

“Crowding in” and the Returns to Government Investment in Low-Income Countries

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Abstract

This paper estimates the effect of government investment on private investment in a sample of 39 low-income countries. Fluctuations in a predetermined component of disbursements on loans from official creditors to developing country governments are used as an instrument for fluctuations in public investment. The analysis finds evidence of “crowding in”: an extra dollar of government investment raises private investment by roughly two dollars, and output by 1.5 dollars. To

understand the implications for the return to public investment, a CES production function with public and private capital as inputs is calibrated. For most countries in the sample, the returns to government investment exceed the world interest rate. However, for some countries that already have high government investment rates, the return to further investment is below the world interest rate.

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1 Introduction

This paper studies the effects of government investment on private investment in developing economies, and draws implications for the optimal rate of government investment. We use fluctuations in past approvals of loans from official creditors to developing country governments to isolate a predetermined source of variation in government investment, and use this as an instrument to estimate the response of private investment to government investment. The analysis finds evidence of “crowding in”: an additional dollar of government investment causes private investment to increase by around 2 dollars, and output by around 1.5 dollars.

The increase in private investment in response to government investment is not, however, a good indicator of the desirability of government investment, which instead depends on the return to public capital. We use a constant elasticity of substitution (CES) production function, with government and private capital inputs, to illustrate the relationship between the return to government capital and the extent of crowding in. Not surprisingly, this relationship depends critically on the elasticity of substitution between the two types of capital. Consider for example the case of extreme complementarity between government and private capital, where the two are required in fixed proportions to produce output. In this case, we should expect to see very strong crowding in of private investment as private capital responds equiproportionately to an increase in government capital. If, however, there are diminishing returns to aggregate capital, this tells us nothing about whether the level of government capital is too high or too low, i.e. whether more government investment is desirable. If, on the other hand, government capital and private capital are substitutes, public investment might “crowd out” private investment even though the return to public capital is high.

We show how our estimates of the effects of government investment on private investment and output can be used to calibrate a production function with a constant elasticity of substitution between government and private

capital. Our calibrations suggest a strong degree of complementarity between government and private capital. Nevertheless, we find that the return to government investment in most countries in our sample is well above the world interest rate, with excess returns between 5-170%. However, for some countries where public investment is high relative to private investment, we find negative excess returns to public investment.

Our paper contributes to a large literature on the optimal scale of public investment. A fundamental challenge in this literature is the difficulty in directly measuring the rate of return on government investments. Many government investment projects are aimed at providing public goods, which are provided free of charge. Absent a price mechanism, measuring the direct economic benefits of these public goods is a challenge. The difficulty in measuring the returns to government investment at the micro level has directed researchers towards a structural macroeconomic approach, that aims to identify the returns to government investment through the equilibrium dynamics associated with government investment shocks. Mostly, the emphasis has been on studying the effects of government investment on private investment.¹

The conceptual framework motivating this focus was introduced by Aschauer [1989], who considers a closed economy model in which government capital increases total factor productivity (TFP).² In the long run, gains from government investment should be reflected in changes in the marginal return to private capital, and hence in changes in private investment. *Within this*

¹A complementary line of work attempts to quantify the social and economic benefits associated with certain types of capital stocks that are often (but not exclusively) publicly provided, such as infrastructure. For example, Calderon et al. [2011] estimate a positive relationship between infrastructure and output using a panel time-series approach; Calderon and Serven [2004] use a similar methodology to establish a positive relationship between infrastructure and growth and infrastructure and equality; Leduc and Wilson [2013] study the short and long term effects of infrastructure investment in the US, and find a positive short-run relationship but no long run effects.

²See also Baxter and King [1993] who consider a similar specification in which the production function is Cobb-Douglas in public and private capital.

conceptual framework, the extent to which government investment “crowds in” private investment in the long run is a sufficient statistic to back out the optimal scope of government investment.³

The empirical literature that followed focused on trying to quantify the equilibrium responses of private investment to changes in government investment. The literature has mostly relied on time-series and cross-country correlations, finding mixed results. In the context of the US, Aschauer [1989] (and a large literature that followed) uses time series estimates of a positive correlation between public and private investment to infer high returns to government investment. However, these estimates remain highly controversial due to plausible endogeneity issues, summarized in Gramlich [1994]. In later work, Leduc and Wilson [2013] consider the effects of exogenous shocks to federal highway spending across US states, and find short-run effects on GDP but no long-run effects. In the context of developing economies, Cavallo and Daude [2011] use panel analysis to explore the empirical correlations between government and private investment, and find a predominantly negative relationship. Using data from India, Serven [1996] finds that, while infrastructure investment crowds in private capital in the long run, other types of government investment (that compete more directly with the private sector) crowd out long run private investment.

There are two shortcomings of this literature which we seek to address in this paper. First, at the conceptual level, most of this literature has failed to take into account the role of the elasticity of substitution between public and private capital when using estimates of crowding in to draw conclusions regarding the desirability of public investment.⁴ As noted above, a large

³In this model, “crowding out” can only result from short-run dynamics, as agents try to inter-temporally smooth the increase in their future income by dissaving.

⁴One notable exception is Fisher and Turnovsky [1998], whose theoretical contribution notes that the equilibrium response of private capital to public investment depends on the elasticity of substitution between government and private capital, as well as congestion effects and distortionary taxation, from which we abstract here. We are unaware of any empirical studies that attempt to calibrate the elasticity of substitution between

crowding in effect may result either from high complementarity between government and private capital, or from high marginal returns to government investment. For a given amount of “crowding in”, under-estimating the degree of complementarity will overestimate the return to government investment.

We illustrate this point more precisely in the following sections, where we assume a production function in which output is produced using labor and a CES aggregate of government and private capital. We use this framework to show the relationship between the extent of crowding in and the returns to government capital, and how this relationship depends on the elasticity of substitution between the two types of capital. In addition, we show how to retrieve the elasticity of substitution from the observed responses of output and private investment to increases in government investment, which in turn allows us to specify the relationship between the extent of crowding in and the optimality of government investment.

The second difficulty is empirical, and is well-understood in the existing literature. Estimating the equilibrium relationship between government and private investment is complicated by unobservable circumstances that affect both government and private investment decisions. For example, a positive correlation between the two might be due to factors that raise the returns to both private and government investment, such as low world interest rates, low oil prices, or even high domestic productivity. On the other hand, a negative correlation may be reflective of countercyclical government spending, with government investment increasing during periods where private investment is low. In either case, the observed simple correlations are not convincing evidence of a causal effect of government investment on private investment.

To address this difficulty, we use a novel instrumental variable approach introduced in Kraay [2012] and Kraay [2013] to estimate of the relationship between government and private investment, in the context of develop-

government and private capital, as we do here.

ing economies. Specifically, we exploit the fact that (a) loans from official creditors (such as the World Bank, and other multilateral and bilateral aid agencies) finance a significant fraction of government spending, particularly in the low-income countries we study, and (b) disbursements of these loans, and the spending they finance, are spread out over many years following the approval of the loans. This permits the isolation of a predetermined source of variation in government investment that reflects only loan approval decisions made in previous years, and well before the associated spending takes place. Under the plausible identifying assumption that loan approval decisions made by official creditors do not anticipate future macroeconomic shocks that matter for private investment and output, this predetermined component of spending can be used as an instrument for government investment.

