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LINKAGE Technical Reference Document

Version 6.0

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Abstract. This paper provides the complete specification of the LINKAGE Model. The latest version of the model is based on Version 6.0 of the GTAP dataset and has been used most recently to assess global free trade in the context of a future round of multilateral trade negotiations (see the World Bank's Global Economic Prospects 2002 and Global Economic Prospects 2004).

The LINKAGE Model is a global dynamic computable general equilibrium model (CGE) with a 2001 base year. In its standard version, it is a neo-classical model with both factor and goods market clearing. It features three production archetypes—crops, livestock, and other—, a full range of tax instruments, price markups, multiple labor skills, vintage capital, and energy as an input combined with capital. Trade is modeled using nested Armington and production transformation structures to determine bilateral trade flows. Tariffs are fully bilateral and the model captures international trade and transportation costs—both direct and indirect (using iceberg trade costs). The latest version of the model also implements tariff rate quotas (TRQs). A recursive framework is used to drive dynamics, with savings-led investment and productivity. The model incorporates adjustment costs in capital markets and trade-responsive endogenous productivity.

The core of the paper is divided into five sections—an introduction, a model overview, a detailed description of each block of the model, a discussion of model dynamics, and a description of the accounting framework underlying the model.

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I Introduction

The LINKAGE Model is a recursive dynamic applied general equilibrium (GE) model.[1] Its heritage dates back to the 1980s. Earlier versions were used to study global trade reform—the Rural/Urban North/South (RUNS) model[2] and the World Agricultural Liberalization Study (WALRAS).[3] The former was initially developed at the Université Libre de Bruxelles and the latter at Stanford University. Both were integrated into separate research programs at the OECD. In the mid 1990s, WALRAS was transformed into the OECD GREEN model used to assess the impacts of greenhouse gas mitigation.[4]

Model implementation in those days still used second generation programming languages—Fortran and C, and initially only ran on mainframe computers. Data collection and transformation were a major part of any GE modeling exercise. Two major advances occurred in the 1990s. First, personal computers became sufficiently powerful to allow all model development and simulation to take place on a desktop. This was accompanied by improvements in software so that model development was done with third generation languages such as GAMS, GAUSS, and GEMPACK. The second major advance was the creation of a unified global database under the direction of Professor Thomas Hertel of Purdue University. Backed by an international consortium of agencies and universities, the Global Trade Analysis Program (GTAP) created a consistent global data set for use in analyzing international economic policy issues.[5] By the mid 1990s, both GREEN and RUNS (and their successors) had been converted to the GTAP database.

The GTAP and GAMS based version of LINKAGE was developed by a team of researchers at the OECD Development Centre in the mid-1990s. It is a direct descendant of the RUNS and WALRAS models with some specification changes as required for particular analyses or to improve specification of specific blocks. Various versions were used to study employment related trade issues[6] and different free trade arrangements.[7] In its most recent incarnation, the model—now residing at the World Bank—has recently been used to assess the impacts of further global trade reform in the context of a new round of multilateral trade negotiations.

The latest version of the LINKAGE Model—LINK6 in shorthand—is based on the most recent release of the GTAP dataset, Version 6.0 released in Summer 2004. GTAP6.0 has improvements from previous releases, and incorporates 87 countries/regions, and 57 sectors, see Annex H. The move from Release 4.0 to 5.0 led to a major expansion of the number of countries in the database. The European Union was divided into its 15 component countries and the data emerging from IFPRI's MERRISSA project was also integrated, i.e. a handful of countries from Southern Africa. Release 5.2 added many countries in Europe and Central Asia, and the release 5.3 included Albania and the Russian Federation. On the goods and services side, the new database had a more detailed breakout of the services sectors. The base year has also moved forward from 1995 to 1997. Release 6.0 added only a few countries, but re-arranged (and deleted the ROW region). Essentially the ROW region was split into regionally-based ROW regions. The new base year has been pushed forward to 2001.

LINK5 introduced some new model specifications. Labor demand specification now allows for any combination of labor to be a substitute or a complement with capital.[8] In the standard model, both of

1 For a general introduction to applied general equilibrium modeling see Derviş, de Melo and Robinson [1982], Shoven and Whalley [1984], Shoven and Whalley [1992], and Francois and Reinhert [1997].
2 See Burniaux [1987], Burniaux and van der Mensbrugghe [1991], and Goldin, Knudsen, and van der Mensbrugghe [1993].
3 See OECD [1990].
4 See Burniaux, Nicoletti and Oliveira-Martins [1992] and van der Mensbrugghe [1994].
5 See Hertel [1997]. Information is also available at www.gtap.org.
6 See Collado, Roland-Holst, and van der Mensbrugghe [1995], and van der Mensbrugghe [1998].
7 See Lee, Roland-Holst and van der Mensbrugghe [1997], and Guerrero and van der Mensbrugghe [1998].
8 The GTAP data set breaks out labor factor payments into skilled and unskilled labor.

GTAP's labor types are combined in an aggregate labor bundle that is a substitute with capital. However, the aggregation routine allows for greater flexibility. For example, GTAP's 'skilled' labor type could be designated as a complement with capital. The two new labor bundles have been designated as 'unskilled' and 'skilled', but this is for heuristic purposes only. The difference between the two labor bundles is their position in the production nest. The model has been adapted to incorporate multiple households, even though the current dataset has only a single representative household per region. The main change this entails is the specification of a more detailed income distribution block. The Armington specification for trade has been modified. Based on work by the Dutch CPB[9], the model allows for short-run versus long-run Armington elasticities, i.e. it introduces the possibility of more flexible long-term responses to changes in import prices (see Annex D). The model also allows for *iceberg* type costs in international trade. These are non-revenue generating costs, which allow for trade to expand if these costs are reduced. In terms of market structure, a price markup has been introduced over average cost. The markup is exogenous at the moment. A reduction in the markup permits assessing the impact of greater competitiveness. Another change has been the introduction of mixed complementarity (MCP) in the model specification. This is not a change in specification *per se*; it simply eliminated the use of min and max functions with the intention (or hope) that this would have better convergence properties. Finally, the dynamics has become somewhat more complex with the introduction of technological change that responds to changes in openness—as measured by the export to output ratio.

Since version 5.3 the model has a few new specification twists. The model now incorporates increasing returns to scale production technology. In the future this will allow the introduction of different market structures. At the moment only three possibilities are allowed—perfect competition, a fixed markup, or contestable markets. Another addition has been the introduction of tariff rate quotas (TRQs) as an additional trade barrier. The latter can easily be used to model standard quotas as well by allowing for the over-quota tariff rate to be prohibitively high. The functioning of labor markets has been expanded to include market segmentation, migration and unemployment. These are detailed in the labor market section. Consumption specification incorporates a transition matrix. Thus the consumption function is defined over a set of consumer goods. The latter are then produced using a "production" function mapping consumer goods to producer goods. The transition matrix has price-sensitive technological coefficients allowing for substitution across inputs.[10] The use of a transition matrix allows for more reasonable consumer behavior. Recently a new exogenous variable has been added, *WTR*, which allows for the accounting of bilateral transfers across regions and across agents. Agents include households, and the government and investment accounts.[11]

II Model overview

The LINKAGE Model is a global, multi-region, multi-sector, dynamic applied general equilibrium model. It is currently implemented in GAMS[12] and its specification is virtually free of references to specific dimensions (region, sector, or time). The model is accompanied by an aggregation facility, which is used to aggregate the extensive GTAP dataset into a tractable dataset for simulation purposes. The output of the aggregation facility is the primary input for the model. The aggregation facility also produces some auxiliary data, such as population, and the model user is expected to provide values for all key elasticities. The dynamic version of the model also requires a series of assumptions, which are to be provided independently of the aggregation facility.[13]

9 See CPB [1999].

10 Currently the transition matrix is square and diagonal, in other words consumer goods are mapped one-for-one to producer goods and the model is equivalent to the previous specification. The next step will be to produce empirically validated consumption transition matrices.

11 This variable is initially set to zero since the GTAP dataset has no bilateral transfers.

12 See www.gams.com.

13 See the LINKAGE User Manual for further details.

The remainder of this section outlines briefly the main characteristics of supply, demand, the dynamics and the policy instruments of the model.

Summary of the Model Specification

Production

All sectors are assumed to operate under cost optimization. By default all production takes place under constant returns to scale but the model allows for increasing returns to scale using fixed production costs. The latter are represented by some fixed combination of capital and labor. Marginal costs are modeled by a series of nested CES production functions, which are intended to represent the different substitution and complementarity relations across the various inputs in each sector. There are material inputs that generate the input/output table, as well as factor inputs representing value added.

Three different production archetypes are defined in the model—crops, livestock, and all other goods and services. The CES nests of the three archetypes are graphically depicted in Figures 1 through 3. Sectors are differentiated by different input combinations (share parameters) and substitution elasticities within each one of the main production archetypes. The former are largely determined by base year data, and the latter are given values by the modeler.

The key feature of the crop production structure is the substitution between intensive cropping versus extensive cropping, i.e. between fertilizer and land (see Figure 1).[14] Livestock production captures the important role played by feed versus land, i.e. between ranch- versus range-fed production (see Figure 2).[15] Production in the other sectors more closely matches the traditional role of capital/labor substitution, with energy introduced as an additional factor of production (see Figure 3). There are two separate labor bundles. One is a direct substitute with capital and the other is a complement. The user can decide how to allocate the two GTAP labor types across the two different labor bundles. In the standard version of the model, both labor types are combined in the so-called 'unskilled' labor bundle that is a substitute with capital and the 'skilled' labor bundle is empty.

In each period, the supply of **primary** factors—capital, labor, and land—is usually predetermined.[16] However, the supply of land is assumed to be sensitive to the contemporaneous price of land. Land is assumed to be partially mobile across agricultural sectors. Some of the natural resource sectors also have a sector specific factor whose contemporaneous supply is price sensitive.

The model includes adjustment rigidities. An important feature is the distinction between *old* and *new* capital goods. In addition, capital is assumed to be partially mobile, reflecting differences in the marketability of capital goods across sectors.[17]

Once the optimal combination of inputs is determined, sectoral output prices are calculated assuming competitive supply (zero-profit) conditions in all markets. (A fixed markup has been introduced in the model allowing for assessing the impacts of greater competitiveness.)

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14 In the original GTAP data set, the fertilizer sector is identified with the crp sector, i.e. chemicals, rubber, and plastics.

15 Feed is represented by three agricultural commodities in the base data set: wheat, other grains, and oil seeds.

16 Capital supply in each period is somewhat influenced by the level of contemporaneous investment if the gap size between periods is greater than 1.

17 For simplicity, it is assumed that old capital goods supplied in second-hand markets and new capital goods are homogeneous. This formulation makes it possible to introduce downward rigidities in the adjustment of capital without increasing excessively the number of equilibrium price to be determined by the model (see Fullerton, 1983).

Consumption and the closure rule

All income generated by economic activity is assumed to be distributed to consumers. A single representative consumer allocates optimally his/her disposable income among the consumer goods and saving. The consumption/saving decision is completely static: saving is treated as a "good" and its amount is determined simultaneously with the demands for the other goods, the price of saving being set arbitrarily equal to the average price of consumer goods.[18]

Government collects income taxes, indirect taxes on intermediate and final consumption, production taxes, tariffs, and export taxes/subsidies. Aggregate government expenditures are linked to changes in real GDP. The real government deficit is exogenous. Closure therefore implies that some fiscal instrument is endogenous in order to achieve a given government deficit. The standard fiscal closure rule is that the marginal income tax rate adjusts to maintain a given government fiscal stance. For example, a reduction or elimination of tariff rates is compensated by an increase in household direct taxation, *ceteris paribus*.

Each region runs a current-account surplus (deficit), which is fixed (in terms of the model numéraire). The counterpart of these imbalances is a net outflow (inflow) of capital, which is subtracted from (added to) the domestic flow of saving. In each period, the model equates gross investment to net saving (equal to the sum of saving by households, the net budget position of the government and foreign capital inflows). This particular closure rule implies that investment is driven by saving.

Foreign Trade

The world trade block is based on a set of regional bilateral flows. The basic assumption in LINKAGE is that imports originating in different regions are imperfect substitutes (see Figures 4 and 5). Therefore in each region, total import demand for each good is allocated across trading partners according to the relationship between their export prices. This specification of imports—commonly referred to as the Armington[19] specification—implies that each region faces a downward-sloping demand curve for its exports. The Armington specification is implemented using a nested CES structure. At the top nest, domestic agents choose the optimal combination of the domestic good and an aggregate import good consistent with the agent's preference function. At subsequent nests, agents optimally allocate demand for the aggregate import good across the range of trading partners.[20]

The bilateral supply of exports is specified in parallel fashion using a nesting of constant-elasticity-of-transformation (CET) functions. At the top nest, domestic suppliers optimally allocate aggregate supply across the domestic market and the aggregate export market. At the second nest, aggregate export supply is optimally allocated across each trading region as a function of relative prices.

Trade measures are fully bilateral and include both export and import taxes/subsidies. Trade and transport margins are also included; therefore world prices reflect the difference between FOB and CIF pricing.

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[18] The demand system used in LINKAGE is a version of the Extended Linear Expenditure System (ELES), which was first developed by Lluch (1973). The formulation of the ELES used in LINKAGE is based on atemporal maximization—see Howe (1975). In this formulation, the marginal propensity to save out of supernumerary income is constant and independent of the rate of reproduction of capital.

[19] See Armington, 1969.

[20] The GTAP data set allows each agent of the economy to be an Armington agent, i.e. each column of demand in the input/output matrix is disaggregated by domestic and import demand. (The allocation of imports across regions can only be done at the national level). For the sake of space and computing time, the standard model specification adds up Armington demand across domestic agents and the Armington decomposition between domestic and aggregate import demand is done at the national level, not at the individual agent level.

Prices

The LINKAGE model is fully homogeneous in prices, i.e. only relative prices are solved for. The price of a single good, or of a basket of goods, is arbitrarily chosen as the anchor to the price system. The price (index) of OECD manufacturing exports has been chosen as the numéraire, and is set to 1 in the base year and all subsequent years. From the point of view of the model specification this has an impact on the evaluation of international investment flows. They are evaluated with respect to the price of the numéraire good. Therefore, one way to interpret the foreign investment flows is as the quantity of foreign saving which will buy the average bundle of OECD manufacturing exports.

Dynamic Features and Calibration

The LINKAGE model has a simple recursive dynamic structure as agents are assumed to be myopic and to base their decisions on static expectations about prices and quantities. Dynamics in LINKAGE originate from three sources: (i) accumulation of productive capital; (ii) the putty/semi-putty specification of technology; and (iii) productivity changes.[21]

Capital accumulation

In the aggregate, the basic capital accumulation function equates the current capital stock to the depreciated stock inherited from the previous period plus gross investment. However, at the sectoral level, the specific accumulation functions may differ because the demand for (old and new) capital can be less than the depreciated stock of old capital. In this case, the sector contracts over time by releasing old capital goods. Consequently, in each period, the new capital vintage available to expanding industries is equal to the sum of disinvested capital in contracting industries plus total saving generated by the economy, consistent with the closure rule of the model.

The putty/semi-putty specification

The substitution possibilities among production factors are assumed to be higher with the *new* than with the *old* capital vintages—technology has a putty/semi-putty specification. Hence, when a shock to relative prices occurs (e.g. tariff removal), the demands for production factors adjust gradually to the long-run optimum because the substitution effects are delayed over time. The adjustment path depends on the values of the short-run elasticities of substitution **and** the replacement rate of capital. As the latter determines the pace at which new vintages are installed, the larger is the volume of new investment, the greater the possibility to achieve the long-run total amount of substitution among production factors.

Dynamic calibration

The model is calibrated on exogenous growth rates of population, GDP per capita, and an autonomous energy efficiency improvement in energy use (known as the AEEI factor). There are various alternatives for calibrating the key growth parameters in the baseline scenario. The model does need some unique instrument per region to achieve a desired per capita GDP growth. The current strategy has three components. First, agricultural productivity is fixed in the baseline using results from recent empirical studies. Second, productivity in the manufacturing and services is divided intro three components. The first component is a uniform shifter. This component is in essence the instrument used to achieve the given per capita GDP growth target. The second component is a sectoral shifter which permits constant

21 Unlike some previous versions of the model, this version of the model does not have a resource depletion module for fossil fuels.

deviations across sectors, for example imposing manufacturing productivity some 2 percent higher than in services. The third component is a shifter determined by sectoral openness. This latter shifter is sensitive to the sectoral export/output ratio. The degree of sensitivity is measured by an elasticity.[22]

III The Neo-classical model in comparative static mode

This section provides a full elaboration of the comparative static equations of the LINKAGE model. In the equations describing the model specification, the following indices are frequently employed. In general, the regional and time indices are omitted unless needed for clarification. The base sectoral, labor and regional indices are specific to the GTAP data set. The other indices are specific to the model specification.

- ***i*** Sectoral index. ***j*** is used as an alias for ***i***. The original sectoral definitions are described in the GTAP annex.
- ***l*** Labor skill
- ***ul*** A subset of *l*, which includes labor substitutable with capital.
- ***sl*** A subset of *l*, which includes labor that is a complement with capital.
- ***k*** An index for consumer goods. Producer goods are indexed by *i*.
- ***f*** An index for other domestic final demand agents (government and investment).
- ***h*** An index for households.[23]
- ***in*** An index for institutions. In the standard model institutions include households (*h*), government current expenditures (*gov*) and investment expenditures (*inv*). The institutions are used as source and destination for bilateral foreign transfers.
- ***r*** Regional index. ***r′*** is used as an alias for ***r***.
- ***v*** Capital vintage.
- ***gz*** Geographic regions—typically rural and urban, and including a national total.
- ***t*** time index.

The sectoral index *i* is identical to the index *j* (in GAMS terminology, they are aliases). Specific labels are used for important subsets of sectors (see below). The energy sector(s) can be re-defined, this is normally taken care of in the aggregation facility. The set of agricultural sectors is split between crop and livestock sectors, which have different production structures (see below). Agricultural and non-agricultural sectors also have different production structures. The regional index is under full user control, and the model requires a residual region (due to Walras' law, see below). The numéraire of the model is the OECD export price index. The time horizon of the model is under full user-control and is defined in the individual scenario input files. The definition of capital vintage is also under user control. The current version of the model allows for only two possibilities: either a single vintage model (with the label *Old*), or a two-vintage model (with the labels *Old* and *New*). The production structure of the model is completely independent of the number of vintages. However, the allocation of output across vintages, and capital market equilibrium equations, have been defined for the two-vintage case. The specific labels for the other final demand accounts are '*Gov*' for the government, and '*Inv*' for investment. The specific labels are necessary for the closure equations. Other labels for important subsets of sectors are the following:

[22] An alternative strategy is to calibrate the dynamics in each region by imposing the assumption of a **balanced growth** path. This implies that capital/labor ratio (in efficiency units) is held constant.

[23] In the standard GTAP dataset there is a single representative household.

Index	*Subset label*	*Description*
i	*cr*	Crops sectors (user-determined)
i	*lv*	Livestock sectors (user-determined)
i	*ag*	Agricultural sectors (the union of the crop and livestock sectors)
i	*ip*	Non-agricultural products (user-determined)
i	*e*	Energy sectors (user-determined)
i	*ft*	Fertilizer sectors (user-determined)
i	*fd*	Feed sectors (user-determined)
i	*ik*	Sectors including in the calibrating productivity (user-determined)
gs	*gz*	Excludes the national total

Production Technology

The aim of the implementation of production technology is to emulate the different degree of substitution and complementarity across the different inputs of production—intermediate goods, capital, labor, land, etc. There are a number of methods for specifying these relations, though two are most often used in applied work. One method uses so-called flexible functional forms, which in some sense can be thought of as a functional (Taylor) approximation of the true technological relations and is calibrated to a given set of own and cross price elasticities. A second method approximates the technological relations using a nested sequence of CES functions. This latter method has the advantage of being simple to implement and to understand, and also has regularity conditions which may be violated by some flexible functional forms.

For the purposes of the standard LINKAGE model, inputs have been divided into several distinct components. These include: intermediate inputs (excluding energy, and in some cases fertilizers or feed), energy intermediate inputs, other special intermediate inputs such as feed and fertilizer, labor, capital, land (in agricultural sectors only), and a natural resource (also referred to as a sector-specific factor).[24] The nested CES structure starts at the top with a combination of an aggregate bundle with most intermediate goods, *ND*, and a value added bundle including an energy bundle (and a fertilizer or feed bundle, in the relevant cases). Typically, the substitution would be low at this level, if not 0. The intermediate demand bundle is decomposed into individual demand for intermediate goods assuming 0 elasticity, i.e. a Leontief specification. The value added bundle, however, is decomposed differently according to the specificity of each sector (i.e. non-agricultural, crops and livestock).

LINKAGE distinguishes three different production structures—crops, livestock, and all other. The key feature of the crop production structure is the distinction between extensive production technology (through the use of more land), and intensive production through the more intense use of agricultural chemicals (e.g. fertilizers). The livestock production structure characterizes the substitution possibilities between range-fed and ranch-fed production. The production structure for all other sectors captures the ubiquitous capital-labor substitution, with energy as an additional factor of production.

For the crops sectors, the value added-energy-fertilizer bundle *VA* is decomposed into labor demand, *L*, on the one hand, and a land-capital-energy-fertilizer bundle on the other hand, *KTEF*. The *KTEF* bundle is further decomposed into a fertilizer bundle *fert* (there is one single fertilizer sector in the current version), and a land-capital-energy bundle, *KTE*. The *KTE* bundle is then decomposed into an energy bundle *XEp* and a capital-land bundle *HKT*. The *HKT* bundle is split into a human capital component, *H*,

[24] In reference to the original 57-sector version of the GTAP 5 data base, the natural resource factor is present in the following sectors: forestry, fisheries, coal mining, crude oil, natural gas, and the other mining sector.

and a physical capital-land bundle. Finally, *KT* is decomposed into its components: capital *Kv*, and land, *T*. This production structure is depicted in Figure 1.

The second production archetype specified in LINKAGE corresponds to livestock production (see Figure 2 for a graphical representation). In these sectors, the value added-energy-feed bundle *VA* is decomposed into a land-feed composite good *TFD*, and a capital-energy-labor composite *KTEL*. The land-feed bundle is decomposed into a feed composite good *feed*, and land *T*. The *feed* aggregate is subsequently decomposed into its different components (e.g. wheat, oilseeds, other grains…). On the other hand, the capital-energy-labor bundle is decomposed into labor *L*, and a capital-energy aggregate *KTE*, which is decomposed into its individual components: energy demand, *XEp*, and capital demand *HK*. The latter is split into its human and physical capital components.

Finally, for the non-agricultural sectors, the value added *cum* energy bundle *VA* is decomposed into labor demand, *L*, on the one hand, and a resource-capital-energy bundle on the other hand, *KTE*. The resource-capital-energy bundle is further sub-divided into an energy composite good, *XEp*, and capital-resource bundle, *KT*. The energy composite good can be subsequently decomposed into various fuel components (e.g. coal, oil, and gas) where relevant. The capital bundle is split into its human and physical capital components. Finally, the capital-resource bundle is decomposed into its elements, capital demand *Kv*, and the sector specific factor *F*, (where appropriate). This production structure is depicted in Figure 3.

The substitution possibilities between factors of production are captured by the elasticities of substitution used in the CES nests. At the tip of each branch in the nest is a basic good. In the case of intermediate goods, both energy and non-energy, the basic good is at the Armington level, and this will be further decomposed into a domestic and an import component. The other basic goods include the factors of production. The model also incorporates multiple capital vintages (a putty/semi-putty specification). Substitution elasticities can vary by the vintage of the capital. To the extent that installed capital is mobile across sectors, the ex-post substitution possibilities are endogenous. Also, the short run and long run substitution possibilities will differ over time. For example, zero substitution elasticity between energy and capital in the short run implies that they are complements, but a positive substitution elasticity in the long run implies that there is long run substitutability between capital and energy.

The subsequent equations define in analytical terms the just described nested structure.[25] The top-level nest is identical for all three archetypes so this will be described first, with the sub-nests described in subsequent sections.

Top-level production nest and market structure

The top-level production nest has final output, *XP*, produced as a combination of aggregate intermediate demand, *ND*, and value added *VA*. Output is produced by different production streams—differentiated by capital vintage (indexed by *v*). Each production stream has an identical production structure, but different technological parameters and substitution elasticities. Thus the top-level equations are based on the output variable *XPv*, which is indexed by vintage. Aggregate intermediate demand is not indexed by vintage since it is assumed that further disaggregation of the *ND* bundle is not vintage-dependent. In other words, the demand for intermediate goods is the same for all vintages per unit of aggregate intermediate demand. The decomposition of the value added bundle is vintage-dependent, for example, the substitution between labor and capital may be lower for installed capital than for new capital.

Equations (P-1) and (P-2) describe respectively the demand for *ND* and *VA*. The demand for each bundle represents some share of output (by vintage), *XPv*. The shares are functions of the relative price of the

[25] See Annex C for a review of the analytical properties of the CES function and its derivatives (Leontief, and Cobb-Douglas).

respective bundle, *PND* or *PVA*, relative to the overall price of the aggregate bundle, *UVCv*. The formulas represent the standard derived demand equations for the CES production function. If the substitution elasticity, σ^p, between value added and intermediate demand is 0, then the production function is the standard fixed-coefficients Leontief function. The production function allows for a uniform technology shifter, *AT*. Note that the composition of both the *ND* and *VA* bundles differs by sector. For example, the *ND* bundle in crop sectors does not include agricultural chemicals—which are part of the *VA* bundle in this case. Note as well the summation in equation (P-1) since the model assumes that the decomposition of the *ND* bundle is not vintage-dependent.

The variable *UVCv* represents so-called unit variable cost (or marginal cost) by vintage. If there are constant-returns-to-scale, *UVCv* also represents unit average cost. Equation (P-3) defines the variable *UVCv*. The CES dual price expression will be used in all price functions rather than the primal accounting equations. The former tend to lead to better convergence properties. Equation (P-4) defines the total unit variable cost, *UVC*, averaging costs over all vintages.

(P-1) $$ND_i = \sum_v \alpha_{i,v}^{nd} \left(AT_i\right)^{\sigma_{i,v}^p - 1} \left(\frac{UVCv_{i,v}}{PND_i}\right)^{\sigma_{i,v}^p} XPv_{i,v}$$

(P-2) $$VA_{i,v} = \alpha_{i,v}^{va} \left(AT_i\right)^{\sigma_{i,v}^p - 1} \left(\frac{UVCv_{i,v}}{PVA_{i,v}}\right)^{\sigma_{i,v}^p} XPv_{i,v}$$

(P-3) $$UVCv_{i,v} = \frac{1}{AT_i}\left[\alpha_{i,v}^{nd}\left(PND_i\right)^{1-\sigma_{i,v}^p} + \alpha_{i,v}^{va}\left(PVA_{i,v}\right)^{1-\sigma_{i,v}^p}\right]^{1/\left(1-\sigma_{i,v}^p\right)}$$

(P-4) $$UVC_i = \sum_i \frac{XPv_{i,v}}{XP_i} UVCv_{i,v}$$

The standard version of the model assumes constant-returns-to-scale and perfectly competitive markets. Alternatives allow for increasing-returns-to-scale and non-competitive market structures. Equation (P-5) defines unit average cost, *AC*. It is the sum of unit variable cost, *UVC*, plus unit fixed costs. The term in bracket measures aggregate fixed cost per firm. It represents the cost of labor and capital where LF^d and KF^d—in fixed proportions—are identified as fixed costs of production. Fixed costs are assumed identical across firms in the same sector. Total fixed cost in the sector are therefore multiplied by the number of firms, *N*, and unit fixed cost is equal to the total divided by output, *XP*. The net price of output, *PX*, is equal to marginal cost, *UVC*, times a price markup represented by the wedge π, equation (P-6). Equation (P-7) defines the gross producer price, *PP*, inclusive of an output tax, τ^p. Finally, equation (P-8) defines gross profits (net of ‘normal’ capital remuneration), Π. Gross profits are equal to the difference in the net output price, *PX*, and average cost, *AC*, multiplied by the level of output.

Under constant returns-to-scale, fixed costs are 0 and therefore average cost is equal to marginal cost. Under perfect competition, the markup is zero and therefore the net output price is equal to average cost and profits are zero. There are two alternative closures available. Under one, the markup is fixed and profits are endogenous. Under a second, profits are zero and the markup is endogenous. This is the case of contestable markets, i.e. increasing returns-to-scale, but the threat of new entry keeps the markup at a level that induces zero profits.[26]

[26] Another possible closure consistent with contestable markets is to allow for entry and exit of firms. However this closure would be hard to justify with a fixed markup. In a future version of the model, the markup will be modeled using the standard specifications of oligopolistic or monopolistic competition.

(P-5) $$AC_i = UVC_i + N_i\left[\sum_l W_{l,i} LF_{l,i}^d + R_{i,Old} KF_i^d\right] / XP_i$$

(P-6) $$PX_i = UVC_i\left(1+\pi_i\right)$$

(P-7) $$PP_i = PX_i\left(1+\tau_i^p\right)$$

(P-8) $$\Pi_i = XP_i\left(PX_i - AC_i\right)$$

The next three sections describe the remaining production equations for each of the three archetype production structures—crops, livestock and manufacturing and services.

Crop Production

The top-level nest was described above. In the crops sector, the next node in the CES nest decomposes the value added bundle, *VA*, into aggregate 'unskilled'[27] labor demand, *ULD*, and a capital-energy-fertilizer-land bundle, *HKTEF*. The key substitution parameter is given by σ^v, which is typically taken to be lower for *old* capital and higher for *new* capital. The assumption is that the latter is more flexible, and can be more easily substituted for labor. The relevant component prices are *UW* for the average sectoral 'unskilled' wage, and *PHKTEF* for the capital-energy-fertilizer-land bundle. Equations (P-9), and (P-10) represent the respective demand for *ULD* and *HKTEF*. The price of the value added bundle, *PVA*, is determined in equation (P-11), again using the CES dual price formula. Determination of the wage rate will be discussed in the section on factor market equilibrium.

(P-9) $$ULD_{cr} = \sum_v \alpha_{cr,v}^l\left(\frac{PVA_{cr,v}}{UW_{cr}}\right)^{\sigma_{cr,v}^v} VA_{cr,v}$$

(P-10) $$HKTEF_{cr,v} = \alpha_{cr,v}^{hktef}\left(\frac{PVA_{cr,v}}{PHKTEF_{cr,v}}\right)^{\sigma_{cr,v}^v} VA_{cr,v}$$

(P-11) $$PVA_{cr,v} = \left[\alpha_{cr,v}^l\left(UW_{cr}\right)^{1-\sigma_{cr,v}^v} + \alpha_{cr,v}^{hktef}\left(PHKTEF_{cr,v}\right)^{1-\sigma_{cr,v}^v}\right]^{1/(1-\sigma_{cr,v}^v)}$$

The key substitution in crops is defined next level, it is the substitution between land (combined with capital and energy), and fertilizers, i.e. between intensive and extensive farming. Equation (P-12) determines the aggregate demand for fertilizers in the crop sector, *fert*, and Equation (P-13) determines the demand for the *HKTE* bundle, i.e. capital, energy, and land, where *Pfert* is the price of chemicals (fertilizers), and *PHKTE* is the price of the *HKTE* bundle. The elasticity of substitution at this level is typically taken to be equal for old capital and new capital. Note that the decomposition of the fertilizer aggregate into its sub-components is assumed to be independent of the vintage. Equation (P-14) determines the price of the *HKTEF* bundle, *PHKTEF*.

27 The two labor bundles are described as 'unskilled' and 'skilled' though the actual distinction is between labor that is a substitute with capital and labor that is a complement with capital. In the standard version of the model, both labor types are part of the 'unskilled' labor bundle.

(P-12) $$fert_{cr} = \sum_{v} \alpha^{fert}_{cr,v} \left(\frac{PHKTEF_{cr,v}}{Pfert_{cr}} \right)^{\sigma^{f}_{cr,v}} HKTEF_{cr,v}$$

(P-13) $$HKTE_{cr,v} = \alpha^{hkte}_{cr,v} \left(\frac{PHKTEF_{cr,v}}{PHKTE_{cr,v}} \right)^{\sigma^{f}_{cr,v}} HKTEF_{cr,v}$$

(P-14) $$PHKTEF_{cr,v} = \left[\alpha^{fert}_{cr,v} \left(Pfert_{cr} \right)^{1-\sigma^{f}_{cr,v}} + \alpha^{hkte}_{cr,v} \left(PHKTE_{cr,v} \right)^{1-\sigma^{f}_{cr,v}} \right]^{1/\left(1-\sigma^{f}_{cr,v}\right)}$$

The *HKTE* bundle is decomposed into an energy bundle on the one hand, *XEp*, and a land-capital bundle, *HKT*, on the other. Equation (P-15) determines the demand for aggregate energy by vintage, *XEp*, where *PEp* is the price of the energy bundle. Equation (P-16) determines the aggregate demand for the capital-land bundle, *HKT*, where *PHKT* is the price of the *HKT* bundle. At this level, new capital is assumed to have a greater elasticity of substitution with energy than old capital. Equation (P-17) is by now the familiar CES dual price expression of the *HKTE* bundle.

(P-15) $$XEp_{cr,v} = \alpha^{e}_{cr,v} \left(\frac{PHKTE_{cr,v}}{PEp_{cr,v}} \right)^{\sigma^{e}_{cr,v}} HKTE_{cr,v}$$

(P-16) $$HKT_{cr,v} = \alpha^{hkt}_{cr,v} \left(\frac{PHKTE_{cr,v}}{PHKT_{cr,v}} \right)^{\sigma^{e}_{cr,v}} HKTE_{cr,v}$$

(P-17) $$PHKTE_{cr,v} = \left[\alpha^{e}_{cr,v} \left(PEp_{cr,v} \right)^{1-\sigma^{e}_{cr,v}} + \alpha^{hkt}_{cr,v} \left(PHKT_{cr,v} \right)^{1-\sigma^{e}_{cr,v}} \right]^{1/\left(1-\sigma^{e}_{cr,v}\right)}$$

The *HKT* bundle is decomposed into the 'skilled' labor bundle and a physical capital-land bundle, with a substitution elasticity of σ^{h}. Equation (P-18) determines the demand for 'skilled" labor bundle, *SLD*, where *SW* represents the average 'skilled' wage. Demand for the capital-land bundle, *KT*, is derived in equation (P-19). And Equation (P-20) provides the price equation for the *HKT* bundle.

(P-18) $$SLD_{cr} = \sum_{v} \alpha^{h}_{cr,v} \left(\frac{PHKT_{cr,v}}{SW_{cr}} \right)^{\sigma^{h}_{cr,v}} HKT_{cr,v}$$

(P-19) $$KT_{cr,v} = \alpha^{kt}_{cr,v} \left(\frac{PHKT_{cr,v}}{PKT_{cr,v}} \right)^{\sigma^{h}_{cr,v}} HKT_{cr,v}$$

(P-20) $$PHKT_{cr,v} = \left[\alpha^{h}_{cr,v} \left(SW_{cr} \right)^{1-\sigma^{h}_{cr,v}} + \alpha^{kt}_{cr,v} \left(PKT_{cr,v} \right)^{1-\sigma^{h}_{cr,v}} \right]^{1/\left(1-\sigma^{h}_{cr,v}\right)}$$

Land, capital, and the sector-specific factor comprise the final bundle.[28] They are aggregated in a CES bundle with a substitution elasticity of σ^k. Equation (P-21) determines the demand for capital, Kv^d, Equation (P-22) determines the demand for land, T^d, and Equation (P-23) determines the demand for the sector-specific factor. The equations integrate changes in efficiency embodied in the factor biased efficiency parameters. Note that the demand for land and the sector-specific factor are aggregated across vintages. Equation (P-24) determines the price of the capital-land bundle, *PKT*.

