the congestion function; and a is $\dfrac{D(1 + m)(1 + \sigma)}{(1 + D)m}$

Equation (73) is a negative exponential form if a equals 1, but this case has no straightforward interpretation. Solow regards equation (73) as the more general form. Dixit's innovations over Mills[35] are in making the land proportion used in housing and transportation endogenous and in using a social welfare function. According to Dixit, a more developed housing production function yields similar results. The rent function:

$$r(u) = \text{const } (C_1 + C_2u)^{-\frac{(1+D)a}{D(a-1)}}, \text{ and} \tag{74}$$

$$\text{traffic density} = \text{const } (C_1 + C_2u)^{-\frac{a}{D(a-1)}} \tag{75}$$

are again negative exponential equations if a equals 1.

Finally:

$$U(u) = \text{const } (C_1 + C_2u)^{-\frac{1}{m(a-1)}}, \tag{76}$$

which says unambiguously that $\dfrac{dU}{du} > 0$: that is, households located

35. Mills, "An Aggregative Model."

farther away have higher utility. The degree of inequality, of course, depends on m. Only $m \rightarrow \infty$ implies equality; hence, more usual values such as $m = 1$ imply considerable inequality.

Dixit's model is important in several respects:

- It is the most developed model of its kind, combining the approaches of Mills and Mirrlees, that is, general equilibrium and optimality.
- More variables are made endogenous than in most models.
- Analytical results are obtained even with congestion.
- The allocation of income between housing and other goods is realistic.

It would be useful to bring capital into a similar model to get a more complete general equilibrium analysis. The curious assumption of fixed leisure hours should be replaced by fixed work hours, but it is difficult to see how such assumptions affect the results.

Dixit did some numerical calculations on his model. He showed the way in which transport cost is crucial in determining optimum city size. Lower transport cost is instrumental in making possible larger cities that allow greater advantage from economies of scale.

Employment Location Models

Most of the models reviewed thus far are concerned primarily with residential location even though they seek to explain urban spatial structure in general. To the extent that they deal with employment location, this is either assumed to be exclusively in the CBD or distributed according to the requirements of housing construction and transportation, as in Mills.[36] Muth investigated wage gradients over space, but his primary concern was with housing. Although a lot more work is required before residential location and housing models can be regarded as satisfactory, enough work has been done to provide some basic understanding of residential and housing patterns in cities. Employment location models, however, are even less advanced; there are really none which can be said to capture the gross employment location patterns. Moreover, much of the work that exists is largely concerned with manufacturing. As

36. Ibid.

pointed out by Gomez-Ibanez, manufacturing employment accounts for only 26 percent of all nonagricultural employment in the United States.[37] There is, however, a considerable amount of work on retail location based on Lakshmanan (1965).[38] This is a particular application of the gravity approach described in the next chapter. The more interesting strands of work on manufacturing employment remain; these are reviewed below.[39]

The paradigm in housing and residential location is a utility-maximizing household; in employment location, the paradigm is a profit-maximizing firm. In general, firms maximize profits according to their production function and spatial variation in costs and revenues. The earliest model is probably that of Weber, originally published in 1929.[4] He assumed that revenues and rents do not vary spatially, production functions have fixed coefficients (that is, no substitution between factors), and transport cost is linear. Isard relaxed some of these assumptions and allowed for substitution between transport and other inputs, as well as for nonlinear transport costs.[41] The optimal location is then found where the transport transformation curve and price line are tangent. Richardson made this general model even more general by allowing for spatial variation in prices and revenues.[42] He does not really have a well-articulated model however; indeed, he asserted that any attempt to construct a general theory of location of nonresidential establishments is bound

37. Jose A. Gomez-Ibanez, "Transportation Policy and Urban Land Use Control," discussion paper D, 75-10 (Cambridge, Mass.: Department of City and Regional Planning, Harvard University, November 1975), p. 35.

38. T. R. Lakshmanan, "A Theoretical and Empirical Analysis of Intraurban Retail Location" (Ph.D. dissertation, Ohio State University, 1965).

39. This section has benefited from the succinct reviews of the literature in Peter Kemper, "The Location of Decisions of Manufacturing Firms within the New York Metropolitan Area" (Ph D. dissertation, Yale University, 1973); and in John E Jackson and Arthur P. Solomon, "Urban and Regional Development: A Critical Review of the Literature" (Cambridge, Mass.: Harvard University, 1977; processed).

40. A. Weber, *The Theory of the Location of Industry* (Chicago: University of Chicago Press, 1965; originally published in German, 1929).

41. W. Isard, *Location and Space Economy: A General Theory Relating to Industrial Location, Market Areas, Land Use Trade and Urban Structure* (New York: The Technology Press of M.I.T., 1956).

42 H. W. Richardson, *Regional Economics: Location Theory, Urban Structure and Regional Change* (New York, Praeger, 1969).

to be unsatisfactory.[43] He found that rent and other factor price surfaces are irregular rather than smooth and therefore concluded there is unlikely to be a unique optimal location for any activity within an urban area. Goldberg arrived at the same conclusion by using the concept of a generalized production function for a profit-maximizing firm. He allowed a tradeoff between economies of scale internal to the firm and agglomeration economies external to it. He found multiple profit-maximizing locations depending on the scale of output desired and the level of external economies.[44]

The profit-maximizing approach has been regarded by others as too narrow in scope. Townroe, for example, pointed out that a location decision should be viewed as only one element in a vector of decisions to be made by a firm.[45] Goldberg's result of multiple optimal locations essentially makes the same point. Others have suggested a catalog of other factors, such as the need for personal (face-to-face) contacts[46] and accessibility to amenities[47] as independent variables which ought to be used in the explanation of employment location. The multiplicity of such suggestions merely points to the intrinsic difficulties of modeling employment location.

Like the equilibrium residential location models, profit-maximizing, optimal location approaches neglect the durability of urban structures in considering the explanatory factors affecting firm location. Unlike the housing and residential location literature in which the historical approach is relatively new, however, the importance of existing land-use patterns and of historical changes in technology has long been recognized in the literature on employment location. This literature seeks to explain changes that occur in existing patterns while taking into account the existence of durable

43. Richardson, *Urban Economics* (Middlesex, England: Penguin Books, 1971), p. 29.

44. Michael A. Goldberg, "An Economic Model of Intrametropolitan Industrial Location," *Journal of Regional Science*, vol. 10, no. 1 (April 1970), pp. 75–79.

45. P. M. Townroe, "Locational Choice and the Individual Firm," *Regional Studies* (April 1969), pp. 15–24.

46. M. Greenhut, *Plant Location in Theory and Practice* (Chapel Hill: University of North Carolina Press, 1956).

47. Zenon S. Malinowski and William N. Kinnard, Jr., *Personal Factors Influencing Small Manufacturing Plant Locations* (Storrs: University of Connecticut, 1961).

capital stock. Hoover took this approach in his early work, as did Allan Pred, who emphasized the changes in technology as the major factor affecting changes in location patterns.[48] He also suggested that different industries should be viewed separately if appropriate explanations are to be obtained. Hoover and Vernon focused on the importance of change in transport technology (from dependence on rail and water transport to roads) to explain the widely observed decentralization of job and industry.[49] Moses and Williamson, mentioned in chapter 1, made much the same point, as did Meyer, Kain, and Wohl.[50] Hoover and Vernon also argued that the introduction of continuous processes requiring large, single-story plants has been a major factor in changing industry location. They also expect that, in time, wage cost and labor considerations will diminish in importance as factors in the location decision.

Given this vast range of ideas about the determinants of employment location, it is not surprising that no tight, self-contained, and satisfactory theories or models exist. The area of such speculation can only be narrowed down with good empirical work that eliminates some of the less important determinants suggested. Indeed, until recently, the lack of progress in this area can be attributed to the lack of appropriate data. As enumerated by Leone, the main data problems have been (*a*) lack of necessary spatial detail on land use within metropolitan areas, (*b*) lack of sufficient establishment detail on employment and production characteristics, and (*c*) lack of time series data that permit changes over time to be traced.[51] Recently, these problems have been somewhat alleviated in the United States because of computerization of country-

48. E. M. Hoover, *The Location of Economic Activity* (New York: McGraw-Hill, 1948); and Allan R. Pred, "The Intrametropolitan Location of American Manufacturers," *Annals of the American Society of Geographers* (1964), pp. 105–80.

49. E. M. Hoover and R. Vernon, *Anatomy of a Metropolis* (Cambridge, Mass.: Harvard University Press, 1959).

50. Leon N. Moses and H. F. Williamson, Jr., "Value of Time, Choice of Models and the Subsidy Issue in Transportation," *Journal of Political Economy*, vol. 71, no. 3 (June 1963), pp. 247–64; and J. R. Meyer, J. F. Kain, and M. Wohl, *The Urban Transportation Problem* (Cambridge, Mass.: Harvard University Press, 1965).

51. Robert A. Leone, "The Role of Data Availability in Intrametropolitan Workplace Location Studies," *Annals of Economic and Social Measurement*, vol. 1, no. 2 (1972), pp. 171–82.

wide data on manufacturing establishments by Dun and Bradstreet. Empirical studies drawing on this data base and others are reviewed briefly below.

Before the Dun and Bradstreet data became available, the microstudies that were conducted used either census data or special survey data. Creamer worked extensively with data from the manufacturing census and traced broad trends from the turn of the century to the present.[52] His was descriptive work with no underlying model, but he confirmed the general trend of decentralization of manufacturing from the central city to the suburbs, along with concentration of office and service activity in the central city. He found further that this decentralization was stronger in the old, established industrial belt in the northeastern United States than in the new industries in the South and Southwest. Kain and Williamson, in separate studies of Chicago, confirmed the trend toward decentralization but found significant employment growth near the airport.[53] Williamson separated the decision to move from the choice of a new location. He ran regressions, separating firms by size, and found significantly different behavior between size classes: smaller firms tend to move shorter distances than larger firms.

Leone and Struyk and James did similar studies; the latter looked at four cities in the United States, while Leone analyzed industrial location in the New York City metropolitan area.[54] They matched the Dun and Bradstreet directories for several years in order to trace the movement of firms over a period of three to four years. Their conclusions were broadly similar. Surprisingly, they found a large amount of movement: about 5 to 10 percent of firms moved every year. The change in employment in nonmovers overshadowed that of the movers, however. Capital-intensive industries tended to

52. D. Creamer, *Manufacturing Employment by Type of Location* (Washington, D.C.: National Industrial Conference Board, 1969).

53. John F. Kain, "The Distribution and Movement of Jobs and Industry," in *The Metropolitan Enigma: Inquiries in the Nature and Dimensions of America's Urban Crisis*, ed J R. Wilson (Cambridge, Mass.: Harvard University Press, 1967); and Harold F. Williamson, Jr., "An Empirical Analysis of the Movement of Manufacturing Firms in the Chicago Area" (Ph D. dissertation, Yale University, 1969).

54. Robert A. Leone, "The Location of Manufacturing Activity in the New York Metropolitan Area" (Ph.D. dissertation, Yale University, 1971); and Raymond J. Struyk and Franklin James, *Intrametropolitan Industrial Location* (Lexington, Mass.: D. C. Heath, 1975).

decentralize, whereas labor-intensive ones that employed low-wage, unskilled labor tended to concentrate in the central city. Struyk and James found that industries tended to cluster together and used shift-and-share analysis in their study. Here the growth of employment in a sector is seen as consisting of two components. One is the national growth rate of the sector; the other is the change of the local share of the national sector. This, however, has not been a particularly successful predictor of the future because it assumes the persistence of past patterns.[55] Struyk and James, and Leone and Struyk in a later study, rejected the incubator hypothesis—as suggested by Hoover and Vernon—that new firms first locate in the center to take advantage of agglomeration economies and move out later.[56] They found instead that new firms are more likely to move within the core than to a suburban ring.

Hamer, Schmenner, and Kemper all suggested a somewhat tighter framework for studying employment location than heretofore used.[57] The paradigm remained the profit-maximizing firm, but each had different concerns. Hamer began with the most general formulation:

$$\Pi j = R_j - C_j, \tag{77}$$

where Πj are profits at location j resulting from R_j revenues and C_j costs, both revenues and costs being location specific for each firm. The firm is therefore seen as a spatial monopolist. Revenue and cost functions can then be drawn for each location and that location and production level which maximizes profits selected. This formulation is too general, and Hamer turned it into a cost minimization problem by assuming that revenue is not specific with regard to location. Further, he assumed that raw materials, business services,

55. See H. James Brown, "Shift and Share Projections of Regional Economic Growth: An Empirical Test," *Journal of Regional Science*, vol. 9, no. 1 (1969), pp. 1–18.

56. Struyk and James, *Intrametropolitan Industrial Location;* Robert Leone and Raymond Struyk, "The Incubator Hypothesis: Evidence from Five SMSAs," *Urban Studies*, vol 13, no. 3 (October 1976), pp. 325–31; and Hoover and Vernon, *Anatomy of a Metropolis*.

57. Andrew M. Hamer, *Industrial Exodus from the Central City* (Lexington, Mass Lexington Books, 1973); Roger Schmenner, "City Taxes and Industry Location" (Ph.D. dissertation, Yale University, 1973); and Peter Kemper, "The Location of Decisions of Manufacturing Firms within the New York Metropolitan Area."

intermediate goods, and transport cost are also invariant with respect to location. Thus, cost minimization is carried out only with respect to land, structure, labor, and public services cost. Hamer concluded from his Boston case study that few factors differ spatially in prices, that substitution possibilities are limited, and that land cost is probably the most significant in the location decision.

Schmenner investigated the effect of differential property and income taxes on industrial location. His model is one of simple regression analysis: he attempted to explain the probability of industry k locating in jurisdiction j by variations in taxes, transport accessibility, and labor force availability. As a proxy for probability he uses:

$$\frac{N_{jk}}{\sum_j N_{jk}}, \tag{78}$$

where N_{jk} is the number of establishments (or labor force) in industry k in jurisdiction j. Alternatively, to investigate the dynamics of change he used changes in location:

$$\frac{\Delta N_{jk}}{\sum_j \Delta N_{jk}} \tag{79}$$

as a measure of probability of change. His explanatory variables were measures of:

Size of jurisdiction	S_j
Property tax differential	T_j
Income tax differential	Y_j
School expenditure differential	P_j
Access to railways	R_j
Access to expressways	H_j
Distance from central city	D_j and
Population density	N_j

where j refers to the jurisdiction. He used Dun and Bradstreet data for four cities in the United States and concluded that (a) property tax differentials have great effects on the location of industries with low profits to capital ratios; (b) income tax differentials have great effects on the location of industries with high profit to capital ratios; (c) highway transport access is highly correlated with the distribution of industry, although it is difficult to disentangle which way the causation flows; and (*d*) space requirements are important in the location decision. He also confirmed the existence of manufacturing density gradients and that access to rail transport is no longer important.

Kemper began with the proposition that firms choose location to maximize profits discounted over time. He made a distinction between the decision of existing firms to change location and that of new firms to find their first location. They choose between alternative locations by comparing the respective advantages of each. For existing firms, the increase in discounted profits should exceed the moving costs; for new firms, start-up costs should be less than discounted profits. Risk and uncertainty are introduced by the consideration of expected discounted profits. As in Hamer, this very general framework is reduced to a manageable one that is susceptible to empirical analysis. Kemper suggested that either the demand for location be estimated by accounting for variation in kinds of firms: that is, estimate probability of firms in, say, one SIC (standard industrial classification) code, choosing a location and explaining it by cost variations over space as done by Schmenner; or, alternatively, the choice of locations with similar characteristics be explained by variations in firm characteristics. Kemper used the latter approach and regarded concentric rings as similar areas. Variations in location characteristics are controlled by running a separate regression for each location; industry characteristics are the explanatory variables. He also used Dun and Bradstreet data, though for the New York City metropolitan area only. He tried various descriptors of location characteristics such as accessibility, availability of inputs, transport, labor costs, and rents. He also tried various measures of industry characteristics such as locally consumed output, exported output, transport requirements, earnings, and employment and growth rates. His estimation model was similar to that of Schmenner:

$$P_i = bX + e, \tag{80}$$

where P_i is the probability of firms choosing location i (that is, the observed proportion of firms in one industry choosing location i); X is the vector of explanatory variables describing industry characteristics; b is the vector of estimated coefficients; and e is the error term.

He then used discriminant analysis to group firms on the basis of the many independent variables to obtain a better idea of the clusters of firm kinds that group in different locations.

His conclusions were not different from others: consumer goods industries, labor-intensive ones, those employing relatively unskilled workers, and those using nonstandardized inputs are all likely to locate in the central city. Large space and heavy transport users tend to locate in suburban rings. He found that the availability of raw material, access to particular transport modes, need for professional or skilled workers, and manufacture of products for local markets was unimportant in the location decision.

Summary

This chapter has reviewed the main strands of the development of urban economic models over the past decade and a half. Little work of this kind existed before this period.

There is a surprising unity of concern among the different models that have been reviewed. Almost all the residential location or housing models investigate optimal residential location—either from the point of view of the household itself or for maximization of social welfare as expressed in a welfare function. Particular attention is paid to the operation of the land market, to the effects of congestion in transport, and, more recently, to the durability of urban structures over time. The literature on employment location looks at location from the viewpoint of the firm seeking to maximize profits. Here, the salient features of these models are brought together to present a better idea of their results.

Muth illustrated how relatively simple economics can be used to understand the structure of urban areas—in particular, the housing market. He maximized utility, which has only housing and other things as arguments; location is not explicitly included in the function. He found that a consequence of market equilibrium is a rent gradient that declines with distance from city center. He also showed how the substitution of capital for land operates to make the housing price gradient much less steep (by an order of magnitude) than the land price gradient. Wingo concentrated on transport and illustrated the complexities of urban modeling by deriving a plausible specification of a transport cost function. He also obtained a decline in rent gradient as a consequence of transport cost. Kain focused on the journey to work as a determinant of residential location but recognized the existence of dispersed employment location. Alonso turned the problem around and emphasized the uniqueness of each urban location. His argument made it invalid to derive aggregated demand functions for urban land. The result of Alonso's argument is that a "pure" location variable must be included in the utility function. The implications of doing this are brought out in later work by Mirrlees, Dixit, and others. They showed that this characteristic of urban land makes inequality inevitable, provided everyone has the same utility function. It can then be inferred that different utility functions would make equality possible.

Mills brought together many strands of work in a general equilibrium model of a city. He concluded that the land-rent profile is crucial to the allocation of activities within a city, that rent declines with distance from the city center, and that factor substitution makes the land-rent profile flatter. He also found that congestion makes the land-rent distance function more convex—a finding corroborated in later work by Solow. Mills demonstrated that even a highly simplified urban general equilibrium model quickly becomes mathematically cumbersome. Beckmann extended earlier work by positing an income distribution function for households and concluded that income increases with distance as had Muth by comparative static analysis along with additional assumptions. Beckmann's work also illustrated how a relatively simple income distribution function makes the mathematics unwieldy. Solow rediscovered the relationship of rent and distance; his contribution, however, is in a lucid exposition of how a step-by-step approach to urban modelbuilding can be followed. In a cost-benefit framework he found that if congestion is neglected, land would be undervalued. This conclusion is taken further by Legey, Ripper, and Varaiya, who concluded that a market city is more spread out unless congestion costs are somehow internalized by the actors in the urban market. Kraus' circular city makes a significant attempt to allow other than radial travel; the resulting model illustrates the costs of adding such simple attributes of reality to a model. Harrison and Kain emphasized the durability of residential and nonresidential capital and the disequilibrium nature of urban growth. In a somewhat different vein, Hanushek and Quigley also focused on the durability of urban structure, but they emphasized the problem of moving costs and the resulting disequilibrium of households from desired equilibrium housing consumption.

Turning to employment location, Weber and Isard posited the profit-maximizing firm searching for an optimal location, but they did not allow spatial variation in costs and revenues except in transport. Richardson allowed for these variations but emphasized their irregular nature. Hoover, Vernon, and Pred stressed the importance of changing technology over time in the explanation of changing industrial location patterns. The empirical work of Leone, Struyk, James, Hamer, Kemper, Schmenner, and others confirms that industrial location has decentralized over time. They all pointed to the need for detailed industry-specific studies for further illumination of the industrial location decision.

Though these models are at a sufficiently theoretical plane, they should be regarded as conceptual building blocks toward more operational models. Each concern exhibited in these models—the tradeoff between land and transport, the uniqueness of location, the effect of transport congestion on city form, the consequences of egalitarian welfare functions, the durability of structures, the complexity of kinds of firm—is a real problem that must first be dealt with at a general level before they can be made operational within policy models.

Higher levels of generality and general equilibrium models as distinguished from particularistic or partial equilibrium models do yield significant insights into urban form. This is precisely what should be expected from highly complex and related phenomena.

3

Operational, or Policy-Oriented, Models

This chapter presents a review of some of the work done on policy-oriented models over the past fifteen years. These models are more difficult to review than the explanatory ones because of their sheer size. In this context it is neither practicable nor beneficial to present all the technical details. Indeed, it is difficult to present these models technically because of the complexity of their notation. Here, my objective is to appreciate the essence of the methodology used in each of these models rather than to achieve a detailed understanding of each. Thus, technical details are provided wherever they are considered necessary for this objective. Symbolic notation is used only when it facilitates exposition.

Most of the models reviewed in this section are large in that they must be operated on a computer. They are spatially disaggregated to a greater or lesser extent, and they allocate activities to geographic zones. They pertain to metropolitan areas, and their concerns are intrametropolitan. Regional models are not considered here.

These policy-oriented models have had a checkered history. They came to prominence in the early 1960s in the United States when concern about the declining central cities was mounting and various planning solutions were being sought. It was thought that these models would help planners in their professional roles as advisers to public decisionmakers and would emphasize objective plan evaluation. It was also expected that they would play an educational role in developing a better theory of urban spatial structure and in giving planners and decisionmakers more systematic ideas of urban areas. In general, these models have not succeeded with respect to the first objective and have been only partially successful with the second. Great disillusionment had set in concerning the use of large-scale, intrametropolitan models in the United States by about 1968. Pack and Pack, however, reported that models are now enjoying a substantial amount of use in regional planning agencies.[1] Intrametropolitan modeling has fared rather better in Great Britain, where American modeling ideas have been adapted and developed into operational models for a number of urban areas. Meanwhile, recent urban modeling in the United States has been done more by economists than others.