Using data on government and private investment in a panel of 39 low-income countries, we find that government investment crowds in private investment: an extra dollar of government investment leads to 2 additional dollars of private investment, and 1.5 additional dollars of output in the subsequent period (though the 95% confidence intervals for both estimates are large). We then use these estimates to calibrate the CES production function. In contrast with the existing literature, which has mostly focused on the Cobb-Douglas case, we find that the data suggest a substantially higher degree of complementarity between government and private capital.

This higher degree of complementarity tends to imply more crowding in. The key point, however, is that for a given amount of crowding in, the implied rate of return on government investment is decreasing with the degree of complementarity. We use our estimated functional form to calibrate rates of return on government investment. Even with high complementarity, we find that, for most countries in our sample, the return to government investment exceeds the world interest rate, with excess returns between 5-170%. However, this conclusion does not apply to countries in our sample

that already have a high ratio of government to private capital stocks.

In the next section we lay out a simple model that can be used to interpret the relationship between estimates of crowding in, the elasticity of substitution, and the return to government investment. Section 3 contains our empirical evidence on the responses of private investment and output to changes in government investment, while Section 4 uses these estimates to calibrate the model of Section 3. The final section offers concluding remarks.

2 Model

Consider a model in which the per capita production function depends on a CES aggregate of government and private capital:

$$Y_t(k_{g,t}, k_{p,t}) = A(\gamma k_{g,t}^\sigma + (1 - \gamma)k_{p,t}^\sigma)^{\frac{\alpha}{\sigma}} \quad (1)$$

where k_g is the per capita government capital stock, k_p is the per capita private capital stock, α is the aggregate capital share, and A represents aggregate productivity. The parameter $\gamma \in [0, 1]$ is a share parameter, and the parameter $\sigma \leq 1$ represents the degree of substitutability between government and private capital. Lower values of σ indicate greater complementarity: the limit $\sigma \rightarrow -\infty$ is the Leontief case where government and private capital are required in fixed proportions. At the other extreme, when $\sigma = 1$, government and private capital are perfectly substitutable. The particular case of $\sigma = 0$ corresponds to the Cobb-Douglas case which is most widely used to motivate and interpret crowding out regressions.⁵

Both private and government capital accumulation follow:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + i_{i,t} \quad (2)$$

⁵Note that the specification $Y = A(k_g)k_p^\alpha$, in which productivity depends on public capital, is essentially the Cobb-Douglas case if $A(k_g) = k_g^\eta$. In the case $\sigma = 0$, output depends on $k_g^\gamma k_p^{1-\gamma}$.

where δ is the depreciation rate and $i_{i,t}$ is investment of type i =(government, private). We assume a small open economy framework: the interest rate is given exogenously by r^* . We also abstract from population growth.

Regardless of the elasticity of substitution, σ , it is optimal to equate the marginal return to government capital with the marginal return to private capital, and to equate both with the rate of return in global markets. To see this, consider the following optimization problem:

$$\max_{k_{g,t}, k_{p,t}} Y_t(k_{g,t}, k_{p,t}) + (1 - \delta)(k_{g,t} + k_{p,t}) - (1 + r^*)(k_{g,t} + k_{p,t}) \quad (3)$$

The first order conditions of this problem yield:

$$\frac{\partial Y}{\partial k_g} = \frac{\partial Y}{\partial k_p} = r^* + \delta \quad (4)$$

These two first-order conditions imply that the optimal ratio of private and public capital is given by:

$$\lambda^* = \frac{k_p^*}{k_g^*} = \left(\frac{1 - \gamma}{\gamma} \right)^{\frac{1}{1-\sigma}} \quad (5)$$

Before continuing, it is worth emphasizing that our assumption that the private and government sectors face the same world interest rate is a simplifying one, and abstracts from two potentially important considerations. First, it is likely that governments face lower financing costs than the private sector, which is typically more credit constrained. In our framework, this wedge would tend to imply a lower cost of capital and higher excess returns to government investment. Second, we are implicitly assuming non-distortionary taxation: the government's cost of funds is given by the world interest rate, and there are no distortions associated with raising those funds. In practice, part of the costs of financing public investment is the distortionary costs of taxation, either for direct financing of government investment or for repayment of government debt in subsequent periods. In our framework this

would tend to imply a higher cost of capital and a lower excess return to government investment.

In reality, the rate of government investment and the public capital stock are unlikely to be chosen according to this optimality condition, but rather reflect a variety of institutional and political economy factors outside the model: there may be over- or under- investment on the part of the government relative to the optimal benchmark. In particular, the excess return to government investment, i.e. $\frac{\partial Y}{\partial k_g} - (r^* + \delta)$, is a good indicator of the extent of over- or under-investment: if $\frac{\partial Y}{\partial k_g} > r^* + \delta$, more government investment is desirable; however, if $\frac{\partial Y}{\partial k_g} < r^* + \delta$, less government investment is desirable. The excess return on government investment, defined as $\frac{\partial Y}{\partial k_g} - (r^* + \delta)$, summarizes the extent to which there is over- or under- investment in government capital.

We do, however, assume that the private sector chooses its investment and capital stock optimally given the behavior of the government. In particular, the second equality in equation 4 describes the optimal behavior of the private sector for a given choice of the government capital stock. This first-order condition for the private sector implicitly defines a relationship between government and private capital, which in turn implies a relationship between government and private investment, i.e. the extent of crowding in or crowding out.

The key conceptual point here is that the extent of crowding in or crowding out of private capital by government capital is, in general, not a sufficient statistic for the return to government investment: simply observing crowding in of private investment alone is not enough to conclude whether or not more government investment is desirable. To see this, we derive expressions for the response of private investment and output to government investment, and show what they tell us about the return to government investment.

Consider an arbitrary benchmark $\bar{i}_{p,t}$, $\bar{i}_{g,t}$, $\bar{k}_{p,t}$, $\bar{k}_{g,t}$ and \bar{Y}_t , which represents the economy's predicted levels of investment, capital and output for time t .