(P-21) $$Kv^d_{cr,v} = \alpha^k_{cr,v}\left(\lambda^k_{cr,v}\right)^{\sigma^k_{cr,v}-1}\left(\frac{PKT_{cr,v}}{R_{cr,v}}\right)^{\sigma^k_{cr,v}} KT_{cr,v}$$

(P-22) $$T^d_{cr} = \sum_v \alpha^t_{cr,v}\left(\lambda^t_{cr}\right)^{\sigma^k_{cr,v}-1}\left(\frac{PKT_{cr,v}}{PT_{cr}}\right)^{\sigma^k_{cr,v}} KT_{cr,v}$$

(P-23) $$F^d_{cr} = \sum_v \alpha^f_{cr,v}\left(\lambda^f_{cr}\right)^{\sigma^k_{cr,v}-1}\left(\frac{PKT_{cr,v}}{PF_{cr}}\right)^{\sigma^k_{cr,v}} KT_{cr,v}$$

(P-24) $$PKT_{cr,v} = \left[\alpha^k_{cr,v}\left(\frac{R_{cr,v}}{\lambda^k_{cr,v}}\right)^{1-\sigma^k_{cr,v}} + \alpha^t_{cr,v}\left(\frac{PT_{cr}}{\lambda^t_{cr}}\right)^{1-\sigma^k_{cr,v}} + \alpha^f_{cr,v}\left(\frac{PF_{cr}}{\lambda^f_{cr}}\right)^{1-\sigma^k_{cr,v}}\right]^{1/\left(1-\sigma^k_{cr,v}\right)}$$

Intermediate Demand

The fertilizer aggregate is decomposed into its relevant components using a final CES nest. Equation (P-25) determines the demand for the individual fertilizer (chemicals) components. The subset represented by the index *ft* spans all the chemical sectors used in agriculture.[29] Demand at this point is specified at the Armington level, i.e. *XAp* represents the demand for agricultural chemicals from both domestic and foreign sources. For simplicity, the split of Armington demand into its various components is done at the national level, not at the agent-level. This implies that the Armington price across agents for the same commodity is uniform. The latter variable is represented by *PA*. However, the domestic sales tax, assumed uniform between domestic and imported goods, is agent-specific and is represented by the variable τ^{Ap} in the production sectors. Note that the model allows for efficiency improvements in the use of agricultural chemicals. Equation (P-26) determines the price of the fertilizer bundle, *Pfert*.

The breakdown of the energy bundle in crops is similar to the split of the fertilizer bundle. This bundle, *XEp*, is decomposed into various energy components, where the index *e* ranges over the energy commodities. Equation (P-27) determines the decomposition of the energy bundle into its respective Armington fuel components, *XAp*. The key inter-fuel substitution elasticity is given by σ^{ep}. Energy price distortions are incorporated in the variable τ^{Ap}. Improvements in energy efficiency are reflected in the variable λ^{ep}. Equation (P-28) defines the aggregate price of energy in production, inclusive of price distortions and improvements in energy efficiency.

Equation (P-29) determines demand for all the non-energy and non-chemical intermediate goods using a standard CES function, where the subset indexed by *nnft* ranges over the appropriate sectors. (A simple

28 The sector-specific resource is normally not part of the crops sector, though it could be through aggregation of the original data.

29 Note that in the GTAP data set, this is a single sector.

Leontief technology can be implemented by setting the substitution elasticity, σ^n, to zero.) The *a* coefficients are closely related to the Leontief coefficients.[30] Equation (P-30) determines the price of the *ND* bundle.

(P-25) $$XAp_{ft,cr} = \alpha_{ft,cr}^{ft}\left(\lambda_{ft,cr}^{ft}\right)^{\left(\sigma_{cr}^{ft}-1\right)}\left(\frac{Pfert_{cr}}{(1+\tau_{ft,cr}^{Ap})PA_{ft}}\right)^{\sigma_{cr}^{ft}} fert_{cr}$$

(P-26) $$Pfert_{cr} = \left[\sum_{ft}\alpha_{ft,cr}^{ft}\left[\frac{(1+\tau_{ft,cr}^{Ap})PA_{ft}}{\lambda_{ft,cr}^{ft}}\right]^{1-\sigma_{cr}^{ft}}\right]^{1/(1-\sigma_{cr}^{ft})}$$

(P-27) $$XAp_{e,cr} = \sum_{v}\alpha_{e,cr,v}^{ep}\left(\lambda_{e,cr}^{ep}\right)^{\sigma_{cr,v}^{ep}-1}\left(\frac{PEp_{cr,v}}{(1+\tau_{e,cr}^{Ap})\,PA_{e}}\right)^{\sigma_{cr,v}^{ep}} XEp_{cr,v}$$

(P-28) $$PEp_{cr,v} = \left[\sum_{e}\alpha_{e,cr,v}^{ep}\left(\frac{(1+\tau_{e,cr}^{Ap})PA_{e}}{\lambda_{e,cr}^{ep}}\right)^{1-\sigma_{cr,v}^{ep}}\right]^{1/(1-\sigma_{cr,v}^{ep})}$$

(P-29) $$XAp_{nnft,cr} = a_{nnft,cr}ND_{cr}\left[\frac{PND_{cr}}{(1+\tau_{nnft,cr}^{Ap})PA_{nnft}}\right]^{\sigma_{cr}^{n}}$$

(P-30) $$PND_{cr} = \left[\sum_{nnft}a_{nnft,cr}\left((1+\tau_{nnft,cr}^{Ap})PA_{nnft}\right)^{1-\sigma_{cr}^{n}}\right]^{1/(1-\sigma_{cr}^{n})}$$

Livestock Production

Livestock production is similarly modeled as a nested structure of constant elasticity of substitution (CES) functions. Figure 2 provides a graphical depiction of the nested CES livestock production structure. The top-level nest was described above. The second level of the nest decomposes the *VA* bundle into the *KTEL* bundle on the one hand, and the *TFD* bundle on the other, the former being a capital, energy, and labor bundle, and the latter being the aggregate of land and feed. These bundles are assumed to be perfect complements, i.e. the substitution elasticity is zero. Equation (P-31) determines the vintage specific demand for the *KTEL* bundle. Equation (P-32) defines the vintage specific demand for the *TFD* bundle. Equation (P-33) determines the price of the *VA* bundle, *PVA*.

(P-31) $$KTEL_{lv,v} = \alpha_{lv,v}^{ktel}VA_{lv,v}$$

(P-32) $$TFD_{lv,v} = \alpha_{lv,v}^{tfd}VA_{lv,v}$$

(P-33) $$PVAf_{lv,v} = \alpha_{lv,v}^{ktel}PKTEL_{lv,v} + \alpha_{lv,v}^{tfd}PTFD_{lv,v}$$

[30] They are not strictly speaking Leontief coefficients since the *ND* bundle is not necessarily in strict proportion with output.

The key substitution in livestock is defined next; it is the substitution between land and feed. Equation (P-34) determines the aggregate demand for feed in each livestock sector, *feed*, and Equation (P-35) determines the demand for land, where *PTFD* is the price of the *TFD* bundle, *Pfeed* is the price of aggregate feed, and *PT* is the sector specific price of land. Note that the decomposition of the feed aggregate into its sub-components is assumed to be independent of the vintage. Land is also aggregated at this level across vintages, and incorporates the land productivity factor. Equation (P-36) determines the price of the *TFD* composite factor.

(P-34) $$feed_{lv} = \sum_v \alpha_{lv,v}^{feed} \left(\frac{PTFD_{lv,v}}{Pfeed_{lv}} \right)^{\sigma_{lv,v}^f} TFD_{lv,v}$$

(P-35) $$T_{lv}^d = \sum_v \alpha_{lv,v}^t \left(\lambda_{lv}^t \right)^{\sigma_{lv,v}^f - 1} \left(\frac{PTFD_{lv,v}}{PT_{lv}} \right)^{\sigma_{lv,v}^f} TFD_{lv,v}$$

(P-36) $$PTFD_{lv,v} = \left[\alpha_{lv,v}^{feed} \left(Pfeed_{lv} \right)^{1-\sigma_{lv,v}^f} + \alpha_{lv,v}^t \left(\frac{PT_{lv}}{\lambda_{lv}^t} \right)^{1-\sigma_{lv,v}^f} \right]^{1/\left(1-\sigma_{lv,v}^f\right)}$$

The *KTEL* bundle is decomposed into aggregate 'unskilled' labor demand on the one hand, *ULD*, and a capital-energy bundle, *HKTE*, on the other. Equation (P-37) determines the demand for aggregate 'unskilled' labor, where *PKTEL* is the price of the *KTEL* bundle. Equation (P-38) determines the aggregate demand for the capital-energy bundle, *HKTE*, where *PHKTE* is the price of the *HKTE* bundle. Equation (P-39) determines the price of the *KTEL* bundle, *PKTEL*.

(P-37) $$ULD_{lv} = \sum_v \alpha_{lv,v}^l \left(\frac{PKTEL_{lv,v}}{UW_{lv}} \right)^{\sigma_{lv,v}^v} KTEL_{lv,v}$$

(P-28) $$HKTE_{lv,v} = \alpha_{lv,v}^{hkte} \left(\frac{PKTEL_{lv,v}}{PHKTE_{lv,v}} \right)^{\sigma_{lv,v}^v} KTEL_{lv,v}$$

(P-39) $$PKTEL_{lv,v} = \left[\alpha_{lv,v}^l \left(UW_{lv} \right)^{1-\sigma_{lv,v}^v} + \alpha_{lv,v}^{hkte} \left(PHKTE_{lv} \right)^{1-\sigma_{lv,v}^v} \right]^{1/\left(1-\sigma_{lv,v}^v\right)}$$

The next step in the nest is the decomposition of the *HKTE* bundle into capital and the energy aggregate. Equation (P-40) determines the demand for energy, *XEp*, and Equation (P-41) determines the demand for capital, *HKT*. The price of the capital-energy bundle, *PHKTE*, is given by Equation (P-42).

(P-40) $$XEp_{lv,v} = \alpha^e_{lv,v} \left(\frac{PHKTE_{lv,v}}{PEp_{lv,v}} \right)^{\sigma^e_{lv,v}} HKTE_{lv,v}$$

(P-41) $$HKT_{lv,v} = \alpha^{hkt}_{lv,v} \left(\frac{PHKTE_{lv,v}}{PHKT_{lv,v}} \right)^{\sigma^e_{lv,v}} KTE_{lv,v}$$

(P-42) $$PKTE_{lv,v} = \left[\alpha^e_{lv,v} \left(PEp_{lv,v} \right)^{1-\sigma^e_{lv,v}} + \alpha^{hkt}_{lv,v} \left(PHKT_{lv,v} \right)^{1-\sigma^e_{lv,v}} \right]^{1/\left(1-\sigma^e_{lv,v}\right)}$$

The *HKT* bundle is decomposed using two additional nests. The first nest splits the bundle into demand for the 'skilled' labor bundle and the *KT* bundle. The second nest decomposed the *KT* bundle into physical capital and a sector-specific factor. Equation (P-43) describes the demand for the 'skilled' labor bundle. Equation (P-44) determines demand for the *KT* bundle. And Equation (P-45) determines the price of the *HKT* bundle.

(P-43) $$SLD_{lv} = \sum_v \alpha^h_{lv,v} \left(\frac{PHKT_{lv,v}}{SW_{lv}} \right)^{\sigma^h_{lv,v}} HKT_{lv,v}$$

(P-44) $$KT_{lv,v} = \alpha^{kt}_{lv,v} \left(\frac{PHKT_{lv,v}}{PKT_{lv,v}} \right)^{\sigma^h_{lv,v}} HKT_{lv,v}$$

(P-45) $$PHKT_{lv,v} = \left[\alpha^h_{lv,v} \left(SW_{lv} \right)^{1-\sigma^h_{lv,v}} + \alpha^{kt}_{lv,v} \left(PKT_{lv,v} \right)^{1-\sigma^h_{lv,v}} \right]^{1/\left(1-\sigma^h_{lv,v}\right)}$$

The final nest on the value added side decomposes the *KT* bundle into demand for physical capital and the sector-specific factor, with a substitution elasticity of σ^k. Equations (P-46) and (P-47) determine respectively the demand for physical capital and the sector-specific factor. The price of the composite bundle, *PKT*, is determined in equation (P-48).

(P-46) $$Kv^d_{lv,v} = \alpha^k_{lv,v} \left(\lambda^k_{lv,v} \right)^{\sigma^k_{lv,v}-1} \left(\frac{PKT_{lv,v}}{R_{lv,v}} \right)^{\sigma^k_{lv,v}} KT_{lv,v}$$

(P-47) $$F^d_{lv} = \sum_v \alpha^f_{lv,v} \left(\lambda^f_{lv} \right)^{\sigma^k_{lv,v}-1} \left(\frac{PKT_{lv,v}}{PF_{lv}} \right)^{\sigma^k_{lv,v}} KT_{lv,v}$$

(P-48) $$PKT_{lv,v} = \left[\alpha^k_{lv,v} \left(\frac{R_{lv,v}}{\lambda^k_{lv,v}} \right)^{1-\sigma^k_{lv,v}} + \alpha^f_{lv,v} \left(\frac{PF_{lv}}{\lambda^f_{lv}} \right)^{1-\sigma^k_{lv,v}} \right]^{1/\left(1-\sigma^k_{lv,v}\right)}$$

Intermediate Demand

The feed aggregate is decomposed into its relevant components using a CES nest. Equation (P-49) determines the demand for the individual feed components, and is analogous to the decomposition of fertilizer in crops (N.B. the introduction of feed-specific efficiency gains). The subset represented by the index *fd* spans all the feed sectors used in agriculture. Equation (P-50) determines the price of the aggregate feed bundle. The breakdown of the energy bundle in livestock is identical to the energy breakdown in the other sectors, and it is determined by Equations (P-51) and (P-52). Demand for the other intermediate goods is described by Equation (P-53), where the subset indexed by *nnfd* ranges over the non-energy and non-feed sectors. Equation (P-54) determines the price of the *ND* bundle.

(P-49) $$XAp_{fd,lv} = \alpha_{fd,lv}^{fd} \left(\lambda_{fd,lv}^{fd}\right)^{(\sigma_{lv}^{fd}-1)} \left(\frac{Pfeed_{lv}}{(1+\tau_{fd,lv}^{Ap})PA_{fd}}\right)^{\sigma_{lv}^{fd}} feed_{lv}$$

(P-50) $$Pfeed_{lv} = \left[\sum_{fd} \alpha_{fd,lv}^{fd} \left[\frac{(1+\tau_{fd,lv}^{Ap})PA_{fd}}{\lambda_{fd,lv}^{fd}}\right]^{1-\sigma_{lv}^{fd}}\right]^{1/(1-\sigma_{lv}^{fd})}$$

(P-51) $$XAp_{e,lv} = \sum_{v} \alpha_{e,lv,v}^{ep} \left(\lambda_{e,lv}^{ep}\right)^{\sigma_{lv,v}^{ep}-1} \left(\frac{PEp_{lv,v}}{(1+\tau_{e,lv}^{Ap})\ PA_{e}}\right)^{\sigma_{lv,v}^{ep}} XEp_{lv,v}$$

(P-52) $$PEp_{lv,v} = \left[\sum_{e} \alpha_{e,lv,v}^{ep} \left(\frac{(1+\tau_{e,lv}^{Ap})PA_{e}}{\lambda_{ej}^{ep}}\right)^{1-\sigma_{lv,v}^{ep}}\right]^{1/(1-\sigma_{lv,v}^{ep})}$$

(P-53) $$XAp_{nnff,lv} = a_{nnfd,lv} ND_{lv} \left(\frac{PND_{lv}}{(1+\tau_{nnfd,lv}^{Ap})\ PA_{nnfd}}\right)^{\sigma_{lv}^{n}}$$

(P-54) $$PND_{lv} = \left[\sum_{nnfd} a_{nnfd,lv} \left((1+\tau_{nnfd,lv}^{Ap})\ PA_{nnfd}\right)^{1-\sigma_{lv}^{n}}\right]^{1/(1-\sigma_{lv}^{n})}$$

Non-Agricultural Production

The third production archetype is non-agricultural. It is more similar to the standard capital/labor substitution model, though as in the other sectors, it has special treatment for energy and the various labor skills. The first set of equations deals with the second part of the nest—the decomposition of the value added bundle, *VA*, into labor demand, *AL*, and a capital-resource cum energy bundle, *HKTE*. The key substitution parameter is given by σ^{v}. The relevant component prices are *UW* for the average sectoral 'unskilled' wage, and *PHKTE* for the capital-resource-energy bundle. Equations (P-55) and (P-56) represent the respective demand for *SLD* and *HKTE*. The price of the value added bundle, *PVA*, is determined in equation (P-57), again using the CES dual price formula.

(P-55) $$ULD_{ip} = \sum_v \alpha^l_{ip,v} \left(\frac{PVA_{ip,v}}{UW_{ip}} \right)^{\sigma^v_{ip}} VA_{ip,v}$$

(P-56) $$HKTE_{ip,v} = \alpha^{hkte}_{ip,v} \left(\frac{PVA_{ip,v}}{PHKTE_{ip,v}} \right)^{\sigma^v_{ip,v}} VA_{ip,v}$$

(P-57) $$PVA_{ip,v} = \left[\alpha^{hkte}_{ip,v} PHKTE^{1-\sigma^v_{jv}}_{ip,v} + \alpha^l_{ip,v} \left(UW_{ip} \right)^{1-\sigma^v_{ip,v}} \right]^{1/\left(1-\sigma^v_{ip,v}\right)}$$

At the next production node, the *HKTE* bundle is decomposed into demand for aggregate energy, *XEp*, and the capital (cum resource) bundle, *HKT*. The key parameter is σ^e, the substitution between energy and capital. The relevant prices are *PEp* for the energy bundle, and *PHKT*, for the capital-resource bundle. Equations (P-58) and (P-59) determine respectively *XEp* and *HKT*. Equation (P-60) determines the price of the *HKTE* bundle, *PHKTE*.

(P-58) $$XEp_{ip,v} = \alpha^e_{ip,v} \left(\frac{PHKTE_{ip,v}}{PEp_{ip,v}} \right)^{\sigma^e_{ip,v}} HKTE_{ip,v}$$

(P-59) $$HKT_{ip,v} = \alpha^{hkt}_{ip,v} \left(\frac{PHKTE_{ip,v}}{PHKT_{ip,v}} \right)^{\sigma^e_{ip,v}} HKTE_{ip,v}$$

(P-60) $$PHKTE_{ip,v} = \left[\alpha^e_{ip,v} \left(PEp_{ip,v} \right)^{1-\sigma^e_{ip,v}} + \alpha^{hkt}_{ip,v} \left(PHKT_{ip,v} \right)^{1-\sigma^e_{ip,v}} \right]^{1/\left(1-\sigma^e_{ip,v}\right)}$$

The *HKT* bundle is decomposed using two additional nests. The first nest splits the bundle into demand for the 'skilled' labor bundle and the *KT* bundle. The second nest decomposed the *KT* bundle into physical capital, land and a sector-specific factor. Equation (P-61) describes the demand for 'skilled' labor bundle. Equation (P-62) determines demand for the *KT* bundle. And Equation (P-63) determines the price of the *HKT* bundle.

(P-61) $$SLD_{ip} = \sum_v \alpha^h_{ip,v} \left(\frac{PHKT_{ip,v}}{SW_{ip}} \right)^{\sigma^h_{ip,v}} HKT_{ip,v}$$

(P-62) $$KT_{ip,v} = \alpha^{kt}_{ip,v} \left(\frac{PHKT_{ip,v}}{PKT_{ip,v}} \right)^{\sigma^h_{ip,v}} HKT_{ip,v}$$

(P-63) $$PHKT_{ip,v} = \left[\alpha^h_{ip,v} \left(SW_{ip} \right)^{1-\sigma^h_{ip,v}} + \alpha^{kt}_{ip,v} \left(PKT_{ip,v} \right)^{1-\sigma^h_{ip,v}} \right]^{1/\left(1-\sigma^h_{ip,v}\right)}$$

The *KT* bundle has three components—capital, *Kv*, land[31] and other sector-specific resources, *F*. The sector-specific resource, *F*, is present in the natural resource sectors, though through aggregation of (or changes to) the initial database could be specified in other sectors as well. The combination of these three factors into the *KT* composite good also assumes a CES specification, though one would typically assume that they are used in fixed proportions. Equation (P-64) determines the demand the sector-specific resource, F^d, and Equation (P-65) specifies land demand. Capital (by vintage), *Kv* is determined in Equation (P-66). The key substitution parameter is given by σ^k. The demands for the sector specific resource and land are summed across vintages since equilibrium on these markets is not vintage specific. Closure of the capital market is vintage related and hence demand for capital needs to be separated by vintage type. The relevant prices of the three factors are *PF* for the sector-specific factor, *PT* for land, and *R* for capital. The CES equations integrate the potential for factor- and sector-specific technological change. Equation (P-67) determines the price of the *KT* bundle, *PKT*.

(P-64) $$F_{ip}^d = \sum_v \alpha_{ip,v}^f \left(\lambda_{ip}^f\right)^{\sigma_{ip,v}^k - 1} \left(\frac{PKT_{ip,v}}{PF_{ip}}\right)^{\sigma_{ip,v}^k} KT_{ip,v}$$

(P-65) $$T_{ip}^d = \sum_v \alpha_{ip,v}^t \left(\lambda_{ip}^t\right)^{\sigma_{ip,v}^k - 1} \left(\frac{PKT_{ip,v}}{PT_{ip}}\right)^{\sigma_{ip,v}^k} KT_{ip,v}$$

(P-66) $$Kv_{ip,v}^d = \alpha_{ip,v}^k \left(\lambda_{ip}^k\right)^{\sigma_{ip,v}^k - 1} \left(\frac{PKT_{ip,v}}{R_{ip,v}}\right)^{\sigma_{ip,v}^k} KT_{ip,v}$$

(P-67) $$PKT_{ip,v} = \left[\alpha_{ip,v}^f \left(\frac{PF_{ip}}{\lambda_{ip}^f}\right)^{1-\sigma_{ip,v}^k} + \alpha_{ip,v}^t \left(\frac{PT_{ip}}{\lambda_{ip}^t}\right)^{1-\sigma_{ip,v}^k} + \alpha_{ip,v}^k \left(\frac{R_{ip,v}}{\lambda_{ip,v}^k}\right)^{1-\sigma_{ip,v}^k}\right]^{1/\left(1-\sigma_{ip,v}^k\right)}$$

Intermediate Demand

There are two nodes concerning the decomposition of intermediate demand. The first relates to non-energy demand as expressed in the aggregate composite bundle *ND*, which, as in other sectors is decomposed by means of a Leontief technology specification. The index *nf* ranges over all non-energy commodities. The second intermediate demand bundle is *XEp*, i.e. aggregate energy demand, and is decomposed by means of a CES function, as in the crops and livestock sectors.

[31] In the base dataset, land only appears in the agricultural sectors. But through aggregation, it could also end up in one of the non-agricultural sectors.

(P-68) $$XAp_{nf,ip} = \alpha_{nf,ip} ND_{ip} \left(\frac{PND_{ip}}{(1+\tau^{Ap}_{nf,ip})\, PA_{nf}} \right)^{\sigma^n_{ip}}$$

(P-69) $$PND_{ip} = \left[\sum_{nf} a_{nf,ip} \left((1+\tau^{Ap}_{nf,ip})\, PA_{nf} \right)^{1-\sigma^n_{ip}} \right]^{1/(1-\sigma^n_{ip})}$$

(P-70) $$XAp_{e,ip} = \sum_{v} \alpha^{ep}_{e,ip,v} \left(\lambda^{ep}_{e,ip} \right)^{\sigma^{ep}_{ip,v}-1} \left(\frac{PEp_{ip,v}}{(1+\tau^{Ap}_{e,ip})\, PA_e} \right)^{\sigma^{ep}_{ip,v}} XEp_{ip,v}$$

(P-71) $$PEp_{ip,v} = \left[\sum_{e} \alpha^{ep}_{e,ip,v} \left(\frac{(1+\tau^{Ap}_{e,ip}) PA_e}{\lambda^{ep}_{e,ip}} \right)^{1-\sigma^{ep}_{ip,v}} \right]^{1/(1-\sigma^{ep}_{ip,v})}$$

Labor demand

The two labor demand bundles, so-called ‘unskilled’ and ‘skilled’, are disaggregated into their respective components using a final CES nesting. Labor biased technological change is incorporated in the model through the λ variable, which is both sector and skill specific. The λ variable is initialized at unit value. Increases in labor productivity are captured by increasing λ. For example, a 2 percent increase in labor productivity would raise λ to 1.02 from its base level. The substitution across ‘unskilled’ labor is given by σ^{ul} and that across ‘skilled’ labor is given by σ^{sl}. Equation (P-72) determines labor demand by skill for those labor types in the ‘unskilled’ bundle. The average sectoral ‘unskilled’ wage, *UW*, is determined in Equation (P-73). The relevant demands for ‘skilled’ labor are derived from equation (P-74), where equation (P-75) reflects the average wage of ‘skilled’ labor by sector.

(P-72) $$LV^d_{ul,i} = \alpha^{ld}_{ul,i} \left(\lambda^l_{ul,i} \right)^{\sigma^{ul}_i - 1} \left(\frac{UW_i}{W_{ul,i}} \right)^{\sigma^{ul}_i} ULD_i$$

(P-73) $$UW_i = \sum_{ul} \left[\alpha^{ld}_{ul,i} \left(\frac{W_{ul,i}}{\lambda^l_{ul,i}} \right)^{1-\sigma^{ul}_i} \right]^{1/\left(1-\sigma^{ul}_i\right)}$$

(P-74) $$LV^d_{sl,i} = \alpha^{ld}_{sl,i} \left(\lambda^l_{sl,i} \right)^{\sigma^{sl}_i - 1} \left(\frac{SW_i}{W_{sl,i}} \right)^{\sigma^{sl}_i} SLD_i$$

(P-75) $$SW_i = \sum_{sl} \left[\alpha^{ld}_{sl,i} \left(\frac{W_{sl,i}}{\lambda^l_{sl,i}} \right)^{1-\sigma^{sl}_i} \right]^{1/\left(1-\sigma^{sl}_i\right)}$$

Income Distribution

The latest version of the model has been modified to handle multiple households.[32] This implies a somewhat more complex treatment of income distribution than the single representative household model. Equations (Y-1)-(Y-4) determine value added by factor, respectively land, sector-specific factor, labor (by skill), and capital remuneration.[33] Land, labor and capital remuneration are net of factor taxes (to be described below). Producers pay respectively *PT*, *W*, and *R* for land, labor and capital services. The relevant factor prices preceded by an *N* represent what households receive. If households receive more than what producers pay, government is subsidizing the relevant factor (e.g. subsidy payments to OECD farmers). Labor payments are accounted for at the skill level indexed by *l*. Both labor and capital incomes include payments to the fixed cost component of total costs in sectors subject to increasing returns. Capital income also incorporates profits.

(Y-1) $$TY = \sum_i NPT_i \, T_i^d$$

(Y-2) $$FY = \sum_i PF_i \, F_i^d$$

(Y-3) $$LY_l = \sum_i NW_{l,i}\left(LV_{l,i}^d + N_i LF_{l,i}^d\right)$$

(Y-4) $$KY = \sum_i \left[\sum_v NR_{i,v} \, Kv_{i,v}^d + NR_{i,Old} N_i \, KF_i^d\right] + \Pi_i$$

Letting *YH* represent aggregate household income, Equation (Y-5) equates household income with the sum across all sources of factor income, with household specific share coefficients. The income distribution share parameters are given by the respective φ parameters, and each must sum to unity across households. Fiscal depreciation of capital, *DeprY*, is subtracted from the total, which may also be adjusted by transfers from the government, as represented by the (exogenous) variable *TRG*. Household income is also adjusted by net financial transfers from abroad, *WTR*. The latter is summed across all regions of origin and destination and across source and destination institutions. The transfers are exogenous in real terms and adjusted by the world price index, *P*. Equation (Y-6) defines fiscal depreciation, where the parameter δ^f is the rate of depreciation. The allocation across households uses the same share vector as for capital remuneration. Disposable income, *Yd*, is household income after tax, where the rate of direct taxation is given by κ^h with a uniform adjustment factor given by χ^k, see equation (Y-7).[34] Equation (Y-8) defines the quantity of disposable income allocated by the LES mechanism (described in the next section). In the default implementation of the model, all of disposable income is allocated across goods and services and saving using a top-level ELES specification. In other words, the decision to save is taken simultaneously with the decision to purchase goods and services. In alternative specifications (see Annex F for an example), the decision to save is taken separately from the allocation of residual income across goods and services, i.e. there is an additional top nest in the disposition of disposable income between savings and consumption. In this case, the LES mechanism is used to allocate the quantity, $Y^{c,}$ which excludes savings, across goods and services.

[32] This is in anticipation of expansion of the GTAP dataset to include more households.
[33] The markups are assumed to be distributed to households using the capital share coefficients.
[34] Closure rules will be discussed in more detail below.

$$YH_{r,h} = \underbrace{\varphi^t_{r,h} TY_r + \varphi^f_{r,h} FY_r}_{\text{Income from natural resources}} + \underbrace{\sum_l \varphi^l_{r,h,l} LY_{r,l} + \varphi^k_{r,h} KY_r}_{\text{Income from labor and capital}}$$

(Y-5)
$$- DeprY_{r,h} + +PGDP_r\, TRG_{r,h}$$
$$+ P\sum_{r'}\sum_{in} WTR_{r',in,r,h} - P\sum_{r'}\sum_{in} WTR_{r,h,r',in}$$

(Y-6) $$DeprY_h = \varphi^k_h \delta^f PGDP\, K$$

(Y-7) $$Yd_h = \left(1 - \chi^k \kappa^h_h\right) YH_h$$

(Y-8) $$Y^c_h = Yd_h$$

Final Demand

Household Final Demand

Household consumption is modeled using a highly articulated demand structure. At the top level of the demand nest, demand for “consumed” commodities is determined by a standard demand function. By default this is the so-called extended linear expenditure system (ELES).[35] This expenditure system is an extension of the more familiar Stone-Geary LES and incorporates household demand for future goods as summarized by household savings decision.[36] The ELES yields a demand for each consumed commodity. These are then mapped to produced commodities using a so-called transition matrix approach. Each consumed commodity, *k*, is made up of one or more produced commodities, *i*. These are combined together using a CES production function (with the possibility of zero substitution). Produced commodities can appear in the production function of one or more consumed commodities. For example, energy can be used to produce transportation services as well as household heating and power services. The final nest splits household demand for produced goods by region of origin.

The generic form of the ELES can be represented by the following utility maximization problem:

$$\max U = \sum_k \mu_k \ln(C_k - \theta_k) + \mu_s \ln\left(\frac{S}{P^s}\right)$$

subject to

$$\sum_k P_k C_k + S = Y \qquad \text{and} \qquad \sum_k \mu_k + \mu_s = 1$$

where *U* represents utility, *C* is a vector of consumer goods, *P* is the vector of consumer prices, *S* represents the value of saving, P^s the relevant price of saving, and *Y* is total income (which is assumed to be completely spent). Equation (D-2) represents the reduced form of household demand for goods and services derived from the ELES, where *XC* represents household demand (defined at the consumer goods level), and *PC* is the relevant household-specific consumer price. Household consumption is the sum of two components. The first component, θ, is called the *subsistence minimum* (or floor consumption).[37] The

35 Annex G describes implementation of the AIDADS demand system as a replacement for the ELES.

36 See Deaton and Muellbauer for an exposition of the LES. See Lluch and Howe concerning the ELES. See Burniaux and van der Mensbrugghe for calibration and implementation of the ELES in applied general equilibrium work.

37 In the model it is adjusted for changes in the population level.

second component is a share of Y^*, which is referred to as the *supernumerary* income. Equation (D-1) defines supernumerary income. It is equal to total disposal income, less aggregate expenditures on the subsistence minima, i.e. it is the residual income after purchasing the subsistence minima. Equation (D-3) determines, as a residual, household saving, S^h. Both the subsistence minima, θ, and the share parameters, μ, are calibrated to a given set of *initial* consumption shares and income elasticities. Equation (D-4) defines the consumer price index in terms of the contemporaneous bundle of consumption.

(D-1) $$Y_h^* = Y_h^c - \sum_k PC_{k,h}\ Pop_h\ \theta_{k,h}$$

(D-2) $$XC_{k,h} = Pop_h\ \theta_{k,h} + \frac{\mu_{k,h}}{PC_{k,h}} Y_h^*$$

(D-3) $$S_h^h = Y_h^c - \sum_k PC_{k,h} XC_{k,h}$$

(D-4) $$CPI_h = \frac{\sum_k PC_{k,h} XC_{k,h}}{\sum_k PC_{k,h,0} XC_{k,h}}$$

The second level of the consumer demand nest maps the consumed commodities, *XC*, to the produced commodities, *XAc*, defined at the Armington aggregate level. Each good *XC* is produced with the CES combination of one or more produced goods, *XAc*. Equation (D-5) reproduces the "production" function. It sums the demand for produced goods across all consumed goods under the assumption that the consumer price (of the produced goods), *PAc*, is uniform across consumption bundles.[38] The substitution across inputs for each consumption good is determined by the elasticity σ^c (which could in principle be zero). The transition matrix coefficients are given by the parameter Γ. Equation (D-6) defines the consumer price of each good *k* using the CES dual price formula. Equation (D-7) defines the household Armington price. It is equal to the national Armington price times a sector- and household-specific sales tax, τ^{Ac}.[39]

(D-5) $$XAc_{i,h} = \sum_k \Gamma_{i,k,h} \left(\frac{PC_{k,h}}{PAc_{i,h}} \right)^{\sigma_{k,h}^c} XC_{k,h}$$

(D-6) $$PC_{k,h} = \left[\sum_i \Gamma_{i,k,h} \left(PAc_{i,h} \right)^{1-\sigma_{k,h}^c} \right]^{1/(1-\sigma_{k,h}^c)}$$

(D-7) $$PAc_{i,h} = \left(1 + \tau_{i,h}^{Ac}\right) PA_i$$

38 For example, this assumes that refined oil used for auto transport is taxed at the same rate as refined oil used in cooking. This is an artifact of the GTAP data set which has consumer taxes at the level of production goods.

39 The standard version of the model assumes the Armington assumption is applied at the national level, not at the level of individual agents.

Other Final Demand

Other final demand in the model concerns government and investment demand for goods and services. Their (Armington) demand is summarized in a single matrix represented by *XAf*, and the institutional accounts are indexed by *f* (for government and investment). Aggregate government and investment expenditures will be discussed below in the section on closure. Demand is modeled using a simple fixed coefficient specification as described by equation (D-8). *FD* represents aggregate demand (by institution), and the share coefficients are α^f. Equation (D-9) describes the aggregate price index, *PFD*; which is simply an average over the tax adjusted Armington prices with the share coefficients as weights.

(D-8) $$XAf_{i,f} = a^f_{i,f} FD_f$$

(D-9) $$PFD_f = \sum_i a^f_{i,f}\left(1+\tau^{Af}_{i,f}\right)PA_i$$

Trade

Import Specification

Imports are modeled using the ubiquitous Armington assumption, which asserts that demand for commodities is a function of their origin[40]. Given a set of preferences, demand for imports would increase with respect to demand for local goods if the price of imports decreases relative to the price of local goods. This idea has typically been captured using the CES specification. The lower is the CES elasticity of substitution, the less imports compete with domestic goods. Conversely, the higher the level of the substitution elasticity, the greater is the competition between imports and locally produced goods. Assuming perfect homogeneity, with the substitution elasticity equal to infinity, the law of one-price holds.

The GTAP data set allows the Armington specification to be implemented at the agent-specific level, i.e. each economic activity in the economy could have its own set of import preferences (degree of substitution and share parameters). This implies that the Armington demand variables, *XAp*, *XAc*, and *XAf*, can each individually be decomposed into domestic and import components. While this is probably a more realistic implementation of the Armington specification, it increases the size of these models quite substantially, also requiring significantly more Armington substitution elasticities. To avoid the dimensionality problem, the Armington specification is implemented at the aggregate (national) level. Hence, total Armington demand is aggregated across all agents in the economy, and this total demand is decomposed into a domestic and an import component.

The decomposition occurs in four steps, i.e. the model uses a nested CES structure.[41] At the top node, the Armington composite good is decomposed into a domestic good, *XD*, and an aggregate import good, *XMT*. The next three nodes decompose aggregate import demand into demand by region of origin, i.e. the variable *XMT*—aggregate import demand—is allocated across all regions of origin thereby creating the bilateral trade flow matrix—*WTF*. To allow for various substitution possibilities across regions of the world, this decomposition occurs using three CES nests. For example, the first nest may split import demand between high-income and developing countries—with perhaps a relatively low degree of substitutability. The next node may further decompose import demand in each of these broad regions

[40] See Armington (1969).