Most urban models focus on land use, the kinds of structures erected on the land, the prices of land and structures, the kinds of households that occupy these structures, and the effect of changes in the transport network on this system. Part of production and employment, often called "basic" or "export based," is considered exogenous. The number and kinds of households in the city and their residence locations are derived from the number and kinds of workers and the locations of their workplaces. This is usually the most important part of large, policy-oriented urban models.

Although by no means exhaustive, this review includes some of the major strands of policy-oriented modeling. I begin with the Lowry model because it was the first of its kind and may still be regarded as the high point of fifteen years of policy modeling experience. Some of the prominent American derivatives of the Lowry model are described in order to suggest the progress of modeling since the parent Lowry model. Most of the distributions generated in these derivative models are based on the gravity concept of interaction. The more recent developments in Great Britain (and some in the United States as well) have used the entropy maximization technique to give a better theoretical basis to this concept. The concept of entropy maximization is therefore introduced in

1. Howard Pack and Janet Rothenberg Pack, "Urban Land-Use Models: The Determinants of Adoption and Use," *Policy Sciences*, vol. 8, no. 1 (1977), pp. 79–101; and "The Resurrection of the Urban Development Model," *Policy Analysis*, vol. 3, no. 3 (Summer 1977), pp. 407–27.

elementary fashion to facilitate understanding of the basis of these models.

The Echenique models discussed here probably represent the highest development of the Lowry framework and are representative of many such efforts in Britain and the United States. Moreover, they are of special relevance because they are among the few comprehensive modeling efforts attempted thus far in the developing countries. The NBER (National Bureau of Economic Research) model reviewed is the most ambitious effort based on the use of behavioral relationships—largely economic—that has yet been developed. This model derives much of its rationale from the classical economic models discussed earlier. The last model reviewed is Edwin Mills' policy-oriented planning model, an optimizing model that uses a mixed integer programming framework.

The Lowry Model and Its Derivatives in the United States

This section is drawn from Lowry's work, which remains the best description of the model to date.[2] The model was developed as part of a study of the Pittsburgh region; its purpose was to aid the regional planning effort. The objectives of this model are best stated in Lowry's words:[3]

> The object of this research has been the development of an analytical model capable of assigning urban activities to sub-areas of a bounded region in accordance with those principles of locational interdependence that could be reduced to quantitative form. The model is not designed to project regional aggregates such as total employment or population, but rather to allocate such aggregates to locations within the region. Properly adapted, it should be useful for the projection of future patterns of land development and for the testing of public policies in the fields of transportation planning, land use controls, taxation, and urban renewal.

The model has three sectors: basic, retail, and household. The

2. I. S. Lowry, *Model of Metropolis* (Santa Monica, Calif.: Rand Corporation, 1964).
3. Ibid., p. 2

basic sector is the export sector, in which employment and location are not affected by local events. The location and employment levels for these activities are assumed to be given. The retail sector has local clients whose employment levels and location are closely tied in with access to local residents. The location of households is powerfully influenced by the residents' place of work. In addition, the location and number of households also depend on the location of retail establishments, and vice versa: in other words, they are interdependent. The structure of the model is therefore quite simple. It follows the methods of social physicists more than it does those of economic theorists. Specifically, it seeks to replicate the urban environment by observing statistical regularities rather than by explaining them. The main principle used in the location of retail enterprises and distribution of households is an analogue to Newton's law of gravity: that is, the level of interaction is directly proportional to the mass of interacting bodies and inversely proportional to the distance between them—usually the square of distance.

The model takes as given the amounts and distributions of basic employment and basic land use. From this information it generates appropriate amounts of retail employment and residential population and distributes these employees and households among the delineated zones of the metropolitan area. The city is divided into a grid composed of one-mile squares; these are the smallest zones that the model handled. The model distinguished four types of land use:

$$A_j = A_j^U + A_j^B + A_j^R + A_j^H, \tag{81}$$

where A_j is total area of tract (zone) j; U refers to unusable land; B to area used by the basic sector; R to the retail sector; and H to the household residential sector. A_j, A_j^B, and A_j^U are given, as is the employment provided by A_j^B: that is, E_j^B. Residential land, A_j^H, is regarded as a residual once all the land devoted to employment has been allocated.

The retail sector is divided into n groups of establishments, each of which has an employment (production) function of its own:

$$E^k = a^k N, \tag{82}$$

where N households in the city generate E^k employment in retail establishments of type k:

$$E_j^k = b^k \sum_{i=1}^{n} \frac{C^k N_i}{T_{ij}^k} + d^k E_j. \tag{83}$$

This is in many ways the main part of the model. It is assumed that retail employment of type k in tract j, E_j^k, depends on the number of households in all parts of the city, but is weighted by the accessibility of their respective residence tracts. Workers in a tract j, E_j, however, are assumed to make short-range shopping trips only; hence, they affect the retail employment in that tract alone. T_{ij}^k is a function of distance between tracts i and j, and N_i is the number of households in tract i. a^k, b^k, c^i, d^k are parameters for each type of employment k. C^k and d^k measure the relative importance of homes and workplaces as origins for a particular type of shopping, k. The expression:

$$\frac{c^k N_i}{T_{ij}^k} \tag{84}$$

is an example of the gravity allocation function characteristically used in such models.

Now:

$$E^k = \sum_{j=1}^{n} E_j^k; \tag{85}$$

thus, b^k is adjusted to satisfy this equation.

Finally, total employment in tract j is expressed as:

$$E_j = E_j^B + \sum_{k=1}^{m} E_j^k, \tag{86}$$

that is, the sum of basic and retail employment.

The land, A_j^R, occupied by the retail sector in each tract is then determined through an exogenously specified employment density coefficient e^k for each type of establishment. Thus:

$$A_j^R = \sum_{k=1}^{m} e^k E_j^k. \tag{87}$$

Total population of households is simply a function of total employment in the city:

$$N = f \sum_{j=1}^{n} E_j. \tag{88}$$

The number of households in each tract is determined by another gravity function relating that tract's accessibility to employment opportunities:

(89) $$N_j = g \sum_{i=1}^{n} \frac{E_i}{T_{ij}}.$$

Total population is, of course, the sum of tract populations which determines the scale factor g.

(90) $$N = \sum_{j=1}^{n} N_j.$$

The model then has some constraints to control establishment size and densities. They are:

(91) $$E_j^k \geq Z^k \quad \text{or} \quad E_j^k = 0.$$

The size of type k establishment must be greater than some number Z^k. Thus:

(92) $$N_j \leq Z_j^H A_j^H$$

places a constraint on maximum residential density for each tract (which may vary from tract to tract).

(93) $$A_j^R \leq A_j - A_j^U - A_j^B$$

restrains the amount of land used by retail establishments to that available.

The model satisfies the necessary conditions for solution (the number of unknowns is equal to the number of equations), and a solution method using the constraint inequalities is suggested.

Much of the effort in adapting this skeletal model to operational use was devoted to the specification of the two gravity allocation procedures in equations (83) and (89): the former allocates households and employees to services, and the latter distributes employees to residences. In the classical gravity function the level of interaction is inversely proportional to a function of distance, that is:

(94) $$T_{ij} = f(r_{ij}^x),$$

where r_{ij} is the distance between tracts i and j. Lowry estimated these functions empirically and used:

$$T_{ij} = f(r_{ij}^{1.33})$$

in equation (89) to allocate workers to residences and used:

(95) $$T_{ij}^k = f(a^k - b^k r_{ij} + c^k r_{ij}^2)$$

in equation (83) to generate service employment from household demand. a^k, b^k, and c^k were estimated separately for each type of employment. The only justification offered for these forms is their empirical validity for the fitted area of Pittsburgh. Lowry took care to attach no importance to the specific functions used.

Even a simple model with few behavioral relationships is quite demanding of data and computer capacity. The city was divided into 456 tracts and it had 1.5 million people divided into 448,000 households and 550,000 jobs. Households were regarded as homogeneous. It had five distinct land uses: basic, residential, retail, unusable, and agricultural or vacant. It was found that retail trade had to be clustered into only three types: neighborhood facilities such as food stores and gasoline services; local facilities such as eating and drinking places and medical and health services; and metropolitan facilities with larger versions of local facilities such as department stores and financial services. Almost all manufacturing was regarded as basic. A great amount of data was needed to generate trip distribution functions. Standards for the use of space had to be derived to generate estimates of area demanded by retail employment. Such data are not easily found in developing countries. Furthermore, gross coefficients (such as space standards for different kinds of retail establishments) are difficult to measure because of (*a*) the greater heterogeneity in kinds of technology and (*b*) factor mixes that are characteristically used in different types of retail establishments in a city of a developing country.

The model was successful in generating plausible codistributions of employment and residential population, given its very simple structure and methodological underpinnings. Lowry himself is cautious in making claims about the model's usefulness; in fact, he regards it as a "first generation" effort leading to better work. Its map of the city is filled partly by hand and partly by its own structure. It is not easy to transplant from one environment to another because the structure of the model is sensitive to the data base on which it is built.

Although the Lowry model was considered to have great promise, it has not been possible to use it operationally in many places. Goldner reviewed the aftermath of the model appreciatively but concluded with the hope that future models be more useful.[4] The de-

4. William Goldner, "The Lowry Model Heritage," *Journal of the American Institute of Planners*, vol. 37, no. 2 (March 1971), pp. 100–10.

scendants of Lowry's model observed the dichotomy of basic and retail sectors, the causal chain from basic employment to residential population to retail employment, and the multiplier relationships of all other employment to basic employment. Among the prominent Lowry derivatives have been TOMM (Time-Oriented Metropolitan Model), begun in 1964 under the direction of John P. Crecine for Pittsburgh; PLUM (Projective Land Use Model), developed originally by Goldner and his associates for the San Francisco Bay area; and, most recently, DRAM (Disaggregated Residential Allocation Model), which is an entropy-maximizing variant of PLUM developed by Stephen Putnam. Although these models have been documented fairly extensively, it is not entirely clear how operational they have been. In any case, the innovations or extensions to the Lowry model suggested in them are briefly reviewed below.

The Lowry model created an "instant metropolis," although Lowry argued that, in effect, the iterative nature of the solution technique could be interpreted as approximating the different stages of city growth. TOMM was developed soon after the Lowry model for the Pittsburgh Community Renewal Program.[5] It was regarded as improving on Lowry in realism, and it introduced the concept of short-term stability in urban land uses, location, and structures. Thus, only a portion of households are free to move and only a portion of urban land use can change in any given period. The model therefore needs base-year values of variables and changes them incrementally over time.

TOMM has greater disaggregation than the Lowry model in a number of respects. It allows for several household kinds and two kinds of workers (blue collar and white collar) and uses census tracts as zones instead of the one-mile square grid adopted by Lowry. Household location is made more realistic by incorporating zoning restrictions and density constraints in each tract. The decision rule for household location, which is analogous to the distribution function of equation (89), includes considerations of site amenities such as schools, quality of structures, level of public services, and it assumes that similar households locate together. Thus, the total location-cost evaluation of a household is said to take into account accessibility, value of site and structure, and site amenities. The total supply of housing in a tract is constant, and prices move in

5. See John P. Crecine, "A Dynamic Model of Urban Structure" (Santa Monica, Calif.: Rand Corporation, 1968; processed).

response to supply and demand. The operation of the model may be described as follows.

TOMM starts with base-year exogenous employment totals for each zone as well as the zonal distribution of the number of households by kind. Also included as initial inputs are zoning constraints, amenities of each tract, an accessibility matrix, and the planned changes in zoning and in the population distribution of each household kind for each zone. The model then generates the endogenous (retail) employment, using production coefficients, and distributes them according to accessibility considerations, establishment size constraints, and the like. Residential location changes are generated, and the whole cycle is iterated until everything is consistent within the set tolerance limits. Once again, the key part of the model is the household allocation function. In this case, TOMM allows for different concepts of distance or time of travel between tracts. These are read in as the accessibility matrix. Different functional forms can be used, for example:

$$T_{ij} = f(r_{ij})\,;f(r_{ij}^{x})\,;f\{[\ln(r_{ij})]^{x}\}\,;f[\exp\,(xr_{ij})], \tag{96}$$

using the same notation as shown earlier in Lowry. The appropriate forms for different classes of households, firms, and cities may be different and ought to be estimated empirically. The amenity, site, and structure cost and other effects are empirically determined for each type of household for each tract. Thus, household allocation is a function of up to ten variables.

The idea in TOMM was essentially to take base-year distributions, estimate a host of parameters by multiple regression techniques, and allocate projected citywide changes in employment, zoning, household location, and the like incrementally according to these parameters. Clearly, it is extremely data intensive; for this reason it was never successfully calibrated on real data.[6] Prototype versions were tested on fabricated data, and it became operational only in that genre. Crecine himself states: "Because of incompleteness, incompatibility and nonexistence of most small area data and lack of time series data TOMM remains in the developmental stage."[7]

6. See Stephen H. Putnam, "Urban Land Use and Transportation Models: A State-of-the-Art Summary" (paper presented at the Second Intersociety Conference on Transportation, Denver, Colo., 1973), p. 22

7. Crecine, "A Dynamic Model," p. 1.

PLUM grew out of the Bay Area Simulation Study and was implemented to provide the land-use allocations and small zone forecasts of population, dwelling units, and employment used by the Bay Area Transportation Study Commission.[8] As with other models of the Lowry type, it begins with zonal forecasts of basic employment in a target year and an inventory of base-year economic and demographic data. It then allocates the employment to residents, determines the associated "population-serving" employment and allocates it to residences, calculates housing units and densities, and, finally, projects their figures into the future. It is much less ambitious than TOMM, which may explain why it has been used operationally. Because PLUM exists in various incarnations, only its basic ideas are reviewed here.

Like TOMM, PLUM uses census tracts rather than Lowry's one-mile square grid. It distinguishes employees from households and population, whereas Lowry regarded the number of households as a proxy for population. Unlike Lowry, it does not disaggregate retail employment into different types. It uses different labor force participation and other parameters, however, for different areas to make distinctions between employees who have different tendencies or preferences. PLUM uses travel time between different zones rather than distances; these times are network times estimated from skim trees.

As inputs PLUM needs disaggregated base-year zonal data on residential population, housing units, basic and retail employment, and land use acreage. In addition, it needs target-year projections for total population, employment, basic zonal employment, and interzonal network times. It then generates zonal disaggregated distributions, as mentioned earlier, for the target year and calculates these distributions for all the intervening years by applying comparative statics between base and target years. Intervening-year distributions are essentially interpolations between the base and target years. The incremental version of PLUM, called IPLUM, generates changes from the base year for the following years rather than calculating the target year initially.

8. See Stephen R. Rosenthal, Jack R. Meredith, and William Goldner, *Plan Making with a Computer Model*, 3 vols. (Berkeley, Calif.: University of California, Institute of Transportation and Traffic Engineering, 1972; report prepared for the U.S. Department of Transportation).

Once again, the central part of PLUM is its set of allocation functions. These were modified "in a continuing attempt to make this crucial modeling concept more realistic. Parameters controlling these allocation functions were disaggregated by county so that differing preferences to commute and to shop could be incorporated into the model."[9] For example, nine work-to-home allocation functions were estimated for San Francisco. In Lowry, the work-to-home allocation function (equation 77) was:

$$N_j = g \sum_{i=1}^{n} \frac{E_i}{T_{ij}}. \tag{97}$$

Here:

$$N_j = g \sum_{i=1}^{n} E_i \cdot \frac{1}{T_{ij}} \cdot A_j, \tag{98}$$

where A_j is a measure of attractiveness of zone j and:

$$T_{ij} = f(r_{ij}^2, \exp(\beta/r_{ij})), \tag{99}$$

where β is a parameter. A_j is a function of vacant acreage, the number of housing units and residential acreage in zone j. The parameters in the T_{ij} function were estimated from observed work trip distributions, but the A_j function was not actually estimated.[10]

Putnam improved on this formulation in DRAM by using entropy maximization techniques developed in Great Britain.[11] He argued that the complete T_{ij} function had never been fitted to any actual data and, moreover, not jointly with a consistent estimation of the A_j function, the measure of attractiveness of zone j. Thus, T_{ij} functions have been calibrated with observed trip data without accounting for the characteristics of different residential zones. Conversely, N_j has been calibrated with attractiveness measures but ignores trip distributions. Putnam used St. Paul-Minneapolis data from 1970 to estimate the T_{ij} and A_j functions jointly. His formulation is

9. Ibid., p. 49.

10. This presentation is taken in a modified form from Stephen H. Putnam, "Urban Land Use and Transportation Models: A State-of-the-Art Summary," *Transportation Research*, vol. 9 (1975), pp. 187–202

11. See Stephen H. Putnam, "Calibrating a Disaggregated Residential Allocation Model—DRAM" (paper presented at the Regional Science Association Annual Conference, London, England, 1976).

similar to the one used by Echenique and others,[12] and is discussed after an introduction to entropy maximization techniques.

DRAM is the most developed application in the United States of Lowry's model, but it does not appear to have actually been applied in any planning situation. It is operational in the sense that it has been estimated on actual data and found to be successful in making forecasts of urban form. Putnam himself only claims that its parameters can be adjusted to yield rather close statistical fits to observed data. He is less sanguine about its success in policy simulations.

In summary, the post-Lowry attempts in the United States have largely been toward greater realism in model results. Thus, different models have attempted greater disaggregation of tracts, finer specifications of household and employment types, incorporation of institutional constraints (such as zoning) and of site amenities, and the recognition of inertia in changes in urban form. Most of these developments have had little theoretical structure and have demanded increasing amounts of detailed data. The only one of these models which appears to have been used in actual planning situations is PLUM (and its variants), but the results are not clear. Balancing this bleak impression are Pack and Pack, who found increasing use of urban development models in planning agencies throughout the United States.[13] This increasing use involved regional models rather than the intraurban models reviewed here. The Packs could find no particular determinants of model adoption and use except for the presence of modelers in the agencies involved.

Most of the operational models have emerged in Great Britain, where it is worth noting that the number of tracts used in most models is in the region of 100, as compared with Lowry's 456. Wilson and Batty provided a good review of the theoretical and practical developments in this field of modeling in Great Britain.[14] As reviewed

12. Marcial Echenique and others, "A Disaggregated Model of Urban Spatial Structure: Theoretical Framework," *Environment and Planning*, vol. 6, no. 1 (January–February 1974), pp. 33–63; and "A Disaggregated Model of a Metropolitan Area: Caracas" (Paper read at Socio Economic Systems Design in Urban and Regional Planning—A Colloquium, Waterloo, Ontario, July 1975).

13. Pack and Pack, "Urban Land-Use Models" and "The Resurrection of the Urban Development Model."

14. A. G. Wilson, *Urban and Regional Models in Geography and Planning* (London: John Wiley, 1974); and Michael Batty, "Recent Developments in Land Use Modelling: A Review of British Research," *Urban Studies*, vol. 9, no. 2 (June 1972), pp. 151–77.

above, there was an explosion of modelbuilding based on the Lowry framework in the United States, but it was largely unsuccessful and few models reached the operational stage.[15] It would appear that American disillusionment resulted in large measure from (a) unrealistic expectations about what could be learned quickly from urban simulation models, (b) serious underestimates of the difficulties of constructing truly useful models, and (c) the lack of an appropriate and long-term financial commitment to their development. Because they are tied to policy and planning requirements of particular communities and studies, virtually all efforts to date have had to deal with unrealistic deadlines and other limitations.[16] The British experience could suggest the opposite: there has been too much money available for the development of these models in the United States. Unwieldy models that generally have not succeeded in being operational for policy use have been the result. There were therefore many disasters and the model "movement" died in 1968. Many obituaries and post-mortems have been written, but D. B. Lee's is perhaps the most insightful.[17]

In Britain, however, descendants of the Lowry model only began appearing around 1968. They were developed by Michael Batty and A. G. Wilson under the auspices of the Centre for Environmental Studies in London. Practical work on the models has been proceeding in concert under the direction of Lionel March and Marcial Echenique and their associates at the Centre for Land Use and Built Form Studies in Cambridge. They have been made operational by the Cambridge group in five towns in England—all of which have a population of less than 500,000. In Britain it is conjectured that constraints of time, money, and computer use led to models that were smaller and therefore easier to put into effect. Even so, the documentation does not clearly explain how they were actually

15. Useful reviews of these developments are found in David Kendrick, Numerical Models for Urban Planning," *Swedish Journal of Economics*, vol. 74, no. 1 (March 1972), pp. 45–67; Garry Brewer, *Politicians, Bureaucrats and the Consultant* (New York: Basic Books, 1973); and Goldner, "The Lowry Model Heritage."

16. G. K. Ingram and others, *The N.B.E.R. Urban Simulation Model*, Report No. PB-198-554-555 (Washington, D.C.: National Technical Information Service, 1971).

17. D. B. Lee, *Requiem for Large Scale Models* (Berkeley: Institute for Urban and Regional Studies, University of California, April 1972).

used for policy purposes. Batty estimated that total resources expended in urban modeling in the last eight years in Britain have not amounted to more than US$600,000.[18] Between twenty and thirty models have been developed during this time. If this estimate is correct, the achievements have been truly remarkable.

The Echenique group has gone on to develop models in Latin America, starting with Santiago and Caracas, and is now developing one in São Paulo. These models, extensions of their work in the five towns in Britain, are essentially Lowry derivatives using Wilson's entropy maximization formalism. The Santiago and Caracas models are reviewed here in some detail, but first a simple introduction to entropy maximization techniques will be useful.