Consider a small shock to the rate of change in $i_{g,t}$ ($\Delta i_{g,t} = (i_{g,t} - i_{g,t-1}) - (\bar{i}_{g,t} - \bar{i}_{g,t-1}) > 0$). In the empirical work that follows, we will capture the benchmark changes $\bar{i}_{g,t} - \bar{i}_{g,t-1}$ and $\bar{i}_{p,t} - \bar{i}_{p,t-1}$ using country and year fixed effects. The first-order Taylor expansion of the equilibrium condition $\frac{\partial Y}{\partial k_p} = r^* + \delta$ implies that the change in private investment $\Delta i_{p,t}$ must satisfy:

$$\frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{p,t+1}^2} \Delta i_{p,t} + \frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{g,t+1} \partial k_{p,t+1}} \Delta i_{g,t} = 0 \quad (6)$$

This condition states that the adjustment in private investment must be such that the marginal product of private capital at $t + 1$ leaves private investors indifferent with respect to investing another unit. Manipulating the above equation yields:

$$\Delta i_{p,t} = - \frac{\frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{g,t+1} \partial k_{p,t+1}}}{\frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{p,t+1}^2}} \Delta i_{g,t} \quad (7)$$

Similarly, the first-order Taylor expansion for output yields:

$$\begin{aligned} \Delta Y_{t+1} &= \frac{\partial Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_g} \Delta i_{g,t} + \frac{\partial Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_p} \Delta i_{p,t} \quad (8) \\ &= \left(\frac{\partial Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_g} - \frac{\partial Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_p} \frac{\frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{g,t+1} \partial k_{p,t+1}}}{\frac{\partial^2 Y_{t+1}(\bar{k}_{g,t+1}, \bar{k}_{p,t+1})}{\partial k_{p,t+1}^2}} \right) \Delta i_{g,t} \end{aligned}$$

where $\Delta Y_{t+1} = (Y_{t+1} - Y_t) - (\bar{Y}_{t+1} - \bar{Y}_t)$.

Lemma 1 *Given the functional form in equation 1, equations 7 and 8 can be written as:*

$$\Delta i_{p,t} = \frac{(\sigma - \alpha)\gamma\lambda}{(\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} \Delta i_{g,t} \quad (9)$$

$$\Delta Y_{t+1} = \quad (10)$$

$$\left(\frac{\gamma}{1 - \gamma} \lambda^{1-\sigma} + \frac{(\sigma - \alpha)\gamma\lambda}{(\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} \right) (r^* + \delta) \Delta i_{g,t}$$

where $\lambda = \frac{k_p}{k_g}$. The excess return on government investment is given by:

$$\frac{\partial Y}{\partial k_g} - (r^* + \delta) = (r^* + \delta) \left(\frac{\gamma}{1 - \gamma} \lambda^{1 - \sigma} - 1 \right) \quad (11)$$

The proof of this lemma is detailed in the appendix, together with other omitted proofs.

Equation 9 captures the extent to which government investment crowds in or crowds out private investment, which naturally depends on the elasticity of substitution between government and private capital. For example, when $\sigma = 1$, corresponding to perfect substitutability, increases in government investment always crowd out private investment. On the other hand, when σ is very negative, corresponding to strong complementarity, private investment is always crowded in by government investment.

It is also clear from comparing Equations 9 and 11 that the extent of crowding in is not informative about the return to government investment. Rather, both depend on the two unknown parameters of the production function, γ and σ , as well as the ratio of private to government capital, λ .

To see the importance of this, consider how the previous literature assumes a Cobb-Douglas production function and uses estimates of “crowding in” to back out the marginal return to government investment. This practice is not innocuous, and tends to over-estimate the returns to government investment (assuming that the actual degree of complementarity is higher), as can be seen from the following lemma:

Lemma 2 *Let β denote an estimated effect of government investment on private investment. For a given estimate $0 < \beta < \lambda$, the excess return on government investment is increasing in σ ; overestimating the substitutability between government and private capital results in overestimating the excess return to government investment.*

Assume, hypothetically, that we estimate that an additional unit of government investment leads to β additional units of private investment - what

are the implications for the excess return to government capital, depending on our prior for σ ? Figure 1 presents the implied marginal product of government capital, for different values of β and different values of σ . The figure illustrates the claim in Lemma 2 (which is proved in the appendix): for positive values of β , assuming a higher σ tends to imply higher rates of return for government investment.

Intuitively, a “crowding in” parameter of $\beta > 0$ simultaneously captures two effects: first, additional government capital raises the marginal product of private capital through increasing overall productivity - this channel would imply that more “crowding in” is associated with higher marginal products of government capital, as illustrated in the figure. The second effect is that additional government capital raises the marginal product of private capital through complementarity - thus, “crowding in” is not necessarily reflective of overall productivity gains, but of higher returns to private capital resulting from complementarity. Naturally, the elasticity of substitution determines the relative importance of these two effects: assuming a high degree of complementarity implies low marginal returns to government investment, because most of response of private capital is attributed to the complementarity channel. In contrast, when government and private capital are more substitutable, more “crowding in” is indicative of higher marginal returns to public investment and overall productivity increases.

The figure also illustrates that not every value of β is consistent with every value of σ . For example, for $\sigma = 1$, crowding out ($\beta \leq 0$) is *necessary*: when government and private capital are perfect substitutes, government capital necessarily crowds out private capital. Intuitively, for $\sigma = 1$, more crowding out (a lower β) implies higher marginal returns to public capital, since more crowding out is indicative of relatively more productive public capital.

In order to make progress in interpreting estimates of crowding in, we need to somehow estimate σ and γ . Unfortunately, conventional approaches to estimating the elasticity of substitution are difficult to apply in the case of

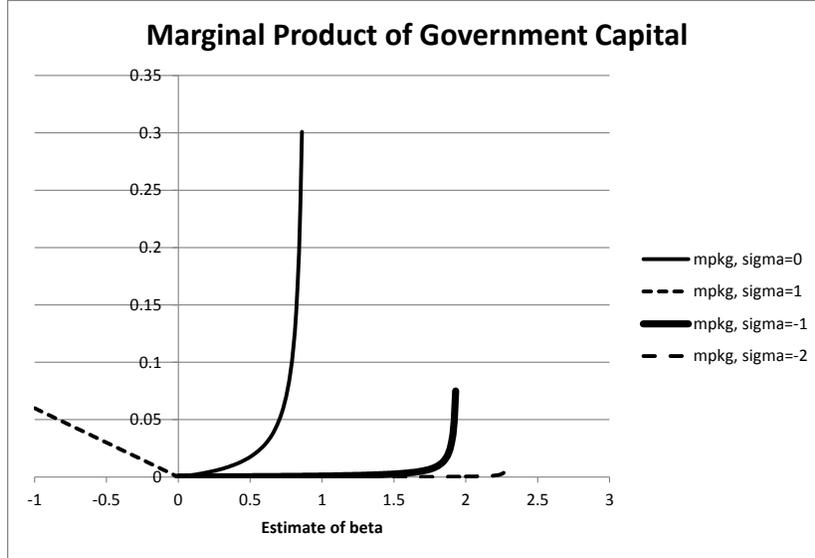


Figure 1: The implied marginal product of public capital for different values of σ , as a function of β .

government and private capital.⁶ The standard approach to estimating the elasticity of substitution between two factors of production is to gather data on their relative use, and data on their relative prices, and then to estimate the proportional response of relative factor use to relative factor prices, which defines the elasticity of substitution. In the case of government capital, it is very difficult to directly observe its price, making this approach infeasible.⁷

Instead, we note that the response of private capital (in Equation 9) and the response of output (in Equation 10) are jointly sufficient to identify γ and σ , given empirical estimates of these two parameters. In particular, let β and β_y denote empirical estimates of the response of private investment and output to government investment. Given λ (which we calibrate based

⁶See, for example, Antras [2004] on how to estimate the elasticity of substitution between capital and labor using aggregate data for the United States.