[41] The four-nested CES is new to Version 6. Previous versions of the LINKAGE model used the more standard two-nested CES. The latter implies that the elasticity of substitution across any pair of trading partners is uniform.

across sub-aggregates—for example between Quad countries on the one hand and high-income Asian countries on the other in the case of imports from all high-income countries, and between East Asian countries and all other developing countries in the case of imports from developing countries. The final node decomposes demand at this level across regions of origin according to the regional concordance of the chosen aggregation (see figure 5 for an illustration). The level of substitution is expected to rise between the top node and the bottom node. It is easy to implement the more traditional two-nested Armington by simply assuming that the degree of substitution is uniform down the bottom three nodes.

Equation (T-1) determines aggregate Armington demand, *XA*. It is the sum across all agents—each production sector, households, and other final demand—of their Armington demand. Equations (T-2) and (T-3) determine the decomposition of the aggregate Armington demand into its two components, respectively, XD^d, the domestic component, and *XMT*, the aggregate import component. The key (Armington) substitution elasticity is given by σ^m, and β^d and β^m represent respectively the domestic and import penetration parameters. The Armington price, *PA*, can be determined using the CES dual price formula, see equation (T-1).

(T-1) $$XA_i = \sum_j XAp_{i,j} + \sum_h XAc_{i,h} + \sum_f XAf_{i,f}$$

(T-2) $$\begin{cases} XD_i^d = \beta_i^d \left(\dfrac{PA_i}{PD_i}\right)^{\sigma_i^m} XA_i & \text{if} \quad \sigma^m < \infty \\ PD_i = PA_i & \text{if} \quad \sigma^m = \infty \end{cases}$$

(T-3) $$\begin{cases} XMT_i = \beta_i^m \left(\dfrac{PA_i}{PMT_i}\right)^{\sigma_i^m} XA_i & \text{if} \quad \sigma^m < \infty \\ PMT_i = PA_i & \text{if} \quad \sigma^m = \infty \end{cases}$$

(T-4) $$\begin{cases} PA_i = \left[\beta_i^d PD_i^{1-\sigma_i^m} + \beta_i^m PMT_i^{1-\sigma_i^m}\right]^{1/\left(1-\sigma_i^m\right)} & \text{if} \quad \sigma^m < \infty \\ XA_i = XD_i^d + XMT_i & \text{if} \quad \sigma^m = \infty \end{cases}$$

If the substitution elasticity between the domestic good and the imported good is infinite, equations (T-2) and (T-3) are replaced by a price identity equating the Armington price with the price of the domestic good and the imported good respectively, i.e. the law of one price. In this case, equation (T-4) is replaced by an equation defining the Armington good as the sum of demand from local production and demand imported from abroad.

At the next level, aggregate import demand, *XMT*, is allocated across the top-tier exporting regions, XM^1. The index *R1* ranges over the top tier regions—for example high income [HIY] and developing [LMY]. There can be more than two top tier regions—these are determined by the user for each individual application. Equation (T-5) specifies the demand for imports by top-tier region of origin, XM^1. All indices are fully specified in this and subsequent equations. The variable *PMT* represents the aggregate (or average) import price in region *r* (across all top-tier trading partners). The price PM^1 represents the (aggregate) import price of importing good *i*, from region *R1*, into region *r*. The key (Armington) substitution elasticity is given by σ^{w1}. Equation (T-6) determines the aggregate import price across all top-tier trading partners, *PMT*, using the CES dual price formula. Both formulas allow for perfect substitution.

$$\text{(T-5)}\quad \begin{cases} XM^1_{r,R1,i} = \beta^1_{r,R1,i}\left(\dfrac{PMT_{r,i}}{PM^1_{r,R1,i}}\right)^{\sigma^{w1}_{r,i}} XMT_{r,i} & \text{if} \quad \sigma^{w1}_{r,i} < \infty \\ PM^1_{r,R1,i} = PMT_{r,i} & \text{if} \quad \sigma^{w1}_{r,i} = \infty \end{cases}$$

$$\text{(T-6)}\quad \begin{cases} PMT_{r,i} = \left[\sum_{R1} \beta^1_{r,R1,i}\left(PM^1_{r,R1,i}\right)^{1-\sigma^{w1}_{r,i}}\right]^{1/\left(1-\sigma^{w1}_{r,i}\right)} & \text{if} \quad \sigma^{w1}_{r,i} < \infty \\ XMT_{r,i} = \sum_{R1} XM^1_{r,R1,i} & \text{if} \quad \sigma^{w1}_{r,i} = \infty \end{cases}$$

At the next tier, the aggregate import composite, XM^1, is broken down into a new set of composite imports designated by XM^2. The relevant composite prices are PM^1 and PM^2. Equation (T-7) determines the demand for the composite bundle XM^2 as a share of XM^1. Because the regions are mutually exclusive, the triple indexing of regions is not needed (i.e. each region *R2* is associated with one and only one region *R1*). The substitution elasticity in the second tier is given by σ^{w2} and is specific to each top-tier region *R1*. Equation (T-8) determines the composite import price of the top-tier, PM^1. The expressions allow for perfect substitutability.

$$\text{(T-7)}\quad \begin{cases} XM^2_{r,R2,i} = \beta^2_{r,R2,i}\left(\dfrac{PM^1_{r,R1,i}}{PM^2_{r,R2,i}}\right)^{\sigma^{w2}_{r,R1,i}} XM^1_{r,R1,i} & \text{for} \quad R2 \in R1 \quad \text{if} \quad \sigma^{w2}_{r,R1,i} < \infty \\ PM^2_{r,R2,i} = PM^1_{r,R1,i} & \text{for} \quad R2 \in R1 \quad \text{if} \quad \sigma^{w2}_{r,R1,i} = \infty \end{cases}$$

$$\text{(T-8)}\quad \begin{cases} PM^1_{r,R1,i} = \left[\sum_{R2\in R1} \beta^2_{r,R2,i}\left(PM^2_{r,R2,i}\right)^{1-\sigma^{w2}_{r,R1,i}}\right]^{1/\left(1-\sigma^{w2}_{r,R1,i}\right)} & \text{if} \quad \sigma^{w2}_{r,R1,i} < \infty \\ XM^1_{r,R1,i} = \sum_{R2\in R1} XM^2_{r,R2,i} & \text{if} \quad \sigma^{w2}_{r,R1,i} = \infty \end{cases}$$

At the final level, aggregate import demand at the second-tier, XM^2, is allocated across trading partners, again using a CES specification. The relevant component price is *PM*. The price *PM* will be described in more detail below, but it does include trade and transport margins, as well as any applicable tariffs. Both the trade and transport margins and the tariffs are specific to both the region of origin and destination of the imports. Equation (T-9) specifies the demand for imports by region of origin, WTF^d. All indices are fully specified in this and subsequent equations. For each good *i*, WTF^d is a square matrix with the imports into region r being read down the rth column.[42] In other words, element (r',r) is import into region r, originating in region r'. The variable PM^2 represents the aggregate (or average) second-tier import price in region r (across all trading partners). The price PM represents the import price (inclusive of trade and transport margins and tariffs) of importing good i, from region r', into region r. The key (Armington) substitution elasticity is given by σ^w. Equation (T-10) determines the aggregate import price, PM^2, using the CES dual price formula.

$$\text{(T-9)} \qquad \begin{cases} WTF^d_{r',r,i} = \beta^w_{r',r,i}\left(\dfrac{PM^2_{r,R2,i}}{PM_{r',r,i}}\right)^{\sigma^w_{r,R2,i}} XM^2_{r,R2,i} & \text{for } r'\in R2 \text{ if } \sigma^w_{r,R2,i} < \infty \\ PM_{r',r,i} = PM^2_{r,R2,i} & \text{for } r'\in R2 \text{ if } \sigma^w_{r,R2,i} = \infty \end{cases}$$

$$\text{(T-10)} \qquad \begin{cases} PM^2_{r,R2,i} = \left[\sum_{r'\in R2}\beta^w_{r',r,i}\left(PM_{r',r,i}\right)^{1-\sigma^w_{r,R2,i}}\right]^{1/\left(1-\sigma^w_{r,R2,i}\right)} & \text{if } \sigma^w_{r,R2,i} < \infty \\ XM^2_{r,R2,i} = \sum_{r'\in R2} WTF^d_{r',r,i} & \text{if } \sigma^w_{r,R2,i} = \infty \end{cases}$$

Tariff rate quotas

This section—new to version 5.2—describes the implementation of tariff rate quotas (TRQs).[43] TRQs were introduced as trade instruments in the Uruguay Round. TRQs were a solution to two problems in agricultural trade—allowing for minimal market access and tariffication. A certain quantity of imports—the in-quota amount—is allowed into a country at relatively low tariffs. Imports over-quota are allowed in principle, but the tariff rate is typically prohibitively high. Thus, there are two tariff rates, τ^{in} and τ^{out}. There is also a quota level designated by WTF^q. And aggregate import demand, WTF^d, will be split into two components—in- and over-quota imports, respectively WTF^{in} and WTF^{out}. There are three possible regimes with a TRQ. If import demand is less than the fixed quota, imports will be taxed at the in-quota tariff rate and over-quota imports are zero. If import demand is equal to the quota level, tariffs are collected at the in-quota rate but *ex ante* excess demand leads to a quota premium rate, τ^{pr}, which guarantees that supply equals demand. The third regime is when the premium exceeds *ex ante* the difference between the over-quota tariff and the in-quota tariff, i.e. the premium is capped at the difference in the two rates. At that point, import demand exceeds the quota level and over-quota imports are positive. Very efficient producers may be able to overcome the high over-quota tariff.

TRQs—since they are based on regime switches—are most easily implemented using mixed complementarity programming (MCP). The latter is based on orthogonality conditions. Equation (T-11) describes the first orthogonality condition. It states that in-quota imports cannot exceed the quota. It is associated with a constraint on the quota premium rate, which has a lower bound of zero. The orthogonality condition can be written as:

42 Normally, the diagonal of the trade matrix is 0. However, due to aggregation of regions, the diagonal may not necessarily be zero.

43 A full description of TRQs can be found in Skully (1999). For implementation in GE models, see Elbehri and Pearson (2000) and Rutherford (2001). The implementation in the LINKAGE model assumes that TRQs are defined on a bilateral basis.

$$\left(WTF^{q}_{r',r,i} - WTF^{in}_{r',r,i}\right)\tau^{pr}_{r',r,i} = 0$$

It represents one of two regimes. If the premium is zero, in-quota imports are less than the quota. If the premium is positive, in-quota imports must equal the quota. Similarly, equation (T-12) describes the upper bound constraint on the quota premium rate. It is capped by the difference in the over- and in-quota rates and is associated with a lower-bound on the over-quota level of imports. The orthogonality condition is:

$$\left(\tau^{pr}_{r',r,i} - (\tau^{out}_{r',r,i} - \tau^{in}_{r',r,i})\right)WTF^{out}_{r',r,i} = 0$$

Similar to above, it reflects two regimes. With the over-quota imports at zero, the tariff premium is less than the difference in the two tariff rates. If over-quota imports are positive, the tariff premium is capped. Equation (T-13) reflects the import quantity identity.

(T-11)	$WTF^{in}_{r',r,i} \le WTF^{q}_{r',r,i}$	$\tau^{pr}_{r',r,i} \ge 0$
(T-12)	$\tau^{pr}_{r',r,i} \le \tau^{out}_{r',r,i} - \tau^{in}_{r',r,i}$	$WTF^{out}_{r',r,i} \ge 0$
(T-13)	$WTF^{d}_{r',r,i} = WTF^{in}_{r',r,i} + WTF^{out}_{r',r,i}$	

Note that the quota premium generates rental income. The income is equal to:

$$PREMY = \tau^{pr}_{r',r,i} WPM_{r',r,i} WTF^{q}_{r',r,i}$$

The income is shared between the importing country and the exporting country. Let χ^{trq} represent the importing country share, than its share of the rental income, assumed to accrue to the government, is:

$$\chi^{trq}_{r',r,i}\tau^{pr}_{r',r,i} WPM_{r',r,i} WTF^{q}_{r',r,i}$$

Export Specification

Export supply is treated in a symmetric fashion as import demand.[44] Domestic producers are assumed to differentiate between local and foreign markets. The constant-elasticity-of-substitution (CET) functional form is specified to implement the (limited) transformation between domestic and foreign markets. Similar to the Armington specification, the CET is implemented in a two-step process where suppliers first distinguish the local market versus the foreign market taken as a whole. In the second step, suppliers optimally allocate production across different foreign markets. Equations (T-14) and (T-15) determine the reduced form supply decisions on the export markets, respectively, XD^s, and *ES*, where the key transformation elasticity is given by σ^x. The relevant component prices are *PD* for domestic supply, and *PET* for aggregate export supply, where the aggregate producer price is given by *PP* (and is determined by production costs). Note that output supply, *XP*, is adjusted by the variable *XMg*, which represents the domestic supply of international trade and transport services. Equation (T-16) essentially determines aggregate output, *XP*, but needs a bit of explanation. In the absence of the CET, aggregate output would be the simple sum of output sold domestically and output sold abroad, i.e.:

$$XP_i = XD^s_i + ES_i$$

With the introduction of the CET, these respective goods are not homogeneous and cannot be added together. Instead, the relevant equation is the CET primal equation:

44 See de Melo and Robinson (1989) for an earlier exposition of trade specification in GE models.

$$XP_i = \left[\left(\gamma_i^d\right)^{-1/\sigma_i^x} \left(XD_i^s\right)^{\left(\sigma_i^x+1\right)/\sigma_i^x} + \left(\gamma_i^e\right)^{-1/\sigma_i^x} \left(ES_i\right)^{\left(\sigma_i^x+1\right)/} \right]^{\sigma_i^x/\left(\sigma_i^x+1\right)}$$

In other words, *XP* is still an aggregate of XD^s and *ES*, but the relevant aggregation function is the CET. The following revenue identity must also hold:

$$PP_i XP_i = PD_i XD_i^s + PET_i ES_i$$

Inserting the reduced form equations (T-14) and (T-15) into the revenue identity, yields the CET dual price formula equation (T-16). Hence, equation (T-16) is equivalent to the supply equilibrium condition.

Perfect transformation, i.e. a transformation elasticity equal to infinity is also a model specification option. In this case, the producer does not differentiate between the domestic and foreign markets and aggregate production is simply the sum of supply to both markets. Further, this implies that the law of one price holds. This means that the two reduced form supply functions in equations (T-14) and (T-15) are replaced by the price identity conditions. The dual price specification of equation (T-16) is replaced by the identity equating total supply to the sum of supplies across the individual markets.

(T-14)
$$\begin{cases} XD_i^s = \gamma_i^d \left(\dfrac{PD_i}{PP_i}\right)^{\sigma_i^x} \left(XP_i - XMg_i\right) & \text{if} \quad \sigma_i^x < \infty \\ PD_i = PP_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

(T-15)
$$\begin{cases} ES_i = \gamma_i^e \left(\dfrac{PET_i}{PP_i}\right)^{\sigma_i^x} \left(XP_i - XMg_i\right) & \text{if} \quad \sigma_i^x < \infty \\ PET_i = PP_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

(T-16)
$$\begin{cases} PP_i = \left[\gamma_i^d PD_i^{1+\sigma_i^x} + \gamma_i^e PET_i^{1+\sigma_i^x}\right]^{1/\left(1+\sigma_i^x\right)} & \text{if} \quad \sigma_i^x < \infty \\ XP_i = XD_i^s + ES_i + XMg_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

Aggregate export supply, *ES*, is allocated across foreign markets using a second CET function. The relevant price for the exporter is the producer price, *PE*, plus the average markup of the export price generated by the TRQ (if present).[45] Equation (T-17) defines the average export price markup. The numerator represents the exporter's share of the quota premium rental income. The export price markup is averaged over all units exported. Equation (T-18) determines export supply, WTF^s. Similar to WTF^d, it represents a trade flow matrix between the originating country *r*, and the destination country, *r*'. (Note the inversion of the indices. Exports from region *r* are read across a row of the trade flow matrix.) The aggregate export supply price—the average across all foreign markets—is represented by the variable *PET*. The individual export supply price, is given by *PE* and represents the producer's export price in region *r*, for supply sold in region *r*', and the export price markup is added to reflect the total unit return to

[45] This formulation implies that the export tax, the transport margin and the import tariff are applied to the producer price. Or, in other words, the export price markup generated by the import quota is not declared by the exporter except as part of its final receipts.

the exporter. The key transformation elasticity is given by σ^z. The aggregate export supply price, *PET*, is determined in equation (T-19), and is the CET dual price.

Similar to the top level CET formulation, an infinite transformation elasticity across foreign markets is allowed as a specification option. In this case, the law of one price holds, as formulated in equation (T-18), i.e. the export price to each region of destination—adjusted by the export price markup—is uniform. The dual price formula is replaced by the supply identity, i.e. aggregate export supply is equal to the sum of export supply to each of the individual markets.[46]

Note that though the model has the option of specifying infinite substitution elasticities between local markets and foreign markets, these elasticities cannot be infinite for *both* exports and imports, due to the presence of trade margins. If top node Armington elasticities were infinite, Equations (T-2) and (T-3) would imply that domestic price (*PD*) and the price of imports (*PMT*) are equal. On the other hand if CET elasticities are infinite, *PD* would be equal to the export price (*PET*). This implies *PET* is equal to *PMT*. However, export prices do not include trade margins, nor tariffs, while import prices do include them (see below). Thus, they cannot be equal, and we cannot impose the law of one price for both imports and exports simultaneously.

$$\text{(T-17)} \qquad \tau^{em}_{r,r',i} = (1-\chi^{trq}_{r,r',i})\,\tau^{pr}_{r,r',i} WPM_{r,r',i} WTF^{q}_{r,r',i} / WTF^{s}_{r,r',i}$$

$$\text{(T-18)} \qquad \begin{cases} WTF^{s}_{r,r',i} = \gamma^{w}_{r,r',i}\left(\dfrac{PE_{r,r',i}+\tau^{em}_{r,r',i}}{PET_{r,i}}\right)^{\sigma^z_{r,i}} ES_{r,i} & \text{if} \quad \sigma^z_i < \infty \\ PE_{r,r',i} = PET_{r,i} - \tau^{em}_{r,r',i} & \text{if} \quad \sigma^z_i = \infty \end{cases}$$

$$\text{(T-19)} \qquad \begin{cases} PET_{r,i} = \left[\sum_{r'} \gamma^{w}_{r,r',i}\left(PE_{r,r',i}+\tau^{em}_{r,r',i}\right)^{1+\sigma^z_{r,i}}\right]^{1/\left(1+\sigma^z_{r,i}\right)} & \text{if} \quad \sigma^z_i < \infty \\ ES_{r,i} = \sum_{r'} WTF^{s}_{r,r',i} & \text{if} \quad \sigma^z_i = \infty \end{cases}$$

Trade Prices

The model incorporates four sets of international (bilateral) prices. *PE* represents the domestic producer price in region *r* for supplying region *r*'. It is the pre-FOB price. *WPE* represents the FOB price in region *r* for supplying region *r*', see equation (T-20). *WPE* incorporates an ad valorem export tax or subsidy, τ^e, which is also indexed by region of origin and destination.[47] Between the originating port in region *r*, and the destination port in region *r*', the price of the commodity is adjusted by a trading partner specific trade and transport margin represented by the *ad valorem* adjustment ζ^t. Equation (T-21) determines *WPM*, which is the CIF price of imports into region *r*', originating in region *r*. The model allows for so-called

46 It is to be noted that this imposes some restrictions on the calibration of the world trade system. Under an infinite elasticity, it is easiest to assume that the aggregate export price, *PET*, is equal to one in the base year, and that the region-specific export prices, *PE*, are similarly all equal to one. This implies that the *world* export price, *WPE*, will not be equal to one in the base year.

47 For example, the US' wheat export enhancement program would yield a different export subsidy for those countries eligible for the subsidy as compared to countries importing US wheat on the world market. Differential export taxes/subsidies may also originate because of commodity aggregation.

iceberg costs. This is represented by the parameter λ^w, which is a non-revenue generating wedge. It is used to represent frictional costs of trade. For example, if λ^w is equal to 0.9 for some transport node, that means that if 100 units leave port *r*, the destination port, *r*', receives only 90 units. Finally, the CIF price, *WPM* is adjusted by tariffs represented by τ^m, which are also differentiated by region of origin.[48] The domestic import price also includes the tariff quota premium rate, τ^{pr}, under a TRQ regime.[49] It is either zero if imports are under quota (or these is no TRQ regime). Some positive rate if the quota is binding. In the case of over-quota imports, the premium is equal to the difference in the two tariff rates and therefore the domestic import price is equal to the CIF price times one plus the over-quota tariff. The resulting price is *PM*, which includes, in the end, export taxes and subsidies, trade and transport margins, and tariffs, see equation (T-22). (Note that the computer implementation of the model only includes the price *WPE*. All the other variables are substituted out using equations (T-20)-(T-22).)

(T-20) $$WPE_{r,r',i} = \left(1+\tau^e_{r,r',i}\right) PE_{r,r',i}$$

(T-21) $$WPM_{r,r',i} = \left(1+\zeta^t_{r,r',i}\right) WPE_{r,r',i} / \lambda^w_{r,r',i}$$

(T-22) $$PM_{r,r',i} = \left(1+\tau^m_{r,r',i}+\tau^{pr}_{r,r',i}\right) WPM_{r,r',i}$$

Demand for International Trade and Transport Services

The volume of demand for international trade and transport services is determined by the volume of bilateral trade. At the world level, it can be calculated in value terms by summing the volume of world trade over all regions and commodities. Equation (T-23) determines the world demand volume for international trade and transport services, *WXMg*, where *WPMg* is its associated price index to be described below.

(T-23) $$WPMg\ WXMg = \sum_r \sum_{r'} \sum_i \zeta^t_{r,r',i} WPE_{r,r',i} WTF^d_{r,r',i}$$

Allocation of the Demand for International Trade and Transport Services Across Regions

The total demand for international trade and transport services will be allocated to regional suppliers based on relative prices. The allocation mechanism is a CES function. Equation (T-24) determines the demand from regional suppliers for international trade and transport services, *AXMg*. The regional share will be determined by the regional price for supplying the services, *APMg*. If the regional price declines with respect to the world price, *WPMg*, then the region will garner a larger share of the world market for these services. The key elasticity is given by σ^{TT}. Equation (T-25) determines the world (or aggregate) price of international trade and transport services, *WPMg*. It is given by the CES dual price formula.

48 Thus allowing for analysis of free trade areas and customs unions.

49 Under the TRQ regime, the variable τ^m is equated with the in-quota tariff rate, there is no separate τ^{in} variable.

(T-24) $$AXMg_r = \alpha_r^{TT} \left(\frac{WPMg}{APMg_r} \right)^{\sigma^{TT}} WXMg$$

(T-25) $$WPMg = \left[\sum_r \alpha_r^{TT} APMg_r^{1-\sigma^{TT}} \right]^{1/\left(1-\sigma^{TT}\right)}$$

Local Supply for Trade and Transport Services

Once the regional demand for aggregate trade and transport services is determined, *AXMg*, this is further decomposed into specific *sectoral* demand for local goods and services, *XMg*.[50] Equation (T-26) determines the production structure of the aggregate regional trade and transport service, *AXMg*. It is a simple fixed coefficient production structure. The regional price of trade and transport service, *APMg*, therefore, is the fixed coefficients dual price, where the relevant price of goods and services is given by the local producer price, *PP* (equation (T-27)).

(T-26) $$XMg_{r,i} = a_{r,i}^{Mg} AXMg_r$$

(T-27) $$APMg_r = \sum_i a_{r,i}^{Mg} PP_{r,i}$$

Goods Market Equilibrium

There are two basic goods in the model, goods produced locally and sold locally, *XD*, and goods produced locally and sold to individual foreign markets, *WTF*. All other goods are composite goods and have been completely defined. Equation (E-1) determines equilibrium on the domestic market for goods produced locally, i.e. it determines price *PD*. Equation (E-2) determines the price of world trade flows, i.e. equilibrium on the international markets, *WPE*. It incorporates the trade friction parameter, λ^w, which represents non-revenue generating *iceberg* costs. (Note that in the code of the model, both the *XD* and *WTF* variables are not indexed by supply and demand. Instead, the equilibrium conditions are substituted in. For example equations (T-2) and (T-14) only contain the single variable *XD*. One of the equations can be assumed to determine the volume *XD*, and the other can be assumed to determine the equilibrium price *PD*. The same holds true for equations (T-9) and (T-19). The substitution of the equilibrium conditions into the other equations reduces the dimension of the resulting model.[51])

(E-1) $$XD_{r,i}^s = XD_{r,i}^d$$

(E-2) $$WTF_{r,r',i}^d = \lambda_{r,r',i}^w WTF_{r,r',i}^s$$

50 It is assumed that these local services are truly domestic and do not contain an import component. See for example equations (T-14) and (T-15).

51 If using a Gauss-Seidel solution algorithm, these equilibrium equations would be replaced by a tâtonnement equation (see Burniaux and van der Mensbrugghe).

Domestic Closure

Domestic closure concerns two aggregate accounts—the government financing balance, and the investment-saving balance. Government derives revenues from direct and indirect taxes, trade taxes, and taxes on factors. Equations (C-1) and (C-2) define respectively nominal and real revenues from import tariffs. Equation (C-3) determines gross government revenues, *YG*. In order, they are derived from the production tax, household income taxes, indirect sales tax on (Armington) consumption in production and final demand, import tariffs, export taxes/subsidies, domestic share of import quota premium rents and factor taxes. Equation (C-4) defines the net financial position of the government (in value terms), S^g. It is the difference between aggregate revenues, and the value of aggregate expenditures on goods and services and transfers to households. It also includes net financial transfers on the current account accruing to the government, *WTR*. Real government saving, RS^g, is equal to nominal saving deflated by the GDP deflator, *PGDP*, see equation (C-5). The volume of government expenditure, FD_{Gov}, is assumed to be a constant share of real GDP (at market price), *RGDPMP*, see equation (C-6).[52] The standard fiscal closure rule is that RS^g is exogenous, i.e. the government has a target for the net fiscal position of the public budget. To achieve this given target, the adjustment factor of the direct tax rate on household income, χ^h, is endogenous. In a long-term model, this seems to be an appropriate closure in order to guarantee the sustainability of the public debt level.[53]

52 In policy simulations, aggregate real government expenditures are typically held constant at their baseline levels.

53 Variations in the closure rule could target the real level of investment. Other fiscal instruments could also be employed to achieve the target.

(C-1) $$TarY_r = \sum_i \sum_{r'} WPM_{r',r,i} \left[\tau^m_{r',r,i} WTF^{in}_{r',r,i} + \tau^{out}_{r',r,i} WTF^{out}_{r',r,i} \right]$$

(C-2) $$RTarY_r = TarY_r / PGDP_r$$

(C-3)
$$\begin{aligned} YG_r \;=\; & \underbrace{\sum_i \tau^p_{r,i}(1+\pi_{r,i}) PX_{r,i} XP_{r,i}}_{\text{Output tax}} + \underbrace{\sum_h \chi^h_r \kappa^h_{r,h} YH_r}_{\text{Income tax}} \\ + & \underbrace{\sum_i PA_{r,i} \left[\sum_j \tau^{Ap}_{r,i,j} XAp_{r,i,j} + \sum_h \tau^{Ac}_{r,i,h} XAc_{r,i,h} + \sum_f \tau^{Af}_{r,i,f} XAf_{r,i,f} \right]}_{\text{Sales tax}} \\ + & TarY_r + \underbrace{\sum_i \sum_{r'} \tau^e_{r,r',i} PE_{r,r',i} WTF^s_{r,r',i}}_{\text{Export tax}} \\ + & \underbrace{\sum_i \sum_{r'} \chi^{trq}_{r',r,i} \tau^p_{r',r,i} WPM_{r',r,i} WTF^q_{r',r,i}}_{\text{Quota rents}} \\ + & \underbrace{\sum_i \sum_l \tau^l_{r,l,i} NW_{r,l,i} (LV^d_{r,l,i} + N_{r,i} LF^d_{r,l,i})}_{\text{Wage tax}} + \underbrace{\sum_i \tau^t_{r,i} NPT_{r,i} T^d_{r,i}}_{\text{Land tax}} \\ + & \underbrace{\sum_i \left[\sum_v \tau^k_{r,i,v} NR_{r,i,v} Kv^d_{r,i,v} + \tau^k_{r,i,Old} NR_{r,i,Old} N_{r,i} KF^d_{r,i} \right]}_{\text{Capital tax}} \end{aligned}$$

(C-4)
$$\begin{aligned} S^g_r \;=\; & YG_r - PFD_{r,Gov} FD_{r,Gov} - \sum_h PGDP_r \, TRG_{r,h} \\ + & P \sum_{r'} \sum_{in} WTR_{r',in,r,Gov} - P \sum_{r'} \sum_{in} WTR_{r,Gov,r',in} \end{aligned}$$

(C-5) $$RS^g = S^g / PGDP$$

(C-6) $$FD_{Gov} = \chi_{Gov} RGDPMP$$

The model does not include an investment schedule that relates the level of investment to its rate of return. (For an alternative, see Annex E.) Instead the investment schedule is flat, and the value of investment is equated to the value of aggregate saving. Equation (C-7) determines the value of foreign saving. It is equal to some exogenous level multiplied by a world price. The world price is a price index of OECD manufactured exports and thus each unit of exogenous foreign saving is essentially equated to the purchase of an average unit of OECD manufactured exports. Equation (C-8) guarantees that both the volume and value of foreign saving flow, aggregated over all regions, sums identically to zero. Equation (C-9) determines the volume of investment, FD_{Inv}. On the left-hand side, the volume of gross investment is multiplied by the price of investment (see equation (D-6)). It is equated with aggregate national saving—from households, S^h, from the public sector, S^g — plus foreign saving, S^f, less depreciation, *DeprY*, and adjusted for net financial bilateral transfers from abroad, *WTR*.[54] Notice that the equation is indexed by r^*. The index ranges over all regions of the model except for 1. This is due to a global Walras' law. The model has one more equation than variable and one of the investment-saving equations is dropped for any one of the regions. Equation (C-10) defines the investment share of GDP at

[54] The variable *WTR* is fully bilateral, whereas the S^f variable is a global adjustment to domestic savings.

market price, *InvSh*. Equation (C-11) defines a world price index, *P*. It is used to evaluate in a common currency the foreign saving flows, S^f, which are fixed in each time period (under the constraint that they sum to zero over all regions.) *P* is defined as a price index of OECD exports. It is also fixed (normally at its base value of 1), and is therefore the numéraire of the model.[55] Equation (C-12) defines an average world rate of return to capital. It is equal to the average aggregate rate of return across regions, weighted by the regional capital stocks.

(C-7) $$S_r^f = P.\overline{S_r^f}$$

(C-8) $$\sum_r S_r^f \equiv 0$$

(C-9) $$\begin{aligned} PFD_{Inv,r*}FD_{Inv,r*} &= \sum_h \left[S_{h,r*}^h + DeprY_h^{r*}\right] + S_{r*}^g + S_{r*}^f \\ &+ P\sum_{r'}\sum_{in} WTR_{r',in,r*,Inv} - P\sum_{r'}\sum_{in} WTR_{r*,Inv,r',in} \end{aligned}$$

(C-10) $$InvSh_r = PFD_{Inv,r}FD_{Inv,r} / GDPMP_r$$

(C-11) $$P = \frac{\sum_{r\in OECD}\sum_{r'}\sum_{i\in Manu} WPE_{r,r',i}WTF_{r,r',i,0}}{\sum_{r\in OECD}\sum_{r'}\sum_{i\in Manu} WPE_{r,r',i,0}WTF_{r,r',i,0}}$$

(C-12) $$WRR = \frac{\sum_r TR_r K_r}{\sum_r K_r}$$

Factor Markets

Labor Markets

Each national economy is divided into two distinct geographic zones, indexed by *gs*. The zones define potentially separate labor markets and are designated by *Rur* and *Urb*, representing respectively rural and urban areas.[56] A third zone, *Tot*, represents the national market. A single elasticity, ω^m, determines the nature of the labor market. If the migration elasticity is infinite, then the labor market is nationally integrated and labor is fully mobile between rural and urban activities. A single economy-wide wage rate, TW_{Tot}, will clear the national labor market. If the migration elasticity is finite, then there is labor market segmentation with migration.[57] Separate market-clearing wage rates will be determined in each labor market. The decision to migrate is a function of the expected relative wages.[58]

55 From the strict point of view of solving a system of equations, equation (C-6) can be thought of as determining the volume of investment for the residual region whose equation is dropped from equation (C-5).

56 In most cases the distinction rural and urban will be blurred since it typically will represent agricultural versus non-agricultural activities. In most economies, the rural sector will have significant non-agricultural activities.

57 Fully segmented markets can be achieved by setting the migration elasticity to 0 and fixing the base year level of migration at 0.

58 Market specific consumer price indices are not available in the standard version of the model since there is a single representative household for the national economy. Thus, only relative nominal wages are used in the migration function.

The equations described below are based on two indices, *gz* and *gs*. The first ranges over the three zones—rural, urban and national. The second, *gs*, is a subset of *gz* and only ranges over the segmented markets—rural and urban.

$$g \in \{Rur, Urb, Tot\}$$

$$gs \in \{Rur, Urb\} \subset g$$

The first three equations describe labor supply in each zone. Rural labor supply is equal to the previous period's labor supply adjusted for (exogenous) natural growth, g^l, from which is subtracted migration, *MIGR*, equation (F-1). Equation (F-2) is a similar equation defining urban labor supply where migration augments the natural growth of urban labor supply. Equation (F-3) determines the national labor supply. It is simply the sum of rural and urban labor supplies. The national labor supply is of course independent of internal migration.

(F-1) $$L^s_{l,Rur} = \left(1 + g^l_{l,Rur}\right) L^s_{l,Rur,-1} - MIGR_l$$

(F-2) $$L^s_{l,Urb} = \left(1 + g^l_{l,Urb}\right) L^s_{l,Urb,-1} + MIGR_l$$

(F-3) $$L^s_{l,Tot} = \sum_{gs} L^s_{l,gs}$$

(F-4) $$AVGW_{l,gz} = \frac{\sum_{i \in gz} NW_{l,i}\left(LV^d_{l,i} + N_i LF^d_{l,i}\right)}{\sum_{i \in gz}\left(LV^d_{l,i} + N_i LF^d_{l,i}\right)}$$

(F-5) $$MIGR_l = \chi^{migr}_l \left(\frac{\left(1 - UE_{Urb}\right) AVGW_{l,Urb}}{\left(1 - UE_{Rur}\right) AVGW_{l,Rur}} \right)^{\omega^m_l} \quad \text{if} \quad \omega^m_l \neq \infty$$

The next two equations determine migration. Equation (F-4) defines the average wage rate, *AVGW*, in all three zones (including the national average). It is the weighted average of the sector-specific (net) wage, weighted by actual sectoral labor demand within each zone. The net wage, *NW*, is the wage rate received by employees (net of the wage tax) and is the natural wage to use in the migration function. Equation (F-5) determines rural to urban migration, *MIGR*. It is a function of the expected urban wage relative to the expected rural wage. Thus the average wage in each sector is multiplied by the probability of finding employment as measured by 1 minus the unemployment rate, *UE*. The migration function is deleted from the model specification if there is an integrated labor market, i.e. if ω^m is infinite.

The model incorporates regime-switching behavior in labor markets. The following discussion describes the basic theory and this is followed by a description of the model equations.

It is relatively straightforward to introduce semi-rigidity in wages and thus creating a wedge between labor supply and demand. Let *WMIN* represent a minimum wage, possibly 0, and *UE* be the unemployment rate. The following orthogonality condition represents two possible regimes:

$$UE.\left(W - WMIN\right) = 0$$

The first regime has the prevailing wage exceeding the minimum wage. In this case unemployment must be zero for the condition to hold. In the second regime, the equilibrium wage is below the minimum wage, in which case the prevailing wage, *W*, is set to the minimum wage and unemployment is positive.