Entropy Maximization

A. G. Wilson introduced the concept of entropy maximization to urban modeling in the context of Lowry-type models. He wanted to find a better theoretical basis for the use of gravity models in distribution and allocation. He explained and developed this in his 1970 publication.[19] The gravity model is based on a Newtonian analogy. A characteristic function of urban models is to find levels of interaction between spatially separated zones, such as the pattern of movements of people or goods between zones. The gravity model posits the interaction between zones i and j as proportional to each of a mass at i, a mass at j, and inversely proportional to some function of the distance (or travel cost) between them—such as equation (72)—in the Lowry model. The gravity analogy deals in aggregates and the formulation is deterministic. Entropy maximization deals directly with the components of the system of interest and obtains interactions as statistical averages. If there is interest in the journey to work, the gravity model takes a residential population in zone i and jobs in zone j as its masses; the entropy-maximizing method deals with individuals, assesses their probability of making a journey,

18. Michael Batty, "In Defense of Urban Modelling," *Journal of the Royal Town Planning Institute,"* vol. 61, no. 5 (May 1975), pp. 184–87.

19. A. G. Wilson, *Entropy in Urban and Regional Modeling* (London: Pion Ltd., 1970).

and obtains statistical averages. This formulation is therefore probabilistic.

My purpose here is to give an intuitive understanding of the entropy maximizing method and its role in urban modeling. Before this can be done, entropy itself needs to be defined and purged of its thermodynamic connotations.

Entropy is a precise mathematical concept measuring the amount of uncertainty represented by a probability distribution. Nothing more nor less can be read into the concept. Given a probability distribution:

$$p_1, p_2, \ldots\ldots\ldots\ldots p_n$$

associated with a random variable,

$$x_1, x_2, \ldots\ldots\ldots\ldots x_n,$$

then:

$$S = -\sum_{i=1}^{n} p_i \ln p_i \tag{100}$$

is the entropy of the system. It is a unique measure of the amount of uncertainty in the given distribution. It measures how uniform a distribution is. Intuition or common sense suggests that a uniform distribution has a large amount of uncertainty. Further, the higher the number of possible states (the larger the n) the more uncertainty there is. The probability of any state occurring is:

$$\frac{1}{n}, \text{ that is, } p_i = \frac{1}{n}. \tag{101}$$

Entropy should therefore be a monotonically increasing function of n. Equation (100) obeys this rule as well as some other conditions that make it a unique, unambiguous measure. Any change toward equalizing the different probabilities will increase the entropy.

Maximizing entropy means that it is desirable to make the distribution as uniform as possible, subject to whatever constraints exist. The most familiar use of entropy is in thermodynamics. The characteristic problem there is that some average level of energy, say $\bar{E}_i$, in a system is known; there are many different quantum levels E_i and probabilities need to be assigned to each of these quantum levels. This problem is solved by maximizing entropy subject to the condition that the expected value of E_i is $\bar{E}_i$. The solution to this constrained maximization problem assigns these probabilities to each quantum level. What the method achieves is that, given the mean value, it provides the most uniform distribution possible.

Another example presents a more intuitive understanding of the entropy-maximization technique. Suppose there are a thousand cars parked bumper to bumper and they occupy the full length of, say, three miles. The total weight of these thousand cars is known. The length and weight of each make of car that may be in this cluster of cars can also be obtained. The problem is: Given only this information, can any inferences be made about the number of cars of each make that are in this cluster? This can be converted into an entropy-maximization problem, and the solution will yield the most likely distribution of makes. If n_i is the number of cars of make i, and N is the total number of cars, then the distribution of:

$$\frac{n_i}{N}, i = 1, \ldots m, \tag{102}$$

where there are m makes, is the distribution that is sought. The total weight and length of all cars and the weight and length of each make is the information that is provided. This information comprises the constraints to the entropy-maximization problem.[20]

The function of the technique, therefore, is to provide the most plausible, unbiased distribution, given rather sketchy information. It ensures that no other information or bias, except that subsumed in the constraint set, occurs in the predicted distribution.

In the context of urban systems, the use of entropy maximization is well illustrated in relation to a system dealing with movements of people during, for example, the journey to work. A state of the system is defined as an assignment of individual persons to the movement channels in the system such that it does not violate any of the constraints on movements. A distribution of the system is a macroproperty of the system: a distribution of total movements regardless of the movements of individual persons. There are three levels of resolution in this analysis. First, the micro-state is the assignment of each individual to particular work trip categories. Second, the mesostate is the number of individuals going from each origin i to each

20. This method of presentation is taken from E. T. Jaynes, *Probability Theory in Science and Engineering* (Dallas: Feld Research Laboratory, 1958).

destination j. Third, there is the macro-state that describes only the number of people working in each destination j and living in each origin i. Many combinations of micro-states can give rise to the same meso-state; many combinations of meso-states can give rise to the same macro-state. This problem is rather similar to the car problem described above. Given a macro-state, entropy maximization can be used to find the most likely distribution of meso-states; given a meso-state, the most likely micro-states can be predicted. Many states of the system can form one distribution. So, on the assumption that all the states are equally probable, the model is based upon the most probable distribution of person movements subject to any constraints.

Generating a journey-to-work distribution through the use of entropy maximization amounts to saying: If the population of a city is asked repeatedly to choose work and residence locations aimlessly (or randomly), though subject to some constraints, the distribution that results most frequently is the distribution that entropy maximization provides. The constraint set can, of course, contain information that divides people according to such things as income classes, race, and kind of employment. This is analogous to the information on length and weight of each make in the car example. The journey-to-work distribution that is obtained is, in this sense, not totally random. Beyond the constraint set, however, the randomness prevails. It is this implication of the entropy maximization technique that is considered the most problematical in the context of urban modeling. Even within such classifications as employment, income, and race, people do not locate themselves aimlessly but instead through the operation of some preference functions (implicit though they may be) and are subject to the external market forces as expressed by prevailing prices. If all this information can be included in the constraint set of the entropy-maximizing procedure, the same result is obtained as with utility maximization. That this is unlikely will now be demonstrated mathematically. In any case, in practice thus far the constraint set has seldom included information from any preference functions. Even the market information included has been rudimentary. Thus, characteristically, entropy-maximization models that use mean values to generate various location distributions can scarcely be regarded as having any predictive value.

Mathematically, entropy maximization does the following. Given a random variable:

$$x_1, x_2, \ldots\ldots x_n, \quad \text{with probability:}$$

$$p_1, p_2, \ldots\ldots p_n,$$

entropy is maximized as follows:

$$S = -\sum_i p_i \ln p_i, \quad \text{subject to} \tag{103}$$

$$\sum p_i f(x_i) = E(f[x]) \quad \text{and} \tag{104}$$

$$\sum p_i = 1 \tag{105}$$

This gives:

$$P_i = \exp(-\lambda - \mu f[x_i]) \quad \text{and} \tag{106}$$

$$e\lambda = \sum \exp(-\mu f[x_i]) \text{ (because } p_i = 1\text{)}, \tag{107}$$

where λ and μ are the Lagrange multipliers.

Therefore, if averages are known, trip distributions can be generated. This is the basic entropy-maximization procedure. Clearly, the number of constraints can be increased to include more information.

The following demonstrates the conditions in which a utility maximizing technique gives similar results. This treatment follows Wilson.[21] In the standard utility maximizing problem, maximize:

$$U = U(x_1, x_2, \ldots\ldots\ldots x_n, M), \quad \text{subject to} \tag{108}$$

$$x_i p_i = M, \tag{109}$$

where U is the utility of the consumer; x_i are amounts of goods consumed; p_i are prices; and M is the income of the consumer. To obtain a solution, U is maximized, first order conditions are obtained, the resulting equation system is solved, and then the optimal quantities are found. Now, if by definition:

$$y_i = \frac{x_i p_i}{M}, \tag{110}$$

then y_i can be a probability distribution. The result is:

$$S = -\sum y_i \ln y_i. \tag{111}$$

21. Wilson, *Entropy in Urban and Regional Modeling.*

We can also write and maximize:

$$U = U\left(\frac{y_1 M}{p_1}, \frac{y_2 M}{p_2}, \ldots, \frac{y_n M}{p_n}, M\right) \quad \text{subject to} \tag{112}$$

$$\sum_i y_i = 1 \tag{113}$$

and find a solution:

$$y_i = y_i\,(p_1, p_2 \ldots, p_n, M), \tag{114}$$

which defines the same system as the standard utility maximization system.

Now maximize:

$$S = -\sum y_i \ln y_i \quad \text{subject to} \tag{115}$$

$$f_k\,(y_1, y_2, \ldots, y_n) = g_k \quad \text{and} \tag{116}$$

$$(k = 1, \ldots m)$$

$$y_i = 1. \tag{117}$$

Wilson suggested that, under certain conditions, maximizing S is equivalent to maximizing:

$$U = S + \sum \mu_k(g_k - f_k) \quad \text{subject to} \tag{118}$$

$$\sum y_i = 1, \tag{119}$$

if μ_k can be regarded as the Lagrange multipliers of the earlier maximizing S problem.

If entropy S plays no role in the utility function, this will exhibit itself in one of two ways: either (a) the parameters μ_k will be large compared with S, thereby reducing S to insignificance; or (b) there will be as many constraints (components of the utility function) as there are variables y_i, in which case the set of constraint equations can be solved directly for the y_is without reference to entropy.

What is really being said here is that if all the information in a utility function is put into constraints in the entropy maximizing problem, the solution will be the same. Specifically, if a utility function orders behavior in a rather deterministic fashion and this can be transformed into a constraint set, there will be little allowance for uncertainty in the system. Hence, maximizing a measure of uncertainty (entropy) would make no difference, and the same answer would be obtained. The argument is this: If the utility function does not have this kind of information, then uncertainty should be taken into account. Entropy maximization does this, whereas utility maximization does not. Therefore, the former is the more general procedure.

The problem is that it may not always be possible to transform the information in a utility function into a usable constraint set for entropy maximization. In fact, if this can be done, there would be no difference in the solution and no meaningful choice. But maximizing entropy in a system that is completely determined by the constraint set serves no purpose. Thus, the correspondence between utility maximization and entropy maximization is illusory: it happens only when entropy maximization loses its meaning.

It is hoped that the explanation above provides an intuitive as well as partial mathematical understanding of the entropy maximization approach. It is intended to give some idea of the role of mean values and of the various parameters that are used in distributions in the Echenique models that are reviewed next. Some basis is also provided for the methodological criticism offered for these models.

The Echenique Models

Some of the principal attempts to apply modeling techniques to cities in developing countries have been made by Marcial Echenique and his associates in cities in Latin America.[22] They have built and calibrated models for Santiago and Caracas and, more recently, for São Paulo. These models are essentially built around a Lowry framework, but the theoretical justification is provided by entropy-maximization–distribution techniques—although each model has its special characteristics and objectives. The Caracas and Santiago

22. This section reviews material in Marcial Echenique, "Urban Development Models: Fifteen Years of Experience" (paper read at the International Conference on Urban Development Models, Cambridge, England, 1975); and "The Applications of Urban Models in South America: The Case of Santiago and Caracas" (paper read at Socio-Economic Systems Design in Urban and Regional Planning—A Symposium, Waterloo, Ontario, July 1975); as well as in Echenique and others, "A Disaggregated Model of Urban Spatial Structure: Theoretical Framework," and "A Disaggregated Model of a Metropolitan Area: Caracas."

models are described below with some care, though technical detail is kept at a minimum.

These models are extensions of the urban stock and activity model developed at Cambridge, which had been applied to New Towns in Britain.[23] The Santiago model expands the original framework by also modeling the inputs to the urban model. This is done by coupling the urban model with a regional model and a detailed transport model. The regional model gives the changes in employment by economic sectors and the associated population change for every time period. The transport model interacts with the stock and activity model and determines the level of accessibility in each zone by a detailed assignment of vehicles to the networks.

The Caracas model attempts to disaggregate the urban model further in order to explore policies related to income distribution and squatter housing. To do this, a rudimentary economic framework is used to model the land market. As in the Lowry model, the socioeconomic groups and income groups of employees are calculated, given the location of basic employment. The income of workers determines the kind of housing and transport that they can afford. They are then distributed to different residential locations according to the operation of a simplified land market, and rents and land values are established as a result of competition between different land uses.

The Santiago model

The structure of the model and a list of inputs and outputs are illustrated in figure 3 (pages 96–97).

The regional model predicts changes in employment by region and sector, while the demographic model translates these into population changes, taking into account demographic variables such as birthrates and deathrates, migration, and the demand for labor in each region. Employment is regarded as proportional to the amount of investment, which is exogenous. In this case only public sector investment is taken into account. Thus, the model assumes a host of multiplier relationships between public sector investment and employment in order to produce the required output to be fed

23. Echenique and others, "A Disaggregated Model of Urban Spatial Structure."

into the intraurban model. The intraurban model is divided into various parts.

The basic employment model distributes the total basic employment given by the regional model through zones and employment sectors. Agricultural employment on the periphery of the city is determined by the agricultural land available, fertility of the soil, and accessibility to consuming zones. It is also subject to maximum density constraints. Similarly, industrial employment is a function of industrial land available in each zone and of the relative accessibility of zones. An additional parameter is introduced to simulate the existing cluster behavior of industry. Administrative and finance employment is merely a function of urban accessibility. Finally, some service employment generated by the rural (agriculture and mining) population is also regarded as basic.

The urban stock and activity model is really the core of the whole exercise. It takes into account the spatial characteristics of the city in order to distribute the population to residential areas and to generate service employment. It takes as inputs the amount of land available for development in each zone, the transport network, the total amount of floor space per employee, and basic employment space standards. The model first distributes the existing stock of floor space, then the location of residential activity, and, finally, the location of service employment generated by the residential activity. This process has to be iterated until the service employment reaches the total given by the regional model. These distributions are generated by entropy-maximization techniques.

The transport model simulates the travel activities of the city, given the distribution of activities and the transport network. Because the distribution of activities depends on the transport network and vice versa, the transport model is linked to the stock and activity model and is iterated to simulate the changes resulting from changes in either. The model has four stages. First, it gives the total number of trips emanating from, and coming to, each zone. Second, it provides the distribution of T_{ij}^k, the number of trips from zone i to zone j distinguished by mode k. The model allows for three modes: bus, motor car, and pedestrian. Third, trips are assigned through the network, defined as a set of modes and links. Each mode has a different network: buses follow preestablished routes, cars avoid congestion, while pedestrians follow the shortest routes because congestion is not allowed for them. Fourth, costs are assigned to each of the modes and routes, fed back into the second step, and

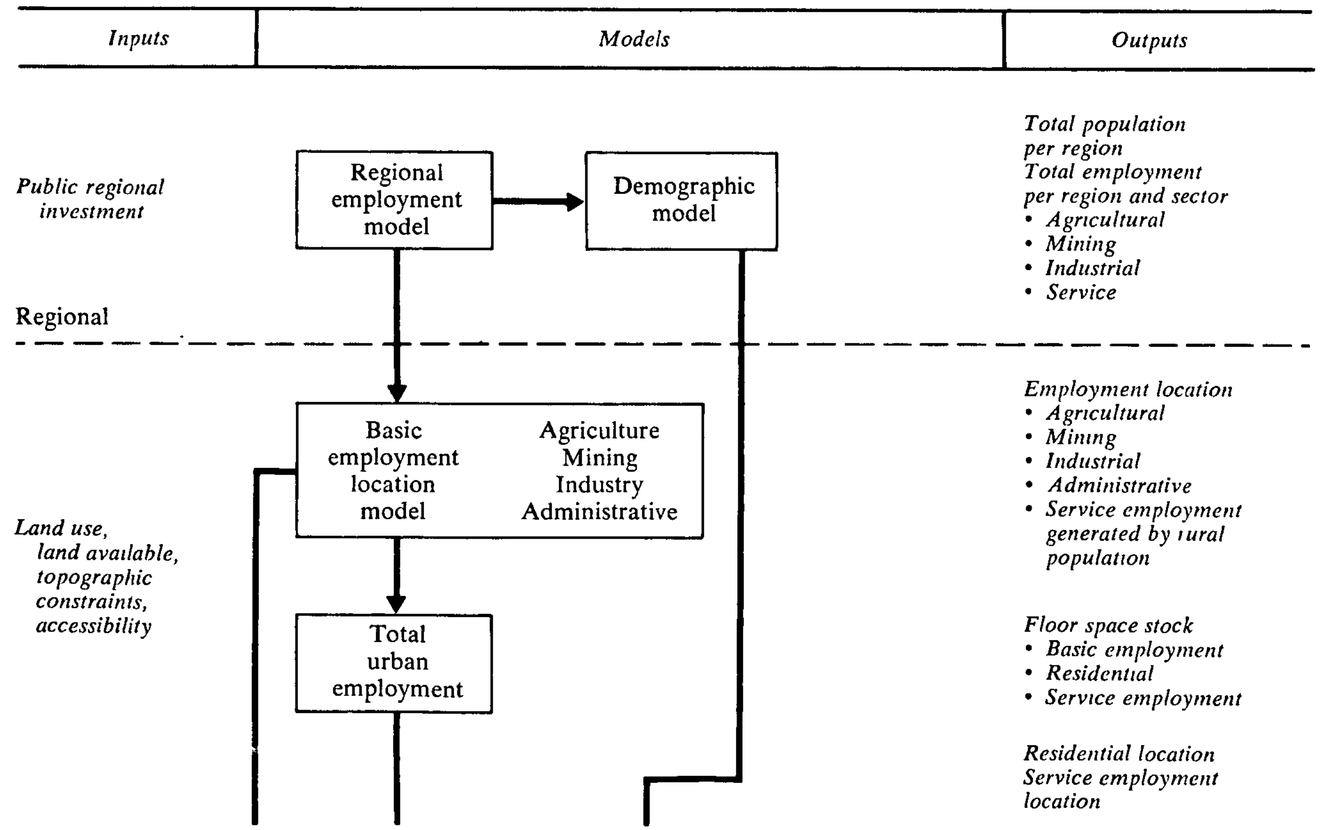
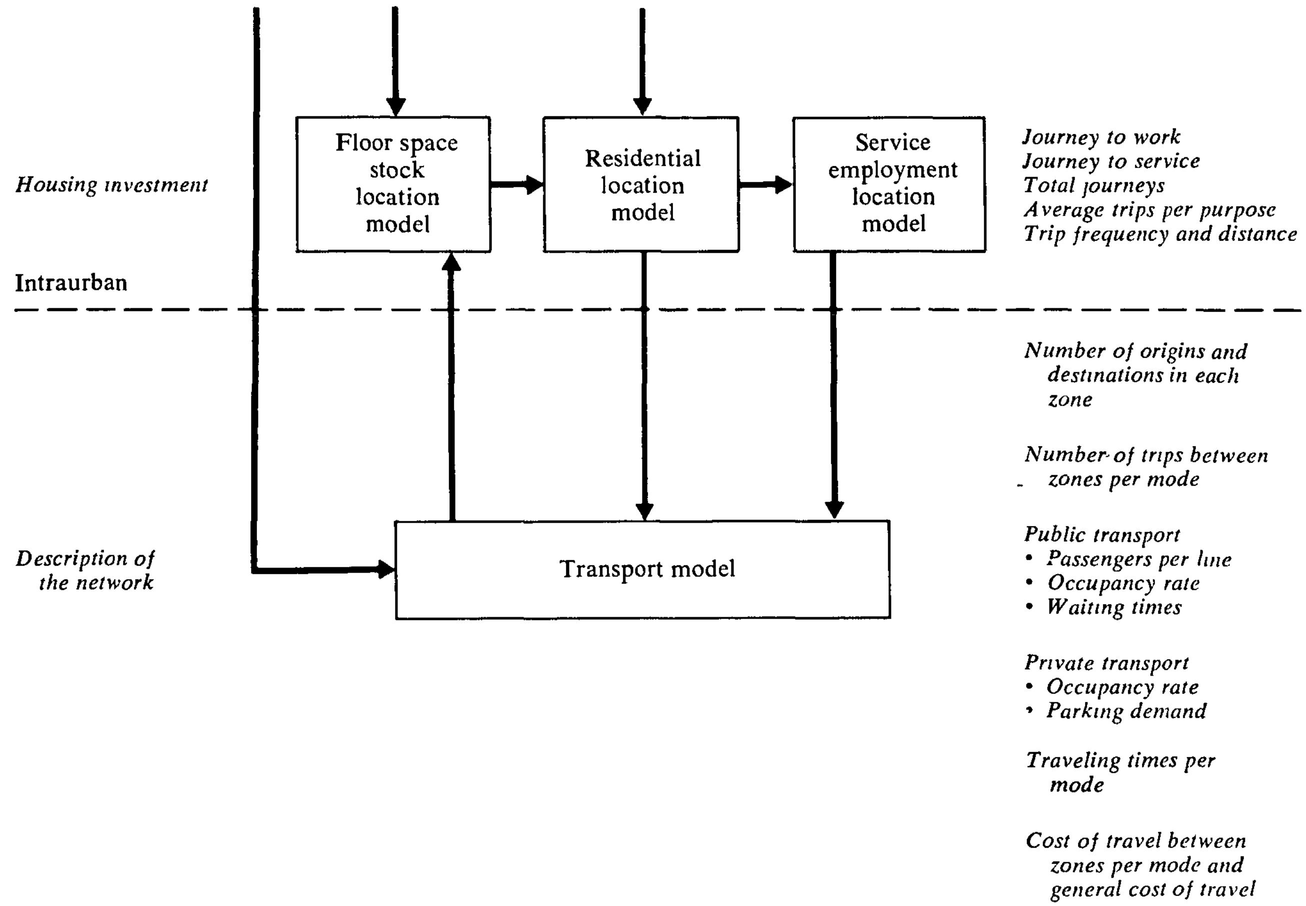


Figure 3. Structure of the Echenique Santiago Model


Housing investment
Floor space stock location model
Residential location model
Service employment location model
Journey to work
Journey to service
Total journeys
Average trips per purpose
Trip frequency and distance
Intraurban
Number of origins and destinations in each zone
Number of trips between zones per mode
Public transport
• Passengers per line
• Occupancy rate
• Waiting times
Private transport
• Occupancy rate
• Parking demand
Traveling times per mode
Cost of travel between zones per mode and general cost of travel
Description of the network
Transport model

iterated until they converge. The output of the transport model is essentially the interzonal accessibility matrix, which is fed back into the activity stock and location model; the whole model is then iterated until convergence occurs.