⁷Indeed, if one takes the simple framework here literally, the relative price of government and private capital is always one, and so there is no variation in the relative price that could be used to identify the elasticity of substitution.

on the average ratio of private and government investment) and $r^* + \delta$ (for which we plug in standard values), we can back out γ and σ by solving:

$$\frac{(\sigma - \alpha)\gamma\lambda}{(\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} = \beta \quad (12)$$

$$\left(\frac{\gamma}{1 - \gamma}\lambda^{1-\sigma} + \beta\right)(r^* + \delta) = \beta_y \quad (13)$$

Then, we use the calibrated parameters for γ and σ to calculate the excess return on government investment using equation 11. In the next section of the paper we develop empirical estimates of β and β_y that are required to implement this calibration exercise.

3 Empirical analysis

In this section we discuss our empirical strategy for estimating the responses of private investment and output to government investment, i.e. β and β_y in Equations 12 and 13. Our goal is to use the empirical estimates of β and β_y to retrieve estimates for γ and σ , which in turn will allow us to draw implications for the returns to government investment.

We estimate a series of regressions of changes in private investment and changes in output on changes in government investment:

$$\frac{I_{P,i,t} - I_{P,i,t-1}}{Y_{i,t-1}} = \beta \frac{I_{G,i,t} - I_{G,i,t-1}}{Y_{i,t-1}} + \mu_i + \zeta_t + \epsilon_{i,t} \quad (14)$$

$$\frac{Y_{i,t+1} - Y_{i,t}}{Y_{i,t-1}} = \beta_y \frac{I_{G,i,t} - I_{G,i,t-1}}{Y_{i,t-1}} + \theta_i + \eta_t + \nu_{i,t} \quad (15)$$

where i and t subscripts denote country and time; $I_{P,i,t}$, $I_{G,i,t}$ and $Y_{i,t}$ are private investment, government investment and output (respectively), all measured in constant local currency units; μ_i and θ_i are country fixed effects; ζ_t and η_t are year fixed effects; and $\epsilon_{i,t}$ and $\nu_{i,t}$ are error terms capturing all other

shocks to private investment and output, respectively. The key parameters of interest are β and β_y . Note that, since changes in private and government investment and changes in output are all normalized by the same variable (the lagged level of output) these parameters capture the change in private investment and output due to a unit change in government investment.

The standard difficulty in statistically identifying β and β_y is that changes in government investment may be correlated with other contemporaneous shocks to private investment and output captured in the two error terms, so that OLS estimation of Equations 14 and 15 will be inconsistent. Moreover, the direction of the bias in the OLS estimates is ambiguous. For example, if government investment increases as a countercyclical policy response during bad times when private investment and output are low, then changes in government investment will be negatively correlated with the error terms and OLS estimates of β and β_y will be biased downwards. If, on the other hand, government investment is procyclical and falls during bad times (when the government cannot borrow to finance spending), then OLS estimates of β and β_y would be biased upwards. Another pedestrian, but likely very important concern is measurement error in government investment, which will bias estimates of β and β_y towards zero. Moreover, this downward bias will be even stronger for the private investment equation, since private investment is measured as total investment from the national accounts less government investment. This implies that when measured government investment is artificially high, measured private investment will be artificially low, inducing a spurious negative correlation between the two.

We address this endogeneity problem using the identification strategy suggested by Kraay [2012] and Kraay [2013] in the context of estimating government spending multipliers. The goal here, as well as in those papers, is to find a predetermined source of variation in government spending that plausibly is uncorrelated with contemporaneous macroeconomic shocks that affect output and investment, and that can be used as an instrument for

changes in government investment. We achieve this by exploiting the fact that (a) a significant fraction of government spending in developing countries is financed by loans from official creditors, and (b) these loans typically disburse over a period of many years following the initial approval of the loan. Drawing on a unique loan-level dataset on nearly 60,000 individual loans from official creditors to developing country governments, Kraay [2013] constructs a measure of predicted disbursements on loans from official creditors that reflects only fluctuations in loan approval decisions made in previous years, but not the current year.⁸ The key identifying assumption is that loan approval decisions in a given year do not anticipate shocks to private investment or output in subsequent years. Given this identifying assumption, subsequent predicted disbursements are uncorrelated with contemporaneous macroeconomic shocks, and are a valid instrument for changes in government investment. Moreover, in countries where official creditor financing of government investment is large, predicted disbursements will also be a strong instrument for changes in government investment.⁹

⁸The predicted disbursements instrument is constructed as follows. We start with a set of approximately 60,000 loans from official creditors to developing country governments, as recorded in the Debtor Reporting System database of the World Bank. For each loan we have the year of the original commitment, and the schedule of subsequent annual actual disbursements. For each loan we construct a 10-year disbursement profile, i.e. the fraction of the original commitment that is disbursed in year 0, 1, 2, ..., 10, with year zero corresponding to the approval year of the loan. We next sort loans into creditor-decade-region-specific bins and average disbursement profiles across loans within bins. We then apply these typical disbursement profiles to individual loans within each bin, and construct predicted disbursements for each loan, i.e. the disbursements that would have occurred had the loan disbursed at the typical rate for all loans in the same bin. The rationale for this step is that it removes any country-specific source of variation in actual disbursements on previously-approved loans, which might respond endogenously to contemporaneous macroeconomic shocks. Finally, we aggregate the predicted disbursements across all loans for each country-year observation, but excluding loans approved in the same year. By construction, the only country-specific variation in this predicted disbursements measure reflects loan approval decisions from previous years, which we assume to be uncorrelated with contemporaneous macroeconomic shocks. For details, see Sections 2 and 3 of Kraay [2013].

⁹In addition, we are also implicitly assuming that the multiplier effect of government consumption expenditures on output and on private investment is zero. This assumption is

We obtain data on government investment and total investment from the IMF’s World Economic Outlook (WEO) database, measured in current local currency units.¹⁰ We convert both to constant local currency units using the overall GDP deflator. Data on GDP in constant local currency units comes from the World Bank’s World Development Indicators. Data on predicted disbursements on loans from official creditors are taken directly from Kraay [2013].

We focus on a set of 39 low-income countries eligible for concessional lending from the International Development Association (IDA, the soft-loan window of the World Bank). We restrict attention to those countries for which at least 15 annual observations on investment are available. These countries are shown in Figure 2, which plots the time-average of government investment and private investment as a fraction of GDP for each country in the sample. Government investment rates vary widely across countries, ranging from as low as 2.5 percent of GDP in Cameroon to nearly 15 percent of GDP in Guyana. There is also a weak negative correlation across countries, with countries with higher government investment rates also having lower private investment rates.