The orthogonality condition is easily implemented using mixed complementarity programming (MCP). It is converted to the following set of conditions:

$$W \geq WMIN \qquad \text{and} \qquad UE \geq 0$$

The minimum wage is driven by the following equation:

$$WMIN = \chi^{wmin} (PS)^{\phi^{ps}} (PABS)^{\phi^{p}} (UE)^{-\phi^{ue}}$$

This equation, depending on the elasticities, can represent a variety of different labor market specifications.[59] If ϕ^{ps} is positive, employees may base their wage demands relative to the average return to factors (where *PS* is the average price of value added). On the other hand, if ϕ^{p} is positive, wages could be linked to the overall price level, i.e. employees are targeting a real wage. Finally, if ϕ^{ue} is positive, this would be consistent with efficiency wages where the discipline of market forces dampens wage demands.[60]

(F-6)
$$\begin{cases} \left(TW_{l,Tot} - WMIN_{l,Tot}\right) . UE_{l,Tot} = 0 & \text{if} \quad \omega^m = \infty \\ TW_{l,Tot} = AVGW_{l,Tot} & \text{if} \quad \omega^m \neq \infty \end{cases}$$

(F-7)
$$\begin{cases} \left(TW_{l,gs} - WMIN_{l,gs}\right) . UE_{l,gs} = 0 & \text{if} \quad \omega^m \neq \infty \\ TW_{l,gs} = TW_{l,Tot} & \text{if} \quad \omega^m = \infty \end{cases}$$

(F-8)
$$WMIN_{l,Tot} = \chi^{wmin}_{l,Tot} PS_{Tot}^{\omega^{ps}_{Tot}} PABS^{\omega^{p}_{Tot}} \left(1 - UE_{l,Tot}\right)^{\omega^{ue}_{Tot}} \quad \text{if} \quad \omega^m = \infty$$

(F-9)
$$\begin{cases} WMIN_{l,gs} = \chi^{wmin}_{l,gs} PS_{gs}^{\omega^{ps}_{gs}} PABS^{\omega^{p}_{gs}} \left(1 - UE_{l,gs}\right)^{\omega^{ue}_{gs}} & \text{if} \quad \omega^m \neq \infty \\ WMIN_{l,gs} = WMIN_{l,Tot} & \text{if} \quad \omega^m = \infty \end{cases}$$

(F-10)
$$UE_{l,gz} = \frac{L^s_{l,gz} - \sum_{i \in gz} \left(LV^d_{l,i} + N_i LF^d_{l,i}\right)}{L^s_{l,gz}} \geq 0$$

Equation (F-6) determines the national wage rate, *TW*. In the case of an integrated market, the national wage rate must be greater than or equal to the minimum wage (possibly 0). This is linked to the orthogonality condition by setting a lower bound on the rate of unemployment (using MCP). Thus if unemployment is zero, the national wage rate is greater than the minimum wage. And if the minimum wage is binding, unemployment is positive. In the case of segmented markets, the national wage is simply set to the average wage, but it has no implication for model behavior. Equation (F-7) represents the relevant wage equations for the segmented markets. If markets are segmented, the market-specific wage rates will be determined by the interaction between labor supply-demand balance and the minimum wage. In the case of a national market, the market-specific wage rates will be set to the national wage rate. Equations (F-8) and (F-9) determine the minimum wage, *WMIN*, for the respective markets. The former determines the minimum wage in the case of an integrated market, and the latter the minimum wage in

59 For further discussion see Agénor et al (2002a) and Agénor et al (2002b).

60 In the equation formulation, 1 less the unemployment rate is used to avoid pushing the minimum wage down to zero. If the original elasticity—i.e. the one with respect to the rate of unemployment—is measured as σ, the elasticity with respect to 1 less the rate of unemployment is equal to:

$$\omega = \sigma(1 - UE_0) / UE_0$$

If σ is 1, then ω is 9 if UE_0 is 10% and ω is 19 is UE_0 is 5%.

the case of segmented markets. There is no national minimum wage in the case of segmented markets. The minimum wage is a positive function of the value added price index and/or the price level, and a negative function of the unemployment rate. Equation (F-10) determines the unemployment rate, *UE*.

Though labor is assumed to be perfectly mobile across sectors within a market segment, inter-sectoral wage differentials are allowed to co-exist reflecting specific institutional features related to the domestic labor markets.[61] In the basic version of the model, the inter-sectoral wage differentials are assumed to be fixed. Equation (F-11) determines the sectoral skill-specific wage rates as a function of the base inter-sectoral wage differentials and changes in the segment-specific wage rate. If there is a national market, the *TW* variable will be the same in all market segments. If there is a minimum wage, and it is binding, the *TW* variable will equal the minimum wage. In all other cases, *TW* clears the relevant market. Equation (F-12) introduces a tax on wages. The variable *W* represents the wage as perceived by employers, whereas *NW* represents the wage received by employees.

(F-11) $$NW_{l,j} = \Phi_{l,j} TW_{l,gs} \quad \text{where} \quad j \in gs$$

(F-12) $$W_{l,j} = \left(1 + \tau^{l}_{l,j}\right) NW_{l,j}$$

Land Market

There are two basic features of the land market. First, the aggregate supply of land is allowed to respond to changes in the real aggregate price of land. Second, the allocation of the supply of land across different economic (agricultural) activities is assumed to respond to relative land prices across these activities using a CET transformation function. Equation (F-13) determines the aggregate supply of land. For finite supply elasticities, the supply curve responds positively to the real aggregate price of land. If the supply elasticity is infinite, the real price of land is fixed.[62] Two functional forms are available for the land supply function—a constant elasticity function or a logistic function. The aggregate supply of land is *TLnd*, with an associated price of *PTLnd*, and a supply elasticity given by η^T. Equation (F-13) determines the supply function for all three cases. The first two equations are for the case of a finite elasticity supply curve. The first is for the constant elasticity version, the second for the logistic supply curve. In the case of the latter, the parameters of the function are calibrated to an initial supply elasticity (i.e. curvature of the supply function) and the initial distance from the asymptote, *LndMAX*. The aggregate (or economy-wide) price of land, *PTLnd*, is determined in equation (F-14). If there is friction in the allocation of land across sectors, i.e. the CET transformation elasticity is finite, then the aggregate land price is determined by the CET dual price aggregator as a function of the sector specific land price, *PT*. If, on the other hand, land is freely mobile across sectors, i.e. the CET transformation elasticity is infinite, the law of one price holds, and the aggregate price of land is determined through the equilibrium condition equating aggregate land demand to its aggregate supply. In the polar case, with a CET elasticity of 0, land is sector specific, and the land market is fully segmented. Equation (F-15) determines the sectoral allocation of land across sectors, T^s, assuming the CET elasticity is finite. The equivalent statement in dual form, if the CET elasticity is infinite, is that the law of one price holds, i.e. the sector-specific land price is uniformly equal

61 In most applications, net wages are uniform across sectors (within a market segment). If independent sources are available for sector-specific labor volumes, it is more than likely the case that intersectoral wage differentials will emerge. These wage differentials can represent a variety of factors—i) labor, even at the same skill level may not be totally homogeneous; ii) sectors may represent combinations of different market institutions, for example formal and informal employment, union vs. non-union; iii) other factors can influence wage differentials across sectors, for example hardship or occupational hazard premium. The implicit assumption made in the model is that all wages move in a coordinated fashion within a labor market segment keeping the differentials constant.

62 In most cases, one would assume that the supply elasticity of land is low.

to the aggregate price. Finally, Equation (F-16) represents sectoral land market equilibrium in the case of a finite CET elasticity. In the case of an infinite elasticity, Equation (F-16) trivially sets supply equal to demand. Equation (F-17) introduces a tax on land remuneration, where *PT* is the price paid by farmers, and *NPT* is the price received by landowners. If the tax is negative, τ^t is a subsidy and the cost to farmers is less than the opportunity cost of the land.

(F-13)
$$\begin{cases} TLnd = \chi^T \left(\dfrac{PTLnd}{PABS}\right)^{\eta^T} & \text{if} \quad 0 \le \eta^T < \infty \text{ and } LndMAX = \infty \\ TLnd = \dfrac{LndMAX}{1 + \chi^T \exp\left(-\gamma^{ts}\left(\dfrac{PTLnd}{PABS}\right)\right)} & \text{if} \quad 0 \le \eta^T < \infty \text{ and } LndMAX < \infty \\ PTLnd = PABS\ PTLnd_0 & \text{if} \quad \eta^T = \infty \end{cases}$$

(F-14)
$$\begin{cases} PTLnd = \left[\sum_i \gamma_i^T PT_i^{1+\omega^t}\right]^{1/\left(1+\omega^T\right)} & \text{if} \quad 0 \le \omega^T < \infty \\ TLnd = \sum_i T_i^d & \text{if} \quad \omega^T = \infty \end{cases}$$

(F-15)
$$\begin{cases} T_i^s = \gamma_i^T \left(\dfrac{PT_i}{PTLnd}\right)^{\omega^T} TLnd & \text{if} \quad 0 \le \omega^T < \infty \\ PT_i = PTLnd & \text{if} \quad \omega^T = \infty \end{cases}$$

(F-16)
$$T_i^d = T_i^s$$

(F-17)
$$PT_i = \left(1 + \tau_i^t\right) NPT_i$$

Sector-specific Factors

The modeling of sector specific factors is straightforward.[63] A constant elasticity supply function is assumed (eventually with a value of infinity). Equilibrium of supply and demand determines the factor price. Equation (F-18) specifies the supply function for the sector specific factor, F^s, with the supply elasticity given by ω^F. Equation (F-19) is the equilibrium condition. In the case of a finite supply elasticity, it determines the sector-specific factor price, *PF*. In the case of an infinite supply elasticity, it trivially equates supply with demand.

63 Given that these factors are typically natural resources, a modeling strategy for the long term would suggest some sort of depletion mechanism in a dynamic framework. The depletion mechanism would need to take into account whether the resource was renewable or not, and in the case of the latter, the potential for investing in new exploration and production.

$$\text{(F-18)} \quad \begin{cases} F_i^s = \chi_i^F \left(\dfrac{PF_i}{PABS}\right)^{\omega^F} & \text{if} \quad 0 \le \omega^F < \infty \\ PF_i = PABS\ PF_{i,0} & \text{if} \quad \omega^F = \infty \end{cases}$$

$$\text{(F-19)} \quad F_i^s = F_i^d$$

Capital Market in a Single Vintage Framework

This section describes sectoral capital allocation under the assumption of a single vintage capital stock. The single vintage model applies particularly to the comparative static version of the model, but can also be used in dynamic mode. What is assumed in the single vintage version of the model is that the aggregate capital stock is fixed and is allocated across sectors using a CET transformation function. A CET elasticity of 0 implies that capital is perfectly immobile and sector specific. A CET elasticity of infinity is the polar case with perfect capital mobility and the rates of return across sectors are equalized.

$$\text{(F-20)} \quad \begin{cases} KS_i^s = \gamma_i^k \left(\dfrac{R_i}{TR}\right)^{\omega^K} K^s & \text{if} \quad 0 \le \omega^K < \infty \\ R_i = TR & \text{if} \quad \omega^K = \infty \end{cases}$$

$$\text{(F-21)} \quad \begin{cases} TR = \left[\sum_i \gamma_i^k (R_i)^{1+\omega^K}\right]^{1/\left(1+\omega^K\right)} & \text{if} \quad 0 \le \omega^K < \infty \\ \sum_i Kv_i^d + N_i KF_i^d = K^s & \text{if} \quad \omega^K = \infty \end{cases}$$

$$\text{(F-22)} \quad \sum_i Kv_i^d + N_i KF_i^d = KS_i^s$$

Equation (F-20) determines the sectoral allocation of capital, KS^s, assuming a finite CET elasticity, where K^s represents the aggregate stock of capital.[64] (Note that we are dropping the vintage index on the sectoral rate of return, R, and that the sectoral capital demand variable is simply K^d, rather than Kv^d.) If the CET elasticity is infinite, then equation (F-20) sets the sector-specific rate of return equal to a uniform economy-wide rate of return, i.e. the law of one price holds. Equation (F-21) determines the aggregate rate of return on capital, TR. If capital is partially mobile, it is the CET dual price of the sector specific rates of return. If capital is perfectly mobile, it is determined via equilibrium, equating total sectoral capital demand to total capital supply. Finally, equation (F-22) determines the sector-specific equilibrium rate of return, R, in the case of partial capital mobility. If capital is perfectly mobile, it trivially sets capital supply equal to capital demand.

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64 K^s represents the normalized value of the capital stock. This will be explained in greater detail below. In theory, the aggregate capital stock is fixed in each time period. In practice, it is somewhat influenced by the level of contemporaneous investment if the periodic step sizes are greater than 1 (see below).

Capital Market Equilibrium in a Multiple Vintage Framework[65]

This section describes sectoral capital allocation under the assumption of multiple vintage capital. Capital market equilibrium under the vintage capital framework assumes the following:

- *New* capital is perfectly mobile and its allocation across sectors insures a uniform rate of return.
- *Old* capital in expanding sectors is equated to new capital, i.e. the rate of return on *Old* capital in expanding sectors is the same as the economy-wide rate of return on new capital.
- Declining sectors release *Old* capital. The released *Old* capital is added to the stock of *New* capital. The assumption here is that declining sectors will first release the most mobile types of capital, and this capital, being mobile, is comparable to *New* capital (e.g. transportation equipment).
- The rate of return on capital in declining sectors is determined by sector-specific supply and demand conditions.

The result of these assumptions is that if there are no sectors with declining economic activity, there is a single economy-wide rate of return. In the case of declining sectors, there will be an additional sector-specific rate of return on *Old* capital for each sector in decline.

To determine whether a sector is in decline or not, one assesses total sectoral demand (which of course, in equilibrium equals output). Given the capital-output ratio, it is possible to calculate whether the initially installed capital is able to produce the given demand. In a declining sector, the installed capital will exceed the capital necessary to produce existing demand. These sectors will therefore release capital on the secondary capital market in order to match their effective (capital) demand with supply. The supply schedule for released capital is a constant elasticity of supply function where the main argument is the change in the relative return between *Old* and *New* capital. Supply of capital to the declining sector is given by the following formula:

$$K^{s}_{i,Old,t} = K^{0}_{i,t}\left[R_{i,Old,t} / R_{i,New,t}\right]^{\eta^k_i}$$

where K^s_{Old} is capital supply in the declining sector, K^0 is the initial installed (and depreciated) capital in the sector at the beginning of the period, and η^k is the dis-investment elasticity. In other words, as the rate of return on *Old* capital increases towards (decreases from) the rate of return on *New* capital, capital supply in the declining sector will increase (decrease). Released capital is the difference between K^0 and $K^{s,Old}$. It is added to the stock of *New* capital. In equilibrium, the *Old* supply of capital must equal the sectoral demand for capital:

$$K^{s}_{i,Old.t} = Kv^{d}_{i,Old,t}$$

Inserting this into the equation above and defining the following variable

$$RR_{i,t} = R_{i,Old,t} / R_{i,New,t}$$

yields the following equilibrium condition:

$$Kv^{d}_{i,Old,t} = K^{0}_{i,t}\left[RR_{i,t}\right]^{\eta^k_i}$$

The supply curve is kinked, i.e. the relative rate of return is bounded above by 1. If demand for capital exceeds installed capital, the sector will demand *New* capital and the rate of return on *Old* capital is equal

[65] The vintage specification of capital is based on work by Fullerton (1983), and Ballard et. al. (1985).

to the rate of return on *New* capital, i.e. the relative rate of return is 1. The kinked supply curve has been transformed into a mixed complementarity (MCP) relation. The following inequality is inserted in the model:

$$K_{i,Old}^{s} = K_{i,t}^{0}\left[RR_{i,t}\right]^{\eta_i^k} \leq K_i^{d,Not} = \chi_{i,Old}^{v} XP_i$$

The right-hand side determines the *notional* demand for capital in sector ***i***, i.e. it assesses aggregate output (equal to demand) and multiplies this by the capital output ratio for *Old* capital. This is then the derived demand for *Old* capital. If the installed capital is insufficient to meet demand for *Old* capital, the sector will demand *New* capital, and the inequality obtains with the relative rates of return capped at 1. If the derived demand for *Old* capital is less than installed capital, the sector will release capital according to the supply schedule. In this case the inequality transforms into an equality, and the relative rate of return is less than 1.[66] Equation (F-23) determines the capital output ratio, χ^v. Equation (F-24) specifies the supply schedule of *Old* capital. In effect, this equation determines the variable *RR*, the relative rate of return between *Old* and *New* capital.

(F-23) $$\chi_{i,v}^{v} = \frac{Kv_{i,v}^{d}}{XPv_{i,v}}$$

(F-24) $$K_{i,t}^{0}\left(RR_{i,t}\right)^{\eta_i^k} \leq \chi_{i,Old}^{v} XP_i \quad \text{and} \quad RR_{i,t} \leq 1$$

There is a single economy-wide rate of return on *New* capital. The equilibrium rate of return on *New* capital is determined by setting aggregate supply equal to aggregate demand. Aggregate demand for new capital is given by:

$$\sum_{i \in Expanding} \sum_{v} Kv_{i,v,t}^{d}$$

where the set *Expanding* includes all sectors in expansion. Since *Old* capital in expanding sectors is equated with *New* capital, the appropriate sum is over all vintages. The aggregate capital stock of *New* capital is equal to the total capital stock, less capital supply in declining sectors:

$$K_t^s - \sum_{i \in Declining} K_{i,t}^{s,Old}$$

where the set *Declining* covers only those sectors in decline. However, at equilibrium, capital supply in declining sectors must equal capital demand for *Old* capital, and capital demand for *New* capital in these sectors is equal to zero. Hence, the supply of *Old* capital in declining sectors can be shifted to the demand side of the equilibrium condition for *New* capital, and this simplification yields equation (F-25) which determines the economy-wide rate of return on *New* capital.

Equations (F-26) and (F-27) determine the vintage and sector specific rates of return[67]. Equation (F-26) defines the rate of return on *Old* capital. If the sector is in decline, the equation yields the equilibrium rate of return on *Old* capital. If the sector is expanding, the relative rate of return is equal to 1, and therefore the *Old* capital rental rate is equal to the *New* capital rental rate. Equation (F-27) equates all *New* capital rates of return to the economy-wide equilibrium rate. Equation (F-28) introduces a wedge between the post- and pre-tax rates of return.

[66] See the LINKAGE User Manual for the implementation of this specification in GAMS.

[67] These are the net rates of return after tax. Thus the relative rate of return variable, *RR*, is defined in terms of the net rate of return.

(F-25)	$\sum_i \sum_v Kv_{i,v,t}^d + N_i KF_{i,t}^d = K_t^s$
(F-26)	$NR_{i,Old,t} = RR_{i,t} TR_t$
(F-27)	$NR_{i,New,t} = TR_t$
(F-28)	$R_{i,v,t} = \left(1 + \tau_{i,v,t}^k\right) NR_{i,v,t}$

Allocation of Output across Vintages

This section describes how output is allocated across vintages. Aggregate sectoral output, *XP*, is equated to aggregate sectoral demand (i.e. the CET combination of *XD* and *ES*). Given the beginning of period installed capital, it is possible to assess the level of *potential* output produced using the installed capital. If this level of output is greater than the aggregate output (demand) level, the sector appears to be in decline, installed capital will be released, *Old* output will be equated with aggregate output (demand), and *New* output is zero. Equation (F-29) equates aggregate output, *XP*, to the sum of output across all vintages. In the case of the single vintage specification, equation (F-29) simply equates *Old* output with total output. In the case of the multiple vintage specification, equation (F-29) determines *New* output as a residual, i.e. the difference between aggregate output and *Old* output. The latter, *Old* output, is determined in equation (F-30). *Old* output is equated to the sectoral supply of *Old* capital, divided by the capital/output ratio.

(F-29)	$XP_i = \sum_v XPv_{iv}$
(F-30)	$XPv_{i,Old,t} = K_{i,t}^0 \left(RR_{i,t}\right)^{\eta_i^k} / \chi_{i,Old,t}^v$

Aggregate Capital Stock in a Recursive Dynamic Framework

In a typical recursive dynamic framework, the time path of the model is solved as a sequence of static equilibria in each year. In other words, the solution in any given year is not a function of forward looking variables, though it may be an explicit function of past variables, though known and therefore exogenous. While there are drawbacks in the recursive dynamic framework, particularly in the modeling of saving and investment behavior, its one key advantage is that it is much easier to set up and solve. There are several backward LINKAGEs linking one period to another: population growth, productivity increases, and capital accumulation. Most of these LINKAGEs can be resolved outside of the modeling framework, or in other words, in between solution periods. For example, the simple one-step capital accumulation function is:

$$K_t = (1 - \delta) K_{t-1} + I_{t-1}$$

With knowledge of the previous period's capital stock and level of investment, it is possible to know, prior to solving the model for the current period, the volume of the capital stock, i.e. it is exogenous. However, if it is desired to solve the model in step sizes greater than 1, then alternative assumptions need to be made about capital accumulation which may require information about contemporaneous variables. Let *n* be the step-size, eventually 1. Then through recursion, the capital accumulation function becomes:

$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n} (1-\delta)^{j-1} I_{t-j}$$

However, if the model is run in step sizes greater than 1, the intermediate values of real investment are not calculated. They can be replaced by assuming a linear growth model for investment:

$$I_t = (1+\gamma^I) I_{t-1}$$

Replacing this in the accumulation function yields:

$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n} (1-\delta)^{j-1} (1+\gamma^I)^{n-j} I_{t-n}$$

With some algebraic manipulation, this formula can be reduced to equation (F-32) below, where the investment growth parameter is implicitly determined by equation (F-31). If n is equal to 1, it is clear that equation (F-32) simplifies to the simple 1 step accumulation function. These two equations are linked to a single contemporaneous variable, FD_{Inv}, the volume of investment.

There are two variables that represent the aggregate capital stock, K and K^s. The first is evaluated in base year prices and is typically some multiple of aggregate GDP. The second is the normalized value of the capital stock, normalized in the sense that it is equated with the value of capital remuneration in the base year. For example, if aggregate value added in the base year is 100, and capital remuneration represents 40 percent of value added, the normalized value of the capital stock in the base year is 40. In other words, its price is 1, since the price, times the volume is equal to its remuneration. The value of the capital stock in base year prices may be something on the order of magnitude of 200 to 400. In the case of 200, the rate of return on capital would be 40/200 or 20 percent. In the case of 400, the rate of return on capital would be 40/400, or 10 percent. The actual value of the capital stock is irrelevant by and large in computing a within period static equilibrium, since it is an indexing issue. It is therefore convenient to normalize the capital stock so that its rate of return is 1 in the base year. However, the actual value of the capital stock does matter in two instances. The first instance is in the calculation of the depreciation allowance. The depreciation allowance is always calculated with respect to the actual value of the capital stock, not some index volume. The second instance where the value of the capital stock matters is in the updating of the capital stock through the investment accumulation function:

$$K_t = (1-\delta) K_{t-1} + I_{t-1}$$

This equation can only hold for the actual level of the capital stock, not the normalized level. Equation (F-33) determines the volume of normalized capital, K^s. The ratio of the normalized capital to the non-normalized volume of capital is constant.

(F-31) $$FD_{Inv,t} = (1+\gamma^I)^n FD_{Inv,t-n}$$

(F-32) $$K_t = (1-\delta)^n K_{t-n} + \frac{(1+\gamma^I)^n - (1-\delta)^n}{\gamma^I + \delta} FD_{Inv,t-n}$$

(F-33) $$K_t^s = \frac{K_0^s}{K_0} K_t$$

Other Equations and Definitions

This section describes some remaining equations and definitions. Equation (M-1) defines real GDP at base year market prices. It is the sum of household consumption, government and investment final demand, demand for local trade and transport services, exports, minus imports. Equation (M-2) defines real GDP at base year factor cost. It is the sum across sectors of the demand for land and natural resources, labor, and capital. All factor variables are adjusted by their relevant productivity factors. Equation (M-3) defines the GDP deflator (at factor cost). Equation (M-4) defines an aggregate absorption price index. It is used in some equations as a deflator. Though there is a global Walras' Law, described earlier, there is also a regional Walras' Law. In other words, in each region there is a redundant equation that can be derived by some combination of the other equations. Equation (M-5) is the dropped equation in each region. It equates total exports (at world prices), including the export of international trade and transport services, plus foreign saving, to total imports (at world prices). It also includes the outflow of the import premium income on own imports and the inflow of the import premium income on own exports.

(M-1)

$$\begin{aligned} RGDPMP_r &= \sum_i \left[\sum_h \left(1+\tau^{Ac}_{r,i,h,0}\right) PA_{r,i,0} XAc_{r,i,h} + \sum_f \left(1+\tau^{Af}_{r,i,f,0}\right) PA_{r,i,0} XAf_{r,i,f} \right] \\ &+ \sum_i \sum_{r'} \left(WPE_{r,r',i,0} WTF^s_{r,r',i} - WPM_{r',r,i,0} WTF^d_{r',r,i} \right) + APMg_{r,0} AXMg_r \end{aligned}$$

(M-2)

$$\begin{aligned} RGDP &= \sum_i AT_i \left[\lambda^t_i NPT_{i,0} T^d_i + \lambda^f_i PF_{i,0} F^d_i + \sum_l NW_{l,i,0} (\lambda^l_{l,i} LV^d_{l,i} + N_i LF^d_{l,i}) \right] \\ &+ \sum_i AT_i \left[NR_{i,Old,0} \left(\sum_v \lambda^k_{i,v} Kv^d_{i,v} + N_i KF^d_i \right) \right] \end{aligned}$$

(M-3)

$$PGDP\ RGDP = \sum_h \left[YH_h + DeprY_h - PGDP.TRG_h \right] - \sum_i \pi_i PX_i XP_i$$

(M-4)

$$PABS = \frac{\sum_i PA_i XA_i}{\sum_i PA_{i,0} XA_i}$$

(M-5)

$$\begin{aligned} 0 \equiv\ & \underbrace{\sum_i \sum_{r'} WPE_{r,r',i} WTF^s_{r,r',i}}_{Exports} + \underbrace{APMg_r AXMg_r}_{Sales\ of\ international\ trade\ services} + \underbrace{P.S^f_r}_{Foreign\ savings} \\ -\ & \underbrace{\sum_i \sum_{r'} WPM_{r',r,i} WTF^d_{r',r,i}}_{Imports} \\ +\ & \underbrace{\sum_i \sum_{r'} \left(1-\chi^{trq}_{r,r',i}\right) \tau^{pr}_{r,r',i} WPM_{r,r',i} WTF^q_{r,r',i}}_{Share\ of\ quota\ rents\ from\ own\ exports} - \underbrace{\sum_i \sum_{r'} \left(1-\chi^{trq}_{r',r,i}\right) \tau^{pr}_{r',r,i} WPM_{r',r,i} WTF^q_{r',r,i}}_{Foreign\ share\ of\ quota\ rents\ from\ own\ imports} \end{aligned}$$

IV Model Dynamics

Endogenous Dynamic Equations

Most of the dynamics occurs outside of the model proper, i.e. in between solutions. One of the exceptions is the capital accumulation function. Before running any policy simulations in a dynamic framework, it is often required to define some sort of reference scenario, or as it is sometimes called, a business-as-usual scenario (BaU). The BaU scenario makes some assumptions about a broad range of dynamic variables—population and labor supply growth rates, the growth rate of factor and energy productivity, and other exogenous variables. If all productivity variables are pre-determined, as well as the population growth rates, the growth rate of real GDP is endogenous. However, the path trend in real GDP growth may be unrealistic, or at least inconsistent with the assumed trend from other studies or prospective outlooks. One way to resolve this dilemma is to make the growth of real GDP exogenous in the reference scenario, and to endogenize some other variable to meet a given GDP target.

There are many alternatives for calibrating the BaU. In the current version of the model the following assumptions are made:

- Agricultural productivity is fixed (and is assumed uniform across factors of production). A share of agricultural productivity is assumed to be related to the sectoral export-output ratio.
- Aggregate GDP growth is given
- Sectoral productivity (outside of agriculture) is only labor-augmenting
- Sectoral productivity is composed of three components—a uniform economy-wide factor (the instrument to achieve the GDP target), a sector-specific factor related to openness, and a constant shifter.

Equation (G-1) determines the growth rate of real GDP (at market price), g^y. In the BaU simulation the growth rate is exogenous, thus the equation can be thought of as determining the uniform productivity factor, i.e. the instrument achieving the GDP target. Equation (G-2) determines the labor-augmenting productivity factor, λ^l. It is equal to the previous period's efficiency factor multiplied by the increase in sectoral productivity. The latter is composed of three components. The first, γ^l, is the uniform productivity shifter intended to target real GDP growth in the baseline. In policy simulations it is exogenous. The second, χ^p, is a sector specific factor intended to capture openness-sensitive changes in productivity. It is described further below. The third, π, is an exogenous shifter. It can be used to provide a constant relative difference across sectors. For example setting manufacturing productivity to be 2 percentage points greater than service sector productivity. Equations (G-2) through (G-4) are indexed by *ik*, these are the sectors with endogenous productivity increases (in the baseline). Normally it excludes agriculture, and possibly the natural resource sectors.

(G-1) $$RGDPMP_t = \left(1+g^y\right)^n RGDPMP_{t-n}$$

(G-2) $$\lambda^l_{l,ik,t} = \left(1+\gamma^l_t+\chi^p_{ik,t}+\pi_{ik,t}\right)^n \lambda^l_{l,ik,t-n}$$

(G-3) $$\chi^p_{i,t} = \phi^p_{i,t}\left(\frac{ES_{i,t}}{XP_{i,t}}\right)^{\eta^p_i}$$

(G-4) $$\chi^p_{ik,t} = \alpha^p_{ik,t}\left(\gamma^l_t+\chi^p_{ik,t}+\pi_{ik,t}\right)$$

Equation (G-3) defines the equation for determining the trade-sensitive productivity component. It is a constant elasticity function of the ratio of aggregate exports to output. The elasticity is given by η^p. The shifter, ϕ^p, is a calibration parameter. It is calibrated in the BaU scenario so that the trade-sensitive portion of sectoral productivity is some share of total productivity. Equation (G-4) is the calibrating equation where α^p is the share, for example 40 percent. Equation (G-4) is only active in the baseline simulation. It essentially calibrates the ϕ^p parameters that are saved in a file for subsequent use in policy simulations. It is possible to shut down the link between openness and productivity by simply setting the sensitivity elasticity to 0.

In subsequent simulations, i.e. in simulations with policy shocks, the uniform shifter, γ^l, is exogenous and the growth of GDP is endogenous. If the openness elasticity is 0, then productivity is completely exogenous in policy shocks. The reference simulation needs additional code in order to save the γ^l and ϕ^p values, which need to be read in for policy simulations.[68]

Productivity in the other sectors (normally agriculture) is exogenous in the baseline simulation (N.B. these sectors are indexed by *ink*). Similar to the sectors with endogenous productivity, a share of productivity in these sectors is assigned to an openness relation. In policy simulations changes to openness are allowed to have an impact on productivity, using the same type of mechanism as described above. Equations (G-5)-(G-8) describe the productivity specification in the sectors with exogenous productivity in the baseline. The factor γ^s is the exogenous productivity for the baseline, for example 2.5 percent. In the baseline simulation, equations (G-5)-(G-8) are not necessary since the sum of χ^p and $(1-\alpha^p)\gamma^s$ is assumed to equal the exogenous γ^s (in other words, $\chi^p = \alpha^p.\gamma^s$). Equation (G-3) is used in the baseline to determine the shift parameters, α^p. In policy simulations, γ^s remains fixed but the χ^p factor is endogenous and responds to changes in the openness relationship according to Equation (G-3).

(G-5) $$\lambda^l_{l,ink,t} = \left[1+\chi^p_{ink.t} + \left(1-\alpha^p_{ink.t}\right)\gamma^s_{ink.t}\right]^n \lambda^l_{l,ink,t-n}$$

(G-6) $$\lambda^k_{ink,v,t} = \left[1+\chi^p_{ink,t} + \left(1-\alpha^p_{ink,t}\right)\gamma^s_{ink,t}\right]^n \lambda^l_{ink,v,t-n}$$

(G-7) $$\lambda^t_{ink,t} = \left[1+\chi^p_{ink,t} + \left(1-\alpha^p_{ink,t}\right)\gamma^s_{ink,t}\right]^n \lambda^t_{ink,t-n}$$

(G-8) $$\lambda^f_{ink,t} = \left[1+\chi^p_{ink,t} + \left(1-\alpha^p_{ink,t}\right)\gamma^s_{ink,t}\right]^n \lambda^f_{ink,t-n}$$

68 The previous standard baseline calibration assumed so-called *balanced* growth. This was implemented by fixing the average capital/labor ratio, in efficiency units, in the baseline scenario. Instead of assuming labor-augmenting productivity, productivity was factor specific. The uniform factor determining capital productivity was used to target GDP growth, and the balanced-growth assumption was used to target labor productivity. The third equation defined the average capital/labor ratio in efficiency units. In policy simulations, it was endogenous.

$$\lambda^k_{ik,v,t} = \left(1+\gamma^k_t + \pi_{ik,t}\right)^n \lambda^k_{ik,v,t-n}$$

$$\lambda^l_{l,ik,t} = \left(1+\gamma^l_t + \pi_{ik,t}\right)^n \lambda^l_{l,ik,t-n}$$

$$\chi^{kl}_t = \frac{\sum_i \sum_v R_{i,v,0} AT_{i,t} \lambda^k_{i,v,t} Kv^d_{i,v,t}}{\sum_i W_{i,0} AT_{i,t} \lambda^l_{i,t} L^d_{i,t}}$$

Exogenous Dynamic Equations

This section describes the dynamics of the model occurring outside of the model specification, i.e. in between model solution periods. The first part of the section deals with the updating of the factor stock variables. Equation (G-9) determines the population level. Equation (G-10) determines the shift factor for the labor supply curve. The growth factor g^L is the projected growth in the overall labor supply. Equation (G-11) determines the shift factor for the land supply curve. Equation (G-12) determines the supply shift factor for the sector-specific factor. The growth factor, g^F, could be made sector-specific in future versions. Since this factor is often associated with natural resources, both renewable and non-renewable, a more appropriate dynamic specification would rely on models of resource depletion in the case of non-renewable resources, and a model of resource balance in the case of renewable resources. Equation (G-13) determines the installed capital in each sector at the beginning of each period. The depreciation rate is uniform across sectors and vintages.

(G-9) $$Pop_t = \left(1+g^{Pop}\right)^n Pop_{t-n}$$

(G-10) $$\chi_t^L = \left(1+g^L\right)^n \chi_{t-n}^L$$

(G-11) $$\chi_t^T = \left(1+g^T\right)^n \chi_{t-n}^T$$

(G-12) $$\chi_{i,t}^F = \left(1+g^F\right)^n \chi_{i,t-n}^F$$

(G-13) $$K_{i,t}^0 = \sum_v (1-\delta)^n Kv_{i,v,t-n}^d$$

The next set of equations refers to the exogenous productivity factors. Equations (G-14)-(G-15) deal with the two remaining factors—land and the sector-specific factor. The productivity factors for land and the sector-specific factor are uniform across non-agricultural sectors. Future versions of the model would allow differentiation. Equation (G-16) determines the change in energy productivity (sometimes called the autonomous energy efficiency improvement, or AEEI). The growth rate of the AEEI is both fuel and sector specific.

(G-14) $$\lambda_{ip,t}^t = \left(1+\gamma_t^t\right)^n \lambda_{ip,t-n}^t$$

(G-15) $$\lambda_{ip,t}^f = \left(1+\gamma_t^f\right)^n \lambda_{ip,t-n}^f$$

(G-16) $$\lambda_{e,j,t}^{ep} = \left(1+\gamma_{e,j,t}^e\right)^n \lambda_{e,j,t}^{ep}$$

Exogenous Variables

Most of the other exogenous variables relate to various taxes and subsidies in the model and other fiscal instruments. The following table lists the exogenous fiscal instruments, which in general are assumed to stay fixed at their base year levels in the BaU scenario.