This modelbuilding exercise had to be abandoned after the governmental changes in Chile in 1973. Consequently, little work was done on it following calibration. Work on the model started in 1970; it was finally operational in 1973.

The simulation was done over an irregular pattern of sixty zones. Data were obtained from a variety of sources: population from the 1970 national survey, employment from 1969 and 1972 origin-destination surveys, floor space from the taxation office, and roads from the Ministry of Public Works and Transport.

All information was made compatible for the use of 1970 as the base year. The regional model was run to simulate the period 1965–70. It was run under three different investment assumptions for prediction up to the year 2000: first, continuation of past investment patterns; second, no investment in Santiago; and, third, no investment in service activity in Santiago.

The results of simulations compared well with actual data for the relevant years. There is, however, no evidence nor information about the efficiency of the model as a predictive device. Because the model was calibrated in 1970, it is not surprising that it simulated the past well.

The Caracas model: a disaggregated model of a metropolitan area

This model is said to combine the macro-scale, or social physics, method with the micro-scale, or economic, method. It attempts to simulate the interaction of supply and demand in the land market. Its basic structure is that of the Lowry model; that is, the location of basic employment (manufacturing, government, and agriculture) is taken as given; employees are then distributed to their residential places; they then generate demand for services, which in turn generates more employment and more services; and the model iterates until equilibrium is reached. This model, however, has a greater degree of disaggregation than others. It distinguishes between various kinds of employment, groups employees by socioeconomic groups and income levels, and allows for two types of housing and transport modes. It consists of five submodels.

First, the employment submodel determines the socioeconomic group according to kind of employment. Each socioeconomic group has an income distribution that determines the kind of housing the employees can afford as well as the transport they use. The model makes distinctions between service, government, industry, and agriculture as sectors of employment. Employees are then grouped into five socioeconomic groups: managerial, professional and technical, clerical, manual, and agricultural workers.

Now:

$$E_i^{xy} = E_i^x p^{E(xy)},$$

where E_i^{xy} is the number of employees working in zone i in employment sector x and belonging to socioeconomic group y; $p^{E(xy)}$ is merely the proportion of employees in sector x who are in socioeconomic group y—a number that is taken from observed data. Thus, E_i^{xy} is obtained, given E_i^x, the number of employees in zone i working in sector x.

The income distribution of each group is then generated, given the mean income of employees in each group y and a parameter of distribution for each group. The distribution is done over five income ranges. The next allocation is of employees in calculated income ranges to kinds of housing and transport. E_i^{zho} is the number of employees working in zone i in income group z living in house h and using transport o. People can live in two kinds of housing: normal or squatter. Transport kinds distinguish between car owners and nonowners. This calculation is done with a knowledge of the mean income of households living in housing kind h and of households using transport kind o. Note that these are all distributions that are generated from known mean values and are analogous to the travel or car example given in the section on entropy maximization. The parameters of distribution are calibrated to approximate the distribution with actual data.

Second, the land submodel distributes land for different uses depending on rent-paying ability and the total supply of land. The model calculates L_j^g, the quantity of land used by activity g in each zone j. The supply of land is considered as a function of total land available in the zone, L_j, and the demand by each activity. The activities are of six kinds: the four employment sectors and the two kinds of housing. Once again this allocation is done from a knowledge of various mean values and the calibration of various parameters. The required data include (a) land standards, which are the average amount of land required by each activity; (b) level of each activity in each zone; (c) mean rent-paying ability of each activity in each zone; and (d) amortization rate of expenditure in land according to activity.

The total land value in each zone is calculated as the sum of all expenditures on land by each of the six activities. Because land values are generated from observed means, this submodel is essentially an accounting device that provides one of the inputs to the calculation of location cost of each activity. It also keeps the land accounts in order.

Third, the residential location submodel determines E_{ij}^{zhok}, the likely distribution of employees working in zone i, living in zone j, belonging to income group z, housing kind h, transport kind o, and using transport mode k. As in other models, this may be regarded as the core of the model.

E_{ij}^{zhok} corresponds, though it is more disaggregated, to N_j, the number of households in each tract j in equation (77) in the Lowry model and in equation (98) in PLUM. In this case:

$$E_{ij}^{zhok} = E_i^{zho} \cdot L_j^h \cdot f\{\exp(-\beta \cdot C_{ij})\}, \tag{120}$$

where E_{ij}^{zho} is the number of workers in zone i, in income group z, in housing type h, and in transport type o; L_j^h is the land in zone j devoted to housing; β is a parameter; and C_{ij} is the location cost of people working in zone i and living in zone j. Note that this formulation is similar to that in PLUM, equation (99). The difference is that $f(\cdot)$ is a more complicated form reflecting the use of entropy maximization and greater disaggregation of households. The location cost variable includes monthly cost of transport to work according to mode, monthly average cost of transport to services according to income groups and mode, and average rent for housing type L in zone j. Each of these location cost components is calculated with the use of various mean values and distribution parameters. Although attractiveness of location zone is treated separately from the transport variable in PLUM, location cost here subsumes some of the attractiveness measures and may therefore be regarded as a more consistent formulation. It was this feature that Putnam incorporated in DRAM.

Once the employees are distributed to residential zones according to income group, housing kind, and mode of transport, the model transforms them into households and then yields the total population in each zone.

Fourth, the transport submodel calculates the transport costs needed for the residential submodel. The cost of the journey to work is merely an average cost per mile for each mode multiplied by the distance and frequency of trips between zone i and j.

The cost of transport to service locations is somewhat more complicated because it has to be calculated according to household location and income groups. Service trips from workplaces also have to be accounted for.

Fifth, the service location submodel calculates:

$$P_{ji}^{sfhok},$$

the number of people living in zone j, traveling to zone i for services, belonging to household income group f, housing type h, having transport type o, and traveling by mode k. The calculation involves a knowledge of mean monthly transport costs to services and the calibration of a distribution parameter. Having calculated the distribution of population making service trips, the model generates:

$$E_i^s,$$

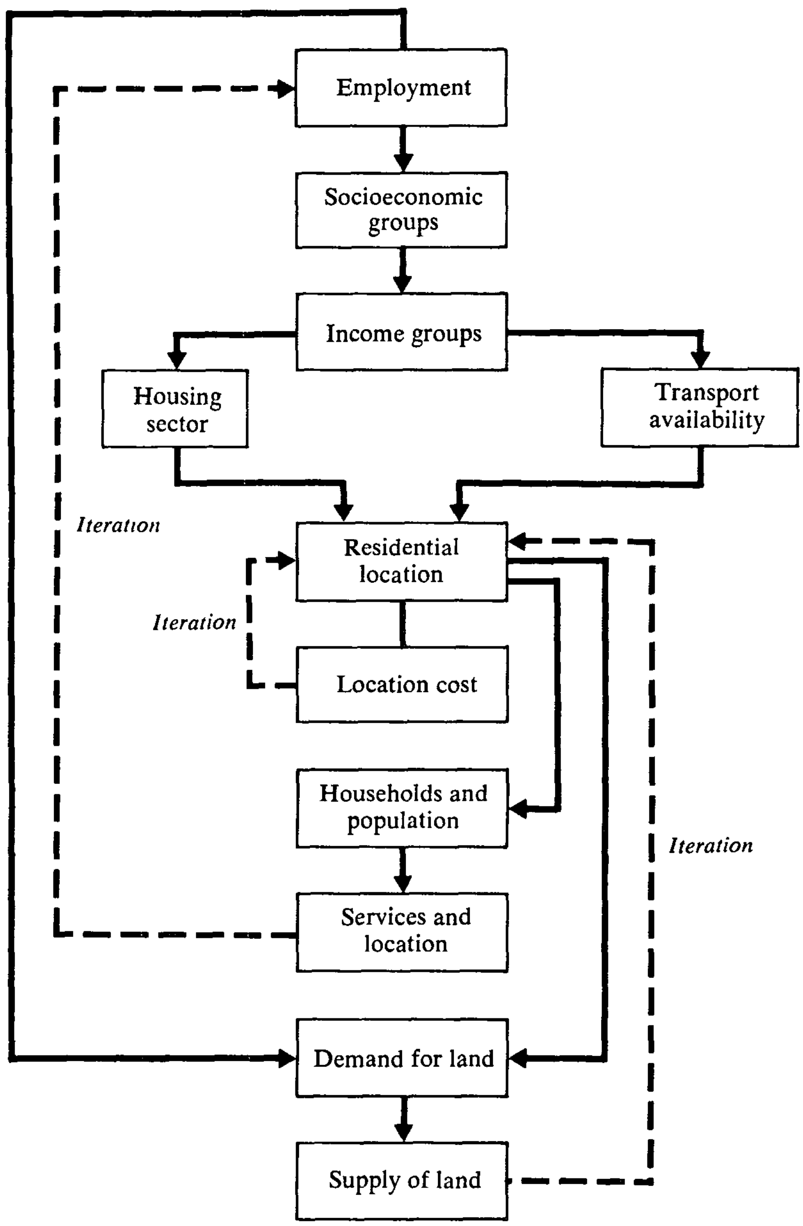
the number of employees required for services in zone i.

SOLUTION. Figure 4 (page 102) illustrates the structure of relations in the model and the iterative process used to solve it. It also illustrates well the central importance of the residential location submodel. A solution of the model requires the following equilibria:

- Employment-population. Given employment, residential population is generated, resulting in a demand for services; hence, service employment is added to original employment and the process iterated until equilibrium is reached.
- Location cost–residential location. This is determined largely by location costs of which one component is rent. The rent is a mean for all residents and therefore depends on the people living in the zone. This process is iterated until equilibrium is reached.
- Residential location–land. The demand for land is generated by mean rent, which determines the amount of land residents are able to buy in competition with other activities. The new land is distributed until there is no further change in the distribution of residents across zones.

The order of solution is as follows: Given employment, first, equilibrium between location cost and residential location is achieved; second, service location and distribution of land is achieved; and,

Figure 4. Caracas Model: Structure of the Iteration Process



third, new increase in employment brings in more residents and the whole process is repeated again and again until full equilibrium is achieved. It is not clear from the summary mathematical statement of the model why the model should be expected to converge to an equilibrium.

APPLICATION TO CARACAS. The city was divided into thirty zones. Service and government employment was concentrated in one zone, industrial employment in three, and agricultural employment in peripheral zones only.

All land with less than 60 percent slope was regarded as available, excluding land for such public purposes as parks, military uses, cemeteries, universities, and roads. The employment structure was found to be 51 percent service, 32 percent manufacturing, 16 percent government and services, and 1 percent agriculture.

Most of the data came from a 5 percent sample survey in 1966 covering some 60,000 people. The data required were of three kinds: basic inputs, coefficients, and output values. The basic inputs are employment by sector in each zone, land available in each zone, and distance matrix for each pair of zones. As was mentioned in the model description, a host of mean values and other coefficients such as land standards, amortization rates, and average transport costs by mode, mean rent, and so forth have to be fed into the model to generate the distributions that the model provides as output. The main outputs provided by the model are E_{ij}^{zhok}, employees working in zone i, living in j, in income group z, living in housing type h, having transport availability o, and using transport mode k; H_j^{fho}, households living in zone j, belonging to income group f, living in housing type h, and having transport availability o; and $C_{ij}^{loczhok}$, cost of location of employees working in zone i and living in j with characteristics z, h, o, k.

Various summations of the above can also be obtained for different variables. Data are therefore required on all these variables to test model output. In addition, transport costs, housing costs, land values, land availability, trip lengths, and the like can also be obtained in disaggregated form.

Simulation of the model yields good results in terms of the closeness of output values to the actual 1966 data.

EVALUATION. The development of the Caracas model was begun in 1968 and was probably finished in late 1973 or early 1974. It is not clear how much it cost nor how many professional man-years were spent on its development. The total financial and professional costs are likely to be substantial, especially if the basic model development cost in Cambridge, England, is included.

The basic criticism of these models is of their model structure and methodology. Despite Wilson's innovative explanation and development of entropy-maximization techniques, their basis for urban simulation remains suspect. The technique essentially takes the mean value of some variable and generates a likely distribution around it, subject to whatever constraints are chosen.

To illustrate, one step in the procedure in the employment submodel is to calculate E_i^{zho}—the number of employees working in zone i, in income group z, living in house type h, and using transport type o. This calculation is carried out by using the value of mean income in income group z, the value of mean income in housing group h, the value of mean income of those in transport group o, and two distribution parameters for distributing housing group h and transport group o. These parameters are calibrated from the base data. Thus, various mean values pertaining to different groups are used along with calibrated distribution parameters to obtain the required E_i^{zho} distribution. A similar procedure is used in all the other distributions obtained in the model. Further, the assumptions underlying various standards such as land standards are essentially those of fixed coefficient-production functions in all activities.

As such, the model merely replicates observed data and really acts as an accounting machine, ensuring that all distributions are consistent. It is therefore not surprising that its simulation is remarkably close to actual values. As Kain and Ingram observed in another context, "At best, such a model can be viewed as a set of reduced form estimates of behavioral structures that are not specified, at worst it is a collection of spurious, accidental or temporary relations between variables. If the latter is the case, the model has no content and could be dangerously misleading in forecasting. If the former is true, then the model is useful as long as the underlying behavioral structure is unaltered."[24] The problem is that it is impossible to judge which is the case. Even if the more optimistic assumption is true, there is no basis for knowing when or for what reasons the unspecified underlying structure changes.

24. John F. Kain and Gregory K. Ingram, "The N.B.E.R. Urban Simulation Model as a Theory of Urban Spatial Structure," in *Urban and Social Economics in Market Planned Economies*, ed. A. A. Brown and others (New York, Praeger, 1974), p. 114.

The usefulness of this model as a policy tool is therefore under serious question. The Caracas model was calibrated for 1966 data. It has not been updated for any later year because of lack of data. If this had been done, there would have been some basis for an informed evaluation. Model simulation for a later year based on the 1966 calibration could have been tested on actual values. On the basis of this test, some faith in the invisible underlying behavioral structure might have been generated. As it is, there is no basis for even this modest assurance, and the lack of post-1966 data merely underscores the difficulty of obtaining data for such a model of a city in a developing country.

The Santiago model never went into actual operation because of the governmental changes in Chile. Because it was developed under governmental sponsorship, it was expected that it would be used for policy purposes. Though it is true that the governmental change in question was of a radical nature, this circumstance nevertheless illustrates an important problem in the use of such a large model for policy purposes. Any good, large model requires time for development as well as for data collection. Three years is almost a minimum for such a model to become operational. Yet policymakers come and go within such periods of time as a matter of routine. Developing countries are particularly prone to rapid change. Models such as the Santiago and Caracas models, which have a rather opaque methodological base, become all the more difficult to explain and sell to successions of competing policymakers. If their bases were less opaque, it might be easier to convince new policymakers of the continuing validity of the models.

It must be pointed out that the Caracas model was developed as an educational device in a university environment. Strictures against its use as a policy tool may therefore be unfair to the authors, although these proffer good advice to potential users.

The basic use of the Echenique models is as forecasting tools for policy purposes. They can provide planners with information on the possible consequences of their actions as well as stimulate thought about new directions. They have no normative content. They do not help in evaluating any consequences. To the extent that the planners have faith in their forecasting structure, the models are clearly useful; to the extent that they do not, the models are unusable. It is then left to the modelers to find ways of defending their model structure and explaining it to planners. This process is easier for models with more comprehensible structures. Models placing greater stress on behavioral relationships can be just as misleading when these relationships are badly estimated, but they are easier to test because these relationships are simpler for the nonmodeler to appreciate or reject. In this respect it is encouraging to note that the Caracas model has as an objective simulation of the land market according to micro-economic concerns, though the actual market simulation is somewhat primitive.

Before concluding this section, it is worth mentioning that Nathaniel Lichfield and Partners have developed similar models somewhat further, as described by Turner.[25] In addition to a few structural modifications, their major extension in the Urban Growth Simulation Model for North Central Texas is the application of evaluative submodels to the output of the main model. These submodels evaluate the effects of alternative policies on the cost of public utilities, air pollution, accessibility to urban resources, and "social deprivation." This is an encouraging development toward making these models more directly relevant for policy concerns.

The NBER Urban Simulation Model

The NBER model is perhaps the most ambitious of all urban modeling to date.[26] It follows a somewhat different family of models pertaining to transport and urban land use. Six of these were built in the late 1950s and through the 1960s for the Puget Sound, Southeastern Wisconsin, Atlanta, Detroit, and the San Francisco Bay area.[27] Some of their unifying characteristics are described below as a prelude to the NBER simulation model.

25 Christopher G. Turner, "The Design of Urban Growth Models for Strategic Land-Use Transportation Studies," *Regional Studies*, vol. 9 (1975); Nathaniel Lichfield and Partners, "Urban Growth Simulation Model." London, 1975. Processed.

26. This section reviews the work described in H. James Brown and others, *Empirical Models of Urban Land Use* (New York: Columbia University Press, 1972); Ingram and others, *The N.B.E.R. Urban Simulation Model* and *The Detroit Prototype of the NBER Urban Simulation Model* (New York: National Bureau for Economic Research, 1972); Kain and Ingram, "The N.B.E R. Urban Simulation Model"; Kain, William C Apgar, and J. Royce Ginn, *Description of the NBER Urban Simulation Model*, vol. 1, Simulation of the Market Effects of Housing Allowances, final report to the U.S. Department of Housing and Urban Development (1976), and Kain and Apgar, *Baseline and Policy Simulations for Pittsburgh and Chicago*, vol. 2, Simulation . . . Housing Allowances

27. They are well reviewed in Brown and others, *Empirical Models of Urban Land Use*.

The objective of these models was to help policymakers in the planning of transport. The models are characterized by the assumption of a unidirectional relationship between land use and transport. Thus, considerable effort is devoted to modeling land use in some detail in order to derive transportation requirements. Regional population and employment forecasts are taken as exogenous. Input-output methods are used to forecast future employment by industry. Retail employment is derived from these forecasts and households are located according to family type. Different models employ various levels of disaggregation for kinds of dwellings, households, and their assignation. These results for projected land use then provide the basis for the design of future transport plans. Some use supply and demand concepts for equilibrating the housing market, taking into account the volume of housing stock and the pattern of filtering with the age of structures. As mentioned in the review of PLUM, the San Francisco models are somewhat different in that they follow the Lowry model structure.

All of these models have heavy data requirements, but they still have few behavioral relationships embedded in them. They are in the genre of mechanistic forecasting models: they find it difficult to cope with technical changes and innovations that affect the structure of cities in a crucial way. Interdependencies are usually modeled in a sequential manner. (The tension between sequential and simultaneous relationships was discussed earlier.)

All of these models were expensive and the major part of the expenses was always devoted to data collection; a relatively small amount was allocated to model development and to analysis of the data collected. The following breakdown of expenses is typical[28]:

	Total cost (millions of dollars)	*Data collection (percent)*	*Analysis and models (percent)*
Atlanta	1.75	36	24
Southeastern Wisconsin	1.99	62	14
San Francisco Bay area	5.54	60	18
Detroit Talus	4.70	46	19
Puget Sound	1.70	—	—

The NBER urban simulation model was embarked on to improve

28. Ibid., p. 94.

on earlier work. The core of the model is firmly based on traditional economic theory with utility-maximizing households and profit-maximizing firms. Its goals, according to its authors, were to (a) enrich economic theory, (b) advance the art of modelbuilding, and (c) evaluate problems of urban growth and decay, evaluate specific problems and policies, and consider broad strategies for dealing with U.S. cities.

This effort has been quite successful in realizing (a) and (b) but somewhat less so in realizing (c), despite the fact that it has now been developed and applied over a period of almost a decade. It would appear to have had adequate financial and intellectual support. The model has now been calibrated in three cities: Detroit, Pittsburgh, and Chicago. A number of policy experiments have also been conducted, primarily the simulation of housing-abandonment phenomena and of housing-allowance programs. The results of these experiments have been largely inconclusive, though many insights into the workings of urban housing markets have been gained in the process.

Before proceeding with further evaluation, it is useful to describe the elements of the model's structure.

The NBER model, in its most recent version, has changed somewhat from the original Detroit prototype but remains similar in its essentials. It incorporates the theoretical approach of the traditional analytic models of residential location and urban spatial structure into a framework with more realistic and less restrictive assumptions. It is more realistic in the following ways.

First, it drops the monocentric assumption of most analytic models and explicitly incorporates multiple workplaces that have a strong influence on choice of residence location. Second, it abandons the long-run equilibrium framework characteristic of analytic models that usually ignore the effects of the durability of capital. The NBER model overcomes this by representing the standing stock of physical capital in the city, changing incrementally, and modeling the supply side of the housing market in some detail. Third, it takes into account externalities such as neighborhood effects and racial discrimination.

It was designed to simulate major changes in urban spatial structure that occur over periods of from ten to fifty years. It simulates effects on the spatial structure of long-term trends in the level and distribution of employment, changes in transport and technology, and increases in income. It provides a description of spatial structure at a point in time and modifies this over a period of years by simulating location and investment decisions of firms, households, and the suppliers of housing.

The model focuses on the behavior of urban housing markets. It represents other urban phenomena, such as industry location and changes in the demographic structure of the population, but the workings of the housing sector remain its central concern. This involves modeling the behavior of housing consumers, suppliers, and the market in some detail. In so doing it is claimed to improve on previous models (most of them of the social-physics variety) that were elaborate statistical descriptions with little or no behavioral content.

The model may be described as having a demand sector, a supply sector, and a market-clearing sector. In its most recent version the activities in these three sectors are carried out in a total of seventeen submodels: eight in the demand sector, six in the supply sector, and three in the market sector.[29] It expands on the Detroit prototype by including more housing bundles and a larger number of more homogeneous zones. It also models tenure choice by households and distinguishes between the demand of households by race and employment status. In its computing technique it maintains a basic list that is a continuous record of each dwelling unit in the sample simulated. The list includes a complete description of the household that occupies the sample dwelling unit. Thus, the model can be seen as essentially a device operating on this basic list, which is the main description of the city modeled. For Pittsburgh and Chicago, the model simulated the behavior of 72,000 and 84,000 households, respectively. The structure of the model is illustrated in figure 5 and that of its iteration process in figure 6 (pages 110 and 111 respectively).

