WPS6216

Policy Research Working Paper 6216

Background Paper to the 2014 World Development Report

An Exploration of the Link between Development, Economic Growth, and Natural Risk

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The World Bank
Sustainable Development Network
Office of the Chief Economist
October 2012

Policy Research Working Paper 6216

Abstract

This paper investigates the link between development, economic growth, and the economic losses from natural disasters in a general analytical framework, with an illustration on hurricane flood risks in New Orleans. It concludes that, where capital accumulates through increased density of capital at risk in a given area, (i) the probability of disaster occurrence decreases with income; (ii) capital at risk—and thus economic losses in case of disaster—increases faster than economic growth; (iii) increasing risk-taking reinforces economic growth. Economic growth and improved protection transfer risks from frequent low-intensity events to rarer high-impact events. In this context, average annual losses from disasters grow with income, and they grow faster than income at low levels of development and slower than income at high levels of development. These findings are robust to a broad range of modeling choices and parameter values, to the inclusion of risk aversion, and to variations in the decision-making framework (including the introduction of prospect theory's decision weights, biases in risk perception and myopic expectations). They show that risk-taking is both a driver and a consequence of economic development, that risk taking should not be indiscriminately suppressed, and that the world is very likely to experience fewer but more costly disasters in the future.

This paper—prepared as a background paper to the World Bank's *World Development Report 2014: Managing Risks for Development*—is a product of the Office of the Chief Economist, Sustainable Development Network. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at shallegatte@worldbank.org.

An exploration of the link between development, economic growth, and natural risk

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Keyword: Development; Economic growth; Risk; Natural disaster; Economic losses.

JEL: O10; O44; Q01; Q54.

1 Introduction

The damages caused by Hurricane Sandy in Haiti and in the US show that poor and rich countries are vulnerable to natural hazards. Large damages in New York City and New Jersey — with a preliminary estimate of direct damages in excess of $50 billion — raise questions on the level of coastal protection in these areas and on the rapid coastal developments that have driven so many households and so much capital and infrastructure in risky areas. In spite of large financial and technical resources, the economic vulnerability to hurricanes remain high in the richest country of the world, and statistical analyses suggest it has been growing in the last hundred years (Pielke et al., 2008).

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This is not an isolated situation. Increasing investments in disaster risk reduction have lead to a significant reduction in human casualties (Kahn, 2005; Kellenberg and Mobarak, 2008), but economic losses from natural disasters have been growing as fast or even faster than economic growth in many countries; see for instance statistical analyses in Barredo (2009), Miller et al. (2008), Neumayer and Barthel (2010), Nordhaus (2010), Pielke et al. (2008), and Bouwer et al. (2007). Climate change is not responsible of this evolution (Schmidt et al., 2009; Neumayer and Barthel, 2010; Bouwer, 2011; IPCC, 2011). The trend in hurricane losses relative to wealth in the US is for instance fully explained by migrations toward hurricane-prone areas and increasing population wealth (Pielke et al., 2008). Globally, there is a trend toward more risk taking: between 1970 and 2010, global population grew by 87%, but the population living in flood plains increased by 114% and in cyclone prone coastlines by 192%. The GDP exposed to tropical cyclones increased from 3.6 percent of total GDP to 4.3 percent over the same period (UN-ISDR, 2011).

This global trend in economic disaster losses is a major concern in many countries. Disaster losses represent an increasing burden on economies and public finances (e.g., Benson and Clay, 2004; Loayza et al., 2009; Fomby et al. 2011; Strobl 2011), and their trend threatens the functioning of insurance and reinsurance markets (e.g., Michel-Kerjan, 2010). Cross-country studies have suggested that economic disaster losses are increasing less rapidly than income, making them easier to manage (Toya and Skidmore, 2007; Mendelsohn et al., 2011). But the trend in economic losses in the US and many other countries suggests that the relationship between economic growth and disaster vulnerability is more complex than these studies suggest, and that the economic vulnerability to disasters may not always decrease with income (Schumacher and Strobl, 2011).