τ^{Ap}	Tax on the (Armington) intermediate demand in production
τ^{Ac}	Tax on the (Armington) household demand for goods and services
τ^{Af}	Tax on the (Armington) other final demand for goods and services
τ^{l}	Tax/subsidy on wages
τ^{k}	Tax/subsidy on capital
τ^{t}	Tax/subsidy on land
τ^{p}	Tax/subsidy on production
τ^{e}	Tax/subsidy on exports
τ^{m}	Tax/subsidy on imports

Fiscal closure has been discussed above. The household direct tax rate shifter, χ^h, is endogenous, and is designated to achieve a given fiscal balance. The (real) level of government saving (deficit), RS^g, is fixed in each time period, and the BaU scenario assumes it remains constant at its base year level. The remaining fiscal variable concerns government transfers to households. This variable is exogenous in each time period, but is assumed to grow at the same rate as the BaU GDP[69]:

$$TRG_t = \left(1+g^y\right)^n TRG_{t-n}$$

The international trade and transport margin, ζ^t, is exogenous, but is allowed to change over time:

$$\zeta^t_{r,r',i,t} = \left(1+\gamma^{tt}\right)^n \zeta^t_{r,r',i,t-n}$$

Empirical evidence shows that the annual change is typically negative and in the order of magnitude of 1 percent or more.

Foreign saving (in volume terms) is exogenous, and in the BaU scenario, it is assumed to be held fixed at base year levels throughout. Other scenarios are possible, the only constraint is that the sum of foreign saving across regions must be identically equal to zero, which is guaranteed within the model specification.

The final exogenous variable is the numéraire. Any price could be chosen as the numéraire. In this model, the OECD export price index, P, has been chosen, and it is set at 1 for all time periods. All values in the model are priced in 2001 dollars.

Dynamic Parameters

Production Parameters

Most of the parameters of the model are fixed at their base year levels. The key parameters are part of the input data set. Most of the other parameters are calibrated conditional on base year data and the set of key parameters. Under the vintage capital specification, the nature of *Old* capital changes as *New* capital gets merged with *Old* capital (in between periods). This changes the production structure over time. In order to reflect these changes, the CES share parameters in the production structure—those dependent on the vintage structure—are re-calibrated between periods to reflect the malleability of *Old* capital. The main

69 In the GTAP dataset it is zero. Model users can adjust the direct tax rate derived from the GTAP dataset. It is then necessary to adjust the value of TRG to keep the SAMs balanced.

idea behind the re-calibration is that the re-calibrated *Old* parameters should be able to re-produce the previous period's aggregate output, with the previous period's aggregated (over vintage) inputs, but using the *Old* substitution elasticities.

The following describes the basic strategy. Assume we have a vintage specific CES node. Let the aggregate bundle be denoted by y_v with a price p_v, and the components be denoted by $x_{v,j}$ with a price $p_{v,j}$ (the index j refers to the number of components in the CES node, not the sector). At equilibrium the following relations must hold:

$$x_{v,j} = \alpha_{v,j}\left(\lambda_j\right)^{\sigma_v}\left(\frac{p_v}{p_{v,j}}\right)^{\sigma_v} y_v$$

$$p_v = \left[\sum_j \alpha_{v,j}\left(\frac{p_{v,j}}{\lambda_j}\right)^{1-\sigma_v}\right]^{1/(1-\sigma_v)}$$

Re-calibration of the CES share parameter involves re-calibration of the *Old* share parameter so that the aggregate components (summed over vintage) can re-produce the aggregate bundle. Let the following relations be defined:

$$Y = \sum_v y_v \qquad P = \sum_v p_v y_v / Y$$

$$X_j = \sum_j x_{v,j} \qquad P_j = \sum_v p_{v,j} x_{v,j} / X_j$$

Variables in capitals represent respectively aggregate volumes and prices, where the aggregations are across vintages. The re-calibration formula is:

$$\overline{\alpha}_{Old,j} = \left(\frac{P_j}{P}\right)^{\sigma_{Old}}\left(\frac{X_j}{X}\right)\left(\lambda_j\right)^{(1-\sigma_{Old})}$$

There are several CES nodes in the production structure with vintage specific share parameters. More detailed information can be found by looking at the GAMS code in the file *recal.gms*.

Consumption Parameters

The ELES consumption function tends to converge towards a Cobb-Douglas utility function over time, particularly in regions with rapid growth. The reason is quite simple. The subsistence minima, as a share of total consumption, decline. This has a rather unfortunate consequence since it implies that income elasticities converge towards 1, which is contrary to all known empirical evidence. Even though the subsistence minima are adjusted by population size, this adjustment is insufficient to fully compensate for economic growth. To partially avoid this convergence problem, the ELES parameters are re-calibrated between periods. The re-calibration is based on the initial (base year) income elasticity estimates, but the latest values for the relevant variables. The following formulas indicate how this is done. The first equation re-calibrates the marginal propensity to consume out of supernumerary income, using the initial income elasticity estimates. The consumption share is based on the previous period's equilibrium solution. The second equation implicitly defines the subsistence minima where all variables are based on the previous period's solution, except for the re-calibrated μ parameter. The equation is linear in the θ parameters and can be solved for by matrix inversion.[70]

[70] In GAMS this is performed by using the equation solver.

$$\mu_{k,h,t} = \eta_{k,h,0} \frac{PC_{k,h,t-n} XC_{k,h,t-n}}{Y^c_{h,t-n}}$$

$$XC_{k,h,t-n} = \theta_{k,h,t} + \frac{\mu_{k,h,t}}{PC_{k,h,t-n}} \left[Y^c_{h,t-n} - \sum_{k'} PC_{k',h,t-n} \theta_{k',h,t} \right]$$

Armington Share Parameters

The model allows for the possibility of changing the share parameters of both of the Armington CES nests. This introduces the possibility of having more flexible long-run responses to changes in import prices, and dampens to some extent the terms-of-trade effects of policy shocks. Put another way, it allows for greater market penetration with less downward movement in prices. Annex D describes in greater detail the mechanism and the updating equations. The incorporation of variable Armington elasticities is under user control. See the LINKAGE User Manual for more details.

The Accounting Framework

One of the most useful features of GE modeling is the tight link between a consistent accounting framework and the underlying consistency of the model. A well-designed GE model relies on a consistent database, and this accounting framework can also be used for diagnostic checking of the model. Despite the rather complex formulas described above, the actual accounting framework is simple and can be succinctly collapsed into fourteen or fewer accounts. Table 1 shows the functional social accounting matrix for the individual regions in the model:

The first row and column refer to the production side of the economy. The column represents the structure of production—intermediate inputs (*XAp*), labor (*L*), capital (*K*) and other factors (including revenues generated by price markups—, plus the taxes— indirect taxes on intermediate demand (*PATax*), taxes on the factors of production (*FTax*), and taxes on final output (*PTax*). The sum of these inputs, multiplied by their appropriate tax adjusted prices is equal to the value of output (*XP*) at the producer price (tax inclusive). The row shows the disposition of domestic output. There are three markets—the domestic market (*XD*), foreign markets (WTF^s) and the market for international trade and transport services (*XMG*). WTF^s is evaluated at world export prices, the producer may receive more or less with the wedge represented by the export tax (*ETax*).

Table 1: Functional Social Accounting Matrix

	Act	Comm	L	K	ITax	ETax	FTax	HH	G	I	Depr	T+T	Tar	ROW	Total
Activities		*XD*				*ETax*						*XMG*		WTF^s	*XP*
Commodities	*XAp*							*XAc*	*XAg*	*Xai*					*XA*
Labor	*L*														*L*
Capital+Other	*K*														*K*
Indirect tax	*PATax*							*CATax*	*GATax*	*IATax*					*ITax*
Export tax									*ETax*						*ETax*
Factor tax	*FTax*														*FTax*
Households			*L*	*K*											*YH*
Government	*PTax*				*ITax*		*FTax*	$\kappa^h\ YH$					*Tariff*		*YG*
Investment								S^h	S^g		*DeprY*			S^f	*I*
Depreciation								*DeprY*							*DeprY*
Intl. T+T														*XMG*	*XMG*
Tariffs		*Tariff*													*Tariff*
Rest of the world		WTF^d													WTF^d
Total	*XP*	*XA*	*L*	*K*	*ITax*	*Etax*	*FTax*	*YH*	*YG*	*I*	*DeprY*	*XMG*	*Tariff*	WTF^d	

Aggregate domestic demand is evaluated at the Armington level, *XA*. It is composed of domestic supply, *XD*, and imports (tariff inclusive). The second column represents total domestic supply of goods. WTF^d represents import demand at world import prices (CIF), and thus includes international trade and transport margins. Tariffs (*Tariff*) are added to the world import price to determine imports at domestic prices. The second row describes the allocation of total domestic supply of goods (*XA*). It is allocated to four accounts—intermediate demand (*XAp*), households (*XAc*), and government (*XAg*) and investment (*XAi*) expenditures on goods and services.

The third and fourth columns indicate the distribution of income, with most of value added flowing to households. The household column indicates the allocation of total household income. A share ($\kappa^h\ YH$) represents direct tax payments, and another share (S^h) is saved. The remainder is spent on private consumption expenditures (*XAc*) including indirect taxes.

Closure is identified with two variables—government savings (S^g) and foreign capital flows (S^f). Both are assumed fixed in any given time period. In the case of the former, direct taxes adjust to meet the fiscal target. In the case of the latter, the real exchange rate adjusts to match the balance of payments constraint. All of the other accounts are simply accounting identities or pass-through accounts.

Table 2 shows a numerical SAM generated from the GTAP dataset. It represents the aggregate SAM of the Quad countries in 2001 as available in Release 6.05 of the GTAP dataset.

A few notes on the numerical SAM:

- The factor tax is negative. This implies that producers are paying less for factor inputs (in some sectors) than the factors are receiving. Land subsidies are an example.

- Aggregate tariff revenue is only $67 billion, compared to an import bill of $4,866 billion. However, the latter includes intra-European and NAFTA trade, both of which are virtually tariff free.

- The GTAP dataset consolidates domestic saving into a single account—the household account. Thus public saving is set to zero, and direct taxes are derived residually.

- The SAM indicates that the Quad countries were net dis-savers, i.e. net capital flows into the region were $233 billion. (N.B. Though the GTAP database starts with officially published sources, the final accounts reflect some adjustment in order to have global accounting consistency.)

- The Quad countries have a negative trade balance of $394 billion for *standard* goods and services. However, since they export $161 billion in international trade and transport services, the final trade balance on goods and services is a negative $233 billion, matching capital inflows.

- The SAM is based on gross investment and savings, i.e. depreciation is not singled out.

Table 2: 2001 Social Accounting Matrix of the Quad countries

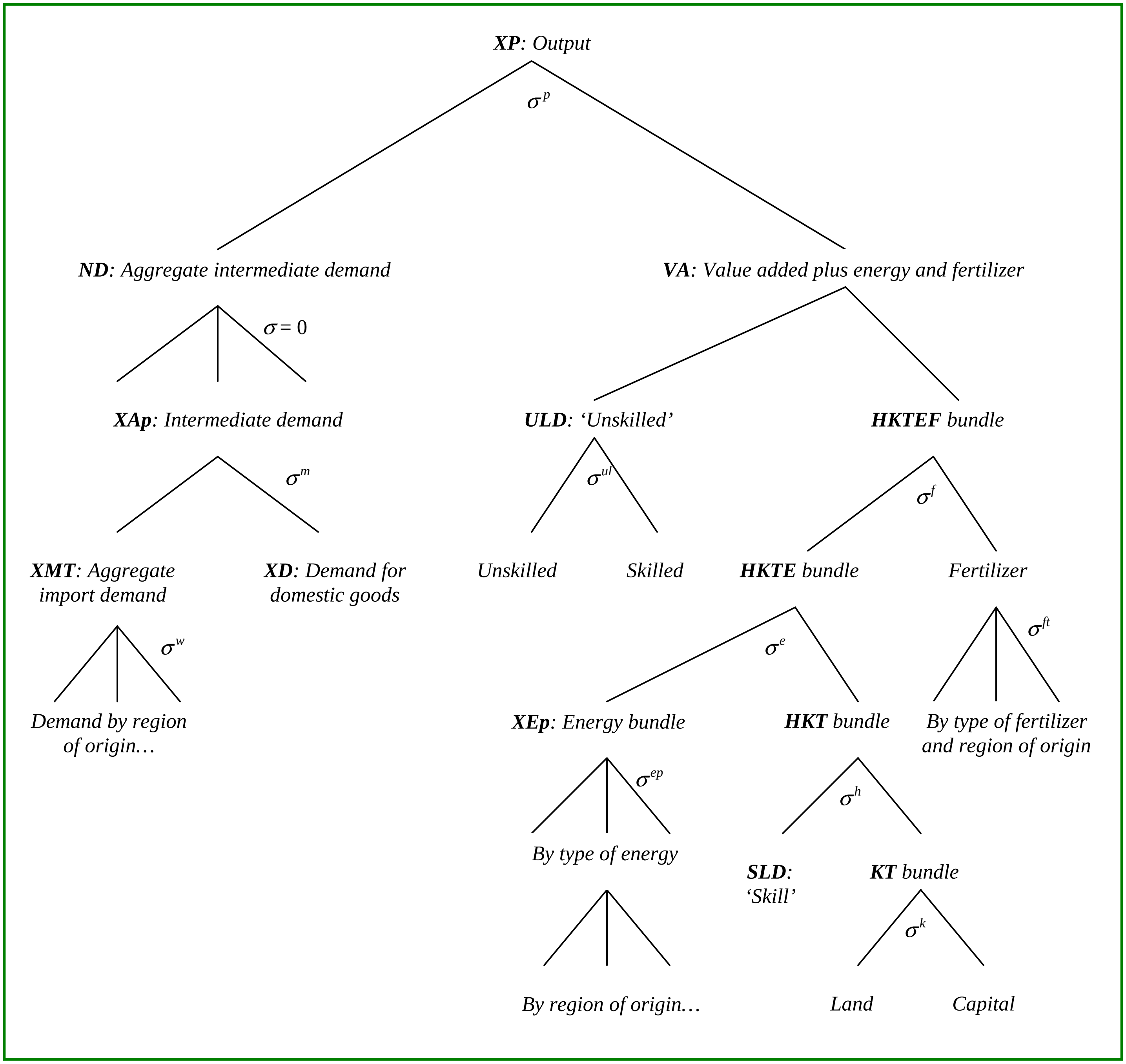
($1997 *billions*)

	Act	*Comm*	*UnSkL*	*SkL*	*K*	*F*	*ITax*	*ETax*	*Ftax*	*HH*	*G*	*I*	*Depr*	*T+T*	*Tariffs*	*ROW*	*Total*
Activities		38,113						4						161		4,472	42,751
Commodities	19,808									14,241	4,144	4,854					43,047
Unskilled labor	7,951																7,951
Skilled labor	5,324																5,324
Capital	8,811																8,811
Other factors	142																142
Indirect tax	212									582	3	126					924
Export tax											4						4
Factor tax	-57																-57
Households			7,951	5,324	8,811	142											22,227
Government	561						924		-57	2,657		0			67		4,151
Investment										4,747	0					233	4,980
Depreciation																	0
Intl. trade margins																161	161
Tariffs		67															67
Rest of the world		4,866										0					4,866
Total	42,751	43,047	7,951	5,324	8,811	142	924	4	-57	22,227	4,151	4,980	0	161	67	4,866	

Source: GTAP Version 6.05

Figures

Figure 1: Production Nesting in the Crop Sectors[71]



[71] The two labor types in the GTAP dataset mapped to two different labor bundles. One is the so-called 'Unskilled' labor bundle that is a substitute with capital. The other is the so-called 'Skilled' labor bundle that is a substitute with capital. In the standard version of the model, both types of labor are mapped to the 'Unskilled' labor bundle. The 'Skilled' labor bundle is further decomposed into specific labor types—this is not shown in Figures 1-3.

Figure 2: Production Nesting in the Livestock Sectors

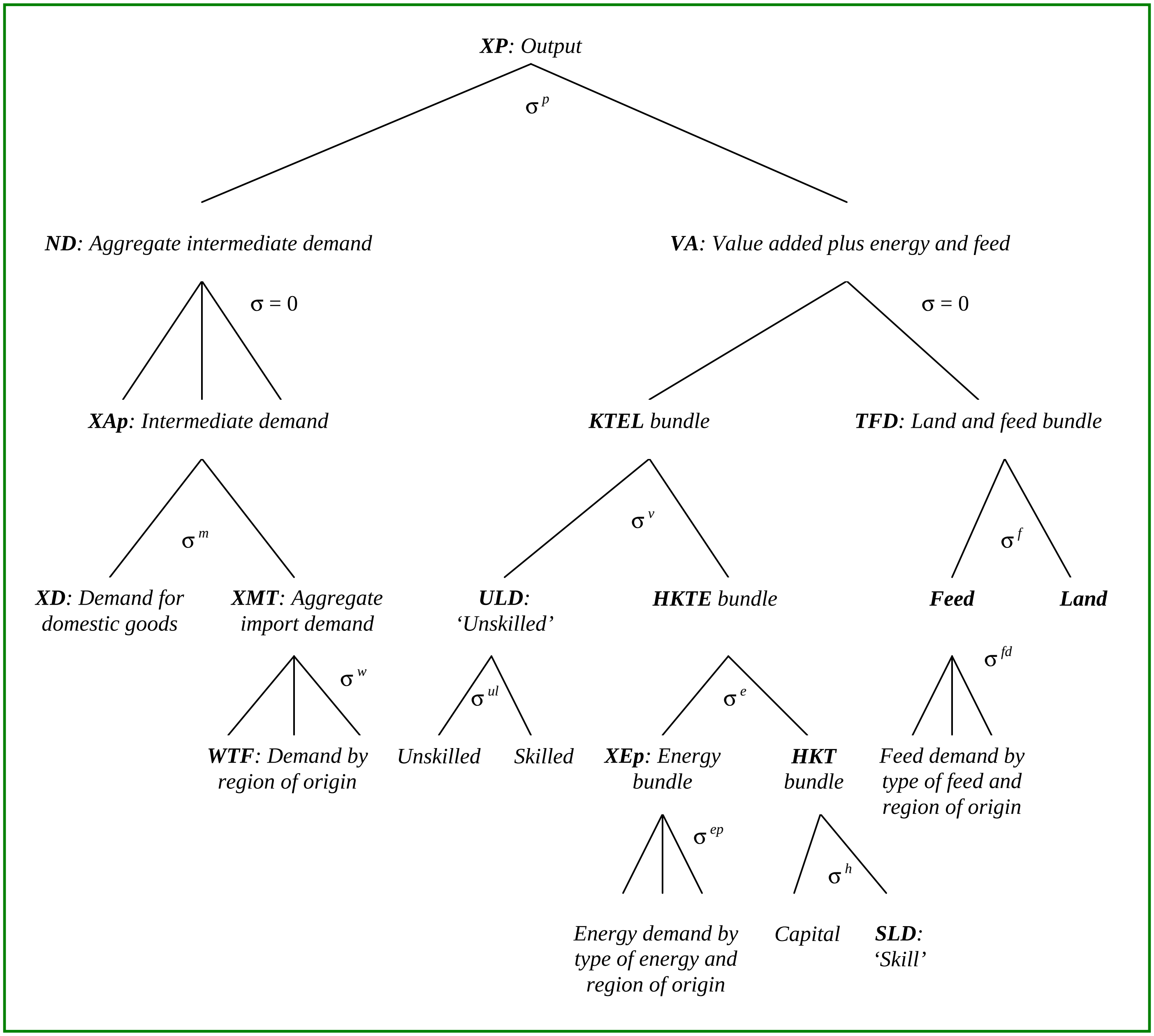


Figure 3: Production Nesting in the Manufacturing and Service Sectors

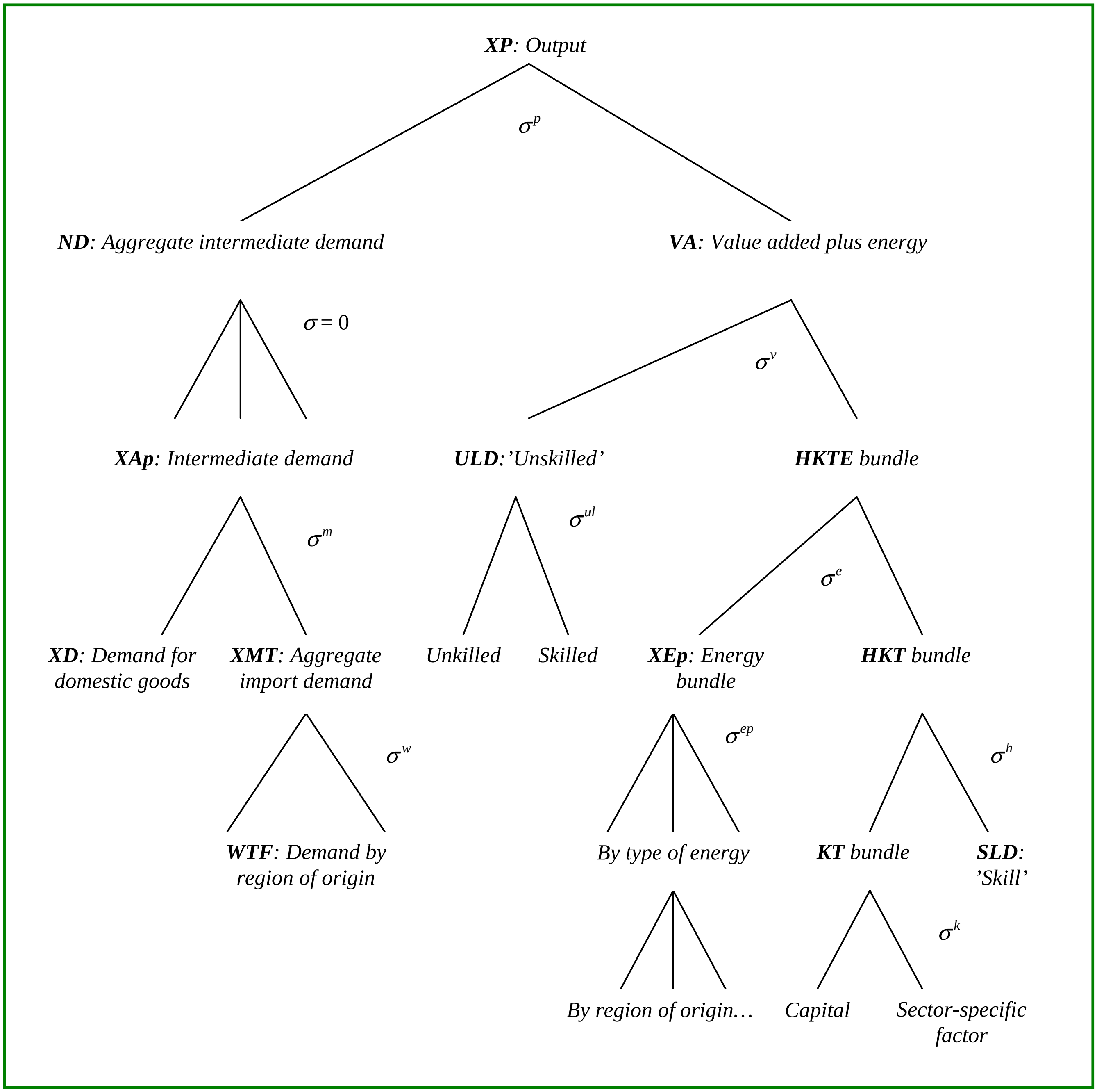
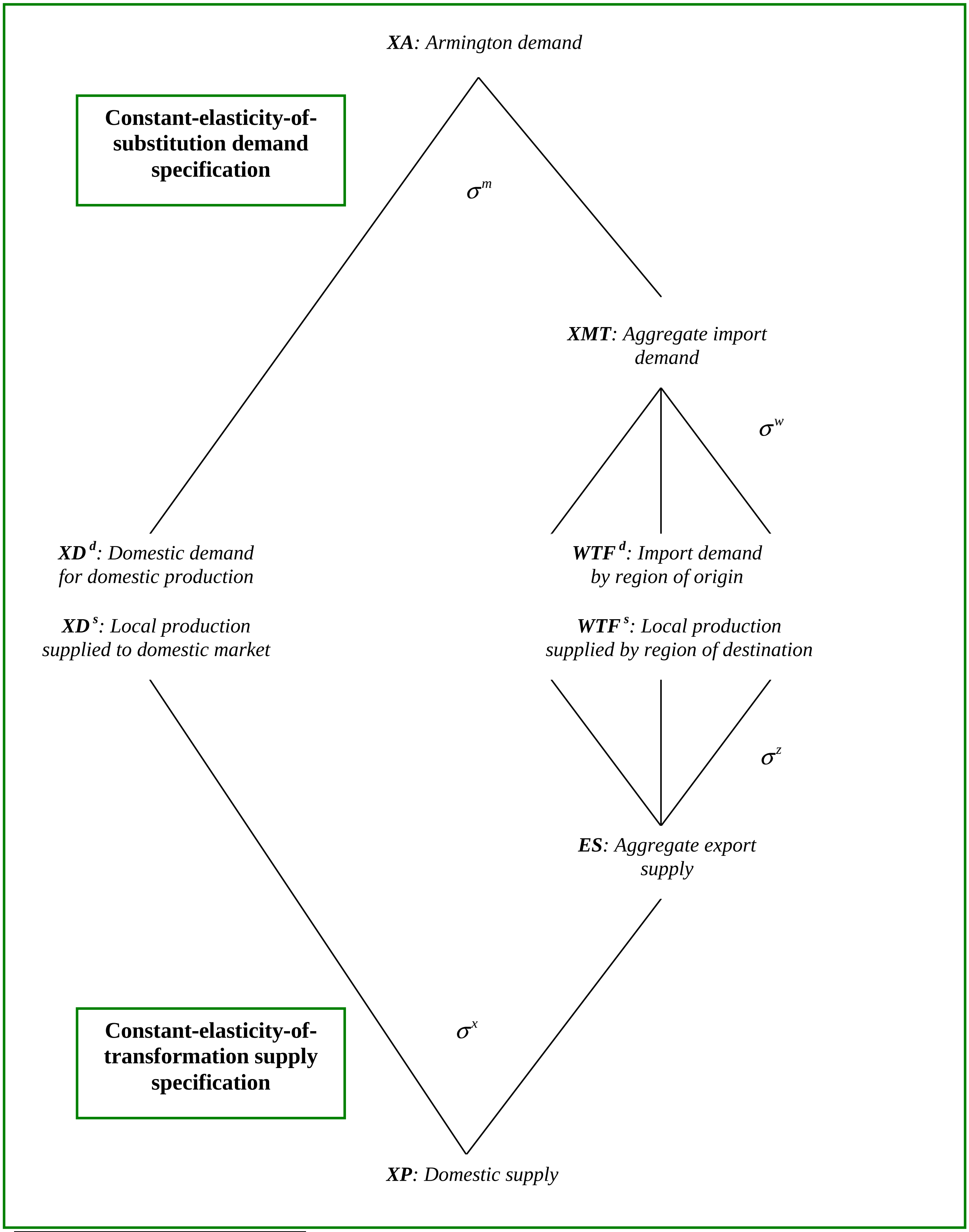
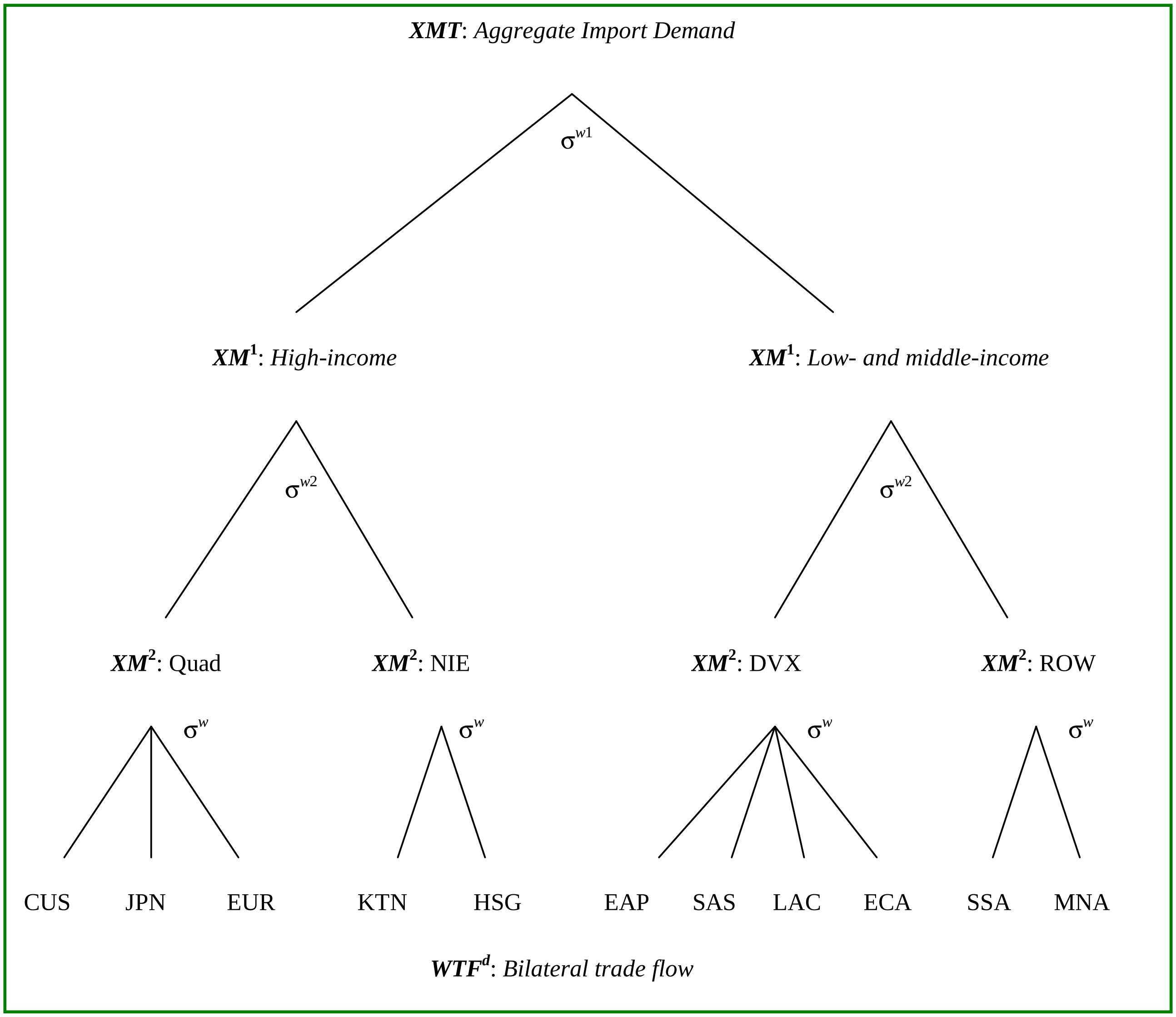


Figure 4: Armington and CET Nests[72]



[72] See additionally figure 5.

Figure 5: **Armington Import Demand Nesting**[73]



[73] The graphic representation is an example of a possible configuration for determining bilateral import flows (*WTF*). Both the first and second tier import bundles, respectively XM^1 and XM^2 are under user-control. In this example, the first tier divides imports between developed and developing countries. Developed country imports are then allocated to the Quad countries on the one-hand and the NIEs on the other. Quad country aggregate imports are allocated to Canada and the U.S. (CUS), Japan (JPN) and Europe (EUR). NIE aggregate imports are allocated to Korea and Taiwan (KTN) and Hong Kong and Singapore (HSG). Developing country aggregate imports are allocated to so-called diverse exporters (DVX) and to the rest (ROW). The former are composed of East Asia and Pacific (EAP), South Asia (SAS), Latin America and the Caribbean (LAC) and Europe and Central Asia (ECA). The rest of the world aggregate is composed of Sub-Saharan Africa (SSA) and the Middle East and North Africa (MNA).

Annex A: Variable Listing

Production Variables

Top level nest and market structure

ND	Demand for aggregate intermediate demand (/x some goods)	*r* x *i*
VA	Demand for value added+energy (/w some goods)	*r* x *i* x *v*
UVCv	Unit variable cost by vintage	*r* x *i* x *v*
UVC	Average unit variable cost	*r* x *i*
AC	Average unit cost	*r* x *i*
PX	Average net producer price	*r* x *i*
PP	Producer price (including output tax)	*r* x *i*
Π	Sectoral profits	*r* x *i*

Crops

ULD	Demand for aggregate 'unskilled' labor	*r* x *cr*
HKTEF	Demand for capital+energy+fertilizer+land bundle	*r* x *cr* x *v*
PVA	Price of value added+energy+fertilizer bundle	*r* x *cr* x *v*
HKTE	Demand for capital+energy+land bundle	*r* x *cr* x *v*
Fert	Demand for fertilizer	*r* x *cr*
PHKTEF	Price of capital+energy+fertilizer+land bundle	*r* x *cr* x *v*
XEp	Demand for aggregate energy bundle	*r* x *cr* x *v*
HKT	Demand for bundle of capital plus land	*r* x *cr* x *v*
PHKTE	Price of capital+energy+land bundle	*r* x *cr* x *v*
SLD	Demand for aggregate 'skilled' labor	*r* x *cr*
KT	Demand for bundle of capital plus land	*r* x *cr* x *v*
PHKT	Price of capital (human and physical)+land bundle	*r* x *cr* x *v*
T^d	Demand for land	*r* x *cr*
F^d	Demand for sector-specific factor	*r* x *cr*
Kv^d	Demand for capital (by vintage)	*r* x *cr* x *v*
PKT	Price for bundle of capital plus land	*r* x *cr* x *v*
XAp	Demand for (Armington) intermediate goods	*r* x *cr* x *j*
PND	Price of aggregate non-energy intermediate goods	*r* x *cr*
PEp	Price of aggregate energy bundle	*r* x *cr* x *v*
Pfert	Price for fertilizer	*r* x *cr*

Livestock

TFD	Demand for land-feed bundle	*r* x *lv* x *v*
KTEL	Demand for capital-energy-labor composite good	*r* x *lv* x *v*
PVA	Price of value added+energy+feed bundle	*r* x *lv* x *v*
T^d	Demand for land	*r* x *lv*
Feed	Demand for feed	*r* x *lv*
PTFD	Price of land feed bundle	*r* x *lv* x *v*

ULD	Demand for aggregate 'unskilled' labor	*r* x *lv*
HKTE	Demand for capital-energy bundle	*r* x *lv* x *v*
PKTEL	Price of labor-capital-energy bundle	*r* x *lv* x *v*
XEp	Demand for aggregate energy bundle	*r* x *lv* x *v*
HKT	Demand for bundle of capital and other factors	*r* x *lv* x *v*
PHKTE	Price of capital+energy bundle	*r* x *lv* x *v*
SLD	Demand 'skilled' labor bundle	*r* x *lv*
KT	Demand for bundle of capital plus other factors	*r* x *lv* x *v*
PHKT	Price of capital (human and physical)+other bundle	*r* x *lv* x *v*
F^d	Demand for sector-specific factor	*r* x *lv*
Kv^d	Demand for capital (by vintage)	*r* x *lv* x *v*
PKT	Price for bundle of capital plus land	*r* x *lv* x *v*
XAp	Demand for (Armington) intermediate goods	*r* x *lv* x *j*
PND	Price of aggregate non-energy intermediate goods	*r* x *lv*
PEp	Price of aggregate energy bundle	*r* x *lv* x *v*
Pfeed	Price of feed	*r* x *lv*

Non-agricultural sectors

ULD	Demand for aggregate 'unskilled' labor	*r* x *ip*
HKTE	Demand for capital+energy bundle	*r* x *ip* x *v*
PVA	Price of value added+energy bundle	*r* x *ip* x *v*
XEp	Demand for aggregate energy bundle	*r* x *ip* x *v*
HKT	Demand for bundle of capital plus other resources	*r* x *ip* x *v*
PHKTE	Price of capital+energy bundle	*r* x *ip* x *v*
SLD	Demand 'skilled' labor bundle	*r* x *ip*
KT	Demand for bundle of capital plus other factors	*r* x *ip* x *v*
PHKT	Price of capital (human and physical)+other bundle	*r* x *ip* x *v*
T^d	Demand for land	*r* x *ip*
F^d	Demand for sector-specific resources	*r* x *ip*
Kv^d	Demand for capital (by vintage)	*r* x *ip* x *v*
PKT	Price for bundle of capital plus other resources	*r* x *ip* x *v*
XAp	Demand for (Armington) intermediate goods	*r* x *ip* x *j*
PND	Price of aggregate non-energy intermediate goods	*r* x *ip*
PEp	Price of aggregate energy bundle	*r* x *ip* x *v*
L^d	Labor demand by skill type	*r* x *l* x *j*
UW	Price of 'unskilled' labor bundle	*r* x *j*
SW	Price of 'skilled' labor bundle	*r* x *j*

The total number of variables/equations in the three production blocks is:

$$NR\left[NS(12+12.NV+NL+NS)+2.NCR(1+NV)+2.NLV(1+2.NV)\right]$$

Where *NR* is the number of regions, *NS* the number of sectors, *NV* the number of vintages, *NL* the number of labor categories, *NCR* the number of crop sectors, *NLV* the number of livestock sectors, *NST* is the number of sectors demanding land, and *NSF* is the number of sectors demanding a sector-specific factor.[74] If we set these dimensions respectively to 15, 20, 2, 3, 5, 1, 5, and 1, the formula evaluates to 1,220 equations per region and a total of 18,300 globally.