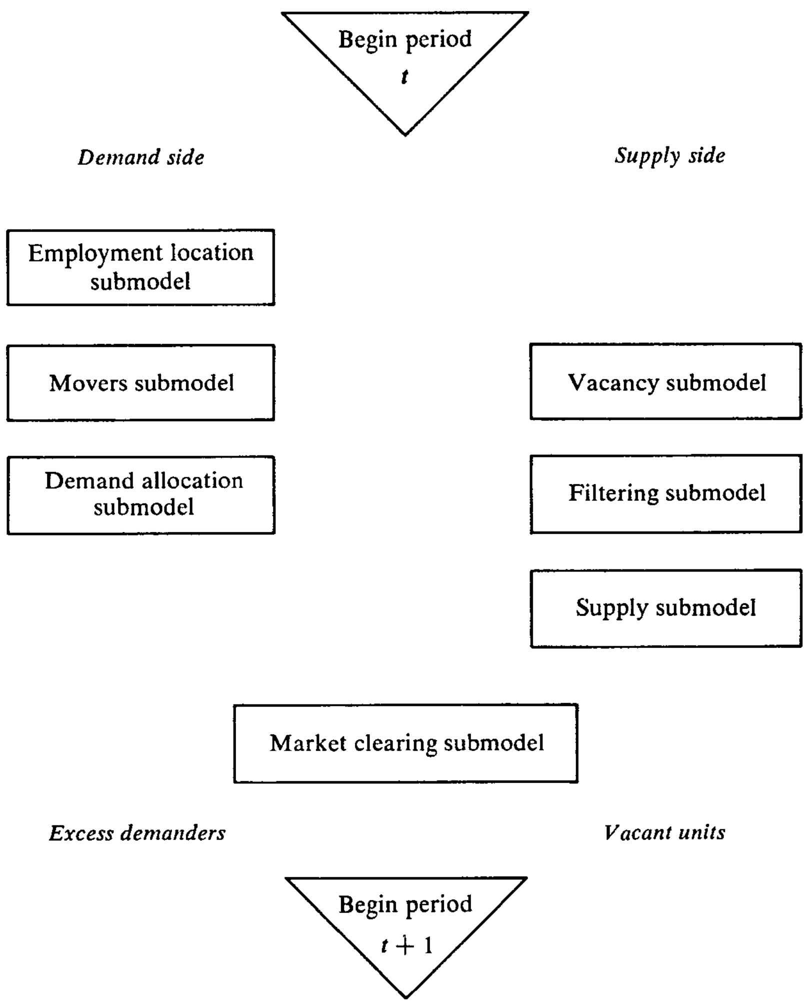
The demand sector

The simulation begins with the exogenous employment submodel, which allocates changes in employment in the export and regional service sectors (analagous to basic employment in Lowry) over eleven different industries and twenty different work zones.[30] These are completely exogenous to the model. Then the population serving employment submodel allocates the change in such employment according to changes in income, residential population, and export

29. Kain, Apgar, and Ginn, *Description of the NBER Urban Simulation Model;* and Kain and Apgar, *Baseline and Policy Simulations.*

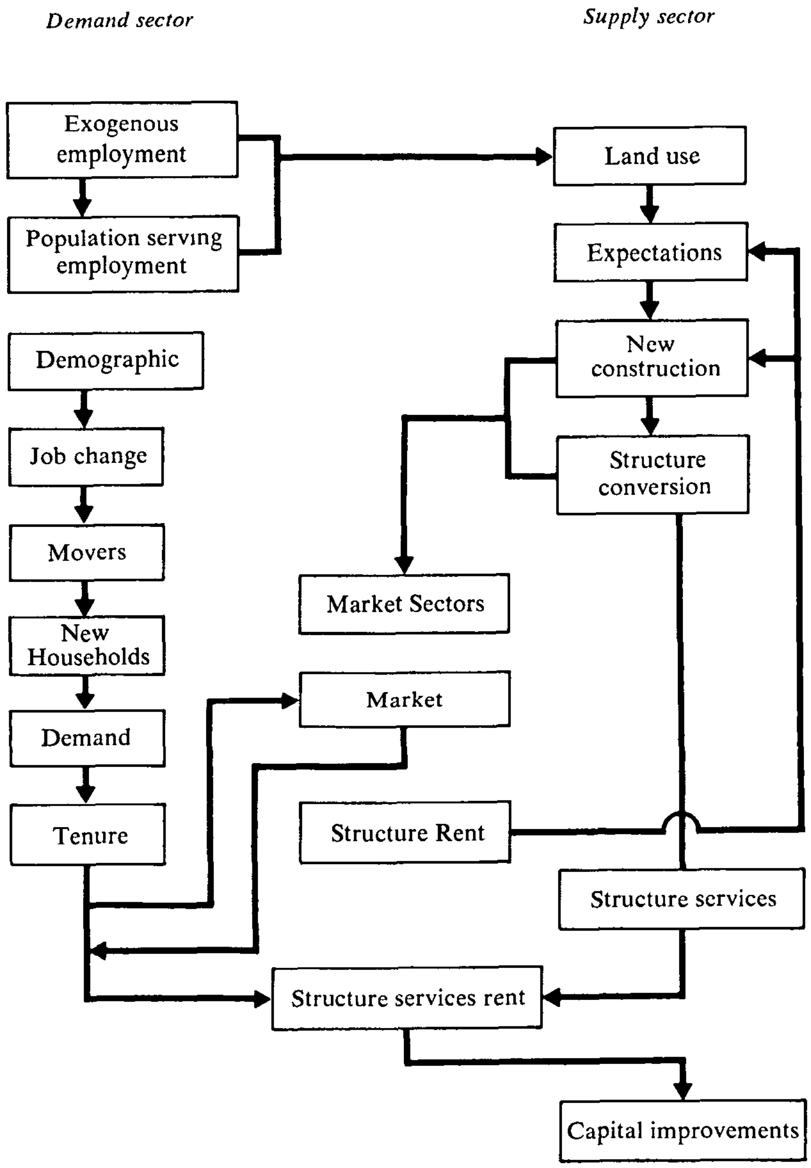
30. The following description follows Kain, Apgar, and Ginn. *Description of the NBER Urban Simulation Model.*

Figure 5. Structure of the NBER *Model*



Source: John F Kain, William C. Apgar, and J. Royce Ginn, *Description of the NBER Urban Simulation Model.* Vol. I, Simulation of the Market Effects of Housing Allowances, final report to the U.S. Department of Housing and Urban Development (1976), p. 18.

Figure 6. The NBER *Model: Structure of the Iteration Process*



employment in each zone. This employment is spread over the entire urban area. These changes are combined with the exogenous changes to give a total job growth that is disaggregated by workplace zone, industry, and occupation; the last is derived by the use of manning tables.

The demographic submodel simulates the life cycle of each household: it gives the age of the head of household and changes the size of family (according to probability matrices) and household income. These operations are performed on households in the basic list.

The job change submodel simulates the labor supply behavior of primary workers. It describes the behavior of primary workers who leave their existing jobs and search for new ones, resulting in employment in a new one (distinguished by industry, occupation, and workplace), unemployment, or exit from the labor force. This feature did not exist in the Detroit prototype.

The movers submodel identifies the households that move as a result of changes in life cycle or job. Tenure choice plays a major role in this identification. Because the basic list maintains a list of households as well as dwelling units, the movers submodel automatically produces the dwelling units available for occupancy, as well as households seeking new housing.

The new households submodel adds to the households seeking accommodation through the formation of new households and immigrants. This is an exogenous submodel.

The demand submodel allocates the 12,000–18,000 households that need housing to one of fifty kinds of housing bundles that include ten structure and five neighborhood kinds. This assignment is also done through the use of probability matrixes estimated to take into account the particular needs of ninety-six household kinds distinguished by twelve life cycle categories, four income kinds, and two races. The gross housing price seen by a household includes rent, money, and time costs of transport. It varies by an accessibility premium for the residence zone, household income, race, and workplace location; race determines discrimination markups. Finally, to complete the demand sector, tenure choice is assigned once the housing bundle is determined for each household.

The supply sector

The land-use submodel updates the existing land use by zone according to kinds of industry and housing. In a fully calibrated model, industry coefficients would convert employment changes into land-use changes—which would be similar to the method used in most Lowry models. Such data were not available, however, so the current NBER version is not programmed to convert employment changes into land-use changes.

The expectations submodel calculates the trend of future rents and neighborhood quality. These are simple extrapolations of the preceding four years. It also calculates demand targets for new construction and conversion for investors according to projected changes in demographics, incomes, and rent.

The new construction submodel calculates the profitability of building each type of structure in each residence zone and arranges each according to the level of profitability.

The structure conversion submodel calculates the net present value of rents, less maintenance costs, and compares them with conversion costs. The most profitable of the units are added to the new construction list and are processed until the demand targets are met.

The structure services submodel is essentially a production function that determines the level of structure services produced by each unit. It derives a marginal cost curve for services produced by each unit.

The capital improvements submodel calculates the appropriate rates of investment and disinvestment for every structure.

The market sector

The market submodel solves a linear programming algorithm for each of the fifty housing submarkets. It minimizes total accessibility cost for each workplace. Thus, shadow prices are produced for each residence zone. These prices may be interpreted to indicate the reduction in accessibility cost caused by the addition of one dwelling unit to a particular zone after households have been assigned to all residence zones in a way that once again minimizes aggregate accessibility cost. They may also be interpreted as location rents.

The structure rent submodel calculates current market rents for each housing bundle in each residence zone to give gross housing prices for each unit. The structure services rent submodel gives the quantities of structure services actually supplied by each structure during that period and the market rents paid for them. These are determined by the interaction of the occupant's structure services demand curve and the dwelling unit's short-run marginal cost curve. The quantity demanded depends on income levels, tenure choice, and the previous year's rent.

Thus, the market sector matches the households in need of housing from the demand sector and the dwelling units available from new construction and conversions. It then allocates the appropriate levels of housing services demanded and supplied.

Embedded in the model structure described above are special features not found in most other models. The model changes employment and residential locations and makes alterations to the housing stock and in the distribution of work trips in an incremental fashion only. This has the effect of letting the existing city structure influence the future, which is a pleasing simulation of reality. This procedure combines the historical approaches of Hanushek and Quigley and Harrison and Kain reviewed in the last chapter.[31] The model does not force equilibrium of each housing submarket in each period. Individual housing submarkets are allowed to have excess supply of units (vacancies) or excess demand that, in turn, affect prices in the next period. The model produces expected housing price by housing kind and zone for each period. These prices affect the behavior of households seeking housing as well as firms producing them. The explicit modeling of the supply side is seldom found elsewhere. The most recent version also models labor market operations, including job-search behavior, which is also an innovation for urban models.

In the Detroit prototype version of the model, the region was divided into nineteen workplaces and forty-four residence zones; the former were aggregates of thirty-two inner residence zones. It distinguished households by size, income, education, and age of the head of household. This resulted in seventy-two household classes. Housing was distinguished by kind of structure, number of rooms, quality, and lot size, resulting in twenty-seven types. There were two modes of travel. The most recent version has almost two hundred residence

31. Eric A. Hanushek and John M. Quigley, "An Explicit Model of Intra-Metropolitan Mobility" (New Haven: Institution for Social and Policy Studies, Yale University, June 1977; processed) and "The Dynamics of the Housing Market: A Stock Adjustment Model of Housing Consumption," *Journal of Urban Economics* (forthcoming); and David Harrison and John F. Kain, "Cumulative Urban Growth and Urban Density Functions," *Journal of Urban Economics*, vol. 1, no. 1 (January 1974), pp. 61–96.

zones (aggregated into fifty residence districts) and twenty workplace zones. Land use was classified as nonresidential, single-family residential, and multifamily residential, as well as undeveloped in the above categories. In addition to the classifications in the Detroit prototype, households were also distinguished by tenure choice, race, occupation and industry of primary worker, workplace zone, and labor force status of primary worker. Housing can be distinguished by fifty housing bundles subsuming ten kinds of structure and five kinds of neighborhood.

A description of some of the processes determining the distribution generated by the model follows.

BEHAVIOR OF THE CONSUMER. The consumer is seen as being essentially the classical utility-maximizing household. In this model the following assumptions are made about the consumer: First, the household has a fixed and predetermined set of demands for travel to known destinations. The journey to work predominates in travel costs and households place monetary value on travel costs. Second, households have preferences for housing attributes that they buy in a finite number of combinations of housing bundles. Third, housing bundle prices vary by location, and these price surfaces are known to consumers who act as price takers.

The consumer's problem is then posed as a problem in cost minimization. Because travel costs are subsumed in gross housing prices, the optimal location for the household is that location that has a housing bundle at the minimum price and that corresponds with its requirements according to needs based on income, household characteristics, tenure choice, race, and taste in the traditional demand analysis way. Only some households look for housing in any period; these include movers, new households, and immigrants.

DETERMINATION OF HOUSING PRICES AND QUANTITIES. As mentioned earlier, the NBER model differs radically from earlier approaches in that it recognizes the durable nature of urban structures and allows city structure to change only marginally in each period. This permits disequilibrium in various housing submarkets. It follows traditional economic theory, however, in assuming that housing production is responsive to market demands and prices, that suppliers are profit maximizing, and that households are price takers in both output and factor markets. It assumes further that (a) housing outputs are heterogeneous and are produced from combinations of existing durable structures, current inputs, and neighborhood attributes; (b) much of the supply in each period is from used structures; and (c) some of the housing attributes are not produced by competitive firms (they may be supplied by local governments). This last assumption has the effect of placing constraints on suppliers' actions.

The production function for housing consequently reflects these assumptions in that it includes existing structures and neighborhood characteristics as inputs in the function. This is a radical departure from the standard practice, which usually includes land, capital, and labor as inputs in a production function.

MARKET CLEARING. The household's selection of a housing bundle is represented in the NBER model by econometrically estimated demand functions expressed as probability matrices. These equations express the probability that a particular household will consume a particular type of housing as a function of the household's socioeconomic and demographic characteristics and the minimum gross price of the bundle. Solution of these functions gives the number of persons employed at each workplace who will demand each kind of housing bundle. The demand for each kind of bundle is summed over all workplaces. Firms then attempt to satisfy this demand in each location on the criterion of profit maximization.

Spatial competition among households in the same housing submarket is represented in this model by a linear programming algorithm that minimizes aggregate travel expenditure for households competing in the same submarket. Thus, the procedure also yields shadow prices for each bundle type in each residence zone in the solution.

The output of the model essentially gives the workplace and location of each household and the bundle of housing it consumes. Further, it produces expected prices for each kind of housing in each residence zone during each period.

Included among the inputs to the model are employment, cost of performing various supply activities (that is, an array of supply costs with a given technology and fixed factor costs), zoning constraints such as limits in residential density, amount of available land in each zone, and income growth.

Evaluation

The NBER model has now been developed over almost a decade, and three versions have been completed. It has now also been used to simulate a number of policy experiments to investigate housing-abandonment phenomena and the effects of housing-allowance programs.

It must be recognized that the NBER model is an admirably ambitious attempt to incorporate urban realities within a basic framework of urban economic theory in order to produce a model of city structure. It is still not clear, however, whether it can be regarded as a reliable policy tool. The authors themselves regard their simulation results as exploratory and stated that "further analysis of the model's output, more numerous simulations, improvements in calibration and additional research will be necessary before the model can be considered a completely reliable tool for analysis of policies.[32] They feel that the primary constraints are the limitations of theoretical and empirical knowledge of housing market behavior and insufficient skill in computer modeling. They also found the lack of the availability of appropiate data to be a severe limitation. They had to use a synthetic sample of time series descriptions by piecing together population census, business census, business patterns, and other state data. It was concluded that data suitable to test the model were lacking.

In the housing allowance experiments, the main conclusions provided by the model simulations were that housing allowances may well cause declines in market rents; that the differential effects on blacks and whites can serve to make little difference on the housing patterns of blacks; and that indirect neighborhood quality effects may well dominate any direct housing effects. The major problems encountered in the base line simulations were the underestimations of certain kinds of structures and general trends of market rents. Some of these underestimates were between 55 and 70 percent and therefore of significant proportions. What is somewhat disturbing is the difficulty of tracing the causes of these underestimates. Given such problems, it is difficult not to question the results of policy experiments as, indeed, the authors themselves have done.

Model calibration would undoubtedly improve if more and better data were available. Is it realistic, however, to expect much better data than was available in Chicago and Pittsburgh without incurring large expenditures? It is doubtful that the resulting marginal improvements would justify the expense. Moreover, because the causes of results obtained are difficult to trace in a model as complex as this, it is even difficult to find criteria for accuracy. The success of

32. Kain and Apgar, p. 3.

the NBER model lies in its pushing the urban modeling frontier a considerable distance and its revealing the limits of modeling. Its use as a policy tool is expensive: apart from development and calibration costs, each complete simulation costs between about US$160 and US$340.

The NBER model has benefited from long-term financial support and has had the participation of some of the best urban practitioners in its conception. As a policy tool it is probably overambitious. As a research enterprise it has had clear-cut objectives. The specification of each submodel has been a project in itself and has illustrated the use of economic theory in modeling. In this respect it has been very instructive and has spawned a host of side studies that have enriched the state of the art of urban studies.

It is probably fair to conclude that the NBER model has been more successful as an analytical or explanatory model than as a policy-oriented model. This was to be expected, because it is too large and its data requirements too intensive and detailed for the time and resource constraints of policymaking. It may well be impossible to construct a model that provides this level of detail and yet is useful for policy purposes. Its usefulness for cities in developing countries lies in the ways it has adapted existing theory to reality. In particular, its enumeration of housing-bundle kinds may be a good approach to the extremely varied bundles found in cities of developing countries. Partial research efforts in these cities could be modeled after parts of the NBER model; applying it fully would be foolhardy.

Mills' Optimizing Programming Model

Another kind of model is reviewed in this section, Mills' optimizing programming model.[33] Although policy-oriented models in particular might be expected to have some normative content, such models have been more the exception than the rule.

The paradigm for normative models in economics is that some market resource allocation first is derived, it is then tested against some welfare criterion, and government policy is then prescribed if this allocation turns out to be suboptimal. The assumption is

33. Edwin S. Mills, "Planning and Market Processes in Urban Models," in *Essays in Honor of W. S. Vickrey*, ed. Ronald Grieson (forthcoming).

always that the government operates in the public interest, whereas everyone else furthers his own interests. In this model Mills explores the issue of the desirable extent of governmental regulation and interference in the urban system. Assumptions are introduced that indicate why it is desirable for government to undertake certain activities. It then specifies the effect of government activity on private resource allocation and the effect of private activity on the use of the public service. It permits calculation of an optimum allocation of both public and private resources. Finally, it demonstrates that competitive markets sustain an optimum allocation of resources if the public sector provides its service in optimum fashion.

The model concentrates on the provision and pricing of transport. It is formulated in a nonlinear programming framework and is a development of earlier models by Mills.[34] Hartwick and Hartwick have articulated a similar model that allows for multiple centers or nuclei as well as intermediate goods.[35]

The model

Unlike other models presented in this chapter, this one makes a larger number of simplified, unrealistic assumptions, but it is still rather complex. The city is seen as a homogeneous plain stretching in all directions from a central export point. The model determines the amount and production technique of each of an arbitrary number of goods and services to be produced at each location in the urban area. Each good that is produced must be exported or transported to the point at which it is consumed. Therefore, this model, unlike others, also takes into account traffic in goods.

Space in the area is represented by a square grid centered on the central export point. All squares at a given distance from the center have identical patterns of production, consumption, and transport. Transport is between squares in the north-south and east-west directions only.

34. Mills, "Markets and Efficient Resource Allocation in Urban Areas," *Swedish Journal of Economics*, vol. 74, no. 1 (March 1972), pp. 100–13; and "Mathematical Models for Urban Planning" (Princeton, N.J.: Princeton University, 1974; processed).

35. Philip G. Hartwick and John M. Hartwick, "Efficient Resource Allocation in a Multinucleate City with Intermediate Goods," *Quarterly Journal of Economics* (May 1974), pp. 340–49.

Produced in the city are $\bar{r}$ goods, of which two are high-income housing and low-income housing. All other goods are exported, and the export amounts are given exogenously. Apart from the center, goods can also be exported from suburban nodes $\hat{u}$ miles from the center. The first set of equations in the model are identities to ensure that (*a*) total exports of each good are at least as much as exogenously prescribed; and (*b*) shipments into each square plus production in the square equal outward shipments plus use as input or final consumption in the square. (It is important for the reader to note that the latter implies in each square a set of $\hat{u}$ times r equations.)

The next set of inequalities builds the transport system. They determine $t_{rk}(u)$, which is the number of unit miles of good r shipped at congestion level k per square at u. The congestion level helps in determining the width of roadway required: higher congestion means a narrower roadway. Once again, this is a set of r times $\hat{u}$ inequalities. In addition, there is one to ensure that land demanded by transport is less than total land for transport. A further set of inequalities and equations ensures that there is only one level of congestion in each square. Finally, there is an inequality to ensure that the land used for all purposes does not exceed the total available.

A feature of the model is that it permits an arbitrarily large number of production techniques used to manufacture goods. This is done by concentrating on the capital-land ratio, which is represented by the height of buildings. A large number of stories signifies a high capital-land ratio. Labor input-output coefficients are independent of building heights.

Such a view of the production process implies that a city's labor force and total output of all goods are determined by the export requirements via the input-output matrix. Transport cost, land, and capital requirements, however, are endogenously determined. The objective function is then to minimize the sum of land, capital, transport, and exports cost needed to produce the required export goods. It is assumed that R_A, rent for land, at the city periphery is uniform, and that the city can therefore be expanded at this uniform unit cost. Similarly, capital can be acquired without limit at a fixed rental rate, R.

The model is solved using mixed integer programming techniques. The only nonlinearity in the model is the group of integers used to represent congestion levels.