This paper investigates the interlinkages between economic disaster losses and development. It considers the fact that higher income makes it possible to invest in better protections against disasters, but may also lead to higher investment in at risk areas. It also considers the role of investments in at risk areas in economic growth, for instance because development of coastal areas is critical for export-led growth or because agglomeration externality and urbanization (often in flood plains) are major drivers of development (Ciccone et al., 1996; Ciccone, 2002; World Bank 2008). Compared with previous investigations of trends in disaster economic losses (e.g., Lewis and Nickerson, 1989; Schumacher and Strobl, 2011), this analysis proposes an explicit modeling of risk taking (how much is invested in risky areas?) and of protection investment (how much is invested in protection?), to characterize the two-way link between economic growth and disaster losses. It does so in a more general framework than previous studies, and it confirms the robustness of its results by investigating the role of risk aversion and different forms of behavioral bias.

Within this framework, it is found that — even with no change in climate conditions and hazard characteristics, and assuming rational decision-making and the absence of wrong incentives from subsidized insurance and post-disaster support — risk-taking generally increases with development. Natural disasters are likely to become more destructive in the future, even relative to income, and even in the presence of risk aversion. Reciprocally, increased risk-taking reinforces economic growth, suggesting that risk taking should not be indiscriminately suppressed. Risk-taking is both a driver and a consequence of economic development. In this context, average annual losses from disasters grow with income, and they grow faster than income at low levels of development and slower than income at high levels of development.

The next section describes the most generic model and the conditions under which disaster losses increase with economic growth. Section 3 presents more detailed results for specific specification of protection costs, and Section 4 investigates special cases of production functions. Section 5 applies this model to New Orleans, demonstrating that reasonable parameter values lead to a situation where economic growth incentivizes risk taking and increases disaster losses. Section 6 shows that these results are robust to the introduction of risk aversion, and Section 7 that they are robust to imperfect decision-making, through the introduction of systematic biases in risk perception and myopic expectations. Section 8 concludes.

2 Development and natural risks

It is generally accepted that richer populations invest more to protect themselves from natural hazards. A richer population, however, may also invest more in at-risk areas, increasing exposure to natural hazards. These two trends have opposite impacts on risk, and the resulting trend in risk is thus ambiguous. This trend is investigated in this section with a simple model.

2.1 A general economic growth framework

We assume there are two categories of capital. Capital R represents the capital related to activities that need to be (or benefit from being) located in areas that are potentially at risk of flooding. There are two categories of such capital. First, some activities directly depend on being in risky areas, such as ports that are located in coastal areas or river flood plains. Second, positive concentration externality is making it profitable to invest in at-risk areas even in sectors that are not directly dependent on being at risk — such as finance in New York City or manufacturing in Shenzhen — to benefit from spill-overs from the industry that need to be located in flood-prone areas (such as ports): (i) lower long-distance transport cost; (ii) infrastructure for local transportation, water, energy, and communication; (iii) large labor market with access to skilled workers; and (iv) access to public services and amenities (art and culture, schools, university, etc.).

Capital S represents the rest of economy, which can be located in safe locations without loss of productivity. These two capitals are inputs in the production function:

$$Y = e^{\gamma t} F(R, S) \tag{1}$$

where t is time, F is a production function and γ is the exogenous growth in total factor productivity. Classically, we assume that $\partial_R F(R, S) > 0$; $\partial_S F(R, S) > 0$; $\partial^2_R F(R, S) < 0$ and $\partial^2_S F(R, S) < 0$ (decreasing returns).

The capital R can be affected by *hazards*, like floods and windstorms. If a hazard is strong enough, it causes damages to the capital installed in at-risk areas, and can be labeled as a *disaster*. We assume that in that case, a fraction X of capital R is destroyed. It is assumed that this is the only consequence of disasters.[2]

These disasters (i.e. hazards that lead to capital destruction) have a probability p_0 of occurring every year, except if protection investments reduce this probability. These protection investments take many forms, depending on which hazard is considered. Flood protections include dikes and seawalls, but also drainage systems to cope with heavy precipitations in urban areas. Windstorm protections consist mainly in building retrofits and stricter building norms, to ensure that old and new buildings can resist stronger winds.