Income Variables

TY	Aggregate land remuneration	*r*
FY	Aggregate sector-specific factor remuneration	*r*
LY	Aggregate labor remuneration (by skill)	*r* x *l*
KY	Aggregate capital remuneration	*r*
YH	Gross household income	*r* x *h*
DeprY	Depreciation allowance	*r* x *h*
Yd	Disposable household income	*r* x *h*
Y^c	Income definition used in the LES function	*r* x *h*

Final Demand Variables

Y^*	Supernumerary income	*r* x *h*
XC	Consumer demand at the level of consumed goods	*r* x *k* x *h*
PC	Consumer prices at the level of consumed goods	*r* x *k* x *h*
Y^*	Supernumerary income	*r* x *h*
XAc	Household (Armington) demand for goods and services	*r* x *i* x *h*
PAc	Household (Armington) demand for goods and services	*r* x *i* x *h*
S^h	Household saving	*r* x *h*
CPI	Consumer price index	*r* x *h*
XAf	Other final (Armington) demand for goods and services	*r* x *i* x *f*
PFD	Aggregate price index for other final demand	*r* x *f*

The total number of variables in the income distribution and domestic final demand blocks is:

$$NR[3 + NL + NH.(7 + 2.NC + 2.NS) + NF.(NS + 1)]$$

where *NH* is the number of households, *NC* is the number of consumer goods and *NF* the number of other final demand accounts (normally 2). Using the same numbers as above, with in addition *NH* set to 1, *NC* set to 20 and *NF* to 2, yields a total number of equations in these two blocks of 2,025, or 135 by region.

Trade Variables

XA	Aggregate Armington demand	*r* x *i*
XD	Domestic demand for domestic production[a]	*r* x *i*
XMT	Domestic demand for aggregate imports	*r* x *i*
PA	Armington price	*r* x *i*
XM^1	First tier import demand bundle	*r* x *R1* x *i*
PMT	Price of aggregate imports	*r* x *i*
XM^2	Second-tier import demand bundle	*r* x *R2* x *i*
PM^1	Price of first-tier import bundle	*r* x *R1* x *i*
WTF	Trade flow matrix[b]	*r* x *r* x *i*
PM^2	Price of second-tier import bundle	*r* x *R2* x *i*

74 This represents an upper bound. The actual number will depend on both user options (e.g. the inclusion of 'highly' skilled labor) and deletion of zero activities.

WTF^{in}	In-quota imports	r x r x i
τ^{pr}	Quota premium rate	r x r x i
WTF^{out}	Over-quota imports	r x r x i
PD	Price of domestic goods sold locally	r x i
ES	Aggregate supply of exports	r x i
XP	Aggregate domestic output	r x i
τ^{em}	Average export price markup due to TRQ income on own-exports	r x r x i
WPE	Bilateral (world) export prices	r x r x i
PET	Price of aggregate exports	r x i
$WXMg$	Volume of world demand for international trade and transport services	1
$AXMg$	Regional supply of international trade and transport services	r
$WPMg$	Aggregate world price of international trade and transport services	1
XMg	Regional sectoral demand for goods and services related to trade	r x i
$APMg$	Regional supply price of international trade and transport services	r

The number of trade variables is:

$$NR\left[2.NS(5+NR+R1+R2)+2\right]+2+4.NTRQ$$

This yields 15,632 variables using the same dimensions as above—assuming the number of trade nodes subject to TRQs, *NTRQ*, is zero. It also assumes that *R*1 and *R*2 are respectively 2 and 4, i.e. there are 2 top-tier import bundles, and each of these is composed of 2 second-tier import bundles. (N.B. As the dimensions increase, the world trade flow matrix becomes sparser and zero activities (both volumes and prices) are deleted from the model definition.)

Domestic Closure Variables

YG	Aggregate government revenue	r
S^g	Government saving (or deficit)	r
RS^g	Real government saving (or deficit)	r
FD_{Gov}	Aggregate volume of government expenditures on goods and services	r
S^f	Foreign saving	r
FD_{Inv}	Aggregate volume of investment expenditures on goods and services[c]	r-1
$InvSh$	Nominal investment as a share of GDP	r
P	Price index of OECD exports	1
WRR	Average world rate of return	1

Factor Market Variables

L^s	Aggregate labor supply	r x l x gz
$AVGW$	Average wage rate	r x l x gz
$MIGR$	Rural to urban migration	r x l
TW	Economy-wide wage by market segment	r x l x gz
$WMIN$	Minimum wage	r x l x gz
UE	Unemployment rate	r x l x gz
NW	After tax wage	r x l x i
W	Sector-specific wage	r x l x i
$TLnd$	Aggregate land supply	r
$PTLnd$	Economy-wide land price	r

T^s	Sectoral land supply	*r* x *i*
PT	Sectoral-specific land price	*r* x *i*
NPT	After tax land price	*r* x *i*
F^s	Supply of sector-specific factors	*r* x *i*
PF	Price of sector-specific factor	*r* x *i*
KS^s	Supply of sectoral capital[d]	*r* x *i*
TR	Economy-wide rental rate	*r*
R	Sector and vintage specific rental rate	*r* x *i* x *v*
NR	Sector and vintage specific rental rate after tax	*r* x *i* x *v*
RR	Relative price of *Old* to *New* capital[e]	*r* x *i*
χ^y	Capital output ratio[f]	*r* x *i*
XPv	Output by vintage	*r* x *i* x *v*
γ^I	Rate of real investment growth	*r*
K	Aggregate capital stock (non-normalized)	*r*
K^s	Aggregate capital stock (normalized)	*r*

The total number of variables for model closure and the factor markets is:

$$NR\left[13 + NL(5.NGZ + 1 + 2.NS) + NS(8 + 3.NV)\right] + 1$$

This yields 6,916 variables, where *NGZ* is the number of geographic zones, typically 3.

Other Variables

RGDPMP	Real GDP at market price	*r*
RGDP	Real GDP at factor cost	*r*
PGDP	GDP deflator (at factor cost)	*r*
PABS	Price index of aggregate domestic absorption	*r*
g^y	Growth rate of real GDP (at factor cost)[g]	*r*
λ^l	Labor productivity factor	*r* x *l* x *ik*
χ^p	Trade-sensitive productivity shifter	*r* x *i*
ϕ^p	Productivity shifter calibration parameter[h]	*r* x *ik*
λ^l	Exogenous labor productivity factor	*r* x *l* x *ink*
λ^k	Exogenous capital productivity factor	*r* x *ink* x *v*
λ^t	Exogenous land productivity factor	*r* x *ink*
λ^f	Exogenous sector-specific factor productivity factor	*r* x *ink*

The number of variables in this block is:

$$NR\left[5 + NK(1 + NL) + NS + NX\left(NL + NV + 2\right)\right]$$

where *NK* is the number of sectors with *endogenous* labor productivity, and *NX* is the number of sectors with *exogenous* productivity. The number of variables in the block using the same dimensions as above (with *NK* equal to 13 and *NX* equal to 7) is 1,890. In policy simulations the productivity calibration parameter is pre-calibrated which reduces the number of variables in the model by *NK* per region.

The total number of equations per region can be summarized by:

$$\begin{aligned} NREQ \quad = \quad & NS\left[31+15.NV+3.NL+NS+2.NH+NF+2.(NR+R1+R2)\right] \\ + \quad & 23+NH(7+2.NC)+NL.(2+5.NGZ)+NF+NK(1+NL)+NX(NL+NV+2) \\ & 2.NCR(1+NV)+2.NLV(1+2.NV) \end{aligned}$$

This total is 2,984. The total number of equations is *NR*.*NREQ*+3, or 44,763.

Notes:

- *a.* *The implementation of the model incorporates the equilibrium condition for the variable XD, therefore, the Armington demand condition can be thought of determining XD, and the CET supply condition can be thought of determining the price of XD, i.e. PD.*
- *b.* *Similar to the explanation in Note a., the model incorporates the equilibrium condition for the variable WTF.*
- *c.* *The investment equation for one region is dropped due to the global Walras' Law.*
- *d.* *Variable only included for models with a single capital vintage.*
- *e.* *Variable only included for model with Old and New capital vintages.*
- *f.* *The capital output ratio is only calculated for Old capital.*
- *g.* *In the baseline (BaU) scenario, the variable g^y is exogenous, and the labor productivity parameter, γ^l is endogenous. In all other scenarios, the reverse holds.*
- *h.* *The calibration parameter is only calibrated in the baseline scenario.*

Annex B: Decomposition of Model Results

This annex provides a list of formulas intended to decompose the results from model solutions. Many of these formulas are standard accounting identities, such as macroeconomic aggregates. There is also a discussion of the various welfare measures and a decomposition of the sources of growth.

Macroeconomic aggregates

Tables B1-3 provide the basic macroeconomic aggregates separated into three parts: volumes, values and price indices. The only adjustment with standard macroeconomic identities is the addition of the exports of international trade and transport services which are separately identified from the exports of goods and other services. The PPP adjusted GDP aggregate is derived from an estimate of the 2001 PPP exchange rate (as available in the World Bank's data banks). The PPP exchange rate is assumed constant over any forecast horizon.

Note that the macroeconomic components for consumption, and government and investment expenditures use variables designated *PAc* and *PAf*. These are not model variables (in order to minimize the dimensionality of the model). They are calculated post-simulation and are equal to the tax-adjusted economy-wide Armington price. The formulas are (where *PAp* is the absorption price for intermediate demand):

$$PAp_{i,j} = \left(1+\tau_{i,j}^{Ap}\right)PA_i$$

$$PAc_{i,h} = \left(1+\tau_{i,h}^{Ac}\right)PA_i$$

$$PAf_{j,f} = \left(1+\tau_{i,h}^{Af}\right)PA_i$$

Table B-1: Macroeconomic aggregates—volumes

Label	Description	Formula
cons	Aggregate consumption	$C = \sum_h \sum_i PAc_{i,h,0} XAc_{i,h}$
gov	Government expenditures	$G = \sum_i PAf_{i,Gov,0} XAf_{i,Gov}$
inv	Investment expenditures	$I = \sum_i PAf_{i,Inv,0} XAf_{i,Inv}$
exp	Aggregate exports	$E = \sum_{r'} \sum_i WPE_{r,r',i,0} WTF_{r,r',i}^{s}$
imp	Aggregate imports	$M = \sum_{r'} \sum_i WPM_{r',r,i,0} WTF_{r',r,i}^{d}$
rgdpmp	Real GDP at market price	$RGDPMP = C + G + I + E - M + APMg_0 AXMg$
rgdpmpppp	Real GDP at market price in international dollars	$RGDPMP\_PPP = RGDPMP.PPP_0$
gdp_pc	Real GDP per capita	$GDP\_PC = RGDPMP / \sum_h Pop_h$
gdpppp_pc	Real GDP per capita in international dollars	$GDP\_PPP\_PC = RGDPMP\_PPP / \sum_h Pop_h$

Table B-2: Macroeconomic aggregates—values

Label	*Description*	*Formula*
cons$	Aggregate consumption	$C\$ = \sum_h \sum_i PAc_{i,h} XAc_{i,h}$
gov$	Government expenditures	$G\$ = \sum_i PAf_{i,Gov} XAf_{i,Gov}$
inv$	Investment expenditures	$I\$ = \sum_i PAf_{i,Inv} XAf_{i,Inv}$
exp$	Aggregate exports	$E\$ = \sum_{r'} \sum_i WPE_{r,r',i} WTF^s_{r,r',i}$
imp$	Aggregate imports	$M\$ = \sum_{r'} \sum_i WPM_{r',r,i} WTF^d_{r',r,i}$
gdpmp	GDP at market price	$GDPMP = C\$ + G\$ + I\$ + E\$ - M\$ + APMg.AXMg$

Table B-3: Macroeconomic aggregates—price indices

cpi	Aggregate consumer price index	$CPI = C\$ / C$
gpi	Government price index	$GPI = G\$ / G$
zpi	Government price index	$ZPI = I\$ / I$
epi	Export price index	$EPI = E\$ / E$
mpi	Import price index	$MPI = M\$ / M$
pgdpmp	GDP deflator at market price	$PGDPMP = GDPMP / RGDPMP$

Factor volumes and prices

Table B-4 provides the formulas used to express aggregate factor volumes and prices. For the labor expressions, the index *l* covers all three possible labor categories. (N.B. The aggregate capital stock is a model variable.)

Table B-4: Aggregate factor volumes and prices

Label	Description	Formula
tlabs	Aggregate labor supply	$TL^s = \sum_l TW_{l,0} L^s_l$
awage	Aggregate wage	$AW = \sum_l TW_l L^s_{l,0} / \sum_l TW_{l,0} L^s_{l,0}$
arent	Aggregate rent	$AR = \sum_i \sum_v R_{i,v} Kv^d_{i,v,0} / \sum_i \sum_v R_{i,v,0} Kv^d_{i,v,0}$
ld	Aggregate sectoral labor demand	$LD_i = \sum_l W_{l,i,0} L^d_{l,i}$
kd	Aggregate sectoral capital demand	$KD_i = \sum_v R_{i,v,0} Kv^d_{i,v}$

World trade prices

Table B-5 contains the formulas for expressing various aggregate trade price indices. Due to the Armington assumption on the demand side and the CET assumption on the supply side, there is no single world price at the sectoral level, i.e. each export/import price is origin and destination specific. The first formula, *WEPI*, calculates the average sectoral world price (by country), summing over all trading partners, at the FOB level. The second formula, *WMPI*, calculates the average sectoral world price, summing over all regions of origin, at the CIF level. The third formula, *AWP*, is an expression for an overall world price index, which is the sum across all regions of origins and destinations at the FOB level.

Table B-5: Aggregate trade prices

Label	Description	Formula
`wepi`	Sectoral export price index at world price	$WEPI_{r,i} = \sum_{r'} WPE_{r,r',i} WTF^{s}_{r,r',i,0} / \sum_{r'} WPE_{r,r',i,0} WTF^{s}_{r,r',i,0}$
`wmpi`	Sectoral import price index at world price	$WMPI_{r,i} = \sum_{r'} WPM_{r',r,i} WTF^{d}_{r',r,i,0} / \sum_{r'} WPE_{r',r,i,0} WTF^{d}_{r',r,i,0}$
`awp`	Sectoral world price index	$AWP_{i} = \sum_{r} \sum_{r'} WPE_{r,r',i} WTF^{s}_{r,r',i,0} / \sum_{r} \sum_{r'} WPE_{r,r',i,0} WTF^{s}_{r,r',i,0}$

Welfare measurement

Table B-6 represents the formulas for the standard measurement of welfare. This involves calculating the expenditure function using different values of utility and prices. Equivalent variation—the standard measure of the change in welfare—in a given year is defined as:

$$EV_{h,t} = E\left(P^{b}_{h,t}, u^{p}_{h,t}\right) - E\left(P^{b}_{h,t}, u^{b}_{h,t}\right)$$

This is equal to the difference in purchasing post-reform and baseline utilities at baseline prices (where the superscript b represents baseline values, and the superscript p represents post-reform values). In other words, it is the amount of income consumers would be willing to forego to achieve the new utility level (at baseline prices), compared with the baseline utility. Using the notation of the Final Demand section of the main text, the expenditure function for the ELES can be expressed as:

$$E(P,u) = \sum_{i} P_{i}\theta_{i} + uP$$

where

$$u = \exp\left(\sum_{i} \mu_{i} \ln(C_{i} - \theta_{i}) + \mu^{s} \ln\left(\frac{S^{h}}{P^{s}}\right)\right)$$

$$P = \exp\left[\sum_{i} \mu_{i} \ln\left(\frac{P_{i}}{\mu_{i}}\right) + \mu^{s} \ln\left(\frac{P^{s}}{\mu^{s}}\right)\right]$$

One of the advantages of the ELES expenditure function is the separability of the price and utility expressions. In the calculation of the expenditure, these can be calculated and stored separately saving storage space and computing time.

In the actual formulas, the expenditure function is calculated for each household, the subsistence minima are adjusted by the household-specific population levels, and the CPI is used as the household savings deflator.

Table B-6: Welfare

Label	Description	Formula
Welfare calculations		
`pindex0`	Expenditure function price index using base year prices	$P_{h,0}^{0} = \exp\left[\sum_k \mu_{k,h,t} \ln\left(\frac{PC_{k,h,0}}{\mu_{k,h,t}}\right) + \mu_{h,t}^{s} \ln\left(\frac{CPI_{h,0}}{\mu_{h,t}^{s}}\right)\right]$
`pindext`	Expenditure function price index using baseline prices	$P_{h,t}^{b} = \exp\left[\sum_k \mu_{k,h,t} \ln\left(\frac{PC_{k,h,t}^{b}}{\mu_{k,h,t}}\right) + \mu_{h,t}^{s} \ln\left(\frac{CPI_{h,t}^{b}}{\mu_{h,t}^{s}}\right)\right]$
`pindex1`	Expenditure function price index using post-reform prices	$P_{h,t}^{p} = \exp\left[\sum_k \mu_{k,h,t} \ln\left(\frac{PC_{k,h,t}^{p}}{\mu_{k,h,t}}\right) + \mu_{h,t}^{s} \ln\left(\frac{CPI_{h,t}^{p}}{\mu_{h,t}^{s}}\right)\right]$
`ut`	Household utility in baseline simulation	$u_{h,t}^{b} = \exp\left(\sum_k \mu_{k,h,t} \ln\left(XC_{k,h,t}^{b} - Pop_{h,t}\theta_{k,h,t}\right) + \mu_{h,t}^{s} \ln\left(\frac{S_{h,t}^{h,b}}{CPI_{h,t}^{b}}\right)\right)$
`u1`	Household utility in post-reform simulation	$u_{h,t}^{p} = \exp\left(\sum_k \mu_{k,h,t} \ln\left(XC_{k,h,t}^{p} - Pop_{h,t}\theta_{k,h,t}\right) + \mu_{h,t}^{s} \ln\left(\frac{S_{h,t}^{h,p}}{CPI_{h,t}^{p}}\right)\right)$
`E1`	Expenditure function—baseline prices and utility	$E\left(P_{h,t}^{b}, u_{h,t}^{b}\right) = Pop_{h,t} \sum_k PC_{k,h,t}^{b}\theta_{k,h,t} + u_{h,t}^{b} P_{h,t}^{b}$
`E2`	Expenditure function—post-reform prices and utility	$E\left(P_{h,t}^{p}, u_{h,t}^{p}\right) = Pop_{h,t} \sum_k PC_{k,h,t}^{p}\theta_{k,h,t} + u_{h,t}^{p} P_{h,t}^{p}$
`E3`	Expenditure function—post-reform prices and baseline utility	$E\left(P_{h,t}^{p}, u_{h,t}^{b}\right) = Pop_{h,t} \sum_k PC_{k,h,t}^{p}\theta_{k,h,t} + u_{h,t}^{b} P_{h,t}^{p}$
`E4`	Expenditure function—baseline prices and post-reform utility	$E\left(P_{h,t}^{b}, u_{h,t}^{p}\right) = Pop_{h,t} \sum_k PC_{k,h,t}^{b}\theta_{k,h,t} + u_{h,t}^{p} P_{h,t}^{b}$
`E5`	Expenditure function—base year prices and baseline utility	$E\left(P_{h,t}^{0}, u_{h,t}^{b}\right) = Pop_{h,t} \sum_k PC_{k,h,0}\theta_{k,h,t} + u_{h,t}^{b} P_{h,t}^{0}$
`E6`	Expenditure function—base year prices and post-reform utility	$E\left(P_{h,t}^{0}, u_{h,t}^{p}\right) = Pop_{h,t} \sum_k PC_{k,h,0}\theta_{k,h,t} + u_{h,t}^{p} P_{h,t}^{0}$

The calculation of cumulative discounted welfare requires some adjustment. Due to the use of the extended linear expenditure system (ELES), savings is included in the evaluation of welfare. Savings represents a stream of future consumption and thus should be extracted from the calculation of cumulative welfare otherwise there would be double counting. To do so, the marginal budget shares, μ, are scaled to sum to one and the expenditure function is re-calculated based on these adjusted marginal budget shares and excluding the savings term. Cumulative welfare is then calculated as:

$$CEV_{h,t_0-1} = \sum_{t=t_0}^{T-1} \beta^{(t-t_0+1)} EV_{h,t}^{a} + \beta^{(T-t_0+1)} EV_{h,T}$$

where t_0 is the first year of reform, and T is the terminal year of evaluation. The sum covers all years until the terminal year, using the adjusted expenditure function, EV^a, i.e. the expenditure function excluding savings. The final term evaluates the discounted equivalent variation in the terminal year which does include saving. The β term is the discount factor equal to 1/(1+r) where r is the discount rate. A relative measure of welfare changes is in many cases more usual since it avoid scaling problems. One such measure is real disposable income. For example, the cumulative welfare gain as a percent of real disposable income is calculated as:

$$CEVP_{h,t_0-1} = \frac{\sum_{t=t_0}^{T-1} \beta^{(t-t_0+1)} EV_{h,t}^a + \beta^{(T-t_0+1)} EV_{h,T}}{\sum_{t=t_0}^{T} \beta^{(t-t_0+1)} \left(YD_{h,t}^b / CPI_{h,t}^b \right)}$$

Growth accounting

Decomposition of the sources of growth from the supply side comes from the following identity which describes real GDP at factor cost:

$$GDP_t = \sum_i \lambda_{i,t}^l W_{i,0} L_{i,t} + \sum_i \lambda_{i,t}^k R_{i,0} K_{i,t}$$

The summation covers sectors and the λ parameters are the factor and sector specific productivity parameters which are linked to productivity growth through the following equations:

$$\lambda_{i,t}^l = \left(1 + \gamma_{i,t}^l\right) \quad \text{and} \quad \lambda_{i,t}^k = \left(1 + \gamma_{i,t}^k\right)$$

Growth in GDP can be decomposed by taking the first difference of the GDP identity and dividing by GDP leading to the following expression:

$$\frac{\Delta GDP}{GDP} = \sum_i \varphi_i^l \frac{\Delta\lambda_i^l}{\lambda_i^l} + \sum_i \varphi_i^l \frac{\Delta L_i}{L_i} + \sum_i \varphi_i^k \frac{\Delta\lambda_i^k}{\lambda_i^k} + \sum_i \varphi_i^k \frac{\Delta K_i}{K_i}$$

The φ parameters represent the relevant shares of sector-specific (efficiency evaluated) factors in GDP:

$$\varphi_i^l = \frac{W_{i,0}\lambda_i^l L_i}{GDP} \quad \text{and} \quad \varphi_i^k = \frac{R_{i,0}\lambda_i^k K_i}{GDP}$$

The second and fourth terms can be further modified by subtracting and adding aggregate factor growth with the final decomposition formula given by:

$$\begin{aligned} \frac{\Delta GDP}{GDP} = & \underbrace{\sum_i \varphi_i^l \frac{\Delta\lambda_i^l}{\lambda_i^l} + + \sum_i \varphi_i^k \frac{\Delta\lambda_i^k}{\lambda_i^k}}_{Productivity} \\ & + \underbrace{\sum_i \varphi_i^l \left[\frac{\Delta L_i}{L_i} - \frac{\Delta L}{L}\right] + \sum_i \varphi_i^k \left[\frac{\Delta K_i}{K_i} - \frac{\Delta K}{K}\right]}_{Reallocation} \\ & + \underbrace{\sum_i \varphi_i^l \frac{\Delta L}{L} + \sum_i \varphi_i^k \frac{\Delta K}{K}}_{Aggregate\ growth} \end{aligned}$$

The first two terms represent respectively the share of GDP growth explained by factor productivity. The third and fourth terms represent the share of GDP growth coming from the reallocation of factors across sectors. And the last two terms explain the part of growth derived from overall factor growth, weighted by the respective share of factors in the determination of output. The output also includes the contribution to growth from the land and sector-specific factor using analogous formulas.

Table B-7: Growth Accounting

Label	Description	Formula
`lalloc`	GDP growth generated by labor re-allocation	$\gamma_t^{l,Alloc} = \sum_i \varphi_{i,t}^l \left[\left(\frac{\sum_l L_{l,i,t}^d}{\sum_l L_{l,i,t-1}^d} - 1 \right) - \left(\frac{\sum_l L_{l,t}^s}{\sum_l L_{l,t-1}^s} - 1 \right) \right]$
`lprod`	GDP growth generated by labor productivity	$\gamma_t^{l,Prod} = \sum_i \varphi_{i,t}^l \left[\left(\frac{\sum_l \lambda_{l,i,t}^l}{\sum_l \lambda_{l,i,t-1}^l} - 1 \right) \right]$
`lgrowth`	GDP growth generated by labor volume growth	$\gamma_t^{l,Growth} = \left(\frac{\sum_l L_{l,t}^s}{\sum_l L_{l,t-1}^s} - 1 \right) \sum_i \varphi_{i,t}^l$
`labshr`	Labor share of real GDP (in efficiency units)	$\varphi_{i,t+1}^l = \sum_l \left(NW_{l,i,0} \lambda_{l,i,t}^l L_{l,i,t}^d \right) / RGDP_t$
`kalloc`	GDP growth generated by capital re-allocation	$\gamma_t^{k,Alloc} = \sum_i \varphi_{i,t}^k \left[\left(\frac{\sum_v Kv_{i,v,t}^d}{\sum_v Kv_{i,v,t-1}^d} - 1 \right) - \left(\frac{K_t^s}{K_{t-1}^s} - 1 \right) \right]$
`kprod`	GDP growth generated by capital productivity	$\gamma_t^{k,Prod} = \sum_i \varphi_{i,t}^k \left[\left(\frac{\sum_v \lambda_{i,v,t}^k}{\sum_v \lambda_{i,v,t-1}^k} - 1 \right) \right]$
`kgrowth`	GDP growth generated by capital volume growth	$\gamma_t^{k,Growth} = \left(\frac{K_t^s}{K_{t-1}^s} - 1 \right) \sum_i \varphi_{i,t}^k$
`kapshr`	Capital share of real GDP (in efficiency units)	$\varphi_{i,t+1}^k = \sum_v \left(NR_{i,v,0} \lambda_{i,v,t}^k Kv_{i,v,t}^d \right) / RGDP_t$

Annex C: The CES and CET Functions

The CES Function

Because of the frequent use of the constant elasticity of substitution (CES) function, this appendix will develop some of the properties of the CES, including some of its special cases. The CES function can be formulated as a cost minimization problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$

subject to

$$V = \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho}$$

where V is the aggregate volume (of production, for example), X are the individual components ("inputs") of the production function, P are the corresponding prices, and a and λ are technological parameters. The *a* parameters are most often called the share parameters. The λ parameters are technology shifters. The parameter ρ is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

A bit of algebra produces the following derived demand for the inputs, assuming V and the prices are fixed:

$$(1) \qquad X_i = \alpha_i (\lambda_i)^{\sigma-1} \left(\frac{P}{P_i} \right)^\sigma V$$

where we define the following relationships:

$$\rho = \frac{\sigma - 1}{\sigma} \Leftrightarrow \sigma = \frac{1}{1-\rho} \qquad \text{and} \qquad \sigma \geq 0$$

$$\alpha_i = a_i^\sigma \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

and

$$(2) \qquad P = \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

P is called the CES dual price, it is the aggregate price of the CES components. The parameter σ, is called the substitution elasticity. This term comes from the following relationship, which is easy to derive from Equation (1):

$$\frac{\partial (X_i / X_j)}{\partial (P_i / P_j)} \frac{(P_i / P_j)}{(X_i / X_j)} = -\sigma$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note, we are assuming that the substitution elasticity is a positive number). For example, if the price of input *i* increases by 10 per cent with respect to input *j*, the ratio of input *i* to input *j* will decrease by (around) σ times 10 per cent.

The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero, in other words, there is no substitution between inputs, no matter what the input prices are. Equations (1) and (2) become:

$$(1') \qquad X_i = \frac{\alpha_i V}{\lambda_i}$$

$$(2') \qquad P = \sum_i \alpha_i \frac{P_i}{\lambda_i}$$

The aggregate price is the weighted sum of the input prices. The Cobb-Douglas function is for the special case when σ is equal to one. It should be clear from Equation (2) that this case needs special handling. The following equations provide the relevant equations for the Cobb-Douglas:

$$(1'') \qquad X_i = \alpha_i \frac{P}{P_i} V$$

$$(2'') \qquad P = A^{-1} \prod_i \left(\frac{P_i}{\alpha_i \lambda_i} \right)^{\alpha_i}$$

where the production function is given by:

$$V = A \prod_i \left(\lambda_i X_i \right)^{\alpha_i}$$

and

$$\sum_i \alpha_i = 1$$

Note that in Equation (1") the value share is constant, and does not depend directly on technology change.

Calibration

Typically, the base data set along with a given substitution elasticity are used to calibrate the CES share parameters. Equation (1) can be inverted to yield:

$$\alpha_i = \left(\frac{P_i}{P} \right)^{\sigma} \frac{X_i}{V}$$

assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalized to 1, simplifying the above expression to a true value share. Let's take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalized to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by α, not the share parameters represented by *a*. This saves on compute time since the *a* parameters never appear explicitly in any equation, whereas a raised to the power of the substitution elasticity, i.e. α, occurs frequently.)

The CET Function

With less detail, the following describes the relevant formulas for the CET function, which is similar to the CES specification.

$$\max \sum_i P_i X_i$$

subject to

$$V = \left[\sum_i g_i X_i^{\nu} \right]^{1/\nu}$$

where V is the aggregate volume (e.g. aggregate supply), X are the relevant components (sector-specific supply), P are the corresponding prices, g are the CET (primal) share parameters, and ν is the CET exponent. The CET exponent is related to the CET transformation elasticity, ω via the following relation:

$$\nu = \frac{\omega + 1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu - 1}$$

Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i \left(\frac{P_i}{P} \right)^{\omega} V$$

$$P = \left[\sum_i \gamma_i P_i^{1+\omega} \right]^{1/(1+\omega)}$$

where the γ parameters are related to the primal share parameters, g, by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i} \right)^{1/\omega}$$

Annex D: Short- versus Long-Run Armington Elasticities

Introduction

In the standard version of the LINKAGE model, both the Armington share parameters and substitution elasticities are fixed at their base year levels. The former are calibrated parameters dependent on base year trade shares and the user-supplied Armington substitution elasticities. The CPB in the Netherlands has introduced a mechanism that updates the share parameters (between solution years) which has the effect of introducing varying substitution possibilities over time, with typically longer-run flexibility.[75] At the limit, this system converges towards a Heckscher-Ohlin specification where all goods are perfectly homogeneous. This annex describes the mechanism for the model's second level nest, but it also applies to the first level Armington nest.

Endogenizing Armington share parameters

Formulation of the second-level nest

The following equations repeat the formulation of the second level nest in the model. The second-level Armington nest allocates aggregate import demand *XMT*, across markets by region of origin. Dropping the sectoral indices we have:

(D-1) $$WTF_{r',r} = \alpha^w_{r',r}\left(\frac{PMT_r}{PM_{r',r}}\right)^{\sigma^w_r} XMT_r$$

The variable *WTF* represents the imports by region *r* originating in region *r*'. The aggregate import price, *PMT*, is derived from the CES dual price equation:

(D-2) $$PMT_r = \left[\sum_{r'} \alpha^w_{r',r}\left(PM_{r',r}\right)^{1-\sigma^w_r}\right]^{1/\left(1-\sigma^w_r\right)}$$

The bilateral import price, *PM*, is the tariff inclusive price of goods originating in region *r*' purchased in region *r*. The CES substitution elasticity is given by σ^w. Under standard models, the share parameters, α^w, are calibrated using base year data and are assumed constant over time. Thus, in order to increase market share, countries necessarily have to accept declining terms of trade. The CPB WorldScan Model introduces *endogenous* share parameters which evolve over time. The basic idea is that once market share is achieved (through price competition for example) it is hard to lose.

First define market share:

(D-3) $$\mu^w_{r',r} = \frac{PM_{r',r}WTF_{r',r}}{\sum_{r'} PM_{r',r}WTF_{r',r}}$$

where μ^w represents the market share of imports into region *r*, from region *r*' (for good *i*), evaluated at domestic, tariff-inclusive, prices.

The motion equation for the CES share parameters is:

[75] See CPB [1999].

(D-4) $$\alpha^{w}_{r',r,t} = \frac{\left(\mu^{w}_{r',r,t-1}\right)^{1-\theta^{w}_{r}} \left(\beta^{w}_{r',r}\right)^{\theta^{w}_{r}}}{\sum_{r'} \left(\mu^{w}_{r',r,t-1}\right)^{1-\theta^{w}_{r}} \left(\beta^{w}_{r',r}\right)^{\theta^{w}_{r}}}$$

The parameter β is calibrated using base year data and the parameter θ in some sense determines the long-run Armington elasticity. The parameter θ must be strictly greater than 0 and less than or equal to 1. If it is equal to 1, then the short-term elasticity is simply equal to the long-term elasticity (and the CES share parameters are constant). As θ approaches 0, the long-run elasticity approaches infinity in which case the model converges towards a Heckscher-Ohlin specification.

The demand equation can be re-written in terms of market share to be:

(D-5) $$\mu^{w}_{r',r} = \alpha^{w}_{r',r} \left(\frac{PMT_{r}}{PM_{r',r}}\right)^{\sigma^{w}_{r}-1}$$

If we substitute expression (D-5) into (D-4) and raise to the power 1/θ, we get:

(D-6) $$\mu^{w}_{r',r,t} = \frac{\left(\mu^{w}_{r',r,t-1}\right)^{\left(1-\theta^{w}_{r}\right)/\theta^{w}_{r}} \left(\beta^{w}_{r',r}\right)}{\left[\sum_{r'} \left(\mu^{w}_{r',r,t-1}\right)^{1-\theta^{w}_{r}} \left(\beta^{w}_{r',r}\right)^{\theta^{w}_{r}}\right]^{1/\theta^{w}_{r}}} \left(\frac{PMT_{r}}{PM_{r'r}}\right)^{\left(\sigma^{w}_{r}-1\right)/\theta^{w}_{r}}$$

Asymptotically this implies:

(D-7) $$\bar{\mu}^{w}_{r',r} = \frac{\beta^{w}_{r',r}}{\left[\sum_{r'} \left(\bar{\mu}^{w}_{r',r}\right)^{1-\theta^{w}_{r}} \left(\beta^{w}_{r',r}\right)^{\theta^{w}_{r}}\right]^{1/\theta^{w}_{r}}} \left(\frac{PMT_{r}}{PM_{r'r}}\right)^{\left(\sigma^{w}_{r}-1\right)/\theta^{w}_{r}}$$

Thus the long term elasticity, $(\sigma^{w}-1)/\theta^{w}+1$ is greater than the short run elasticity σ so long as θ is less than 1. The β parameters are calibrated using base year data:

(D-8) $$\beta^{w}_{r',r} = \frac{\mu^{w}_{r',r} \left(P^{m}_{r',r}\right)^{\left(\sigma^{w}_{r}-1\right)/\theta^{w}_{r}}}{\sum_{r'} \mu^{w}_{r',r} \left(P^{m}_{r',r}\right)^{\left(\sigma^{w}_{r}-1\right)/\theta^{w}_{r}}}$$

It is to be noted that both the α and β parameters are normalized to sum to 1. Under the normal calibration routine, the aggregate price index, *PMT*, is user-determined, and the α parameters are calibrated as a function of the component prices and the aggregate price index. Because of the normalization rule imposed here, the aggregate price index is an outcome of the calibration routine, not an input. The sequence for calibration is:

i. calculate the value shares (μ parameters, D-3)
ii. calculate the β parameters (D-8)
iii. calculate the CES share parameters (α parameters, D-4)
iv. calculate the aggregate CES price *P* (D-2)
v. calculate the aggregate CES volume *XMT* (by dividing aggregate value by *P*)

Solving for the long-run share

Equation (D-7) is a rather messy non-linear system of implicit equations in terms of the long-run share parameters. Moreover, the aggregate price index, *PMT* is itself a function of the α parameters, which in turn depend on the μ parameters. A system of equations can be entered into a non-linear solver (for example Excel or GAMS), which can solve for the equilibrium shares. The following system has been coded in both Excel and GAMS and is available from the author.

$$PMT_r^{1-\sigma_r^w} = \sum_{r'} \alpha_{r',r}^w \left(PM_{r',r}\right)^{1-\sigma_r^w}$$

$$\alpha_{r'r}^w Y^{\theta^w} = \left(\mu_{r',r,t-1}^w\right)^{1-\theta_r^w} \left(\beta_{r',r}^w\right)^{\theta_r^w}$$

$$\mu_{r',r}^w Y \left(PM_{r'r}\right)^{\left(\sigma_r^w-1\right)/\theta_r^w} = \beta_{r',r}^w \left(PMT_r\right)^{\left(\sigma_r^w-1\right)/\theta_r^w}$$

$$Y^{\theta_r^w} = \sum_{r'} \left(\mu_{r',r}^w\right)^{1-\theta_r^w} \left(\beta_{r',r}^w\right)^{\theta_r^w}$$

The exogenous inputs are σ, θ, β, and *PM*. The endogenous outputs are *PMT*, α, μ, and *Y*. The factor *Y* is used to simplify the middle two expressions. Note that the β parameters have to be re-calibrated for each change of σ and/or θ. A 'shock' represents perturbing one (or more) of the component prices and to observe the change in the long-run equilibrium value share (μ).