We focus on these low-income countries because of the importance of official creditor financing of government spending in this sample. To see this, consider the over-time average of total disbursements on loans from official creditors as a fraction of total government spending, and as a fraction of total government investment. Among these 39 low-income countries, the median values of these ratios are 18 and 63 percent, respectively, suggesting

important because disbursements on loans from official creditors finance both government consumption and government investment expenditures, and so our predicted disbursements instrument is correlated with fluctuations in both types of spending. If government consumption spending raised investment or output, this would violate the exclusion restriction. See Kraay [2012], Section 6 for a more complete discussion.

¹⁰We are grateful to Toh Kuan of the WEO team for kindly making this data available to us. The government investment series is not available in the public WEO database, but is maintained by the WEO team as part of their larger internal database. This is to our knowledge the largest existing dataset for public investment.

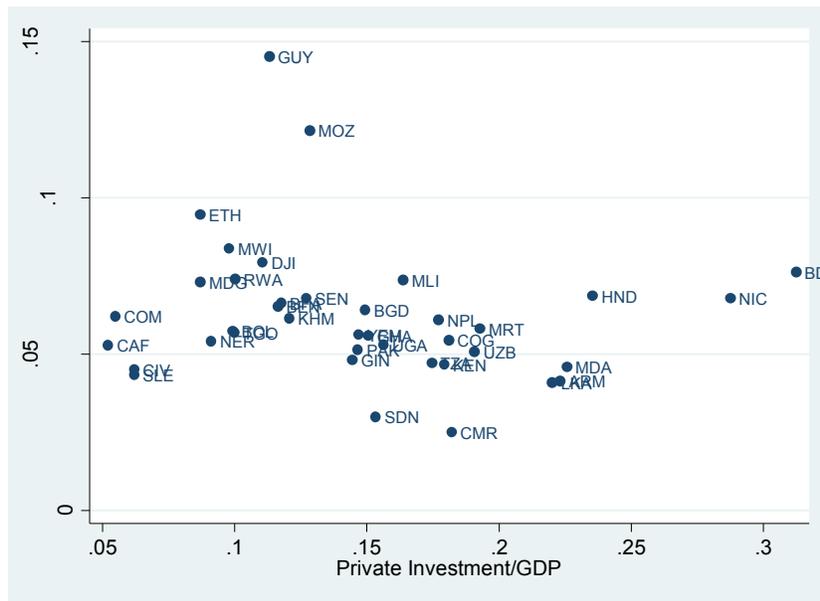


Figure 2: This graph plots the over-time average of government investment as a fraction of GDP (on the vertical axis) against the over-time average of private investment as a fraction of GDP (on the vertical axis). Total investment and total government investment are drawn from the IMF's World Economic Outlook database. Private investment is the difference between the two. Annual averages are calculated over all years between 1980 and 2012 for which data are available.

a very substantial role for official creditor financing of government spending. This will be important for the strength of the first-stage relationship between changes in predicted disbursements on loans from official creditors and changes in government investment.¹¹

Our main empirical results are in Tables 1 and 2, which report estimates of β and for β_y obtained through a number of different variants on our basic empirical specification. Consider first Table 1, which presents estimates of the response of private investment to government investment. The first three columns report results for the full set of 916 annual country-year observations, while the second three columns report results for the subset of these countries located in Sub-Saharan Africa. Panel A reports ordinary least squares (OLS) estimates, Panel B reports two-stage least squares (2SLS) estimates, and Panel C reports the first-stage regression of changes in government investment on changes in the predicted disbursement instrument.

In the full sample of observations and in the Sub-Saharan Africa sample, we find OLS estimates of β that are negative, small, and insignificantly different from zero (columns (1) and (4)). The 2SLS estimates are substantially larger, in the vicinity of 2. As expected given the importance of official creditor financing of government investment, the first-stage relationship between changes in government investment and changes in predicted disbursements is positive, and highly significant, particularly in the full sample where the first-stage F-statistic is 13.9. On the other hand, the relationship is weaker in the Sub-Saharan Africa sample, possibly due to greater measurement error in the government investment data, with the first-stage F-statistic of 7.3 falling short of the rule of thumb of 10. We therefore base inferences on weak-instrument consistent confidence sets reported at the bottom of the table. These are unfortunately quite wide, but exclude zero, suggesting a

¹¹Interpreting these ratios is complicated by the fact that we do not observe the extent to which individual loans finance government investment versus government consumption. However, the scale of disbursements on these loans clearly is large relative to overall government spending.

Table 1: The response of private investment to government investment.

	All IDA Countries			IDA Countries in Sub-Saharan Africa		
	All Observations	Excluding Influential Observations	Control for Lagged Dependent Variable	All Observations	Excluding Influential Observations	Control for Lagged Dependent Variable
<i>Panel A: Ordinary Least Squares (Dependent Variable is Change in Private Investment)</i>						
Change in Government Investment	-0.0858 (0.125)	-0.113 (0.116)	-0.0780 (0.126)	-0.0900 (0.167)	-0.108 (0.148)	-0.0778 (0.166)
<i>Panel B: Two-Stage Least Squares (Dependent Variable is Change in Private Investment)</i>						
Change in Government Investment	1.881* (1.066)	2.298* (1.297)	1.892* (1.059)	1.891 (1.526)	1.668 (1.204)	1.939 (1.557)
<i>Panel C: First-Stage Regression (Dependent Variable is Change in Government Investment)</i>						
Change in Predicted Disbursements	0.290*** (0.0778)	0.270*** (0.0750)	0.290*** (0.0775)	0.248** (0.0918)	0.326*** (0.0732)	0.246** (0.0915)
First-Stage F-Statistic	13.94	12.98	14.00	7.30	19.88	7.24
Weak Instrument Consistent 95% Confidence Interval for β	[0.583, 4.863]	[0.772, 6.275]	[0.594, 4.878]	[0.137, 9.526]	[0.292, 5.219]	[0.180, 9.854]
Number of Observations	916	908	916	611	607	611
Notes: This table reports the results from a series of regressions of changes in private investment on changes in public investment. All changes are in constant local currency units and are scaled by lagged GDP. The sample consists of IDA-eligible countries (first three columns) and IDA-eligible countries in Africa (second three columns). Panel A reports OLS estimates, Panel B reports 2SLS estimates, and Panel C reports the corresponding first-stage regressions. Weak instrument-consistent confidence intervals are based on the Moreira Likelihood Ratio statistic. Changes in predicted disbursements on loans from official creditors are used as an instrument for changes in government investment in Panel B. Heteroskedasticity-consistent standard errors clustered at the country level are indicated in parentheses. * (**) (***) indicates significance at the 10 (5) (1) percent level.						

Table 2: The response of output to government investment.