Solution

The model is solved for a hypothetical U.S. city with about 1 million population. It has five sectors ($\bar{r} = 5$) for the following: office activities, retail firms, manufacturing firms, low-income housing, and high-income housing. $\bar{u}$ equals 11, implying an urban area of about 250 square miles; $\bar{s}$ equals 20, implying that the highest building has 20 floors; and income groups are the lower half of the population and the richer half: R_A equals $4,000 an acre, and R equals $1 million, the rental rate per $10 million worth of capital value.

An entire input-output matrix was constructed for the five sectors. Approximate values of automobile travel and movement of goods were used. Time costs varied with income.

With such a simplified model there are 788 variables excluding slack variables. Of these, 30 are integer valued. There are 219 constraints in the example. The model was solved on an IBM 360/91 computer and took six minutes, or $70, of computer time and used about 400 K of the core of the computer. (The capacity of the core is about 1,100K.)

The solution produces a city rather typical of a U.S. city, as intended. The central square is devoted to office activities, in seventeen-story buildings, and transport is at the highest congestion level. The next square has retail firms, some low-income housing, and some office activities. The city center is ringed by low-income housing as is typical of U.S. cities. All other squares have some of all activities, but the suburban export nodes ($\hat{u} = 7$) have no manufacturing firms.

The congestion pattern is unsatisfactory: high in square zero, low in square one, and high again after square five. Land rent falls rapidly near the center and then slowly, which is typical of most cities.

Evaluation

In many ways this model represents an interesting comparison with the Echenique model. Conceptually, the two do rather similar things. Both specify some basic sector exogenously, around which a city is generated. Echenique and colleagues specify this sector as one that is unrelated to local or residential location, whereas Mills specifies exports. Both models distribute activities and transport by maximizing some function subject to some constraints. Both involve the guessing of a large number of mean values and parameters specifically. This model must be fed with the following: (a) total export of each good; (b) input-output matrix coefficients (input of good q per unit output of good r using production technique s); (c) land standard for transport at each congestion level; (d) capital-land ratio for transport; (e) unit cost of exporting each good from squares at each distance; (f) total cost of shipping goods per unit at each congestion level; (g) rental rate of peripheral lands; and (h) rental rate of capital.

Mills simplifies his model by regarding the city as symmetric; he thereby reduces the spatial problem to a unidimensional one. His model permits generalization by allowing the identification of each square uniquely; under such circumstances, however, computation becomes cumbersome.

The crucial conceptual difference between the two models is in their objective functions. Entropy maximization merely does some "most probable" distribution within given constraints, whereas costs are minimized in Mills' model. The latter is more appealing because it makes it easier to understand what is being minimized. It is therefore possible to appreciate the reason behind the distributions that are generated. Furthermore, in the policymaking context, the policymaker understands such a procedure readily. With entropy maximization, at best the modeler himself has some idea of the generating force in his model. It is easier for a modeler to interact with a policymaker who also understands what is going on.

The meanings of parameters in this model also are easily understood, whereas in the entropy-maximization approach they are merely some distribution parameters that are calibrated. As a result, interaction with a policymaker could yield changes in parameters and in the objective function that could easily be incorporated into Mills' model, whereas it would be difficult to do so in the Echenique efforts.

A caveat is necessary here. Difficulty in intuitively understanding the procedure in a model does not necessarily mean that its results are any less reliable. It does mean, however, that it is more difficult to evaluate and that its results demand a greater degree of faith.

The main drawbacks in Mills' model are its monocentric assumption and its unimodal transport. As has been argued earlier in this book, such assumptions may have some validity in the context of the United States but almost none where developing countries are concerned. Further, its production functions are of the fixed coefficient kind that do great violence to urban reality. Mills recognizes these defects and suggests the following by way of extensions of the model:

First, introducing a second mode of transport that has a different right of way, such as a subway, is easy because it merely requires another set of input-output coefficients that can be used to represent the second mode. Buses, rickshaws, and pedestrians would be more complex because the same roadway would be used.

Second, Mills suggests that economies of scale are possible to include by the use of more integer variables and constraints specifying threshold levels of production. He conjectures, however, that such modifications would require identification of each square; the computations required could well exhaust even a large computer.

In its present form, the model can be used easily to test policies such as land pricing, transport pricing, land taxation, and land-use controls with manageable modifications. In each case, the primary output of the model is the amount and production technique of each good to be produced in each square, the origin and destination of each shipment, and the congestion level in each square. This suffices as the profile of a city.

The number of variables and constraints in this model is large despite its conceptual simplicity. It needs one of the newer, large computers, such as the IBM 360/91, which are often not available in developing countries.

In conclusion, such a model is regarded as an interesting departure from other standard approaches and has some advantages over them. Its drawbacks, however, do not make it a fruitful approach to the cities of developing countries at present.

Summary

In this chapter I have reviewed policy-oriented models that are regarded as representative of the main strands of urban modeling. Each of the models reviewed employs a different approach and none is regarded as successful policy models—particularly for possible use in cities of developing countries.

The objective of each of the models is to distribute activities spatially within a city. A typical output would be the allocation of residential and employment location by zones in the city. This would be disaggregated by socioeconomic kinds of households, kinds of residential structures, kinds of employment, and so forth.

The Lowry model is the parent of many models; its offspring are numerous. Its conceptual contribution was to posit a basic sector of activity that is exogenously given and that is the driving force behind a city. This activity requires households to supply labor; these households, in turn, require retail services. The provision of retail services requires more households; and the process is iterated until equilibrium is reached. The principle behind the generation of location distributions is the gravity model of interaction. The original Lowry model was partly filled in by hand to produce plausible employment-residential codistributions. Lowry himself was not sanguine about its potential usefulness, but his contribution must be regarded as the single most important and influential one in noneconomic urban modeling. The Lowry model descendants in the United States have only had limited success and have been subject to heavy criticism from within the profession. Consequently, this has led to disillusionment.

More innovative developments have been carried out in Great Britain, where Wilson has given a more respectable theoretical base to the gravity model through the method of entropy maximization. Echenique and his associates have developed models based on Lowry and entropy maximization and have attempted to apply them to South American cities. Although these models have reproduced city structures fairly faithfully after careful calibration, their value as planning or predictive tools remains suspect. They have scant behavioral underpinnings, which produces skepticism about their use as predictive devices. Moreover, they are heavily data intensive, which limits their applications in developing countries.

The NBER model can be regarded as the economists' answer to urban planners. This model has a behavioral structure based on microeconomic theory and borrows much from the analytical models of Muth and Alonso. It makes them more realistic because it drops the monocentric assumption and takes into account the existence of disequilibrium in markets and the effects of the durability of capital structures and of externalities such as neighborhood effects. Despite these pleasing features, it can scarcely be called an operational model. It is unwieldy to operate, it is not evaluative, and there are some indications that it has developed into a research process rather than a product. Its data requirements are even more intensive than the models of Echenique and his associates. Much can be learned, however, from the formulation of its submodels and the way in which it has been developed over the years in attempting to answer different questions.

The last model discussed—Mills' programming model—differs from the others in that it is an optimizing model and has much more simplifying assumptions. It regards the city as monocentric and allows a unimodal form of transport. Despite its simple and unrealistic assumptions, it becomes mathematically complex and large. It has 788 nonslack variables, of which 30 are integer valued, and 219 constraints. Its solution requires nonlinear programming techniques and late generation computers.

Large models of urban areas are expensive in data as well as in technical expertise, particularly for developing countries, and their use is of dubious value. The essential conclusion of this section is a somewhat paradoxical one: although large-scale urban models may be of some use as research methods that explore urban form and structure and their underlying rationale, they are of little use as practical policy devices. This is more the case in the context of developing countries, where change is so rapid that models are soon rendered obsolete for policy purposes. The next chapter suggests some fruitful approaches that are more modest in scope—though not without their own limitations.

4

Some Fruitful Approaches to Urban Modeling

In this chapter, I review some work that employs approaches that, though different in each case, seem to provide hope for useful urban modeling. My intention here is merely to illustrate and suggest some fruitful approaches. I must emphasize that the models discussed in this chapter are not considered directly transferable to cities in developing countries. They are too rooted in their particular institutional settings for such transferability, as indeed they should be.

I begin with a description of some of the work of the Transportation and Location Analysis Group in the Master Planning Commission of Stockholm, reported in Lundquist and Andersson. Their model, an evaluative one, is particularly suited to the Swedish environment, in which the government has considerable control over urban form.

The second model discussed is the Urban Institute housing model, described in various publications of that institution. This model reflects the U.S. institutional system's focus on the urban housing market. It was designed originally to help the U.S. Department of Housing and Urban Development to test such policies as the housing subsidy scheme. It is therefore policy oriented and different in structure from the other models discussed.

Finally, I describe two explanatory models that seek to find behavioral relationships explaining residential patterns from city to city. They are presented to illustrate why the development of policy-oriented and explanatory models should go hand in hand to increase our understanding of cities. The relationships and parameters found in a model of the Apps or Wheaton kind described later can then be fed into policy-oriented models as inputs. Such models are examples of the useful way in which a data-gathering effort could be organized systematically.

All of these models are described in skeletal form and in semitechnical detail to facilitate understanding.

The Andersson-Lundquist Stockholm Model

The Stockholm Master Planning Commission established a group in 1970 to "provide metropolitan planning with a unified system of transportation and location planning."[1] The group realized at the outset, however, that realistically they could not aim at the "construction of a complete model that could simultaneously solve all the spatial planning problems of a growing metropolitan area." They decided to opt for a system with a limited number of related submodels. Their realistic approach was concerned with evaluating specific policy alternatives for Stockholm. The main policy variables were the provision of critical transport links and residential housing.

The urban problem is therefore viewed as one that can be analyzed by means of dynamic interdependent investment planning. Scarce resources are to be allocated to physical investments that will provide a maximum of social and economic welfare. The high degree of complexity of urban structure is caused essentially by strong interdependencies over space and time.

The Stockholm group attacked this problem by using hierarchically decomposable models. First, a city should be divided into a small number of relatively large zones. The interactions between these zones should be studied and approximate plans derived at this

1. See L. Lundquist, "Transportation Analysis and Activity Location in Land Use Planning—with Application to the Stockholm Region" (paper read at the Conference on Dynamic Allocation in Space, Stockholm, 1973), and "Integrated Location—Transportation Analysis: A Decompositional Approach," *Regional and Urban Economics*, vol. 3, no. 3 (1973), pp. 233–62; and Åke E. Andersson, "Integrated Transportation and Location Theory and Policy" (Stockholm: Stockholm Master Planning Commission, n.d.; processed).

level of aggregation. Further disaggregation can then be attempted, and the results of that feedback can be used for further iteration. The rationale for this approach lies in technical realism. Interdependencies that are believed to exist in the urban system can be modeled only with nonlinear mathematics, which are manageable only if the problem size is kept small. It is thereby possible to have a mathematically complex model at an aggregate level and then have a sequential transition to disaggregated linear models of larger size.

According to the authors of the Stockholm model, an urban modeling exercise should aim to subsume the following: (a) planning under uncertainty, (b) normative welfare criteria; (c) individual behavior, (d) spatial and sectoral disaggregation, and (e) explicit treatment of interdependencies, indivisibilities, and economies of scale.

Uncertainty about the future is a characteristic feature in the planning of cities in developing countries. This is currently caused by their explosive growth rates, and in many places is compounded by political instability. The latter is particularly important in the formulation of the normative welfare criteria used in an optimizing model. If there is an interest in the implementation of planning ideas, the welfare criteria used must reflect those of the political policymakers. These criteria, however, can be expected to be myopic (because of election cycles, for example). Then, if the interest of the modeler is in longer-term welfare, the results obtained from the use of different welfare criteria should be robust. In this way, uncertainty about the future is better handled.

In the modeling framework, therefore, both uncertainty and normative welfare criteria can be tackled by using sensitivity analysis rather than by attempting to incorporate stochastic elements in the model. Thus, a case is made for the need for an integrated optimization-simulation framework.

In the Stockholm model, the city is composed of building stock, transport systems, and recreation land. The interplay between the structure of the transport network and the urban location pattern is fundamental. This interplay is mutual and should not be represented by oneway relationships. As was stressed in the NBER model, basic physical infrastructure (building stock and transport links) change slowly and activities therefore have to be allocated within these constraints over the short term.

Thus, the first level of interaction is between building stock and design of the transport network; this is explored in Model 1. The second level of interaction is between activity location and transportation behavior; this is subsumed in Model 2. The next level, Model 3, can disaggregate the activity analysis even further, both sectorally and spatially. The tradeoff in modeling is essentially between greater disaggregation and representation of relationships. The three-level Stockholm model frame work is illustrated in figure 7 (page 130). The model scheme and the number of submodels was determined according to the "efficiency of model structure." Here, "efficiency" includes the degree of approximation of the observed interdependency pattern, the rate of convergence in feedback loops, and the computational feasibility of the submodels.

Model 1

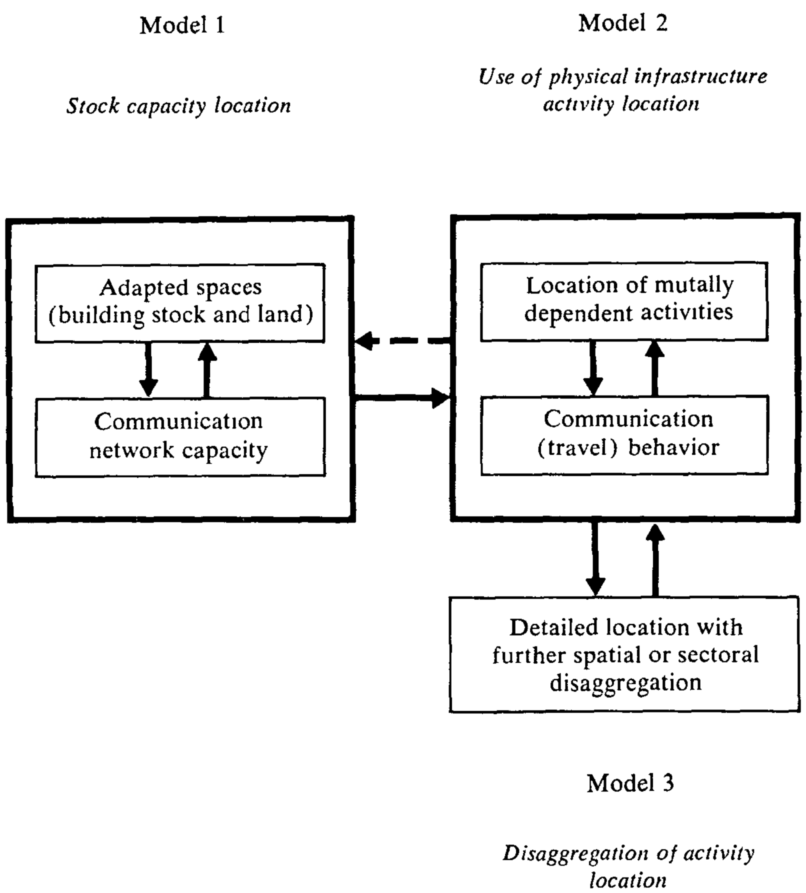
In specifying Model 1, which can take as inputs regionwide data from a nationwide interregional model, the first task is to specify a welfare function. The variables of interest that can enter a welfare function are:

- accessibility: A high level of accessibility provides good opportunities for interaction and is promoted by compact urban structures.
- environment: This can include such things as space standards measured by level of pollution, degree of segregation, and population density often (though not necessarily) promoted by decentralized urban structures.
- costs of investment: These can include availability of resources, operating costs, and so forth.

In the Stockholm model, the welfare function is essentially a weighted sum of indexes of accessibility and the unease of congestion. The welfare function may be written as $W = \alpha I + \beta C$, where I is the index of the level of interaction; C is the index of the unease of congestion; and α and β are the weights to be used.

The measure of interaction used, I, is specifically related to the Stockholm region, in which a uniform labor market, good social integration, and good possibilities of interaction between workplaces were observed to exist. Thus, I is basically computed as the average distance of interaction. Divide Stockholm into N disjoint zones and

Figure 7. The Three-Level Stockholm Model Framework



Source. L. Lundquist, "Transportation Analysis and Activity Location in Land Use Planning—with Application to the Stockholm Region" (paper read at the Conference on Dynamic Allocation in Space, Stockholm, 1973), p. 27.

make the following definitions:

- B_i is amount of building stock in zone i, $i=1,2...N$.
- $d_{ij}(\bar{t})$ is the friction of distance between zone i and zone j, given transport network $(\bar{t})$
- t_k is a zero-one variable denoting whether a transport link k exists ($t_k=1$ if link exists), $k=1,2...M$. (Thus there is a finite number of links, M, that can possibly exist.)
- $\bar{t}$ is a vector denoting existing transport structure $(t_1, t_2...t_m)$.

Now:

$$I = \sum_{i=1}^{N} \sum_{j=1}^{N} B_i d_{ij}(\bar{t}) B_j / \bar{B}^2, \tag{121}$$

where $\bar{B}$ is total building stock and is given for a time period $(\bar{B} = \sum_i^N B_i)$. The expression is essentially a quadratic function of zonal building stocks with interactions between all zones considered through the use of $d_{ij}\,(\bar{t})$.

If A_i is defined as area of zone i and the cost of congestion is then formulated as:

$$C = \sum_i^N \left\{ \left(\frac{B_i}{A_i} \right) B_i \right\} / \bar{B}, \tag{122}$$

Then B_i/A_i is a measure of density in a zone and C is a weighted sum of densities in the city.

The constraints to the model are basically resource constraints related to availability of capital and labor. These are derived from a nationwide interregional model that gives their regional magnitudes. The model may be written as follows. Minimize:

$$W = \alpha I + \beta C, \tag{123}$$

subject to

$$\alpha + \beta = 1, \tag{124a}$$

$$\sum_{i=1}^{N} B_i = \bar{B}, \tag{124b}$$

$$\sum_{i=1}^{N} \Delta B_i \cdot k_i^B + \sum_{j=1}^{M} \Delta t_j \cdot k_j^T \leq K, \tag{124c}$$

$$(124d) \qquad \sum_{i=1}^{N} \Delta B_i \cdot l_i^B + \sum_{j=1}^{M} \Delta t_j \cdot l_j^T \leq L,$$

$$(124e) \qquad B_i = B_i^0 + \Delta B_i, \quad \text{and}$$

$$(124f) \qquad \Delta B \geq 0.$$

Here α and β are the weights assigned to each cost. These weights can be varied to test the effects of different degrees of importance attached to issues of environmental and accessibility concern. The first constraint merely ensures that the weights sum to unity. The second constraint (124*b*) ensures that the sum of building stock in each zone at the end of the current time period is equal to the total, $\bar{B}$, demanded for the whole region. Constraints (124*c*) and (124*d*) specify the capital and labor resource constraints. ΔB_i are the additions to building stock in zone i, and Δt_j the additions (0 or 1) to the existing transport structure in the current time period. k_j^B, k_j^T are the capital coefficients and l_i^B and l_j^T, the labor coefficients: that is, the amount of capital or labor required to produce one unit of building stock or transport link. These were apparently derived from the estimation of Cobb-Douglas production functions for the area. The fifth constraint (124*e*) shows that building stock at the end of the current period B_i is the sum of existing stock B_i^0 and the addition ΔB_i. The last constraint (124*f*) ensures that some building activity does take place in the period.

The solution to this problem yields an optimal combination of building stocks and transport network structure. The chosen objective function is a highly nonlinear, combinatorial one which cannot be dealt with by conventional optimization techniques. Thus, a heuristic, tree-searching procedure is suggested to find a solution. In a given situation the total increase in building stock can be divided into a finite number of building projects. These are then located sequentially and the optimal transport network is found in each case. This itself is determined by a tree search over transport links. Because the technique is heuristic, an optimal solution can never be guaranteed. Basically, the procedure is a systematic trial and error process: in principle, there is only a finite number of building projects and transport links, so there is only a finite number of possibilities that can be considered. Hence, an optimal solution can be found.

Stockholm was divided into twelve geographical zones for the purposes of this model. Twenty-five transport projects were considered, and the building demand was disaggregated into six building projects. Model solutions give level of building stock in each zone and the existence or nonexistence of transport links between zones. The model is flexible in the following ways:

- structure: The specification of the objective function and the constraints can be varied to test the robustness of results. In particular, values of α and β can be systematically varied to observe the effects of differing priorities.
- exogenous data: Capital and labor coefficients can be varied to represent different technologies. The given, desired stocks can be varied as well as the projections of growth rate.
- heuristic procedures: The solution procedure can be varied to test for sensitivity of results.

These are precisely the kinds of decision variables that can be explored in the context of cities in developing countries. The objective function can easily be modified to reflect problems of such cities; investment and maintenance costs, for example, would probably be more important than certain kinds of environmental problems. The most appealing part of this (general) model is its low requirement of data: a city needs to be divided into only a few zones. As is evident from equation (123), the kind of parameters and data needed are at such aggregated levels that it should be easy to estimate or guess them for cities in developing countries. Furthermore, the robustness of results can be tested by varying the data.

Lest this sound too optimistic, it should be recalled that the model is highly complex in a mathematical sense and that the solution is not straightforward. High levels of skill are required—and these are not often available in developed countries, let alone the less developed ones. In addition, the input requirements and the objective function formulation are such that the modeler should be intimately acquainted with the city being modeled, particularly because many judgments are involved.

Model 2

Model 1 is used merely to allocate building stock capacity and transport network links by zone; it says nothing about the location of residences and employment. Model 2 is designed to do this, but within the same zonal structure as Model 1. An exogenously projected growth of production activities and a matching number of households should be allocated to maximize the socioeconomic objectives. Ideally, these activities should be allocated by taking into account mutual interactions over space and time, but this presents a computational difficulty because these interlinkages introduce nonlinearities and nonconvexities in modelbuilding. The analysis of the demand and supply of transport should be done simultaneously with the location of activities. Here, this interaction is approximated by an iterative scheme. Activities are spatially allocated, given a transport network; the network is then changed to take into account the location of activities, and so on, until convergence is reached.