Example

Table D-1 presents the results from a simple partial equilibrium model of a price shock in a three-region CES. The initial import shares are 36.3, 50.4, and 13.3 percent respectively. The shock is a permanent 5 percent decline in the price of exports from the middle exporter. The table indicates the short- and long-term impacts of the price decline under various assumptions about the two key elasticities. Focusing on the value of the short-run elasticity of 2.5, the immediate impact is a rise in market share to 52.3 percent from the base 50.4 percent (see the final column). This is true irrespective of the value of θ, which doesn't affect the share parameters until the second period. In a standard model with time-invariant share parameters, this is also the long-term change in market share. With a low value for θ (0.1), i.e. very sensitive share parameters, the long-term market share is nearly 69 percent, i.e. the long-term elasticity is 16, over six times higher than the short-term elasticity. A Cobb-Douglas specification implies constant (value) shares, irrespective of price movements. Going from right to left, it is obvious that the long-run elasticity increases as θ declines. From top to bottom, the long-run elasticity increases with σ. Table D-2 summarizes the value of the long-run elasticity as a function of σ and θ.

Table D-1: Long-term market share as a function of short- and long-run Armington elasticities

	Theta (θ)									
Sigma (σ)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	50.4%	50.4%	50.4%	50.4%	50.4%	50.4%	50.4%	50.4%	50.4%	50.4%
1.5	56.8%	53.6%	52.5%	52.0%	51.7%	51.5%	51.3%	51.2%	51.1%	51.0%
2.0	62.9%	56.8%	54.7%	53.6%	53.0%	52.5%	52.2%	52.0%	51.8%	51.7%
2.5	68.7%	59.9%	56.8%	55.2%	54.2%	53.6%	53.1%	52.8%	52.5%	52.3%
3.0	73.9%	62.9%	58.9%	56.8%	55.5%	54.7%	54.1%	53.6%	53.2%	53.0%
3.5	78.6%	65.9%	60.9%	58.3%	56.8%	55.7%	55.0%	54.4%	54.0%	53.6%
4.0	82.6%	68.7%	62.9%	59.9%	58.0%	56.8%	55.9%	55.2%	54.7%	54.2%
4.5	86.0%	71.4%	64.9%	61.4%	59.3%	57.8%	56.8%	56.0%	55.4%	54.9%
5.0	88.8%	73.9%	66.8%	62.9%	60.5%	58.9%	57.7%	56.8%	56.1%	55.5%

Table D-2: Long-term Armington elasticities

	Theta (θ)									
Sigma (σ)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.5	6.0	3.5	2.7	2.3	2.0	1.8	1.7	1.6	1.6	1.5
2.0	11.0	6.0	4.3	3.5	3.0	2.7	2.4	2.3	2.1	2.0
2.5	16.0	8.5	6.0	4.8	4.0	3.5	3.1	2.9	2.7	2.5
3.0	21.0	11.0	7.7	6.0	5.0	4.3	3.9	3.5	3.2	3.0
3.5	26.0	13.5	9.3	7.3	6.0	5.2	4.6	4.1	3.8	3.5
4.0	31.0	16.0	11.0	8.5	7.0	6.0	5.3	4.8	4.3	4.0
4.5	36.0	18.5	12.7	9.8	8.0	6.8	6.0	5.4	4.9	4.5
5.0	41.0	21.0	14.3	11.0	9.0	7.7	6.7	6.0	5.4	5.0

Annex E: Alternative trade specifications

Introduction

This Annex describes three alternative trade specifications for the LINKAGE model. The first has to do with the disaggregation of domestic absorption between domestic and imported goods. The default version of the model assumes that there is a single economy-wide agent deciding on the decomposition. The alternative is to assume that each domestic agent is an Armington agent. In principle this allows the import content of domestic absorption to be agent-specific and allows for agent-specific substitution elasticities. It does increase the size of the model substantially, particularly for normal aggregations, say for example a 20 x 20 model.

The other two alternatives also deal with the top level disaggregation of absorption, but still assuming an economy-wide Armington agent. The standard version of the LINKAGE model uses a standard nested CES structure for modeling trade. This formulation has some drawbacks, particularly in dynamic scenarios, because it fails to capture one of the key stylized facts over the last few decades, that is that the trade to GDP elasticity has been between 1.5 and 2.5. In these models, the trade to GDP elasticity tends to hover around 1 since in an Armington structure trade growth is essentially a function of relative price changes. In dynamic scenarios, particularly in the absence of trade shocks, relative price changes are modest—and this has also been true empirically, at least at the macro level. This section derives two alternative specifications for trade both of which include an income component. This allows the trade system to capture a growth component to trade unrelated to relative price changes. Neither of the formulations captures the true dynamics of trade growth and should only be seen as reduced form attempts to capture some stylized facts. The first section describes a very simple modification to the standard CES function by inserting shift parameters, similar in many respects to the LES function. The second section describes the implementation of the AIDS (almost ideal demand system) functional form. It is a more generalized flexible functional form with an income variable. However, with large shocks it does allow market shares to become negative so it needs to be used with some caution.

Agent-specific Armington behavior

Having an economy-wide Armington agent simplifies extensively the code and reduces model size. At the same time it imposes some potentially high and perhaps unrealistic restrictions on demand behavior. The GTAP data set carries a full set of import demands for each domestic agent and thus it is easy to implement agent-specific Armington behavior. This section will not show all of the changes to the model equations, but simply indicate how they change.

Production

Production is modeled as a series of nested CES functions. Intermediate demand at the Armington level of aggregation, *XAp*, is broken out into specific components, for example energy, feed and all other goods in livestock. In the default version of the model, all agents face the same Armington price, *PA*, albeit with perhaps an agent-specific indirect tax. The changes to the demand equations therefore only require replacing the tax-adjusted price variable with an agent-specific Armington price designated as *PAp*. For example, the demand for fuels in the energy bundle is given by the following equation:[76]

$$XAp_{e,j} = \alpha_{e,j}^{ep} \left(\frac{PEp_j}{(1+\tau_{e,j}^{Ap})PA_e} \right)^{\sigma_j^{ep}} XEp_j$$

In the agent-specific Armington specification, the equation would be:

[76] Ignoring for the moment capital vintages and the energy productivity parameter.

$$XAp_{e,j} = \alpha_{e,j}^{ep}\left(\frac{PEp_j}{PAp_{e,j}}\right)^{\sigma_j^{ep}} XEp_j$$

In this expression, the variable *PAp* represents the sector-specific (*j*) Armington price of consuming fuel *e*.[77] To summarize, the variable *XAp* represents the same variable in both model specifications, but in the new specification it is determined using the agent-specific Armington price, *PAp*.

The decomposition into domestic and import components is done at the agent level. This introduces two new variables, *XDp* and *XMp*, respectively domestic and import intermediate demand defined by producing sector and ranging over all input goods. Three new equations are added to the model to complete it. Equations (P-78) and (P-79) define for each cell of the input-output table the decomposition of the Armington bundle into its domestic, *XDp*, and import component, *XMp*, respectively. The component prices are agent-specific due to indirect taxes, otherwise each agent faces an economy-wide domestic, *PD*, and import, *PMT*, price.[78] The third equation defines the cell-specific Armington price using the standard CES price aggregation formula.

(P-78) $$XDp_{ij} = \alpha_{ij}^{dp}\left(\frac{PAp_{ij}}{(1+\tau_{ij}^{Dp})\ PD_i}\right)^{\sigma_{ij}^{mp}} XAp_{ij}$$

(P-79) $$XMp_{ij} = \alpha_{ij}^{mp}\left(\frac{PAp_{ij}}{(1+\tau_{ij}^{Mp})\ PMT_i}\right)^{\sigma_{ij}^{mp}} XAp_{ij}$$

(P-80) $$PAp_{ij} = \left[\alpha_{ij}^{dp}\left((1+\tau_{ij}^{Dp})\ PD_i\right)^{1-\sigma_{ij}^{mp}} + \alpha_{ij}^{mp}\left((1+\tau_{ij}^{Mp})\ PMT_i\right)^{1-\sigma_{ij}^{mp}}\right]^{1/(1-\sigma_{ij}^{mp})}$$

Final demand

The changes for final demand emulate those for intermediate demand. Each agent faces an agent-specific Armington price, which replaces the tax-adjusted economy-wide Armington price in all demand equations. For households the following equations are added for the decomposition:

(D-10) $$XDc_{ih} = \alpha_{ih}^{dc}\left(\frac{PAc_{ih}}{(1+\tau_{ih}^{Dc})\ PD_i}\right)^{\sigma_{ih}^{mc}} XAc_{ih}$$

(D-11) $$XMc_{ih} = \alpha_{ih}^{mc}\left(\frac{PAc_{ih}}{(1+\tau_{ih}^{Mc})\ PMT_i}\right)^{\sigma_{ih}^{mc}} XAc_{ih}$$

(D-7') $$PAc_{ih} = \left[\alpha_{ih}^{dc}\left((1+\tau_{ih}^{Dc})\ PD_i\right)^{1-\sigma_{ih}^{mc}} + \alpha_{ih}^{mc}\left((1+\tau_{ih}^{Mc})\ PMT_i\right)^{1-\sigma_{ih}^{mc}}\right]^{1/(1-\sigma_{ih}^{mc})}$$

Household Armington demand, *XAc*, is decomposed into its domestic, *XDc*, and import component, *XMc*, with respectively equations (D-10) and (D-11). The household specific Armington price, *PAc*, is defined

[77] It would have been possible to keep the same expressions across trade specifications by defining a variable *PAp* in the standard specification and adding the equation:

$$PAp_{e,j} = (1+\tau_{e,j}^{Ap})PA_e$$

to the model specification. The original formulation is used to conserve on model size.

[78] The import price is the average price of imported commodity *i* over all trading partners. The decomposition of imports across regions of origin is not agent-specific.

in equation (D-7'). Equations (D-12) through (D-14) provide the same breakout for the other final demand components.

$$\text{(D-12)} \qquad XDf_{if} = \alpha_{if}^{df} \left(\frac{PAf_{if}}{(1+\tau_{if}^{Df})\ PD_i} \right)^{\sigma_{if}^{mf}} XAf_{if}$$

$$\text{(D-13)} \qquad XMf_{if} = \alpha_{if}^{mf} \left(\frac{PAf_{if}}{(1+\tau_{if}^{Mf})\ PMT_i} \right)^{\sigma_{if}^{mf}} XAf_{if}$$

$$\text{(D-14)} \qquad PAf_{if} = \left[\alpha_{if}^{df} \left((1+\tau_{if}^{Df})\ PD_i\right)^{1-\sigma_{if}^{mf}} + \alpha_{if}^{mf} \left((1+\tau_{if}^{Mf})\ PMT_i\right)^{1-\sigma_{if}^{mf}} \right]^{1/(1-\sigma_{if}^{mf})}$$

The next few equations determine the aggregate demand for domestic production and imports. In the standard specification these are determined by the economy-wide Armington agent. In this specification, they are added up across the different Armington agents. Equations (T-2) and (T-3) are replaced with (T-2') and (T-3'). Equations (T-1) and (T-4), determining respectively *XA* and *PA*, are dropped from the model since they are not meaningful in this specification.

$$\text{(T-2')} \qquad XD_i^d = \sum_j XDp_{ij} + \sum_h XDc_{ih} + \sum_f XDf_{if}$$

$$\text{(T-3')} \qquad XMT_i = \sum_j XMp_{ij} + \sum_h XMc_{ih} + \sum_f XMf_{if}$$

The decomposition of *XMT* across regions of origin is the same in both model specifications. Several other equations in the model are modified to reflect different accounting identities. These include the government revenue equation (C-1), the GDP identity (M-1), and the aggregate absorption price index (M-4).

In summary, the following variables are added to the model: *XDp*, *XMp*, *PAp*, *XDc*, *XMc*, *XDf*, *XMf*, *PAf*, and *PA* and *XA* are deleted. For each region the number of net additional variables is given by the following formula:

$$NS(3.NS + 2.NH + 3.NF - 2)$$

For a 20-sector, 1-household, 2-sector final demand model, this implies a net additional 1,320 variables translating into 19,800 variables for a 15-region model and over 26,000 additional variables for a 20-region model.

A more generalized CES function

The idea behind the generalized CES function comes from the LES. The standard preference function for the CES is the following:

$$XA_i = \left[b_i^d\,(XD_i^d)^{\rho_i^m} + b_i^m\,(XMT_i)^{\rho_i^m} \right]^{1/\rho_i^m}$$

By adding shifters to each of the components, the preference function looks more like the LES function (with non-unitary substitution elasticities):

$$XA_i = \left[b_i^d\,(XD_i^d - \theta_i^d)^{\rho_i^m} + b_i^m\,(XMT_i - \theta_i^m)^{\rho_i^m} \right]^{1/\rho_i^m}$$

Minimizing the cost of obtaining the Armington composite good subject to the preference function above yields equation (T-2') and (T-3'), with the same definitions as in the original equations, but with the addition of the shift parameters. It is clear that if the shift parameters are zero, the two equations collapse to the standard optimality conditions of the CES function.[79] The definition of the aggregate Armington price, *PA*, is identical to that defined in equation (T-4).

(T-2')
$$XD_i^d = \theta_i^d + \beta_i^d \left(\frac{PA_i}{PD_i}\right)^{\sigma_i^m} \left[XA_i - \left(\theta_i^d PD_i - \theta_i^m PMT_i\right)/ PA_i\right]$$

(T-3')
$$XMT_i = \theta_i^m + \beta_i^m \left(\frac{PA_i}{PMT_i}\right)^{\sigma_i^m} \left[XA_i - \left(\theta_i^d PD_i - \theta_i^m PMT_i\right)/ PA_i\right]$$

Calibration is done with inputs for the 'income' elasticities. The share parameters are calibrated using the following formulas (and with the variables initialized to their base-year levels):

$$\beta_i^d = \eta_i^d \left(\frac{PA_i}{PD_i}\right)^{1-\sigma_i^m} \frac{PD_i XD_i^d}{PA_i XA_i}$$

$$\beta_i^m = \eta_i^m \left(\frac{PA_i}{PMT_i}\right)^{1-\sigma_i^m} \frac{PMT_i XMT_i}{PA_i XA_i}$$

The 'income' elasticity parameters are given by η^d and η^m respectively.[80] The shift parameters can be calibrated from the demand formulas though the system is over-determined. For simplification, it is currently assumed that the domestic shift parameter is zero and equation (T-2') is used to calibrate the import shift parameter.

The allocation of aggregate imports across region of origin remains the same as in the standard version of the model—using further CES nests.

The Almost Ideal Demand System (AIDS) for modeling import demand

A more flexible functional form, such as the AIDS function, has been used to model import specification.[81] It has two advantages over the standard CES nesting. First, it allows with a single nest, a flexible representation of own- and cross-price effects. And second, it has an integrated income term allowing for non-unitary income effects.

Equation (T-28) represents the key import demand relation for the AIDS function:

(T-28)
$$s_{r',r,i} = \alpha_{r',r,i} + \sum_{r''} \gamma_{r',r'',r,i} \ln\left(PM^a_{r'',r,i}\right) + \beta_{r',r,i} \ln\left(XA_{r,i}\right)$$

The variable *s* represents the market share of region *r*' in aggregate demand for good *i* in region *r*. The diagonal element represents the combined demand of domestic production for domestic consumption (*XD*), as well as the diagonal component of the bilateral trade flow matrix ($WTF_{r,r}$).[82] The share is a function of three components. The first is a shift parameter (essentially calibrated to the initial market share information). The second is the change in market share due to changes in relative import prices across trading partners (for example to what extent the market share of region *r*' reacts to changes in the

79 Also note that if the substitution elasticity is 1, the formulas collapse to the standard LES specification.

80 They may be adjusted from the user-inputted values to ensure demand regularity conditions are satisfied.

81 See for example Robinson, Soule and Weyerbrock (1991).

82 At the country level, the diagonal part of the bilateral trade flow matrix will typically be zero, but it will most likely be positive for regional aggregations.

price of imports from region *r*"). The final term is an expansion term relating import share as a function of overall demand, *XA*.[83] The aggregate price index, *PA*, is defined by the following expression:

$$\text{(T-29)} \qquad \ln(PA_{r,i}) = \alpha^0_{r,i} + \sum_{r'} \alpha_{r',r,i} \ln(PM^a_{r',r,i}) + 0.5 \sum_{r'} \sum_{r''} \gamma_{r',r'',r,i} \ln\left(PM^a_{r',r,i}\right) \ln\left(PM^a_{r'',r,i}\right)$$

Equation (T-9), which determines bilateral trade flows (at the bottom node of the standard trade nest), is replaced with the following expression:

$$\text{(T-9')} \qquad WTF^d_{r',r,i} = s_{r',r,i} \frac{PA_{r,i} XA_{r,i}}{PM^a_{r',r,i}} \quad for \quad r \neq r'$$

In other words, the bilateral flow is given by the endogenously determined share term. The off-diagonal bilateral price, PM^a, is equal to the tariff-inclusive bilateral price of imports[84]:

$$\text{(T-30)} \qquad PM^a_{r',r,i} = PM_{r',r,i} \quad for \quad r \neq r'$$

The diagonal component of the AIDS matrix—i.e. demand for domestically produced goods as well as intra-regional trade—is decomposed using a single CES nest. This component, designated by *XDM*, is derived from the following expression:

$$\text{(T-31)} \qquad XDM_{r,i} = s_{r,r,i} \frac{PA_{r,i} XA_{,r,i}}{PM^a_{r,r,i}}$$

The price of the component, *PDM*, is equal to the relevant AIDS price, PM^a:

$$\text{(T-32)} \qquad PDM_{r,i} = PM^a_{r,r,i}$$

The component is composed of two composite goods—the domestically produced good, *XD*, and the intra-regional bilateral flow, $WTF_{r,r,i}$. Equations (T-33) and (T-34) determine the split of the *XDM* bundle using a CES specification:

$$\text{(T-2')} \qquad XD_{r,i} = \alpha^{dd}_{r,i} \left(\frac{PDM_{r,i}}{PD_{r,i}} \right)^{\sigma^b_{r,i}} XDM_{r,i}$$

$$\text{(T-9'')} \qquad WTF_{r,r,i} = \alpha^{dm}_{r,i} \left(\frac{PDM_{r,i}}{PM_{r,r,i}} \right)^{\sigma^b_{r,i}} XDM_{r,i}$$

Finally, the composite price, *PDM*, is defined by equation (T-33):

$$\text{(T-33)} \qquad PDM_{r,i} = \left[\alpha^{dd}_{r,i} (PD_{r,i})^{1-\sigma^b_{r,i}} + \alpha^{dm}_{r,i} (PM_{r,r,i})^{1-\sigma^b_{r,i}} \right]^{1/(1-\sigma^b_{r,i})}$$

Calibration is significantly more complicated than the nested CES structure—first because it requires more import data, and second, because the system of equations for calibrating the parameters of the AIDS function is over-determined.

The first step is relatively straightforward, and that is the calibration of the α^{dd} and α^{dm} parameters for equations (T-2') and (T-9"). The standard CES calibration formulas can be used assuming the user provides the substitution elasticity σ^b.

83 There is nothing inherent in the AIDS model which would prevent the share parameter from being negative or greater than 1.

84 Note that in the model code, all bilateral trade prices are expressed in relation to *WPE*, the bilateral price FOB. This is to save on memory.

The second step is also relatively easy. Given user-specified income elasticities, η^{AIDS}, the β parameters in equation (T-28) can be calibrated.[85] The β parameters can be calculated using the following expression:

$$\beta_{r',r,i} = s_{r',r,i}(\eta^{AIDS} - 1)$$

The third step requires use of an optimization package. The basic idea is to minimize a loss function subject to all of the conditions of the AIDS function, including the expressions for the own- and cross substitution elasticities. The loss function minimizes the sum of squared errors between the initial (user-supplied) matrix of elasticities and the resulting matrix of elasticities. The system of equations includes (T-28) and (T-29) to calibrate the α and α^0 parameters. One degree of freedom is lost because the α coefficients must sum to one:

$$\sum_{r'} \alpha_{r',r,i} = 1$$

Regularity also requires that the γ coefficients sum to zero and that they be symmetric[86]:

$$\sum_{r''} \gamma_{r'',r',r,i} = 0$$

$$\gamma_{r'',r',r,i} = \gamma_{r',r'',r,i}$$

The substitution elasticities are related to the other parameters and variables using the following expression:

$$\sigma^q_{r',r'',r,i} = 1 + \frac{\gamma_{r',r'',r,i}}{s_{r',r,i}s_{r'',r,i}} + \frac{\beta_{r',r,i}\beta_{r'',r,i}}{s_{r',r,i}s_{r'',r,i}} \ln(XA) \quad for \quad r' \neq r''$$

The substitution elasticities are given by σ^q.[87] The user inputs an initial matrix of substitution elasticities and the optimization routine takes into account the constraints listed above to minimize the following loss function:

$$\sum_{r'} \sum_{r'' \neq r'} \left(\sigma^q_{r',r'',r,i} - \sigma^{q,0}_{r',r'',r,i}\right)^2$$

where $\sigma^{q,0}$ is the initial input.[88] The optimization is performed for each (r,i) pair. In other words, the calibration is limited to a single bilateral flow at a time. It would be possible to do all calibration simultaneously, but this would change the loss function.

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85 The income elasticities may need to be adjusted to make sure the demand regularity conditions are satisfied.

86 The symmetry condition can be limited to the upper-triangular part of the γ matrix.

87 The matrix of elasticities can also be limited to the upper-triangular part because of symmetry. The own-elasticities, i.e. the diagonal components would have the same expression with addition of the following term: $-1/s_{r',r,i}$.

88 The loss function sum can be limited to the upper-triangular part of the elasticity matrix.

Annex F: Saving and Investment Behavior in LINKAGE

In the standard implementation of the model, household saving is derived from the ELES expenditure system and in essence is determined by residual, see equation (D-3). Investment is driven by saving. It is equated to domestic saving—both private and public—plus some level of foreign saving, which is exogenous in each period. In other words international capital flows are pre-determined. The next section shows how an investment schedule can be introduced into the model specification, which allows for at least partial international capital mobility. The last section introduces a savings function independent of the ELES. Either or both of these changes can be implemented independently of each other.

An investment function

The first modification to the standard model is to add an investment function. The following function makes investment (relative to GDP) a positive function of the price of a savings goods (to be discussed below), P^w, and the local rate-or-return, *TR*, with respect to the average world rate of return, *WRR*, and an accelerator term represented by the growth of GDP:

$$\text{(C-5')} \qquad InvSh_r = \lambda_r^I \left[\chi_r^{Inv} \left(\frac{P^w}{P} \right)^{\varepsilon_r^I} \left(\frac{TR_r}{WRR} \right)^{\varepsilon_r^{rr}} \left(1 + g_r^{GDP} \right)^{\varepsilon_r^Y} \right] + (1 - \lambda_r^I) I_{r,-1}$$

Thus investment in region r increases with the world price of savings (equivalent to the inverse of a global interest rate), the domestic rate of return, relative to an average world rate of return, and GDP growth. The equation is specified as a partial adjustment equation, allowing for different short- and long-term responses. The degree of adjustment is determined by the variable λ^I.

This equation replaces the original equation (C-5), and de-facto determines the investment level, with equation (C-7) determining the level of foreign capital flow, which equates desired domestic investment with available resources. (Note that equation (C-5') is specified over all regions whereas the original equation (C-5) was defined over (*r*-1) regions. The difference is that there is an additional variable, P^w.) The price, P^w, is a global equilibrium price, which clears the global savings/investment balance. In other words, the price P^w, ensures that equation (C-6) holds.

The interpretation (provided by Paul Armington, see Burniaux and van der Mensbrugghe (1994)), is that savings represents a claim on future consumption. Let that be represented by C_f, and the forward price of consumption is P^w. Nominal savings is then given by:

$$S^h = C_f P^w$$

The investment supply schedule, represents the supply of claims to future consumption. One could also write P^w as 1/(1+*r*), where *r* is the interest rate. Thus if *r* rises, i.e. P^w falls, savings will increase and investment will decrease. As shown in Figure E-1, savings is a negative (positive) function of the forward price of consumption (the interest rate), and the volume of investment is a positive (negative) function of the forward price of consumption (the interest rate).

The intra-temporal market for savings is an integrated global market, i.e. there is a single global interest rate which insures investment/savings balance at the global market. The capital markets are nonetheless segmented, i.e. the allocation of savings only affects capital flows at the margin. Installed capital is immobile and there is no mechanism in this specification which guarantees that rates of returns converge across regions.

A savings function

Incorporating a savings function in LINKAGE requires somewhat more work than adding an investment function due to the simultaneous determination of consumption and savings in the ELES framework. In the modified framework, the savings/consumption decisions are nested. At the top nest, consumers allocate their disposable income between household savings and aggregate consumption. At the second nest, aggregate consumption is allocated across goods and services using a sub-utility function in goods and services alone. This is modeled using an LES specification.[89] The simplest savings function is that savings is a constant proportion of income:

$$S^h = \mu^s Y$$

Recent empirical evidence suggests that savings behavior is somewhat more complex (see for example Masson et al (1998) and Loayza et al (2000)). For example, Loayza et al estimate a savings behavior as a function of 11 variables:

- Real per capita GDP
- Real growth rate of per capita GDP
- Real interest rate
- M2/GNP
- Terms of trade
- Urbanization ratio
- Old dependency ratio
- Young dependency ratio
- Government saving/GDPI
- Private credit flow/GDPI
- Inflation rate

They find that level and growth of per capita income have a significant and positive impact on savings. For example, a one percentage point increase in the growth rate leads to a 45 basis point (bp) increase in the savings rate in the short run, and nearly 1.1 percentage points in the long run. On the other hand, their results regarding financial variables suggest that financial liberalization does not necessarily lead to higher savings. The coefficient on the real interest rate is negative suggesting that the income effect outweighs the substitution effect. Financial deepening, as measured by the ratio of M2 to GNP, has an insignificant coefficient, and increases in private credit depress savings. The three demographic variables all have significantly negative coefficients. The results on the dependency ratio variables are consistent with the life-cycle hypothesis, and the coefficient on urbanization could reflect greater diversification possibilities in urban areas and thus a reduction in precautionary savings. The dependency ratio variables could interact with the growth variables, with the latter raising savings, but tempered by growing youth and elderly populations. Their final finding is that macroeconomic uncertainty (as captured by the inflation rate) is positively correlated with savings, i.e. greater inflation leads to higher savings. They suggest that this is due to an increase in precautionary savings.

In the revised specification of savings behavior in the LINKAGE Model, we have focused our attention on four channels—GDP growth, young and old and dependency ratios and the real interest rate. The behavior of savings has the following functional form:

[89] We have also experimented with the AIDADS functional form. The latter is a generalization of the LES but the marginal shares (μ) are no longer constant, but a function of other variables in the model. This allows for more interesting (and plausible) substitution relations across goods (include gross complementarity). The specification works as expected but there are yet to be resolved issues regarding the calibration of the AIDADS parameters.

$$\mu^s = \frac{S^h}{Y} = \alpha^s + \beta^s \mu^s_{-1} + \beta^g \ln\left(\frac{GDP/Pop}{GDP_{-1}/Pop_{-1}}\right) + \beta^y\left(\frac{Pop^{l15}}{Pop^{g15}}\right) + \beta^e\left(\frac{Pop^{g65}}{Pop^{g15}}\right) + \beta^r\left(\frac{TR}{CPI}\right)$$

This functional form expresses the savings rate as a linear function of four factors, excluding the persistence term (or lagged savings rate). The coefficients represent short-term impacts, with the long-term impacts equal to the relevant coefficient divided by the term ($1-\beta^s$). The first factor is the growth of per capita GDP (as measured by the first difference in logs). The second is the youth dependency ratio where Pop^{l15} represents the number of persons under 15 and Pop^{g15} is a proxy for the working age population and represents the number of persons aged between 15 and 65. The third term is the elderly dependency ratio where Pop^{g65} is the number of persons greater than 65. The final term is the average real rate of return of capital.

Table E-1 provides the values for these parameters from the Loayza et al paper. The estimates reflect different sample definitions—the full sample, low- and middle-income countries alone and only OECD countries. There are significant differences in the estimates across samples. Generally, the coefficients are higher for developing countries than for OECD countries (in the short-term). Thus the growth impact is some 50 percent higher for developing countries, and the dependency ratios are some 4 to 6 times as important. The higher long-term multiplier for OECD countries leads to some convergence in the long-term impact, and in fact, the growth factor appears greater in OECD countries overtime. We will often assume that the coefficient on the real interest rate factor is zero. In theory, there is ambiguity about the sign of this factor. In the estimates, the sign flips between the full and segmented samples, and have no statistical significance in the case of the latter.

Table F-1: Estimated impacts of various variables on the private saving rate[a]

	Persistence term	*Growth coefficient*	*Youth dependency ratio*	*Elderly dependency ratio*	*Real interest rate*
Full sample	0.587*	0.450*	-0.299*	-0.655*	-0.253*
Sample of low and middle-income countries	0.476*	0.425*	-0.279*	-1.370*	0.002
Sample of OECD countries	0.674*	0.285*	-0.068	-0.218	0.020
Long-term impacts[b]					
Full sample	2.4	1.090	-0.724	-1.586	-0.613
Sample of low and middle-income countries	1.9	0.811	-0.532	-2.615	0.004
Sample of OECD countries	3.1	0.874	-0.209	-0.669	0.061

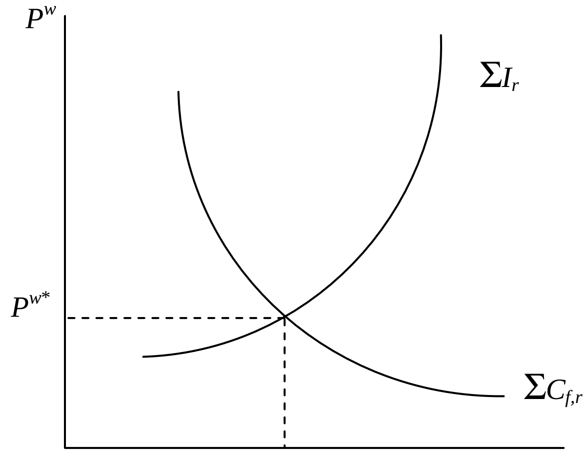
Notes: a. Asterisks represent coefficients with t-statistics greater than 2.
b. The numbers in the persistence column refer to the long-term multiplier.
Source: Loayza et al (2000), Table 5, p. 176.

The standard model requires are few other modifications. Equation (Y-8) requires a new definition of the LES expenditures. The LES allocation function is limited to disposable income net of savings, equation (Y-8'). Similarly, the level of saving is not determined residually, but as a share of disposable income, where the share variable is driven by the equation above.

(Y-8') $Y^c_h = Yd_h - S^h_h$

(D-3') $S^h_h = \mu^s_h Yd_h$

Figure F- 1: Global market for claims to future consumption



C_f: Savings volume
I: Investment volume

Annex G: AIDADS and its Implementation in LINKAGE

The standard demand system (at the top nest) is an ELES, or an LES if the savings function alternative is used. The LES has certain properties that have been criticized in the past and several alternatives have been developed based on more flexible functional forms.[90] Two are relatively well-known: the almost ideal demand system (AIDS)[91] and some version of the CRESH function based on implicit additivity.[92] A third, AIDADS, has received more attention recently[93] and was initially proposed by Rimmer and Powell.[94] It is a relatively natural extension to the LES function, the latter being a special case of the AIDADS function. The insight of Rimmer and Powell was to allow the marginal propensity term of the LES to be a function of other variables, rather than be a constant as in the LES. This allows for more complex demand behavior, as well as providing better validation for observed changes in consumption patterns.

Basic formulation

AIDADS starts with the implicitly additive utility function given by:[95]

$$\text{(G-1)} \qquad \sum_k U_k \left(XC_k, u \right) \equiv 1$$

Assume the following functional form for the utility function:

$$\text{(G-2)} \qquad U_k = \mu_k \ln\left(\frac{XC_k - \theta_k}{Ae^u} \right)$$

where

$$\text{(G-3)} \qquad \mu_k = \frac{\alpha_k + \beta_k G(u)}{1 + G(u)}$$

with the restrictions

$$\sum_k \alpha_k = \sum_k \beta_k = 1$$

$$0 \le \alpha_k \le 1$$

$$0 \le \beta_k \le 1$$

$$\theta_k < XC_k$$

Cost minimization implies the following:

$$\min \sum_k PC_k XC_k$$

subject to

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90 Another alternative, developed in the standard LINKAGE framework, is a nested structure, which allows for more complex substitution and complementarity relations. This can be rather data intensive and perhaps less transparent than a direct approach using a more flexible functional form.

91 Thoroughly elaborated in Deaton and Muellbauer (1980).

92 First elaborated by Hanoch (1975) and popularized in its Constant Difference of Elasticities (CDE) form in the GTAP model. For the latter see Hertel et al. (1991) and Hertel (1997).

93 Hertel and co-authors have been using AIDADS with the GTAP model. See for example Yu et al. (2002).

94 See Rimmer and Powell (1992a), Rimmer and Powell (1992b) and Rimmer and Powell (1996).

95 The notation used below is the same as in the model description. The regional and household subscripts have been dropped for expositional purposes.

(G-4) $$\sum_k \mu_k \ln\left(\frac{XC_k - \theta_k}{Ae^u}\right) \equiv 1$$

The first order conditions lead to:

(G-5) $$\lambda \frac{\partial U_k}{\partial XC_k} = PC_k = \lambda \frac{\mu_k}{XC_k - \theta_k} \Rightarrow \lambda\mu_k = PC_k XC_k - PC_k\theta_k$$

summing over *k* and using the fact that the μ_k sum to unity implies:

(G-6) $$\lambda = \sum_k PC_k XC_k - \sum_k PC_k\theta_k = Y^c - \sum_k PC_k\theta_k = Y^*$$

where Y^c is aggregate expenditure, and Y^*, sometimes referred to as supernumerary income, is residual expenditure after subtracting total expenditure on the so-called subsistence minima, θ.

Re-inserting equation (G-6) into (G-5) yields the consumer demand equations:

(G-7) $$XC_k = \theta_k + \frac{\mu_k}{PC_k} Y^* = \theta_k + \frac{\mu_k}{PC_k}\left[Y^c - \sum_{k'} PC_{k'}\theta_{k'}\right]$$

Equation (G-7) is virtually identical to equation (D-2) in the main text.[96] Similar to the linear expenditure system (LES), demand is the sum of two components—a subsistence minimum, θ, and a share of supernumerary income. Unlike the LES, the share parameter, μ, is not constant, but depends on the level of utility itself. AIDADS collapses to the LES if each β parameter is equal to the corresponding α parameter, with the ensuing function of utility, *G*(*u*), dropping from equation (G-3).