It is assumed that better defenses reduce the probability of disasters, but do not reduce their consequences.[3] This is consistent with many types of defenses. For instance, seawalls can protect an area up to a design standard of protection but often fail totally if this standard is exceeded; building norms allow houses to resist up to a certain wind speed, but when this wind speed is exceeded, houses are completely damaged and require total rebuilding. This modeling choice is made without loss of generality, if there is no risk aversion.[4] In that case, indeed, reducing the probability of a disaster or the consequences in case of disaster is equivalent.

Better defenses are also more expensive, and the annual cost of defenses C increases when the remaining disaster probability p decrease.[5] The function $C(p, R)$ is assumed twice differentiable, $C(p_0, R) = 0$ (the probability of occurrence is p_0 in the absence of protections), $\partial_R C(p, R) \geq 0$ (it is as or more expensive to protect more capital),

[2]Disaster fatalities and casualties are not considered in this simple model, assuming that early warning, evacuation and emergency services can avoid them, which is consistent with the observation that disaster deaths decrease with income, at least above a certain income level (Kahn, 2005; Kellenberg and Mobarak, 2008). Human losses could be taken into account if it is assumed that fatalities and casualties can be measured by an equivalent economic loss, which is highly controversial; see a discussion in Viscusi and Aldy (2003). The analysis remains valid with indirect losses, but only if the latter increase linearly with direct losses (which is inconsistent with some analyses, see Hallegatte 2008 and Henriet et al. 2012).

[3]This is equivalent to the self-protection of Ehrlich and Becker (1972).

[4]The role of risk aversion is investigated in Section 6.

[5]The probability p here includes both the probability that an event exceeds protection capacities, and the defense failure probability, even for weaker events.

$\partial_p C(p,R) \leq 0$ (the cost increase when the probability decrease), $\partial_p^2 C(p,R) \geq 0$ and $C(0,R) = +\infty$ (the marginal cost is increasing and it is impossible to reduce the probability to zero) and $\partial_{pR}^2 C(p,R) \leq 0$ (the cost of protecting more capital increases when the probability decreases).

Any given year, the economic surplus π is given by:

$$\pi = e^{\gamma t} F(R,S) - C(p,R) - L - r(R+S) \tag{2}$$

where r is the interest rate, L is the losses from disasters, and is given by a random draw with probability p. If a disaster occurs, losses are equal to XR, i.e. a fraction X of the capital located in the risky area is destroyed. Any given year, the expected loss $\mathbb{E}[L]$ is equal to pXR and the expected output is equal to:

$$\mathbb{E}[\pi] = e^{\gamma t} F(R,S) - C(p,R) - (r + pX)R - rS \tag{3}$$

Note that in this equation, disaster losses appear as an additional cost of capital at risk, in addition to the interest rate r.

We also define the *risk-free situation* as a situation in which there is no risk, either because $p_0 = 0$ (no hazard), because $X = 0$ (no vulnerability), or because $C(p,R) = 0$ (costless protections). In the risk-free situation, there are still two capitals R and S, but none of them is at risk.

2.2 Optimal choice of p, R, and S

We assume that a social planner — or an equivalent decentralized decision-making process — decides which amounts of capital R and S are to be located in the risky and safe areas, and the level of protection (p and $C(p,R)$) that is to be built. Its program is:[6]

$$\begin{array}{l} \max_{p,R,S} \ \mathbb{E}[\pi] \\ s.t. 0 \leq p \leq p_0 \end{array} \tag{4}$$

We assume first that there is no risk aversion and we assume that the expected surplus is maximized. From the social planner's perspective, doing so is acceptable if disaster losses remain small compared with aggregated income, consistently with the Arrow-Lind theorem for public investment decisions (Arrow and Lind, 1970). As discussed in Mahul and Ghesquiere (2007), this theorem holds only if some conditions are met, including if disaster losses can be pooled among a large enough population (e.g., a large country), and

[6]This model is more general than the model of Schumacher and Strobl (2011). In the latter, the only decision concerns protection investments that mitigate disaster consequences, and there is no benefit from taking risks and thus no trade-off between safety and income. This model also differs from Hallegatte (2011) in that it is more general on the shape of production and protection cost functions, and it introduces the interest rate to account for the consumption–investment trade-off.

with many other uncorrelated risks, i.e. in the presence of comprehensive insurance coverage or post-disaster government support, or if disaster losses can be smoothed over time thanks to savings and borrowing (i.e. self-insurance) or reinsurance. In other terms, the optimal pathways determined by this analysis are valid assuming that the social planner ensures that individual losses remain small thanks to temporal smoothing and redistribution or insurance across individuals. Section 6 investigates the case with risk aversion.