	All IDA Countries			IDA Countries in Sub-Saharan Africa		
	All Observations	Excluding Influential Observations	Control for Lagged Dependent Variable	All Observations	Excluding Influential Observations	Control for Lagged Dependent Variable
<i>Panel A: Ordinary Least Squares (Dependent Variable is Change in Output)</i>						
Change in Government Investment	0.190* (0.0994)	0.148* (0.0757)	0.0911 (0.0830)	0.184 (0.122)	0.127 (0.0928)	0.0982 (0.107)
<i>Panel B: Two-Stage Least Squares (Dependent Variable is Change in Output)</i>						
Change in Government Investment	1.418* (0.797)	1.248 (0.819)	1.267 (0.783)	2.580** (1.177)	1.684* (0.831)	2.439* (1.305)
<i>Panel C: First-Stage Regression (Dependent Variable is Change in Output)</i>						
Change in Predicted Disbursements	0.290*** (0.0778)	0.255*** (0.0720)	0.269*** (0.0786)	0.248** (0.0918)	0.308*** (0.0663)	0.211** (0.0967)
First-Stage F-Statistic	13.94	12.57	11.71	7.3	21.59	4.74
Weak Instrument Consistent 95% Confidence Interval for β	[-0.046, 3.933]	[-0.336, 4.160]	[-0.293, 4.035]	[0.346, 11.943]	[0.109, 5.713]	[-0.168, 25.653]
Number of Observations	916	907	916	611	606	611
Notes: This table reports the results from a series of regressions of changes in real GDP on changes in public investment. All changes are in constant local currency units and are scaled by lagged GDP. The sample consists of IDA-eligible countries (first three columns) and IDA-eligible countries in Africa (second three columns). Panel A reports OLS estimates, Panel B reports 2SLS estimates, and Panel C reports the corresponding first-stage regressions. Weak instrument-consistent confidence intervals are based on the Moreira Likelihood Ratio statistic. Changes in predicted disbursements on loans from official creditors are used as an instrument for changes in government investment in Panel B. Heteroskedasticity-consistent standard errors clustered at the country level are indicated in parentheses. * (**) (***) indicates significance at the 10 (5) (1) percent level.						

significant positive effect of government investment on private investment, i.e. crowding in. The fact that the 2SLS estimates are much larger than the OLS estimates is consistent with a downward bias in the former, due to some combination of (a) measurement error in government investment, and (b) countercyclical behavior of government investment.

In columns (2) and (5) we explore the sensitivity of our results to the presence of influential observations. We use the procedure suggested by Hadi [1992] to identify influential observations in the first-stage and reduced-form regressions, and then re-estimate the OLS, 2SLS and first-stage regressions in the slightly-reduced sample that excludes these observations. Removing influential observations has little effect on our findings in the full sample, but it dramatically improves the strength of the first-stage relationship in the Sub-Saharan Africa sample, where the first-stage F-statistic jumps to 19.9. As a result, the confidence set for the response of private investment to government investment shrinks considerably, although it is still uncomfortably wide. The point estimate of the effect falls slightly to 1.67.

In columns (3) and (6) we perform an important robustness check that addresses concerns about the validity of the exclusion restriction that justifies the instrument. Our identifying assumption is that loan approval decisions made by official creditors in a given year are not correlated with future macroeconomic shocks that matter for private investment and output. However, loan approval decisions might very well respond to contemporaneous macroeconomic shocks in the borrowing country. If these shocks are correlated over time, then this would violate the exclusion restriction. To address this concern, we include lagged private investment as a control variable to pick up any persistence in shocks to private investment. To conserve space we do not report the coefficient on the lagged dependent variable, which generally is not significantly different from zero. Consistent with this, our estimates of the response of private investment to government investment do not change much, and remain in the vicinity of 2.

Table 2 reports our estimates of the response of output to government investment, β_y , and has the same structure as Table 1. Across all specifications, the OLS estimates are small and positive, ranging from 0.1 to 0.2, and are weakly significantly different from zero in the full sample. The first-stage relationship between changes in government investment and changes in predicted disbursements are of course identical to those in Table 1 in the full sample (columns (1) and (4)), and only slightly different in the no-influential observations sample and controlling for the lagged dependent variable. The 2SLS estimates of the output response to government investment vary somewhat across specifications, and range from 1.2 to 2.6. However, the weak-instrument consistent confidence sets are quite wide in several specifications, and include zero in four out of six cases. Overall, we can summarize these results as suggesting a (rather imprecise) estimate in the vicinity of about 1.5.¹²

4 Structural interpretation

In this section, we use our empirical estimates of β and β_y to calibrate our CES production function using equations 12 and 13, and draw implications for the rates of return on government investment. To back out the parameters γ and σ from the coefficients β and β_y , we use $\alpha = 0.33$, $r^* + \delta = 0.13$ (which

¹²It is useful to note that these quite large effects of government investment on output are consistent with considerably smaller estimates of overall government spending multipliers. Kraay [2012] and Kraay [2013] reports estimates of overall government spending multipliers in low-income countries that are in the vicinity of 0.5. Under the assumption that government consumption expenditures have no effect on output, it is straightforward to show that the estimated overall government spending multiplier is $\frac{\beta_y \gamma_i}{\gamma_i + \gamma_c}$, where γ_i and γ_c are the slopes of the first-stage regressions of changes in government investment and government consumption on changes in the predicted disbursements instrument. As long as disbursements on loans from official creditors raise both government consumption and investment expenditures, i.e. and are both positive, the overall government spending multiplier will be smaller than the effect of government investment on output that we estimate here. In our full sample of observations, $\gamma_i = 0.29$ and $\gamma_c = 0.36$, consistent with an estimated overall spending multiplier less than half of our estimate of 1.5 for β_y .

can be thought of as a 10% annual rate of depreciation and a 3% world interest rate), and $\lambda = \frac{k_p}{k_g} = 3$, which is the average ratio of private to government investment in our sample.¹³

Given that our estimates of β and β_y are quite imprecise, we use a range of estimates that roughly fall within the confidence intervals for these parameters. The theory also restricts the range of admissible estimates of β and β_y , as some values are inconsistent with $\gamma \in (0, 1)$ and $\sigma \leq 1$. The admissible range for β requires $\beta < \lambda$; thus, we consider values of β between 1 and 2.5, and values of β_y between 0.5 and 2.

Table 3 presents the calibration results for different combinations of β and β_y . The third and fourth columns present the calibrated values of σ and γ , respectively, given the values of β and β_y . Recall that the Cobb-Douglas specification, which is typically used in the literature, is nested within the CES framework with the parameter $\sigma = 0$. Our estimates suggest a high degree of complementarity ($\sigma < 0$). This has important implications for the mapping between the extent of “crowding in” and the implied rate of return on government investment.

The fifth and sixth columns use the calibrated values of σ and γ to calibrate the excess returns to government investment using equation 11, for different values of λ : the average value in the sample ($\lambda = 3$), and the 10th percentile ($\lambda = 1$). As evident from equation 11, the excess return depends on the ratio of government and private capital stocks, λ . As could be expected, the excess return to government investment is always higher in countries with a higher λ (i.e., where government capital is relatively scarce). The calibration suggests that for the “average” country, the excess returns to government investment range from 5% to 170%. These estimated excess

¹³We are implicitly assuming here that (a) the available time series for each country is sufficiently long that government-private differences in initial capital stocks are sufficiently depreciated that they do not matter, and (b) that flows of government and private investment result in increases in corresponding capital stocks. As emphasized by Pritchett [2000] the latter is a non-trivial assumption.