Elasticities

This section develops the main expressions for the income and price elasticities. These formulas will be needed to calibrate the initial parameters of the AIDADS function.

Income elasticity

To derive further properties of AIDADS requires specifying a functional form for *G*(*u*). Rimmer and Powell (1996) propose the following:

(G-8) $$G(u) = e^u$$

The first step is to calculate the marginal budget share, ρ, defined as:

$$\rho_k = PC_k \frac{\partial XC_k}{\partial Y^c}$$

The following expression can be derived from equation (G-7):

$$\frac{\partial XC_k}{\partial Y^c} = \frac{Y^*}{PC_k}\frac{\partial \mu_k}{\partial Y^c} + \frac{\mu_k}{PC_k}\frac{\partial Y^*}{\partial Y^c} = \frac{Y^*}{PC_k}\frac{\partial \mu_k}{\partial u}\frac{\partial u}{\partial Y^c} + \frac{\mu_k}{PC_k}$$

Thus:

(G-9) $$\rho_k = \mu_k + Y^* \frac{\partial \mu_k}{\partial u}\frac{\partial u}{\partial Y^c}$$

Expression (G-9) can be expanded in two steps—first evaluating the partial derivative of the share variable, μ, with respect to utility, and then the more difficult calculation of the partial derivative of *u* with respect to income. The share formula is:

[96] The subsistence parameters are not adjusted by population levels in the Annex equations.

$$\mu_k = \frac{\alpha_k + \beta_k e^u}{1 + e^u}$$

Its derivative is:

(G-10) $$\frac{\partial \mu_k}{\partial u} = \frac{\left(1+e^u\right)\left(\beta_k e^u\right) - \left(\alpha_k + \beta_k e^u\right)e^u}{(1+e^u)^2} = \frac{e^u(\beta_k - \alpha_k)}{(1+e^u)^2}$$

Utility and income are combined in implicit form and thus we will invoke the implicit function theorem to calculate the partial derivative of *u* with respect to *Y*. First, insert equation (G-7) into equation (G-4):

$$\sum_k \mu_k \ln\left(\frac{XC_k - \theta_k}{Ae^u}\right) = \sum_k \mu_k \ln\left(\frac{\mu_k Y^*}{Ae^u PC_k}\right) = 1$$

Expanding the latter expression yields:

(G-11) $$f\left(u, Y^c\right) = \sum_k \mu_k \ln\left(\frac{\mu_k}{PC_k}\right) + \ln\left(Y^*\right) - \ln(A) - u = 1$$

which provides the implicit relation between Y^c and *u*. The implicit function theorem states the following:

(G-12) $$\frac{\partial u}{\partial Y^c} = -\frac{\partial f}{\partial Y^c}\left[\frac{\partial f}{\partial u}\right]^{-1}$$

The partial derivative of *f* with respect to Y^c is simply:

(G-13) $$\frac{\partial f}{\partial Y^c} = \frac{1}{Y^*}$$

The partial derivative of *f* with respect to *u* is:

(G-14)
$$\begin{aligned}
\frac{\partial f}{\partial u} &= -1 + \sum_k \left[\frac{\partial \mu_k}{\partial u}\ln\left(\frac{\mu_k}{PC_k}\right) + \mu_k \frac{PC_k}{\mu_k} PC_k \frac{\partial \mu_k}{\partial u}\right] \\
&= -1 + \frac{e^u}{(1+e^u)^2}\sum_k \left[\left(\ln\left(\frac{\mu_k}{PC_k}\right) + 1\right)(\beta_k - \alpha_k)\right] \\
&= \frac{e^u}{(1+e^u)^2}\left[\sum_k (\beta_k - \alpha_k)\ln\left(XC_k - \theta_k\right) - \frac{(1+e^u)^2}{e^u}\right] \\
&= \frac{e^u}{(1+e^u)^2}\Omega^{-1}
\end{aligned}$$

where

(G-15) $$\Omega = \left[\sum_k (\beta_k - \alpha_k)\ln\left(XC_k - \theta_k\right) - \frac{\left(1+e^u\right)^2}{e^u}\right]^{-1}$$

The second line uses equation (G-10). In the third line, equation (G-7) substitutes for the expression in the logarithm, and the adding up constraint allows for the deletion of non-indexed variables. Substituting (G-13) and (G-14) into (G-12) yields:

(G-16) $$\frac{\partial u}{\partial Y^c} = -\frac{\Omega}{Y^*}\frac{\left(1+e^u\right)^2}{e^u}$$

Substituting (G-10) and (G-16) into (G-9) yields:

$$\rho_k = \mu_k - (\beta_k - \alpha_k)\,\Omega$$

The income elasticities are derived from the following expression:

(G-18) $$\eta_k = \frac{\partial XC_k}{\partial Y}\frac{Y^c}{XC_k} = \frac{\partial XC_k}{\partial Y}\frac{Y^c}{XC_k}\frac{PC_k}{PC_k} = \frac{\rho_k}{s_k}$$

where s_k is the average budget share:

(G-19) $$s_k = \frac{PC_k XC_k}{Y} = \frac{PC_k\theta_k}{Y} + \mu_k\frac{Y^*}{Y}$$

It can also be written as:

(G-20) $$s_k = \mu_k + \left(\frac{PC_k\theta_k - \mu_k\sum_{k'} PC_k{}'\theta_{k'}}{Y}\right)$$

Thus the income elasticity, η, is equal to the ratio of the marginal budget share, ρ, and the average budget share, *s*.

Price elasticity

The matrix of substitution elasticities is identical to the expression for the LES and has the form:

(G-21) $$\sigma_{kk'} = [\mu_{k'} - \delta_{kk'}]\frac{\mu_k Y^*}{s_k s_{k'} Y^c}$$

where

$$\delta_{kk'} = \begin{cases} 1 & k = k' \\ 0 & k \neq k' \end{cases}$$

It is clear that the matrix is symmetric. The matrix of substitution elasticities is also equal to:

(G-22) $$\sigma_{kk'} = [\mu_{k'} - \delta_{kk'}]\frac{\mu_k Y^*}{s_k s_{k'} Y^c} = \frac{(XC_k - \theta_k)}{XC_k}\frac{(XC_{k'} - \theta_{k'})}{XC_{k'}}\frac{Y^c}{Y^*} - \frac{\delta_{kk'}}{s_{k'}}\frac{(XC_k - \theta_k)}{XC_k}$$

The compensated demand elasticities derive from the following:

(G-23) $$\xi_{kk'} = s_{k'}\sigma_{kk'} = [\mu_{k'} - \delta_{kk'}]\frac{\mu_k Y^*}{s_k Y^c}$$

Finally, the matrix of uncompensated demand elasticities is given by:

(G-24) $$\varepsilon_{kk'} = \xi_{kk'} - s_{k'}\eta_k = [\mu_{k'} - \delta_{kk'}]\frac{\mu_k Y^*}{s_{k'} Y} - s_{k'}\eta_k$$

The uncompensated demand elasticities can also be written as:

(G-24') $$\varepsilon_{kk'} = -\frac{\mu_k}{s_k Y^c}\left[PC_{k'}\theta_{k'} + \delta_{kk'}Y^*\right] + \frac{s_{k'}}{s_k}(\beta_k - \alpha_k)\Omega$$

The first term on the right-hand side is always negative. The second term differs from the LES expression for the uncompensated demand elasticities.[97] We can see from expression (G-24') that the AIDADS specification allows for both gross complementarity and substitution. As well, it allows for luxury goods,

[97] Recall that for the LES, the α and β terms are equal and thus the second term drops.

i.e. positive own-price demand elasticities should the second term be positive and greater than the first term.

Implementation

Implementation of AIDADS is somewhat more complicated than the LES since the marginal propensity to consume out of supernumerary income is endogenous, and utility is defined implicitly. The following four equations are needed for model implementation:

(D-1) $$Y_h^* = Y_h^c - \sum_k PC_{k,h}\theta_{k,h}$$

(D-2) $$XC_{k,h} = \theta_{k,h} + \frac{\mu_{k,h}}{PC_{k,h}} Y_h^*$$

(D-2') $$\mu_{k,h} = \frac{\alpha_{k,h} + \beta_{k,h} e^{u_h}}{1 + e^{u_h}}$$

(D-2") $$u_h = \sum_k \mu_{k,h} \ln\left(XC_{k,h} - \theta_{k,h}\right) - 1 - \ln\left(A_h\right)$$

Equations (D-1) and (D-2) are identical to their LES (ELES) counterparts with the exception of the population adjustment on the subsistence minima. Equation (D-2') determines the level of the marginal propensity to consume out of supernumerary income, μ, which is a constant in the case of the LES (ELES). It requires however the calculation of the utility level, *u*, which is defined in equation (D-2").

Calibration

Calibration requires more information then the LES. Where the LES has 2*n* parameters to calibrate (subject to consistency constraints), AIDADS has 3*n* parameters (less the consistency requirements)—α, β and θ. The calibration system includes equations (D-1), (D-2), (D-2') and (D-2") which have 2+2*n* endogenous variables (Y^*, θ, μ, and A). There are no equations for calibrating the α and β parameters. If we have knowledge of the income elasticities, we can add the following equations:

(G-25) $$\Psi = \frac{1}{\Omega} = \left[\sum_k \left(\beta_k - \alpha_k\right) \ln\left(XC_k - \theta_k\right) - \frac{\left(1 + e^u\right)^2}{e^u}\right]$$

(G-26) $$\eta_k = \frac{\rho_k}{s_k} = \frac{\mu_k - \left(\beta_k - \alpha_k\right)\Omega}{s_k} = \frac{\mu_k}{s_k} - \frac{\left(\beta_k - \alpha_k\right)}{s_k \Psi}$$

There are an additional 1+*n* equations, solving for Ψ and α. There is need for an additional *n* equations. Assuming we have knowledge of at least *n* price elasticities, for example the own-price elasticities, we can add the following equation:

(G-27) $$\varepsilon_{kk} = -\frac{\mu_k}{s_k Y}\left[P_k \theta_k + Y^*\right] + \left(\beta_k - \alpha_k\right)\Omega$$

The α and β parameters are not independent, the following restrictions must hold:

(G-28) $$\sum_k \alpha_k = 1$$

(G-29) $$\sum_k \beta_k = 1$$

The system is under-determined, there are 5+4*n* equations and 3+4*n* variables. One solution, is to make the own-price elasticities endogenous. In this case, we are adding *n* variables, but then the system is over-determined. We can minimize a loss function with respect to the price elasticities:

(G-30) $$L = \sum_k \left(\varepsilon_k - \varepsilon_k^0\right)^2$$

where ε^0 represents an initial guess of the own-price elasticities and the calibration algorithm will calculate the endogenous ε in order to minimize the loss function, subject to constraints (G-25)-(G-29) and the model equations (D-1), (D-2), (D-2') and (D-2").[98] The exogenous parameters in the calibration procedure include *PC*, *XC*, *s*, *Y*, η, ε^0 and *u*.

98 In its current implementation, the calibration procedure takes the income elasticities and own-price elasticities as they are derived from the calibration of the LES demand system. It would be relatively straightforward to input an independent set of own-price elasticities.

Annex H: The GTAP dataset, Version 6

Dimensions

The current version of the LINKAGE Model is based on Version 6 (pre-release 4) of the GTAP dataset. An aggregation facility is used to aggregate the GTAP data and format it for use for the LINKAGE Model. Version 6 (pre-released in September, 2004) has a 2001 base year. Functionally, it is identical to release 5.

The new dataset includes 87 countries/regions, an increase from 45 in Release 4.0, 66 in release 5.0, and 78 in release 5.4 (Table H-1). The changes to the regions include:

Rest of Oceania	A regional composite of many of the Pacific Islands previously in ROW.
Rest of East Asia	Three countries previously in ROW.
Rest of Southeast Asia	A handful of countries previously in ROW.
Rest of North America	Three economies previously in ROW.
Central America	The countries of Central America previously in XCM.
Rest of Free Trade Area of Americas	Countries of the Caribbean currently negotiating the FTAA and previously in XCM.
Rest of the Caribbean	All other economies of the Caribbean previously in XCM.
Rest of Europe	A handful of small countries and territories previously in ROW.
South Africa	Extracted from the previous XSF region.
Rest of Southern African Development Community	Other countries of SADC some previously in XSF, and others in XSS.

The new GTAP data set has three new countries—South Africa, Tunisia and Madagascar, and 9 new composite regions. Several composite regions have been dropped including Central America and the Caribbean, Rest of Southern Africa and Rest of the World. The countries/territories in the old Rest of the World Region have been allocated to other regions of the new regional configuration.

The sectoral dimension in Version 6 is the same is in version 5—there are 57 sectors (Table F-2).

GTAP and LINKAGE

This section describes the relation between the GTAP database, and the data for the LINKAGE Model. All data in GTAP are in value terms (2001 dollars). The data for the LINKAGE model is likewise carried over in value terms, therefore in the calibration routine (*cal.gms*), the relevant volumes (e.g. *XAp*), are divided by their corresponding price (often 1). All the tax variables represent the income generated by the taxes, not the tax rates. They need to be calibrated by appropriate formula to generate the tax rates.

In the GTAP database the last index is always the region. For the LINKAGE model the indices are reversed for all variables (i.e. the regional index becomes the first index), except for the trade flow variables. We maintain that the trade matrices should be read as exports along the rows, and imports down the columns, but the sectoral index comes last, not first.

Equations (H-1)-(H-4) determine the domestic and import input-output tables (in value) at market prices and their corresponding (cell-specific) domestic and import taxes. Note that *PDTax* and *PMTax* represent revenues, not tax rates. The standard version of the model does not use the domestic and import input/output matrix. Instead they are aggregated together to form an Armington level input-output table, as are the tax matrices.[99]

[99] Thus we define:

(H-1) $XDp_{r,i,j} = VDFM_{i,j,r}$

(H-2) $PDTax_{r,i,j} = VDFA_{i,j,r} - VDFM_{i,j,r}$

(H-3) $XMp_{r,i,j} = VIFM_{i,j,r}$

(H-4) $PMTax_{r,i,j} = VIFA_{i,j,r} - VIFM_{i,j,r}$

Equations (H-5)-(H-9) determine the value added vectors. There are five factors in the GTAP database. The aggregation facility permits the collapse of these five into just two (capital and labor) for the LINKAGE model. It also allows for the creation of a third labor category, highly skilled labor, whose remuneration is extracted from payments to skilled labor.

(H-5) $T^d_{r,i} = EVFA_{Land,i,r}$

(H-6) $L^d_{r,UnSkLab,i} = EVFA_{UnskLab,i,r}$

(H-7) $L^d_{r,SkLab,i} = EVFA_{SkLab,i,r}$

(H-8) $K^d_{r.i} = EVFA_{Capital,i,r}$

(H-9) $F^d_{r,i} = EVFA_{NatlRes,i,r}$

Equations (H-10)-(H-21) describe the domestic and import final demand vectors (in value) at market prices and their corresponding domestic and import taxes. Similar to intermediate demand these matrices are added together to compose their respective Armington level aggregates. Equations (H-10)-(H-13) determine household final demand. Equations (H-14)-(H-17) determine public expenditures on goods and services. Equations (H-18)-(H-21) determine investment expenditures on goods and services. Note that the latter is extracted from the column labeled 'CGD' of GTAP's input-output table.

(H-10) $XDc_{r,i} = VDPM_{i,r}$

(H-11) $CDTax_{r,i} = VDPA_{i,r} - VDPM_{i,r}$

(H-12) $XMc_{r,i} = VIPM_{i,r}$

(H-13) $CMTax_{r,i} = VIPA_{i,r} - VIPM_{i,r}$

$$XAp_{r,i,j} = XDp_{r,i,j} + XMp_{r,i,j}$$

$$PATax_{r,i,j} = PDTax_{r,i,j} + PMTax_{r,i,j}$$

where *XAp* is the Armington input-output matrix, and *PATax* is the matrix of indirect taxes on intermediate Armington purchases.

(H-14) $XDg_{r,i} = VDGM_{i,r}$

(H-15) $GDTax_{r,i} = VDGA_{i,r} - VDGM_{i,r}$

(H-16) $XMg_{r,i} = VIGM_{i,r}$

(H-17) $GMTax_{r,i} = VIGA_{i,r} - VIGM_{i,r}$

(H-18) $XDi_{r,i} = VDFM_{i,CGD,r}$

(H-19) $IDTax_{r,i,j} = VDFA_{i,CGD,r} - VDFM_{i,CGD,r}$

(H-20) $XMi_{r,i} = VIFM_{i,CGD,r}$

(H-21) $IMTax_{r,i,j} = VIFA_{i,CGD,r} - VIFM_{i,CGD,r}$

Equations (H-22)-(H-25) determine the trade matrix variables. *WTF* is the world trade matrix in world prices (assumed to be 1), FOB, i.e. before the inclusion of the trade and transport margins, but including any export tax imposed by the country of origin. Equation (H-23) describes the value of the export tax. Equation (H-24) determines the cost of the trade and transport margins. And, Equation (H-25) describes the value of the tariffs (which should logically be read down the columns, since it is the importing country that imposes the tariff).

(H-22) $WTF_{r,r',i} = VXWD_{i,r,r'}$

(H-23) $ETax_{r,r',i} = VXWD_{i,r,r'} - VXMD_{i,r,r'}$

(H-24) $TMarg_{r,r',i} = VIWS_{i,r,r'} - VXWD_{i,r,r'}$

(H-25) $Tar_{r,r',i} = VIMS_{i,r,r'} - VIWS_{i,r,r'}$

Equation (H-26) describes the supply of international trade and transport services by each country (N.B. that this enters the current account balance as an export.)

(H-26) $XMarg_{r,i} = VST_{i,r}$

Equations (H-27) and (H-28) complete the determination of the flow of income in each domestic economy, representing respectively, household saving (i.e. consolidated savings of the household, government, and business sectors), and the depreciation allowance.

(H-27) $Saveh_r = SAVE_r$

(H-28) $deprY_r = VDEP_r$

Equation (H-29) defines the value of the base year capital stock.

(H-29) $KStock_r = VKB_r$

The latest version of the model also uses two additional tax instruments—taxes/subsidies on production and factor inputs. The former is used as a check since it can be calculated from all other data by residual. Equation (H-30) represents the production tax/subsidy.[100] Equations (H-31)-(H-35) represent the taxes on the factors of production.

[100] The LINKAGE model assumes these are taxes if they are positive, hence the change in sign.

(H-30) $TaxXP_{r,i} = -OSEP_{i,r}$

(H-31) $TaxFact_{r,Land,i} = -FBEPF_{Land,i,r}$

(H-32) $TaxFact_{r,UnSkLab,i} = -FBEPF_{UnskLab,i,r}$

(H-33) $TaxFact_{r,SkLab,i} = -FBEPF_{SkLab,i,r}$

(H-34) $TaxFact_{r,Capital,i} = -FBEPF_{Capital,i,r}$

(H-35) $TaxFact_{r,NatlRes,i} = -FBEPF_{NatlRes,i,r}$

Table H-1: Regional Concordance

1	AUS	**Australia**
2	NZL	**New Zealand**
3	XOC	**Rest of Oceania** *American Samoa* (asm), *Cook Islands* (cok), *Fiji* (fji), *French Polynesia* (pyf), *Guam* (gum), *Kiribati* (kir), *Marshall Islands* (mhl), *Federated States of Micronesia* (fsm), *Nauru* (nau), *New Caledonia* (ncl), *Norfolk Island* (nfk), *Northern Mariana Islands* (mnp), *Niue* (niu), *Palau* (plw), *Papua New Guinea* (png), *Samoa* (wsm), *Solomon Islands* (slb), *Tokelau* (tkl), *Tonga* (ton), *Tuvalu* (tuv), *Vanuatu* (vut), *Wallis and Futura Islands* (wlf)
4	CHN	**China**
5	HKG	**Hong Kong (China)**
6	JPN	**Japan**
7	KOR	**Republic of Korea**
8	TWN	**Taiwan (China)**
9	XEA	**Rest of East Asia** *Macao* (mac), *Mongolia* (mng), *North Korea* (prk)
10	IDN	**Indonesia**
11	MYS	**Malaysia**
12	PHL	**Philippines**
13	SGP	**Singapore**
14	THA	**Thailand**
15	VNM	**Vietnam**
16	XSE	**Rest of Southeast Asia** *Brunei Darussalam* (brn), *Cambodia* (khm), *Lao PDR* (lao), *Myanmar* (mmr),*Timor-Leste* (tmp)
17	BGD	**Bangladesh**
18	IND	**India**
19	LKA	**Sri Lanka**
20	XSA	**Rest of South Asia** *Afghanistan* (afg), *Bhutan* (btn), *Maldives* (mdv), *Nepal* (npl), *Pakistan* (pak)
21	CAN	**Canada**
22	USA	**United States**
23	MEX	**Mexico**
24	XNA	**Rest of North America** *Bermuda* (bmu), *Greenland* (grl), *Saint Pierre et Miquelon* (spm)
25	COL	**Colombia**
26	PER	**Peru**
27	VEN	**Venezuela, Republica Bolivariana de**
28	XAP	**Rest of Andean Pact** *Bolivia* (bol), *Ecuador* (ecu)
29	ARG	**Argentina**
30	BRA	**Brazil**
31	CHL	**Chile**
32	URY	**Uruguay**
33	XSM	**Rest of South America** *Falkland Islands* (flk), *French Guiana* (guf), *Guyana* (guy), *Paraguay* (pry), *Suriname* (sur)
34	XCA	**Central America** *Belize* (blz), *Costa Rica* (cri),*El Salvador* (slv), *Guatemala* (gtm), *Honduras* (hnd), *Nicaragua* (nic), *Panama* (pan)
35	XFA	**Rest of Free Trade Area of Americas** *Antigua & Barbuda* (atg), *Bahamas* (bhs), *Barbados* (brb), *Dominica* (dma), *Dominican Republic* (dom), *Grenada* (grd), *Haiti* (hti), *Jamaica* (jam), *Puerto Rico* (pri), *Saint. Kitts & Nevis* (kna), *Saint Lucia* (lca), , *Saint. Vincent and the Grenadines* (vct), *Trinidad & Tobago* (tto), *United States Virgin Islands* (vir)
36	XCB	**Rest of the Caribbean** *Anguilla* (aia), *Aruba* (abw), *Cayman Islands* (cym), *Cuba* (cub), *Guadeloupe* (glp), *Martinique* (mtq), *Montserrat* (msr), *Netherlands Antilles* (ant), *Turks and Caicos Islands* (tca), *British Virgin Islands*(vgb)
37	AUT	**Austria**
38	BEL	**Belgium**
39	DNK	**Denmark**
40	FIN	**Finland**
41	FRA	**France**

42	DEU	**Germany**
43	GBR	**United Kingdom**
44	GRC	**Greece**
45	IRL	**Ireland**
46	ITA	**Italy**
47	LUX	**Luxembourg**
48	NLD	**Netherlands**
49	PRT	**Portugal**
50	ESP	**Spain**
51	SWE	**Sweden**
52	CHE	**Switzerland**
53	XEF	**Rest of European Free Trade Area (EFTA)** *Iceland* (isl), *Liechtenstein* (lei), *Norway* (nor)
54	XER	**Rest of Europe** *Andorra* (and), , *Bosnia and Herzegovina* (bih), *Faroe Islands* (fro), *Gibraltar* (gib), *Macedonia* (mkd, *former Yugoslav Republic of*), *Monaco* (mco), *San Marino* (smr), *Serbia and Montenegro* (scg)
55	ALB	**Albania**
56	BGR	**Bulgaria**
57	HRV	**Croatia**
58	CYP	**Cyprus**
59	CZE	**Czech Republic**
60	HUN	**Hungary**
61	MLT	**Malta**
62	POL	**Poland**
63	ROM	**Romania**
64	SVK	**Slovakia**
65	SVN	**Slovenia**
66	EST	**Estonia**
67	LVA	**Latvia**
68	LTU	**Lithuania**
69	RUS	**Russian Federation**
70	XSU	**Rest of the former Soviet Union** *Armenia* (arm), *Azerbaijan* (aze), *Belarus* (blr), *Georgia* (geo), *Kazakhstan* (kaz), *Kyrgyz Republic* (kgz), *Moldova* (mda), *Tajikistan* (tjk), *Turkmenistan* (tkm), *Ukraine* (ukr), *Uzbekistan* (uzb)
71	TUR	**Turkey**
72	XME	**Rest of the Middle East** *Bahrain* (bhr), *Iran* (irn), *Iraq* (irq), *Israel* (isr), *Jordan* (jor), *Kuwait* (kwt), *Lebanon* (lbn), *West Bank and Gaza* (pse), *Oman* (omn), *Qatar* (qat), *Saudi Arabia* (sau), *Syrian Arab Republic* (syr), *United Arab Emirates* (are), *Republic of Yemen* (yem)
73	MAR	**Morocco**
74	TUN	**Tunisia**
75	XNF	**Rest of North Africa** *Algeria* (dza), *Egypt* (egy), *Libyan Arab Jamahiriya* (lby)
76	BWA	**Botswana**
77	ZAF	**South Africa**
78	XSC	**Rest of South African Customs Union** *Lesotho (lso), Namibia (nam),Swaziland (swz)*
79	MWI	**Malawi**
80	MOZ	**Mozambique**
81	TZA	**Tanzania**
82	ZMB	**Zambia**
83	ZWE	**Zimbabwe**
84	XSD	**Rest of Southern African Development Community** *Angola* (ago), *Democratic Republic of the Congo* (cod, *formerly Zaïre*), *Mauritius* (mus), *Seychelles Islands* (syc)
85	MDG	**Madagascar**
86	UGA	**Uganda**
87	XSS	**Rest of Sub Saharan Africa** *Benin* (ben), *Burkina Faso* (bfa), *Burundi* (bdi), *Cameroon* (cmr), *Cape Verde* (cpv), *Central African Republic* (caf), *Chad* (tcd), *Comoros* (com), *Congo* (cog), *Côte d'Ivoire* (civ), *Djibouti* (dji), *Equatorial Guinea* (gnq), *Eritrea* (eri), *Ethiopia* (eth),

Gabon (gab), *Gambia, The* (gmb), *Ghana* (gha), *Guinea* (gin), *Guinea-Bissau* (gnb), *Kenya* (ken), *Liberia* (lbr), *Mali* (mli), *Mauritania* (mrt), *Mayotte* (myt), *Niger* (ner), *Nigeria* (nga), *Rwanda* (rwa), *Saint Helena* (shn), *Sao Tome & Principe* (stp), *Senegal* (sen), *Sierra Leone* (sle), *Somalia* (som), *Sudan* (sdn), *Togo* (tgo)

Table H-2: Sectoral Concordance

1	PDR	**Paddy rice**
2	WHT	**Wheat**
3	GRO	**Cereal grains, n.e.s.**
4	V_F	**Vegetables and fruits**
5	OSD	**Oil seeds**
6	C_B	**Sugar cane and sugar beet**
7	PFB	**Plant-based fibers**
8	OCR	**Crops, n.e.s.**
9	CTL	**Bovine cattle, sheep and goats, horses**
10	OAP	**Animal products n.e.s.**
11	RMK	**Raw milk**
12	WOL	**Wool, silk-worm cocoons**
13	FRS	**Forestry**
14	FSH	**Fishing**
15	COA	**Coal**
16	OIL	**Oil**
17	GAS	**Gas**
18	OMN	**Minerals n.e.s.**
19	CMT	**Bovine cattle, sheep and goat, horse meat products**
20	OMT	**Meat products n.e.s.**
21	VOL	**Vegetable oils and fats**
22	MIL	**Dairy products**
23	PCR	**Processed rice**
24	SGR	**Sugar**
25	OFD	**Food products n.e.s.**
26	B_T	**Beverages and tobacco products**
27	TEX	**Textiles**
28	WAP	**Wearing apparel**
29	LEA	**Leather products**
30	LUM	**Wood products**
31	PPP	**Paper products, publishing**
32	P_C	**Petroleum, coal products**
33	CRP	**Chemical, rubber, plastic products**
34	NMM	**Mineral products n.e.s.**
35	I_S	**Ferrous metals**
36	NFM	**Metals n.e.s.**
37	FMP	**Metal products**
38	MVH	**Motor vehicles and parts**
39	OTN	**Transport equipment n.e.s.**
40	ELE	**Electronic equipment**
41	OME	**Machinery and equipment n.e.s.**
42	OMF	**Manufactures n.e.s.**
43	ELY	**Electricity**
44	GDT	**Gas manufacture, distribution**
45	WTR	**Water**
46	CNS	**Construction**
47	TRD	**Trade**
48	OTP	**Transport n.e.s.**
49	WTP	**Sea transport**
50	ATP	**Air transport**
51	CMN	**Communication**
52	OFI	**Financial services n.e.s.**
53	ISR	**Insurance**
54	OBS	**Business services n.e.s.**
55	ROS	**Recreation and other services**
56	OSG	**Public administration and defense, education, health services**
57	DWE	**Dwellings**

References

Agénor, Pierre-Richard, Alejandro Izquierdo and Hippolyte Fofack (2002a), "IMMPA: A Quantitative Macroeconomic Framework for the Analysis of Poverty Reduction Strategies," *mimeo*, The World Bank, June 11, 2002.

Agénor, Pierre-Richard, Reynaldo Fernandes, Eduardo Haddad and Dominique van der Mensbrugghe (2002b), "Analyzing the Impact of Adjustment Policies on the Poor: An IMMPA Framework for Brazil," *mimeo*, The World Bank, December 3, 2002.

Armington, Paul (1969), "A Theory of Demand for Products Distinguished by Place of Production," *IMF Staff Papers*, Vol. 16, pp. 159-178.

Ballard, Charles L., Don Fullerton, John B. Shoven and John Whalley (1985), *A General Equilibrium Model for Tax Policy Evaluation*, National Bureau of Economic Research, The University of Chicago Press, Chicago, IL.

Burniaux, Jean-Marc (1987), *Le Radeau de la Méduse: Analyse des dilemmes alimentaires*, Economica, Paris.

Burniaux, Jean-Marc, Giuseppe Nicoletti and Joaquim Oliveira-Martins (1993), "GREEN: A Global Model for Quantifying the Costs of Policies to Curb CO_2 Emissions," *OECD Economic Studies: Special Issue on The Economic Costs of Reducing CO_2 Emissions*, No. 19, Winter 1992, pp. 49-92.

Burniaux, Jean-Marc, and Dominique van der Mensbrugghe (1994), "The RUNS Global Trade Model," *Economic & Financial Modelling*, Autumn/Winter, pp. 161-282.

Collado, Juan Carlos, David Roland-Holst and Dominique van der Mensbrugghe (1995), "Latin America Employment Prospects in a More Liberal Trading Environment," in *Social Tensions, Job Creation and Economic Policy in Latin America*, David Turnham, Colm Foy and Guillermo Larrain, editors, OECD, Paris.

CPB Netherlands Bureau for Economic Policy Analysis (1999), *WorldScan: the Core Version*, CPB, The Hague.

Deaton, Angus, and John Muellbauer (1980), *Economics and Consumer Behaviour*, Cambridge University Press, Cambridge, UK.

de Melo, Jaime, and Sherman Robinson (1989), "Product Differentiation and the Treatment of Foreign Trade in Computable General Equilibrium Models of Small Economies," *Journal of International Economics*, Vol. 27, pp. 47-67.

Derviş, Kemal, Jaime de Melo and Sherman Robinson (1982), *General equilibrium models for development policy*, A World Bank Research Publication, Cambridge University Press, New York, NY.

Elbehri, Azis and Ken R. Pearson (2000), "Implementing Bilateral Tariff Rate Quotas in GTAP using GEMPACK," *Global Trade Analysis Project* (GTAP), Technical Paper No. 18.

Francois, Joseph and Kenneth Reinert (1997), *Applied Methods for Trade Policy Analysis: A Handbook*, Cambridge University Press, New York, NY.

Fullerton, Don (1983), "Transition Losses of Partially Mobile Industry-specific Capital," *Quarterly Journal of Economics*, Vol. 98, February, pp. 107-125.

Goldin, Ian, Odin Knudsen and Dominique van der Mensbrugghe (1993), *Trade Liberalization: Global Economic Implications*, OECD/The World Bank, Paris.

Hertel, Thomas W., Everett B. Peterson, Paul V. Preckel, Yves Surry and Marinos E. Tsigas (1991), "Implicit Additivity as a Strategy for Restricting the Parameter Space in CGE Models," *Economic and Financial Computing*, Vol. 1, No. 1, pp. 265-289.

Hertel, Thomas W., editor (1997), *Global Trade Analysis: Modeling and Applications*, Cambridge University Press, New York, NY.

Howe, Howard (1975), "Development of the Extended Linear Expenditure System from Simple Savings Assumptions," *European Economic Review*, Vol. 6, pp. 305-310.

Lee, Hiro, David Roland-Holst and Dominique van der Mensbrugghe (1997), "APEC Trade Liberalization and Structural Adjustment: Policy Assumptions," *APEC Discussion Paper Series*, APEC Study Center, Graduate School of International Development, Nagoya University, March.

Lluch, Constantino (1973), "The Extended Linear Expenditure System," *European Economic Review*, Vol. 4, pp. 21-32.

Loayza, Norman, Klaus Schmidt-Hebbel and Luis Servén (2000), "What Drives Private Saving Across the World?" *The Review of Economics and Statistics*, May, 82(2), pp. 165-181.

Masson, Paul R., Tamim Bayoumi and Hossein Samiei (1998), "International Evidence on the Determinants of Private Saving," *The World Bank Economic Review*, 12(3), pp. 483-501.

OECD (1990), *OECD Economic Studies: Special Issue on Modelling the Effects of Agricultural Policies*, No. 13, Winter 1989/90.

Rimmer, Maureen T. and Alan A. Powell (1992a), "An Implicitly Directly Additive Demand System: Estimates for Australia," *Impact Project Preliminary Working Paper*, No. OP-73, October.

Rimmer, Maureen T. and Alan A. Powell (1992b), "Demand Patterns across the Development Spectrum: Estimates of the AIDADS System," *Impact Project Preliminary Working Paper*, No. OP-75, October.

Rimmer, Maureen T. and Alan A. Powell (1996), "An implicitly additive demand system," *Applied Economics*, 28, pp. 1613-1622.

Robinson, Sherman, Mary Soule, and Silvia Weyerbrock (1991), "Import Demand Functions, Trade Volume, and Terms-of-Trade Effects in Multi-Country Trade Models," *mimeo*, Department of Agricultural and Resource Economics, University of California at Berkeley.

Rutherford, Thomas F. (2001), "Implementing Tariff Quotas in an MCP Model," *mimeo*, University of Colorado, Boulder, April.

Shoven, John B. and John Whalley (1984), "Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey," *Journal of Economic Literature*, Vol. XXII(3), September, pp. 1007-51.

Shoven, John B. and John Whalley (1992), *Applying General Equilibrium*, Cambridge Surveys of Economic Literature, Cambridge University Press, New York, NY.

Skully, David (1999), "The Economics of TRQ Administration," *International Agricultural Trade Research Consortium*, Working Paper No. 99-6.

van der Mensbrugghe, Dominique (1994), "GREEN: The Reference Manual", *OECD Economics Department Working Papers*, No. 143, OECD, Paris.

van der Mensbrugghe, Dominique (1998), "Trade, Employment and Wages: What Impact from 20 More Years of Rapid Asian Growth?" in *The Future of Asia in the World Economy*, Colm Foy, Francis Harrigan, and David O'Connor, editors, OECD, Paris.

van der Mensbrugghe, Dominique, and Ramiro Guerrero (1998), "Free Trade Arrangements in the Americas: Quid for Agriculture?" paper presented at a joint IPEA/IDB conference on *FTAA and MERCOSUR: The Brazilian Economy and the Process of Regional and Subregional Integration*, Brasilia, 5-6 October.

Yu, Wushing, Thomas W. Hertel, Paul V. Preckel and James S. Eales (2002), "Projecting World Food Demand Using Alternative Demand Systems," *Global Trade Analysis Project (GTAP) Working Paper*, Number 21.