If $p = p_0$, then there is no protection in place — because protections are too expensive — and the situation is highly simplified: disaster risk reduces by a fixed fraction the productivity of the capital at risk. Classically, this reduces the amount of such capital without influencing its growth rate on the balanced growth pathway. In the following, we assume that $p < p_0$.

First order conditions lead to the optimal values of p, R, and S:

$$e^{\gamma t}\partial_R F(R,S) - \partial_R C - (pX + r) = 0 \quad (5)$$

$$e^{\gamma t}\partial_S F(R,S) - r = 0 \quad (6)$$

$$\partial_p C = -XR \quad (7)$$

While the marginal production of capital S is r, the marginal production of capital R is $\partial_R C + pX + r$, i.e. the cost of capital r plus the capital losses due to disasters pX plus the incremental cost of protection $\partial_R C$. The term $\partial_R C + pX + r$ is what we define as the *risk-adjusted cost of capital*, and it is larger than the risk-free cost of capital, to account for natural risks.[7]

Since $pX > 0$, $\partial_R C > 0$, and $\partial_R F$ is decreasing, the first equation shows that the presence of risk ($X > 0$ and $C > 0$) leads to a reduction in R.

Proposition 1 *The presence of risk and the possibility to protect lead to a reduction in the capital that is located in the risky area, compared with the risk-free situation.*

Taking the derivative of the three equations with respect to time t, one gets:

$$\gamma e^{\gamma t}\partial_R F + [e^{\gamma t}\partial^2_R F - \partial^2_R C]\partial_t R + e^{\gamma t}\partial^2_{RS} F \partial_t S = (X + \partial^2_{pR} C)\partial_t p \quad (8)$$

$$\gamma e^{\gamma t}\partial_S F + e^{\gamma t}\partial^2_S F \partial_t S + e^{\gamma t}\partial^2_{RS} F \partial_t R = 0 \quad (9)$$

$$\partial_t p \partial^2_p C = -(X + \partial^2_{pR} C)\partial_t R \quad (10)$$

Equation (10) shows that if $(X + \partial^2_{pR} C) < 0$, then an increase of at-risk capital (everything else being unchanged) leads to a decrease in protection (i.e. an increase in

[7] Equivalently, one can define the risk-adjusted marginal productivity of capital as the marginal productivity of capital reduced by the cost of protection and disaster capital losses: $e^{\gamma t}\partial_R F(R,S) - \partial_R C - pX$.

the probability of occurrence), because the cost of protection then increases more rapidly with R and p than the avoided disaster losses. If $X + \partial^2_{pR}C = 0$, then the probability of occurrence is independent of R, and thus constant over time even in presence of economic growth.

If we have

$$(X + \partial^2_{pR}C) > 0 \; , \tag{11}$$

then having more capital at risk leads to an increase in protection, and the probability of occurrence p and the amount of capital at risk R evolve in opposite direction. In the following, we assume that this condition is met.

Replacing $\partial_t p$ in Eq. (8) and replacing $e^{\gamma t}\partial_R F$ by $(r + pX + \partial_R C)$ gives:

$$\partial_t R = \frac{\gamma(r(1 - \frac{\partial^2_{RS}F}{\partial^2_S F}) + pX + \partial_R C)}{-e^{\gamma t}(\partial^2_R F - \frac{(\partial^2_{RS}F)^2}{\partial^2_S F}) + \partial^2_R C - \frac{(X+\partial^2_{pR}C)^2}{\partial^2_p C}} \tag{12}$$

Since $\frac{\partial^2_{RS}F}{\partial^2_S F} < 1$, the capital at risk increases over time when:

$$-e^{\gamma t}(\partial^2_R F - \frac{(\partial^2_{RS}F)^2}{\partial^2_S F}) > -\partial^2_R C + \frac{(X + \partial^2_{pR}C)^2}{\partial^2_p C} \tag{13}$$

If this inequality is not satisfied, then development and growth leads to a reduction in capital at risk. It probably corresponds to locations that are too costly to protect against natural disasters and where the optimal solution is "strategic retreat", i.e. the abandonment of human settlements.