Table 3: Calibration results

Empirical estimates		Calibrated parameters			Excess return to public investment			
β	β_y	σ	γ	Average ($\lambda=3$)	10th percentile ($\lambda=1$)	threshold λ	% above threshold	
1	0.5	-0.35	0.4	0.25	-0.04	1.35	82.05	
1	1							
1	1.5							
1	2							
1.5	0.5	-1.19	0.17	0.17	-0.10	2.06	53.85	
1.5	1	-0.67	0.49	0.66	-0.01	1.02	89.74	
1.5	1.5	-0.54	0.65	1.17	0.11	0.67	100.00	
1.5	2	-0.48	0.73	1.7	0.22	0.51	100.00	
2	0.5	-3.18	0.02	0.11	-0.13	2.54	48.72	
2	1	-1.72	0.22	0.6	-0.09	1.59	71.79	
2	1.5	-1.43	0.4	1.11	-0.04	1.18	84.62	
2	2	-1.32	0.5	1.5	0	1.00	89.74	
2.5	0.5	-10.48	0.00	0.05	-0.13	2.92	38.46	
2.5	1	-4.96	0.007	0.54	-0.13	2.30	51.28	
2.5	1.5	-4.13	0.03	1.05	-0.13	1.97	53.85	
2.5	2	-3.8	0.06	1.56	-0.12	1.77	53.85	

Notes: The calibrated values of the parameters of the CES production function σ and γ (equation 1), as a function of the empirical estimates of the response of private investment to government investment (β) and the response of output to government investment (β_y). The excess returns to government investment for $\lambda = 3$ and $\lambda = 1$ implied by this calibration are calculated using equation 11, with $r^* + \delta = 0.13$. The last two columns present the threshold level of $\lambda = \frac{k_p}{k_g}$, above which there are positive excess returns to government investment, and the percent of countries in our sample for which λ is above that threshold. The highlighted row presents the calibrated values for our point estimates of β and β_y .

returns may be overstated, for several reasons. First, as previously noted, our estimation of the costs of financing abstract from distortions associated with taxation. Second, the excess returns are scaled by $r^* + \delta$ - a smaller value of r^* or δ would imply lower excess returns (though the sign would not change). Third, our estimates of β_y rely on the assumption that shocks to government investment affect output only through the accumulation of public and private capital. If there are other indirect “stimulus” effects, our estimates of β_y may be overstated; as illustrated in Table 3, for a given β , higher values of β_y imply higher excess returns.

Despite high calibrated excess returns to government investment in the “average” country, the excess returns for countries with $\lambda = 1$ tend to be negative. The seventh column presents the threshold values of λ , above which the calibrated marginal return to government investment is higher than the world interest rate. This threshold ranges from 0.5 to about 2.5. The implication is that for countries with $\lambda > 2.5$, we can say with some confidence that there are positive excess returns to government investment; for countries with $\lambda < 0.5$, we can say with some confidence that there are negative excess returns to government investment. The last column presents the percent of countries in our sample for which the calibrated excess returns are positive. The number ranges between 38%-100%. For most parameter values, there are at least some countries for which the calibrated excess returns are negative. Table 4 presents calibrated excess returns by country, for the point estimates $\beta = 2$ and $\beta_y = 1.5$.

These findings suggest that, for most developing countries, there is underinvestment in government capital. However, for some countries such as Ethiopia, Central African Republic, Comoros, Guyana, Malawi and Mozambique, there may be excess government capital.

Country	code	λ (ratio of private and public investment)	Excess returns (%)
Armenia	ARM	5.39	505.51
Burundi	BDI	4.09	253.29
Benin	BEN	1.78	22.32
Burkina Faso	BFA	1.77	21.83
Bangladesh	BGD	2.33	54.61
Bolivia	BOL	1.73	19.79
Central African Republic	CAF	0.99	-4.64
Côte d'Ivoire	CIV	1.38	5.83
Cameroon	CMR	7.31	1074.88
Congo, Rep.	COG	3.32	147.06
Comoros	COM	0.88	-6.59
Djibouti	DJI	1.39	6.38
Ethiopia	ETH	0.92	-5.93
Ghana	GHA	2.70	83.42
Guinea	GIN	3.00	112.08
Guyana	GUY	0.78	-8.28
Honduras	HND	3.43	160.52
Kenya	KEN	3.85	216.11
Cambodia	KHM	1.96	31.51
Sri Lanka	LKA	5.39	506.36
Moldova	MDA	4.91	401.68
Madagascar	MDG	1.19	0.26
Mali	MLI	2.22	47.16
Mozambique	MOZ	1.06	-3.11
Mauritania	MRT	3.32	146.60
Malawi	MWI	1.17	-0.36
Niger	NER	1.68	17.61
Nicaragua	NIC	4.24	277.37
Nepal	NPL	2.90	102.38
Pakistan	PAK	2.85	97.66
Rwanda	RWA	1.35	5.00
Sudan	SDN	5.13	448.69
Senegal	SEN	1.87	26.78
Sierra Leone	SLE	1.43	7.71
Togo	TGO	1.75	20.96
Tanzania	TZA	3.70	194.92
Uganda	UGA	2.95	107.07
Uzbekistan	UZB	3.75	202.69
Yemen, Rep.	YEM	2.61	76.37

Table 4: The estimated excess returns by country, for the point estimates $\beta = 2$ and $\beta_y = 1.5$.

5 Conclusion

This paper studies the returns to government investment in developing countries. We estimate the extent to which government investment crowds in private investment, and the effect that it has on output. We use predicted disbursement of loans from official creditors as an instrument, and find that government investment has positive and significant effects on both private investment and output. Our point estimates suggest that one dollar of government investment is associated with 2 additional dollars of private investment, and 1.5 additional dollars of output in the subsequent period.

While, at first glance, “crowding in” tends to suggest favorable implications regarding the desirability of government investment, a structural interpretation of these findings suggests that the implications are rather mixed. We use our empirical estimates to calibrate a CES production function, and use the estimated functional form to derive implications for the marginal return to government investment. For the “average” country in our sample, the calibration suggests underprovision of government capital, and high rates of return associated with additional government investment. However, for countries that already have high rates of government investment (such as Ethiopia, Central African Republic, Comoros, Guyana, Malawi and Mozambique), even these favorable estimates suggest that the rate of return on government investment is below the world interest rate. Since our estimates for the “average” country imply rather high rates of return, the negative excess return to government investment in these countries is a rather robust conclusion.