The essentials of Model 2 are now described with even less rigor than those of Model 1. The variables of interest are clustered into one vector:

$$X^T = (L^T, R^T, Q^T), \tag{125}$$

where L^T is a vector of the zonal supply of developed land; R^T is a vector of residential activity by zone; and Q^T is a vector of production activities by zone. The objective of the model is to allocate these variables by zone.

The objective function is of the form:

$$\alpha X^T D X + (1 - \alpha) X^T Y X,$$

where D is an interaction matrix; Y is a congestion matrix; and α is the tradeoff parameter. This function is therefore the sum of two quadratic indicators. Interaction costs between two activities are weighted by the intensity of interaction between them, and congestion is measured in a similar way.

The constraint set includes exogenously determined growth rates of production levels, the availability of labor and capital, the balance of supply and demand in the land and housing markets, the updating of building stock (taking depreciation into account), and limits on the amount of change that can take place in one period. Employment and residential housing are related through a set of equations balancing labor requirements for production with labor contributed by housing units.

The model has a quadratic programming structure with a quadratic objective function and linear constraints. Global optimum cannot be guaranteed because of the existence of nonconvexities. Various initial solutions must be tried.

The model was tried for the Stockholm region using seven zones.

All economic activity was grouped into three production activities: manufacturing industry, retail service, and public services. The data and parameters used were obtained in various ad hoc ways. For example, they were obtained from regional projections (production levels, total labor force, and spatial requirements), national data (technological coefficients), and fictive values (friction of distance, interaction intensities, and regional variation of technology). The model gives the location of economic and residential activities given the overall constraints and objective function.

Once again (as in Model 1) it may be observed that the model appears to be of manageable size without a great amount of information requirement. Its structure is not described as a model to be followed, but more as a realistic approach worth learning from. The delineation of activities and the specification of the objective function can easily be modified to reflect cities in developing countries. It should be noted that the core of the model lies in the specification of the objective function. The information embodied in the constraints is usually not difficult to obtain. Specification of interzonal interaction costs and congestion costs is, however, another matter. The transport network is embedded in these characterizations.

Models 1 and 2 are conducted at a relatively high level of aggregation. For many planning situations, further spatial and sectoral disaggregation is required. Sectoral disaggregation should be carefully done because the model structure requires a certain amount of homogeneity (in capital and labor coefficients, for example) in the industries that are clustered into one sector. At the household level, there is little behavioral content in this model except for that subsumed in the objective function. Model 3 does not appear to have been developed yet, but it would include more behavioral detail at the household and firm level. It would, however, be at a highly disaggregated level, so it is likely to suffer from the normal information and other difficulties discussed in the context of other policy-oriented models in chapter 3.

Use in planning

In a planning context, models such as Models 1 and 2 would appear to be particularly useful because they evaluate possible courses of action in a way that can easily be discussed with a policymaker. Moreover, the policymaker's objective function can be probed by varying the coefficients in the objective function. It can also be useful in letting policymakers know what their objective functions really are when these functions are unarticulated but implicit from the appreciation of particular sets of results. Such sensitivity tests can also yield robust courses of action that score high on different weighting schemes and would be likely to survive political changes.

These models are also at a level of aggregation at which policy decisions are often made. For example, should a road be built or not built; should a new bridge be constructed or not? Once such a course of action about transport links and levels of desired building stock by zones has been suggested, further disaggregation can either be left to normal market processes or to a disaggregated model at the next level. The latter requires a deeper knowledge of peoples' preferences with regard to housing and location: one method of investigating this is provided in the Apps model described later in this chapter.

In conclusion, the solution procedure of the Stockholm model is by no means simple, for the model itself is mathematically complex.

The Urban Institute Housing Model

The Urban Institute housing model illustrates an approach to modeling that augurs well for the future and for applicability to cities in developing countries.[2] It is a partial model in the sense that it restricts its attention to the workings of the housing market and neglects other aspects of urban structure. The view taken in the design of this model is that explanations of changes in housing supply and quality are more likely to be found in long-term movements of housing cost, demographic changes, and income trends rather than in short-term changes in such indicators as interest rates, housing

2. This section reviews the work described in Frank DeLeeuw, "The Distribution of Housing Services," Working Paper no. 208-6 (Washington, D.C : Urban Institute, 1972; DeLeeuw and Raymond J. Struyk, "The Urban Institute Housing Model: Application to Chicago, Illinois," Working Paper no. 208-26 (Washington, D.C.· Urban Institute, 1975), and *The Web of Urban Housing* (Washington, D.C.: Urban Institute, 1975); DeLeeuw and others, "The Market Effects of Housing Policies," Working Paper 208-23 (Washington, D.C.: Urban Institute, 1974); and Struyk, "A Simulation Model of Urban Housing Markets in Developing Countries," Working Paper no. 5062-1 (Washington, D.C.: Urban Institute, 1976).

starts, and housing prices. The model therefore traces the behavior of the housing market over relatively long periods of time and is run over ten-year intervals. This strategy implicitly takes into account the durable nature of urban stock, since it is reasonable to expect changes to take place over a decade.

The model represents the population of an urban area by a relatively small number—between thirty and fifty—of "model" dwellings; it represents market behavior by the interaction of these model households and dwellings over a ten-year interval. Households choose among all available dwellings, taking into account their income and family kind, dwelling quality and price, and neighborhood characteristics. Owners of existing dwellings decide how much to upgrade or depreciate their dwelling in order to maximize expected profits. Builders are prepared to construct new dwellings demanded at a price that covers costs and a normal profit. Governments can affect market outcomes in a wide variety of ways, ranging from complex subsidy formulas to rent controls and outright prohibition of certain dwellings in certain locations.

The model has been applied to six different metropolitan areas to obtain key parameters of household and landlord behavior. An attempt has also been made by Struyk[3] to apply the model to the housing market in a hypothetical city of a developing country. It is recognized that conveying a credible description of urban housing markets requires that the model account for a host of details and characteristics of these markets that cannot be captured in a simple way. For example, there are differences between new stock and existing stock in the same way that there are differences between housing submarkets of different qualities and locations. Particular attention is devoted to two characteristics of housing that make it different from other goods: durability and neighborhood effects.

The model dwellings resemble the actual housing stock at the start of the decade by location and level of housing services. Model households resemble the actual population of a metropolitan area in income distribution and in demographic and racial composition. The model, in effect, is a description of the process by which households and dwellings become matched within an urban area. The other agents in the model are the building industry, which supplies the new dwellings, and the government, which can regulate the

3. Struyk, "A Simulation Model."

housing market in a variety of ways. Thus there are four economic agents participating in the model: (a) households that decide which dwellings to select; (b) owners of existing dwellings who decide what quantities to supply at what cost; (c) builders who construct new dwellings; and (d) government that constrains or facilitates outcomes in various ways.

The time frame of the model, as stated, is a decade: that is, the model predicts the situation of end-of-decade households and dwellings, given the housing stock at the start of the decade and given accessibility costs, construction costs, and certain other information during the decade.

A solution to the model is a set of locations, quantities, and prices with which everyone is satisfied and has no incentive to change.

The model is firmly based on past theoretical work and extends it by bringing various strands together in a comprehensive housing model. The authors of the model identify three strands of past work as of particular relevance:

Transportation cost–housing tradeoff: The model draws on Alonso, Muth, and their followers, all of whom stress the relation between transportation cost and housing demand and the implications for rent gradients and location patterns.[4]

Filtering: The second strand of the literature that is followed deals with quality differences among existing dwellings, their changes, and the occupancy patterns accompanying them. This literature is reviewed in Grigsby, and is more recently developed in Ohls and Sweeney.[5]

Neighborhood effects: The development of the literature on the effects of neighborhood racial composition and wealth on household choices and household prices has been influential in the inclusion of zonal wealth and racial composition in household utility functions in this model. The work of Becker, Bailey, and Schelling

4. See especially W. Alonso, *Location and Land Use: Toward a General Theory of Land Rent* (Cambridge, Mass.: Harvard University Press, 1964), and Richard F. Muth, *Cities and Housing* (Chicago, University of Chicago Press, 1969).

5. William Grigsby, *Housing Markets and Public Policy* (Philadelphia. University of Pennsylvania Press, 1957; James C. Ohls, "Public Policy toward Low Income Housing and Filtering in Housing Markets," *Journal of Urban Economics,* vol. 2, no. 2 (April 1975), pp. 144–71; and James L. Sweeney, "A Commodity Hierarchy Model of the Rental Housing Market," *Journal of Urban Economics,* vol. 1, no. 3 (July 1974), pp. 288–323.

has been influential in formulating theory on the question of race and that of Ellickson and Hamilton on the treatment of wealth and fiscal burdens; Kain and Quigley and Schnare have provided empirical measures of the effects of neighborhood attributes on housing prices.[6]

Thus, the construction of this model shows admirably the link between explanatory models of the relatively simple type and policy-oriented models that typically tend to be rather more complex. It is distinguished from other such models by the following characteristics: (a) the scope of the ideas incorporated into the model, ranging from accessibility-housing cost tradeoffs to constraints on market outcomes, imposed by the existing stock to the influence of neighborhood, race, and wealth on household choice; (b) the relatively small size of the entire model compared with other simulation models; (c) the empirical application of exactly the same theoretical model to six areas and an attempted extension to a city in a developing country; and (d) the range of housing, income, and land-use policies that can easily be analyzed within the framework of the model.

The model

The model structure will now be described in semitechnical detail. Space does not permit a full description and a nontechnical description would not be useful here. Close attention is paid to the meanings of particular parameters used.

The model is driven by two kinds of behavior: utility maximization by households and profit maximization by owners of existing dwellings:

6. Gary S. Becker, *The Economics of Discrimination* (Chicago: University of Chicago Press, 1957); Martin J. Bailey, "Notes on the Economics of Residential Zoning and Urban Renewal," *Land Economics,* vol. 35, no. 3 (August 1959), pp. 288–90; T. J. Schelling, "On the Ecology of Micro Motives," *The Public Interest,* no 25 (Fall 1971), pp. 59–98; Bryan Ellickson, "Jurisdictional Fragmentation and Residential Choice," *American Economic Review,* vol. 61, no. 2 (May 1971), pp. 334–39; Bruce Hamilton, "Property Taxation's Incentive to Fiscal Zoning," in *Property Tax Reform,* ed. G. Peterson (Washington, D.C.: Urban Institute, 1973); John F. Kain and J. M. Quigley, "Measuring the Value of Housing Quality," *Journal of the American Statistical Society,* vol. 65, no. 330 (June 1970), pp. 532–80; and Ann Schnare, "Externalities, Segregation and Housing Prices (Washington, D.C.: Urban Institute, 1974).

HOUSEHOLD UTILITY FUNCTIONS. Each household maximizes utility:

$$(126) \qquad U_{ij} = H_{ij}X_{ij}Z_1Z_2Z_3 \ldots . \quad \ldots . \quad \ldots . ,$$

where U_{ij} is the utility of the jth dwelling to the ith household; H refers to housing services; X to other goods and services; and Z_1, Z_2, and Z_3 are neighborhood or zone characteristics.

Now:

$$(127) \qquad H_{ij} = [Q_j - \gamma_1\alpha_i M_i/P_n]^{\alpha_i} \quad \text{and}$$

$$(128) \qquad X_{ij} = [(M_i - Q_jP_j) - \gamma_1(1 - \alpha_i)M_i]^{1-\alpha_i},$$

where Q_j is quantity of housing services offered by dwelling j; γ_1 is a parameter affecting the degree to which households will alter their housing choice in response to a price discount; α_i is a parameter expressing the strength of housing preferences as against preference for other goods for households of kind i; M_i is household i's model income (after adjustments for taxes and transfers); P_n is the price per unit of service of newly constructed dwellings; and P_j is the price per unit of dwelling j.

Model income is:

$$(129) \qquad M_i = (M_i^a)^{.6}(\bar{M})^{.4},$$

where M_i^a is the actual annual income of household i and $\bar{M}$ is the median income of all households of kind i. The exponents .6 and .4 smooth the income distribution sufficiently to validate the unitary income elasticity assumption implicit in the utility function.

Note that if $\gamma_1 = 0$, the utility function reduces to a utility function of the Cobb-Douglas kind. Here other goods are merely represented as income minus housing with the implication that the price of other goods is normalized at 1. $\gamma_1 > 0$ has the effect of positing a minimum level of housing services; only an excess over it gives positive utility. These minimum levels vary with income, unlike a constant term that would be found in a Stone-Geary utility function. The value of γ_1 is the critical determinant of the degree of substitutability in demand among housing submarkets. A high γ_1 raises the minimum level of housing and other goods that are acceptable and therefore narrows the options available to a household.

The neighborhood characteristics are represented by Z_1, Z_2, and Z_3; each has its own functional form:

$$(130) \qquad Z_1 = (200 - T_j)^{.5+\alpha_i-\alpha_1},$$

where Z_1 represents accessibility; T_j average travel time in a month; α_i is once again the strength of housing preferences for household type i; and α_1 is the value of α_i for households of type 1 (white, nonelderly families). $200 - T_j$ is an approximation of the monthly leisure time available to a worker in zone j. The exponent .5 is based on studies suggesting that people value leisure time at about half their wage rate. Z_2 represents the average net rent (gross rent less operating costs) of a zone relative to the average net rent in a standard metropolitan statistical area (SMSA). More precisely:

$$Z_2 = \left[\frac{\{(P_j - P_o)\,Q_j\}_{\text{zone}}}{\{(P_j - P_o)\,Q_j\}_{\text{SMSA}}}\right]^{.01\gamma_2}, \tag{131}$$

where P_j is the price per unit of housing services; P_o is minimum operating costs per unit; and Q_j is quantity of housing services. The numerator is the zonal average and the denominator is the SMSA average. γ_2 is a parameter expressing willingness to pay in exchange for living in a wealthy zone. The higher γ_2, the higher the utility people get by living in a wealthy neighborhood. Consequently, they are willing to pay more for this privilege. Finally, Z_3 is a zonal characteristic capturing the effect of racial composition of neighborhoods on utility:

$$Z_3 = R_{ij} + [1{,}000/(100\,\gamma_3 + 1)], \tag{132}$$

where R_{ij} is the proportion of households in the zone of dwelling j belonging to the same racial group as household i; and γ_3 is a parameter expressing the strength of preferences for racial homogeneity.

The larger γ_3, the more sensitive utility is to changes in R_{ij}. For example, if $\gamma_3 = 0$, Z_3 can vary from only 1,000 to 1,001 while if $\gamma_3 = 1$, Z_3 can vary from 100 to 101, a much larger percentage variation.

Thus, the utility function takes account of utility gained by households due to quantity of housing services, quantity of other goods, accessibility of neighborhood they live in, average wealth or income of neighborhood, and racial composition of neighborhood, each expressed in appropriate functional form.

SUPPLY FUNCTIONS FOR EXISTING DWELLINGS. The supply curve for existing dwelling j is specified as follows:

$$Q_j = \left[\beta_1 + \tfrac{2}{3}\beta_2\left(\frac{P_j - P_o}{P_e}\right)\right] Q_o, \tag{133}$$

where Q_j is the level of housing services currently provided by dwelling j; Q_o is the level provided ten years ago; P_j is the price per unit of housing services offered by dwelling j; P_o is operating costs per unit; and P_c is capital costs per unit of service for a new dwelling. All prices are in flow terms: that is, on a monthly basis. β_1 and β_2 are parameters to be determined empirically.

This expression is derived from profit maximization subject to a production function. The profit maximization procedure is actually maximization of expected profits. This embodies within it the behavior of the landlord over a long time period. β_1 can be interpreted as some measure of depreciation over a decade. β_2 is a parameter in the production function and determines the slope of the supply function.

Solution

In essence, once plausible parameter values are given for $(\gamma_1, \gamma_2, \gamma_3)$, β_1 and β_2 and the α_is, the model can be used to simulate the housing market.

The key exogenous variables in the model are:

- P_n, the price of new housing, which includes P_c and P_o, the capital cost and operating cost components
- P_o, the operating cost component, which is very important for it affects strongly the number of removals from the housing stock
- Q_m, the minimum quantity standards for new housing
- Y_i, the household incomes
- Q_o, the quantity of initial housing services
- T_j, the travel time associated with each zone

The endogenous output of the model comprises:

- prices and quantities for each existing and new dwelling
- number of new dwellings and number of removals
- assignments of households to dwellings, zonal averages of prices, quantities, incomes, and racial proportions

Zonal averages of net rents and of racial proportions are fed back into the model and affect household behavior as explained earlier. Ultimately, all the endogenous output is determined by exogenous variables, by government regulations, and by the behavioral parameters for households and owners.

A model solution is said to be found when four conditions are met. First, each household occupies that dwelling unit which provides the maximum utility to the household among all available choices. Second, each dwelling unit can be occupied only by one household or be vacant. Third, total profits over all dwellings must be maximized subject to the above two conditions being satisfied. Fourth, change in racial proportions in each zone are minimized over the simulation period but are subject to the three preceding conditions.

The solution algorithm model is quite complex and is essentially a systematic procedure for searching over vectors of possible dwelling prices in order to find a solution. Many solution algorithms for simulations of market behavior approximate actual market processes in a Walrasian manner. Thus, prices and quantities are often seen to adjust successively to find a market-clearing solution. In the solution algorithm here, this is not the case; only the final solution reflects the outcome of all market transactions. Although the solution algorithm is too complex to be described here, it may be intuitively described as sliding up and down the housing supply curve in search of the optimal solution that satisfies the four conditions mentioned above. According to the authors, it is always the case that the first two conditions are met, but they are less sure of the latter two. Sometimes this occurs because there is no feasible solution, as when there is some household too poor to afford any of the price-quantity combinations offered.

Data requirements and inputs

One of the features of this model is its great flexibility. It can be run with a range of data quality. On the one hand, the estimation of model parameters could be done after the availability of extensive and detailed data. The estimation of each one of the parameters—α, S, γ_1, γ_2, γ_3, β_1, β_2—needs detailed econometric work and each can become a research project by itself. Similarly, the construction of model households, household incomes, model dwellings, travel times, and so forth all need data that are readily available in the U.S. census and Federal Housing Administration surveys. Such data are not, in general, so readily available in cities of developing countries. On the other hand, all the parameters and exogenous inputs could be educated guesses and the model could still be used for policy purposes. These guesses could then be successively improved. For example, rough notions of income distribution can be used to construct model households; similarly, model dwellings can be constructed from impressions of the variety of existing dwellings. This can be done primarily because the model is small in size: it is difficult to construct plausible data when the model requires thousands of households, but it can be done by hand when it requires only between thirty and fifty. Indeed, the entropy maximization models use similar averages and then generate the distributions in an attempt to be more realistic. Here, no attempt is made to squeeze more information from the data than exists and averages remain averages.

Outputs

The model is quite clear in identifying the parameters, the exogenous variables, and the endogenous variables. Ideally, the parameters should be estimated and should reflect the behavior of households. The exogenous variables are the ones that are played with in order to simulate government policies. For example, a housing allowance program would be simulated by changing the Y_is, while a construction subsidy would be simulated by changing the P_n (the price of new households). Quantity constraints are subsumed by Q_m. The model gives us the effect of different variables by forecasting changes in endogenous variables.

Evaluation

The Urban Institute housing model has been calibrated for six metropolitan areas of the United States: Durham, North Carolina; Washington, D.C.; Chicago; Portland (Oregon); Pittsburgh; and Austin, Texas. Except for a few instances, parameter values for different cities are not very different. This gives substance to the belief that they do reflect behavioral relationships. The forecasting abilities of the model have been tested by using the parameter values of the other four cities on Washington and Chicago with encouraging results: approximately 18 percent error in prediction. The composite results have been used to simulate urban areas for policy testing. The simulations allow for variations in policies, kinds of cities, and behavioral parameters. Policy experiments have been successfully conducted to estimate the effects of four kinds of housing policy: housing allowance, income redistribution with no earmarking, construction subsidies, and changes in minimum standards of new construction. Thus, the model can be used to explore the results of a rather wide variety of policies—both those aimed at affecting the demand side directly or the supply side. More recently, tenure choice has been introduced into the model.

Application in a developing country

The suggestion here is not that the Urban Institute housing model is ripe for application to a city in a developing country. Indeed, it is based too much on the character of the U.S. housing market for such a transplantation. In a pilot study, Struyk demonstrated some of the ways in which the model can be applied to a hypothetical city in a developing country.[7] He retained the assumption that the housing market is basically competitive, but he put more stress on segmentation. He also allowed for different transport modes, but these vary mechanically by income level. Each model household is assigned a travel cost according to its income level which determines the mode used. The extent of site services provided (roads, water, and so forth) was introduced as an additional neighborhood characteristic. In his simulations, the role of government examined included legislated changes in mortgage rates, changes in minimum and maximum quality of housing, and imposition of maximum quantity controls on construction of new housing.

Although this first attempt at extending the model to a city in a developing country is an admirable one, illustrating the flexibility of the model and limited data needs, its institutional structure remains heavily that of the United States and particularly that of a relatively free housing market, albeit a segmented one. This extension is particularly good at demonstrating how the model can be run on (usually) available data as a first approximation.

Shortcomings

Lest an impression is given that the Urban Institute housing model is ideal, its shortcomings need to be spelled out. First, as its name implies, it is restricted to a description of the housing market and has nothing to say on other aspects of urban structure such as employment location and other nonhousing land use. Some account of

7. Struyk, "A Simulation Model."

the employment interaction between location and residence is taken through the inclusion of travel times and accessibility costs in the household budget equation and in the derivation of neighborhood characteristics. Second, locational detail is restricted to the division of a city into between five and ten zones. Given the computational complexity of the model, this is difficult to improve on. Third, the model gives results at the end of ten-year periods but supplies no information on the path taken to reach those results. Although this depiction is a realistic one in that it takes into account the durability of urban structures, as a policy simulation device it falls short on providing results seen as relevant by policymakers who usually have much shorter time horizons. Fourth, according to the authors the uncertainty surrounding the supply parameters is uncomfortably large. Fifth, the solution procedure is complex, expensive, and rather opaque. A typical run of the model costs about $120 to run with approximately forty model households. Increasing the number of these households to about seventy-five would apparently quadruple costs. Thus, expanding the model is expensive. It took about ten professional man-years over a period of about three years to make the model operational at a total cost in excess of $600,000. These man-years have, moreover, been those of some of the best people in the field. Each city application cost about $20,000–40,000 and took a period of about four months to put into effect. This presumes the existence of a U.S.-type data base that can readily be adapted to the model. Struyk's simulation of New Urbania, however, a hypothetical city in a developing country, shows that it is possible to run the model on data that are currently available in the developing countries.