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A Proofs

A.1 Proof of Lemma 1

The first derivative of Y with respect to k_p is given by:

$$\frac{\partial Y}{\partial k_p} = (1 - \gamma)\alpha A(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 1} k_p^{\sigma - 1} \quad (16)$$

In the ratio of the second derivatives, the constants $\alpha A(1 - \gamma)$ will cancel out. We thus ignore them and take the derivatives:

$$\frac{\frac{\partial^2 Y}{\partial k_g \partial k_p}}{(1 - \gamma)\alpha A} = \left(\frac{\alpha}{\sigma} - 1\right)\gamma(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 2} k_p^{\sigma - 1} \sigma k_g^{\sigma - 1} \quad (17)$$

$$\frac{\frac{\partial^2 Y}{\partial k_p^2}}{(1 - \gamma)\alpha A} = \left(\frac{\alpha}{\sigma} - 1\right)(1 - \gamma)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 2} k_p^{2(\sigma - 1)} \sigma \quad (18)$$

$$+ (\sigma - 1)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 1} k_p^{\sigma - 2} \quad (19)$$

Taking the ratio of the two yields:

$$\frac{\frac{\partial^2 Y}{\partial k_g \partial k_p}}{\frac{\partial^2 Y}{\partial k_p^2}} = \quad (20)$$

$$\frac{(\frac{\alpha}{\sigma} - 1)\gamma(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 2} k_p^{\sigma - 1} \sigma k_g^{\sigma - 1}}{(\frac{\alpha}{\sigma} - 1)(1 - \gamma)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 2} k_p^{2(\sigma - 1)} \sigma + (\sigma - 1)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 1} k_p^{\sigma - 2}}$$

Canceling out the terms $(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 2}$ and $k_p^{\sigma - 1}$:

$$= \frac{(\frac{\alpha}{\sigma} - 1)\gamma \sigma k_g^{\sigma - 1}}{(\frac{\alpha}{\sigma} - 1)(1 - \gamma)k_p^{\sigma - 1} \sigma + (\sigma - 1)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)k_p^{-1}} \quad (21)$$

Multiplying the numerator and the denominator by k_p yields:

$$= \frac{(\frac{\alpha}{\sigma} - 1)\gamma \sigma k_g^\sigma \frac{k_p}{k_g}}{(\frac{\alpha}{\sigma} - 1)(1 - \gamma)k_p^\sigma \sigma + (\sigma - 1)(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)} \quad (22)$$

Denote $\lambda = \frac{k_p}{k_g}$. Then, $k_p = \lambda k_g$, and the above expression can be rewritten as:

$$= \frac{(\frac{\alpha}{\sigma} - 1)\gamma \sigma k_g^\sigma \lambda}{(\frac{\alpha}{\sigma} - 1)(1 - \gamma)\lambda^\sigma k_g^\sigma \sigma + (\sigma - 1)(\gamma k_g^\sigma + (1 - \gamma)\lambda^\sigma k_g^\sigma)} \quad (23)$$

Canceling out k_g^σ yields:

$$= \frac{(\frac{\alpha}{\sigma} - 1)\gamma \sigma \lambda}{(\frac{\alpha}{\sigma} - 1)(1 - \gamma)\lambda^\sigma \sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} \quad (24)$$

$$= \frac{(\alpha - \sigma)\gamma \lambda}{(\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} \quad (25)$$

Multiplying the above expression by -1 yields the expression in equation 12.

To derive equation 13, note that equating the expression in equation 16

with $r^* + \delta$ yields:

$$\alpha A(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 1} = \frac{r^* + \delta}{1 - \gamma} k_p^{1 - \sigma} \quad (26)$$

It follows that:

$$\frac{\partial Y}{\partial k_g} = \gamma \alpha A(\gamma k_g^\sigma + (1 - \gamma)k_p^\sigma)^{\frac{\alpha}{\sigma} - 1} k_g^{\sigma - 1} = (r^* + \delta) \frac{\gamma}{1 - \gamma} \lambda^{1 - \sigma} \quad (27)$$

Thus:

$$\Delta Y = (r^* + \delta) \frac{\gamma}{1 - \gamma} \lambda^{1 - \sigma} + \frac{\partial Y}{\partial k_p} \frac{\partial k_p}{\partial i_g} = (r^* + \delta) \left(\frac{\gamma}{1 - \gamma} \lambda^{1 - \sigma} + \beta \right) \quad (28)$$

A.2 Proof of Lemma 2

First, consider the process of backing out γ given σ and λ , using the relationship in equation 12:

$$\frac{(\sigma - \alpha)\gamma\lambda}{(\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)} = \beta \quad (29)$$

$$(\sigma - \alpha)\gamma\lambda = \beta((\alpha - \sigma)(1 - \gamma)\lambda^\sigma + (\sigma - 1)(\gamma + (1 - \gamma)\lambda^\sigma)) \quad (30)$$

$$= \beta((\alpha - \sigma)\lambda^\sigma - \gamma(\alpha - \sigma)\lambda^\sigma + (\sigma - 1)\lambda^\sigma + (\sigma - 1)\gamma(1 - \lambda^\sigma)) \quad (31)$$

$$= \beta((\alpha - \sigma)\lambda^\sigma + (\sigma - 1)\lambda^\sigma - \gamma((\alpha - \sigma)\lambda^\sigma + (1 - \sigma)(1 - \lambda^\sigma))) \quad (32)$$

$$= \beta((\alpha - 1)\lambda^\sigma - \gamma(\alpha\lambda^\sigma + (1 - \sigma) - \lambda^\sigma)) \quad (33)$$

$$= \beta((\alpha - 1)\lambda^\sigma - \gamma(1 - \sigma - \lambda^\sigma(1 - \alpha))) \quad (34)$$

It follows that:

$$\gamma((\sigma - \alpha)\lambda + \beta(1 - \sigma - \lambda^\sigma(1 - \alpha))) = \beta(\alpha - 1)\lambda^\sigma \quad (35)$$

$$\gamma = \frac{\beta(\alpha - 1)\lambda^\sigma}{(\sigma - \alpha)\lambda + \beta(1 - \sigma - \lambda^\sigma(1 - \alpha))} \quad (36)$$

And:

$$1 - \gamma = \frac{(\sigma - \alpha)\lambda + \beta(1 - \sigma)}{(\sigma - \alpha)\lambda + \beta(1 - \sigma - \lambda^\sigma(1 - \alpha))} \quad (37)$$

$$\frac{\gamma}{1 - \gamma} = \frac{\beta(1 - \alpha)\lambda^\sigma}{(\alpha - \sigma)\lambda + \beta(\sigma - 1)} = \frac{\beta(1 - \alpha)\lambda^\sigma}{\alpha\lambda - \sigma(\lambda - \beta) - \beta} \quad (38)$$

An estimate of $\gamma \in (0, 1)$ requires that the denominator is positive; this is true for σ sufficiently low (provided that $\lambda > \beta$). The estimated excess return to public investment is then:

$$(r^* + \delta) \left(\frac{\beta(1 - \alpha)\lambda}{\alpha\lambda - \sigma(\lambda - \beta) - \beta} - 1 \right) \quad (39)$$

This expression is increasing in σ ; thus, underestimating the degree of complementarity between public and private capital is overestimating the rate of return on public investment.