Overview

The recommendation here is that the approach embodied in this model—the logical structure and its policy applicability—is a good one to follow. It was built from a base of clear objectives; explicit recognition was made of the theoretical bases used. Such clarity is rarely found in urban models. Its ten-year span is also a sensible one in view of the durable aspects of city structure. A user must, however, beware of the temptation to transfer the model as it is to a city in a developing country for it is firmly rooted in the U.S. experience and institutional system. Its methodology is what should be transplanted.

Models of Housing Demand at a Disaggregated Level

Paradoxically, fruitful policy-oriented models are likely to be at higher degrees of aggregation, whereas explanatory models may have to be disaggregated. Policymaking is time bound, and policies are characteristically made at an aggregated level. People's behavior varies in all kinds of ways according to socioeconomic characteristics, family cycle concerns, and the like. It is not really surprising, therefore, that models which study this behavior are likely to be more detailed than those which prescribe or test policies. Moreover, the results of detailed explanatory models make it easier to feed policy-oriented models with more realistic data, parameters, and coefficients that otherwise are often merely educated guesses.

Reviewed here is some analytical or explanatory work that is regarded as embodying fruitful approaches. The first is a study of housing demand in Reading, England, by Patricia Apps; and the second is one of housing demand and residential location by William Wheaton.[8] The structure of both models is based on the economic theory of market demand, which states that the individual's consumption patterns are the result of his preferences, income constraints, and housing prices. They are also based on the assumption that differences in preferences may be caused by various factors such as social status, household size, and the stage in life cycle of the household. Apps was more concerned with measuring housing demand per se, whereas Wheaton focused on residence location. Both took explicit account of the fact that individuals and households differ from each other and that their choices reflect these differences.

Basically, Apps' model predicts changes in housing demand resulting from changes in factors such as number of households, income, social status distribution, and size or composition of household.

8. Patricia Apps, "A Residential Model: 1. Theory," *Environment and Planning,* vol. 5, no. 5 (September 1975), pp. 619–32, "A Residential Model: 2. Implicit Prices for Housing Services," *Environment and Planning,* vol 5, no. 6 (November–December 1973), pp 705–19, and "A Residential Model: 3. Demand Equations for Housing Services," *Environment and Planning,* vol. 6, no 1 (January–February 1974), pp. 11–32; and William C. Wheaton, "Income and Urban Location" (Ph.D. dissertation, University of Pennsylvania, 1972).

The analysis is conducted at two levels. At the first level, housing is seen as a single commodity, and a set of income-consumption (or Engel) curves are derived for each household type. At the next level, there is a two-dimensional set of Engel functions: one for different kinds of households, the other for different housing commodities. Working through the conventional consumer theory of utility maximizing households with budget constraints, the Engel curve is obtained:

$$C_h = \Pi_h X_h = C_1(M, \Pi_1, \ldots \Pi_n), \tag{134}$$

where C_h, the allocation of expenditure on housing (commodity X_h) with price Π_h, is specified for household i of kind k; M is household income; and Π_i, $i = 1, \ldots, n$ are the relative prices of other commodities. The form of the function for household kind k for different incomes reveals preferences for that household type.

At the next level, utility is a function of housing characteristics considered as a set of commodities. We then have, for household j of kind k:

$$\begin{aligned} U = {} & U_1(X_{11}, X_{12} \ldots X_{1m}) + U_2(X_{21}, X_{22}, \ldots X_{2m}) \\ & + \ldots + U_h(X_{h1}, X_{h2}, \ldots X_{hm}) + \ldots \\ & U_n(X_{n1}, X_{n2}, \ldots X_{hm}), \end{aligned} \tag{135}$$

where U_i, $i = 1$ to n is the utility provided by the ith group of commodities; and U_h is the utility provided by housing, with $X_{h1} \ldots X_{hm}$ being the m housing characteristics.

The second level of Engel curves for housing consumption and income are then:

$$\begin{aligned} C_h &= \sum_{f=1}^{m} \Pi_{hf} X_{hf} = \sum_f C_{hf} \\ C_{hf} &= C_{hf}(C_h, \Pi_{h1}, \Pi_{h2}, \ldots, \Pi_{hm}), \end{aligned} \tag{136}$$

where Π_{hf} is the price of housing commodity f; X_{hf} is the amount of housing commodity f consumed by household j of kind k; and C_{hf} is the expenditure on housing commodity f, now expressed as a function of total housing expenditure and the relative prices of different housing commodities.

Given data on total housing expenditures and characteristics of housing such as (a) location variables including employment accessibility, shopping accessibility, and quality of neighborhood schools; (b) dwelling characteristics such as age of structure, type of structure, and condition of structure; and (c) space of dwelling, equation (136) can be estimated by multiple regression methods to obtain Π_{h},s (the implicit prices of different housing services).

Finally, these prices having been obtained, demand functions for each of these characteristics can be estimated using household kinds as the independent variables.

This procedure reveals household consumption patterns for different housing characteristics subject to income and price constraints. A by-product of this model is that the system is easier to handle because the number of housing characteristics is much smaller compared to housing kinds. This is particularly true of developing countries where the variety of housing kinds is even greater—from straw shacks to tall apartment buildings.

Apps was quite successful in obtaining the housing characteristic prices from careful regression analysis, but she was not able to obtain good demand functions in the next stage. There is the normal problem of linear assumptions: housing characteristics are assumed to be noninteracting. Apps speculated that deficiencies in the data were causing her problems.

Apps' work is only one of a number of studies that have dealt with hedonic price estimations as reviewed by Ball.[9] A recent such study in the United States, in New Haven, was by King.[10] One theoretical problem in this kind of work, and one which may have caused Apps difficulty in obtaining demand functions, is that there is an identification problem between the supply and demand side. The observations used are of characteristics and bundle prices, whereas the hedonic prices are unobserved. In a sense they are the transaction prices, and demand and supply are difficult to disentangle. This argument is put forward in some detail by Sherwin Rosen.[11] Notwithstanding these difficulties, this strand of work is quite informative and is regarded here as a fruitful approach.

9. Michael J. Ball, "Recent Empirical Work on the Determinants of Relative House Prices," *Urban Studies*, vol. 10, no. 2 (June 1973), pp. 213–31.

10. A. Thomas King, "The Demand for Housing: Integrating the Roles of Journey to Work, Neighbourhood Quality and Prices," in *Household Production and Consumption*, ed. Nestor Terleckyj (New York: Columbia University Press, 1975).

11. Sherwin Rosen, "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, vol. 82, no. 1 (January 1974), pp. 34–35.

As should be evident from the description of this model, it is extremely data intensive. Apps used a good 10 percent sample survey of Reading and a host of other data sources to build all the indexes of housing characteristics. The appeal of this procedure is that it reveals the preferences of consumers. It is this kind of knowledge that is necessary for the disaggregation of results obtained from models of the Lundquist-Andersson kind. In particular, some appreciation of the differences in behavior of different income groups is obtained. This information is necessary for robust planning so that plans do not get subverted by disgruntled groups whose preferences have been grossly violated—as has happened often in relocation of low-income groups in various cities. The question of data availability remains, however, though detailed consumer household surveys are now being made with increasing frequency.

A related approach that combines Lancastrian consumer theory with Alonso's bid-rent theory was developed by Wheaton in an attempt to estimate utility functions directly.[12] As in Apps, housing demand is viewed as demand for subgoods or characteristics. Housing is therefore a hedonic good in the spirit of Rosen's definition of hedonic goods: houses do not possess final consumption attributes but rather are purchased as inputs into self-production functions for ultimate characteristics.[13] This is a particularly appropriate way of looking at the vast range of abodes in developing countries. Indeed, as has been suggested by Sudra and Turner, shelter should replace houses in the terminology and should be considered a process, not a good.[14] The components of shelter are combined by the household to yield utility.

Wheaton started with consumer i's utility function:

$$U_i = U_i(S, Z), \tag{137}$$

where S is a vector of shelter attributes:

$$S = S(X_1, X_2, \ldots X_n), \tag{138}$$

and Z denotes all other goods. The budget constraint may be written:

$$M = C + T + R, \tag{139}$$

12. Wheaton, "Income and Urban Location."

13. Rosen, "Hedonic Prices and Implicit Markets."

14. Tomás Z. Sudra and J F. C. Turner, "Housing Priorities Demand of Lower Income Households in Mexico City," Urban and Regional Reports no. 76-5 (Washington, D.C.: World Bank, June 1976; processed).

where M is household income; T is travel expenditure; R is rent; and C is expenditure on all other things.

Regarding the price of Z as unity, equation (137) may be rewritten:

$$U_{ij} = U_i(S_{ij}, M_i - R_{ij} - T_{ij}), \tag{140}$$

which is the utility of household i at location j, written in partially indirect form. Given a pattern of rents, the consumer selects some S_j that maximizes income. This is standard consumer theory. Now, to derive consumers' bid rent functions, write:

$$R_{ij} = M_i - T_{ij} - U_i^{-1}(S_j, U_i^0), \tag{141}$$

where

$$U_i^0 = U_i(S_j, M_i - R_{ij} - T_{ij}) \tag{142}$$

is utility at some specified level and the solution is analytically defined by U_i^{-1}. The problem at hand is to find a set of U_i so that each unit location j goes to the highest bidder and so that each consumer has the maximum bid over one and only one unit. The outer envelope of bid rents at equilibrium-utility levels is the market rent surface.

In summary, the classical consumer approach to hedonic goods takes rents and allows consumers to decide on location according to utility maximization. Equilibrium is achieved by adjusting rents. The bid approach takes utility levels and allows owners to decide on location so as to maximize rents. Equilibrium is achieved by adjusting utility levels.

To proceed further, two critical assumptions have to be made. First, households can be aggregated such that households with similar socioeconomic characteristics can be grouped and assumed to have similar tastes within the group. Second, the shelter market is at a short-run equilibrium.

The next step is to find an appropriate functional form for the utility function. Wheaton chose the C.E.S. form:

$$U = -\sum \beta_i X_i^{-\alpha_i} - (C + \epsilon)^{-\alpha_0}, \tag{143}$$

where ϵ is the stochastic term. This can be transformed to:

$$M - R - T = (-U - \sum_i \beta_i X_i^{-\alpha_i})^{-1/\alpha_0} - \epsilon. \tag{144}$$

This function was chosen because it is not homothetic, it does not have equal elasticities of substitution, and because it does have indifference curves that asymptote to minimum consumption levels. Wheaton regarded this last property as quite important in the selection of functional form. For the poor in developing countries, the minimum level of utility necessary from shelter may be so low as to warrant neglect. An equation such as (144) can be estimated by nonlinear methods, and two parameters, α_i and β_i, are obtained for each shelter attribute X_i. The level of utility is embodied in the constant term. Wheaton estimated such functions from data on San Francisco, but the nonlinear estimation method was quite complex.

Such an approach can be taken to model consumers' preferences for residential location and shelter. Apart from estimation, the critical problem in such an approach is the definition of homogeneous groups and the finding of data that are sufficiently intensive. Each group must have enough observations to make estimation feasible. Further detailed information is required on housing characteristics, household characteristics, quantities of services, neighborhood characteristics, and household expenditure. Thus, this approach, although recommended as fruitful, suffers from data and estimation problems. Its principal virtue with regard to developing countries is that it takes into account the wide variety of preferences, incomes, and the like that exist there. The results of such models are particularly important because planners and policymakers do not generally have intuitive notions of the preferences of people outside their class and income experience. With huge slum and squatter populations, such organized information would be invaluable in cities of developing countries. Good estimates of such magnitudes as income and price elasticities of demand for housing and transportation are the kind of inputs that both the Urban Institute housing and Stockholm models need for useful operation.

5

Modeling Poor Cities: What Should Be Done?

How cities in developing countries should be modeled depends basically on the objectives of the exercise. The lessons from the experiences of the past twenty years of urban modeling in developed countries is more explicit in showing us what not to do than in what should be done. One of the main lessons learned, however, is that expectations of the capabilities of modeling should be modest at this stage. Consequently, limited objective models are more likely to be operationally useful than comprehensive models. Models should be tailored to fit the task required of them.

Much of the demand for models of urban structure comes from the need for help in the identification, design, and evaluation of projects in urban areas. Identification of projects requires models that predict or prescribe the future structure of a city in response to other changes in the economy. Design and evaluation require cost-benefit analyses, and urban models can help in tracing some relations as well as in optimization. To feed such models, explanatory or analytical models are needed to promote basic understanding of the urban process. Behavioral relationships and parameters can then be used in the policy-oriented models that may ultimately be needed.

It is the major conclusion of this review that policy-oriented models should be of the "sketch-planning" kind—that is, small and manageable—whereas the analytical or explanatory models can be larger and disaggregated. The reasons for this have been documented throughout this work. Some issues of concern are worth emphasizing, however. A useful policy model needs to have a fast turnaround time to make it more responsive to policy needs and issues. It is only then that modelers and decisionmakers can interact and test for various alternatives. Policymakers have had little involvement in models developed in the United States. It is therefore not surprising that these models have not been much used for policy purposes. Thus, if the development of policy-oriented models is now a serious goal, this major shortcoming must be remedied. It is clear, however, that models of the "sketch-planning" kind will not always be adequate for aid in the design and evaluation of urban projects. They are mainly of help in articulation and testing of gross alternatives. Once this is done, more detailed models can be used to make consistency checks and, perhaps, to trace more detailed relationships.

The Stockholm model and the Urban Institute housing model are two approaches that meet some of these requirements. They are both small and manageable, though mathematically complex. The Stockholm model attempts to have a structure conducive to planner-modeler interaction and is therefore a step in the right direction. It was also developed with fairly specific objectives. Its policy applications were in helping gross decisions such as whether to build or not to build a bridge. That is a realistic approach in the sense that policy decisions are usually about large investments rather than about detailed residential locations. Policymakers are interested in gross impacts rather than in disaggregated allocation. This view of the policy-making process is more relevant in countries not having command economies, as is the case for almost all developing countries.

The Urban Institute housing model is based more on behavioral relationships, but it uses the idea of telescoping a large system into a small, more manageable, and comprehensible one. Its data requirements are flexible in that it can be operated on educated guesses as well as on hard data. Thus, in developing countries, where speed in decisionmaking needs to be encouraged and where data are scarce, this kind of model can be valuable when used in an incremental fashion. Systematic policy planning can be begun with rough data and the model successively refined as better and better data become available.

One other aspect of the Urban Institute model deserves comment. It seeks to be a general behavioral model so that the same structure can be calibrated for different cities. It is not self-evident that such an approach can work across continents and cultures. This needs more research on the explanatory model side and should be conducted concurrently. The presumption here is that within the United States, where the institutional structure is essentially common across cities, this is not an unjustified approach. If a model is to be useful, it is important that attention be given to the particular institutional structure of the country concerned. Thus, adequate account must be taken of the larger public sectors in developing countries and of the consequent constraints on the private market. Similarly, the higher segmentation of markets in developing countries has to be recognized. Local nationals can be expected to be more conversant with such institutional differences than are expatriates. Thus, it is suggested that local participation in modeling is of the utmost importance. This should aid in interaction with policymakers as well.

Urban models should be seen as a process rather than as products. The use, for example, of a model of the Urban Institute type, which can involve successive improvements in data as well as model, is clearly a process. This process is educational for modelers as well as policymakers. If viewed this way, modeling will suffer less from short time horizons. This makes it even more important that local nationals be involved in modeling efforts.

I have mentioned that policy models require assistance from explanatory or analytic models. This help may be from the specification and estimation of behavioral relations or it could be data oriented. Continuing research aimed at achieving greater understanding of existing patterns as well as of future changes is necessary for the development of good policy models. Such research might help in appreciating what cities are for and why they exist. The Apps and Wheaton models, which are very disaggregated, have been suggested as fruitful approaches to understanding the behavioral relations implicit in housing. In such efforts it is almost inevitable that extremely disaggregated data are needed because the interest is in the whys and wherefores of behavior. This is affordable in this context because the usefulness of such modeling is not time bound. Specifically, the kind of information severely lacking in developing countries is of the following nature.

Transport

Some ideas on transport in cities in developing countries were discussed in the Introduction. Detailed information on the following are needed to continue that discussion.

Analysis of movements

The hustle and bustle characteristic of a city in a developing country has to be systematized. Information is needed on the origin-destination pattern, traffic variability at different times, and, as a result, the importance of the journey to work.

Costs of different modes

Detailed information on operating (and fixed) costs of different modes—walking, bicycles, automobiles, buses—is needed. In addition, the income classes that use these modes have to be determined in order to calculate time costs. Using some (explicit) notion of efficiency, is the prevailing pattern efficient?

Congestion

Movement in central areas of cities in developing countries is often painfully slow. To what extent is this relevant as a transport cost?

Such information is crucial to the understanding of location patterns in these cities. Various conjectures concerning the movement of people were offered in the Introduction; they can be tested with the help of such information. Mean values, cost parameters, and even preference patterns can be derived to be used as inputs in operational models.

Housing

Again, certain kinds of information are required for modeling cities in developing countries.

Production

Considerable information is already available on production. Further quantitative information is needed on the different segments of the housing sector to be able to construct adequate representations of the supply side of this sector.

Preference structures

This is a rather muddy area of research that needs further theoretical as well as empirical clarification. It is necessary to disentangle the effects of incomes from those of tastes in people's choice of housing. The Apps and Wheaton kinds of work are examples of some useful approaches. For a more intuitive grasp of housing preferences and patterns as they change with income and time, Homer Hoyt's studies of residential neighborhoods in the United States provide good examples.[1]

Housing-transport tradeoffs

Studies of preferences and income variation with regard to housing will also provide information on the effect of transport cost on location choices. It is clear from existing models that different urban location patterns occur depending on the elasticity of demand for space and the imputed cost of time. More measurement in this area would provide valuable insights in the design of urban economic models.

Housing for the poor has become a much discussed problem. In most cities in developing countries a majority of the population lives in slums, squatter settlements, and shanty towns. This housing is often regarded by planners as deficient and as a situation to be remedied. Such housing comprises a significant magnitude, and hard information on it is not yet available. Such information is therefore crucial to the good design of the housing sector in urban models.

Industry and Employment

The following areas require study for relevant material pertaining to industry and employment.

The informal sector

Although more and more information on the content of the informal sector is slowly being made available, little progress is being made on its spatial characteristics. It would be useful to know the

1. Homer Hoyt, *The Structure and Growth of Residential Neighborhoods in American Cities* (Washington, D.C.: Government Printing Office for the U.S. Federal Housing Administration, 1939); and *Where the Rich and the Poor Live: The Location of Residential Areas Occupied by the Highest and Lowest Income Families in American Cities* (Washington, D.C.: Urban Land Institute, 1966).

kind of movements of goods and people it generates. What degree of flexibility does it have to the urban structure? To what extent is it innovative in overcoming traditional spatial problems of transport costs, for example?

Small-scale industry

Bergsman and colleagues have performed a valuable service in the United States by attempting to find clusters of activity that always seem to go together.[2] A similar analysis of patterns in developing countries would provide a great amount of interesting information on the nature of economic activity in these cities. The connection between activities within these clusters would provide clues on the reasons for agglomeration.

Links with the hinterland

It has often been suggested that the degree of primacy of large cities in developing countries is higher than elsewhere. This, coupled with the vast literature on the dual economies of these countries, would give some impression of self-sufficiency in these cities. In particular, models of the Lowry kind and others often take some basic export sector as given and as the real driving force in the model structure; it is therefore important to identify the nature and extent of linkages that cities in developing countries have with their immediate hinterland and the more distant areas in the country.

The informal sector was mentioned in the Introduction. Its existence, it has been suggested, makes many market activities in developing countries different in nature from those in Western cities. Its employment patterns have a large effect on location patterns—both residential and employment. Thus, information on its spatial characteristics is essential to the design of useful urban models.

Each of these is a substantive research project in itself. Moreover, as outlined in the Introduction, account must be taken of the differences among cities within the developing world. Information gathering

2. J. Bergsman, P. Greenston, and R. Healey, "The Agglomeration Process in Urban Growth," *Urban Studies,* vol. 8, no. 3 (October 1972), pp. 263–88; and "A Classification of Economic Activities Based on Location Patterns," *Journal of Urban Economics,* vol. 2, no. 1 (January 1975), pp. 1–29.

must be done within some kind of framework to be most useful. That is merely another term for modeling.

The modeling agenda suggested above is eminently practical. It involves concurrent investment in the accretion of knowledge as well as in policy aids. Some of this research is messy but necessary if the present understanding of poor cities is to be expanded in order to articulate sensible policies. Given the current state of the art, it is likely to be more useful to investigate and model particular aspects of urban phenomena rather than to attempt to make large, all-purpose models. Much can also be learned from the ways in which cities grow and change autonomously, a process that is particularly vibrant in developing countries.

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INVESTMENT IN DEVELOPING COUNTRIES has been biased, it has been alleged, toward urban areas at the expense of the countryside, but economic research may well suffer from the opposite bias. The marked lack of research into the growth and structure of cities in developing countries contrasts with the substantial body of work on agricultural and rural development.

The paucity of work relevant to urban areas in developing countries has necessitated this review of work on models that have arisen mainly in Europe and North America. *Urban Economic and Planning Models* takes as its themes the central questions: How can these methods be transferred to cities in developing countries? What are the possibilities of applying current urban modeling techniques to these cities?

Problems intrinsic to the modeling of all urban areas are considered, with special attention to difficulties encountered in building models of cities in poor countries. Several varieties of model are evaluated on the basis of their usefulness as predictive and planning tools and in light of the rapidly changing conditions of these cities. Some productive understandings emerge about the modeling of poor cities.

Rakesh Mohan is an economist in the Urban and Regional Economics Division of the World Bank.

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