To interpret this inequality, we can disregard for now the interactions between R and S (i.e. assuming that $\partial^2_{RS}F = 0$) and assume that protection costs are independent of R (i.e. $\partial^2_R C = 0$ and $\partial^2_{pR}C = 0$). In this case, the capital at risk increases over time when:

$$-e^{\gamma t}\partial^2_R F > \frac{X^2}{\partial^2_p C} \tag{14}$$

This inequality is verified if the marginal productivity of the capital at risk decreases more rapidly with R than the marginal cost of protection increases with p. In this same situation, the probability of disaster decreases with economic growth.

Proposition 2 *If $X + \partial^2_{pR}C > 0$ and if the concavity of the production function is high enough — or if the convexity of the protection costs is high enough — then economic growth leads to an increase in capital at risk (i.e. an increase in losses when a disaster occurs), and a decrease in the probability of occurrence. If the concavity is lower, then economic growth leads to a decrease in capital at risk (and an increase in disaster probability).*

It is interesting to note that — counter-intuitively — the capital at risk increases over time when the concavity is high (i.e. returns of capital at risk are rapidly decreasing), and decreases otherwise.

Note that if development leads to a reduction in capital at risk R and an increase in p, then at one point the economy reaches a situation where there is no protection and $p = p_0$. In such a situation, as already stated, capital at risk R grows at the same rate than in the risk-free situation.

The capital R in the risk-free situation is referred to as R^s, and its evolution is:

$$\partial_t R^s = \frac{\gamma(r(1 - \frac{\partial^2_{RS} F}{\partial^2_S F}))}{-e^{\gamma t}(\partial^2_R F - \frac{(\partial^2_{RS} F)^2}{\partial^2_S F})} \tag{15}$$

Assuming that the capital R increases with economic growth, i.e. that condition (13) is verified, then the comparison of Eqs. (12) and (15) explains how the protection influences the evolution of capital:

- The term $pX + \partial_R C$ in the numerator is the impact of risk and protection marginal costs on marginal productivity; it increases the numerator and accelerates the absolute growth in R.

- The term $\partial^2_R C$ in the denominator is the decreasing or increasing return on protection; if the returns on protection are constant (e.g. $C(p, R) = RC(p)$ or $C(p, R) = C(p)$), then this term does not exist; if the returns are decreasing (i.e. costs are convex and $\partial^2_R C > 0$), then this term increases the denominator and slows down the growth in R; if the returns are increasing, then the growth in R is accelerated.

- The term $-\frac{(X+\partial^2_{pR} C)^2}{\partial^2_p C}$ in the denominator is the impact of the change in protection that is provided if more capital is installed in at risk areas (if the probability of occurrence is fixed, this term does not appear). Since $\partial^2_p C > 0$, this term is negative and reduces the denominator and thus accelerate the growth in R.

2.3 Trend in average annual losses

The average economic losses due to disasters are equal to $\mathbb{E}[L] = pXR$.

$$\partial_t \mathbb{E}[L] = XR\partial_t p + pX\partial_t R \tag{16}$$

It can be rewritten:

$$\partial_t \mathbb{E}[L] = \partial_t p \left[\partial_p C - \frac{p\partial^2_p C}{1 + \frac{\partial^2_{pR} C}{X}} \right] \tag{17}$$

Since $\partial_p C < 0$ and $\partial_p^2 C > 0$, and under condition (11), then $\mathbb{E}[L]$ and p evolve in opposite directions.

Proposition 3 *If $X + \partial_{pR}^2 C > 0$, then average annual disaster losses increase when the probability of occurrence decreases over time.*

This result highlights the need to consider the combination of exposure (R) and probability (p) to investigate risks. In particular, a reduction in the probability of occurrence does not mean that average losses decrease; on the opposite, this general analysis suggests that under mild conditions a decrease in the probability of occurrence leads to an increase in average losses, because of the increase in capital at risk.

Importantly, this analysis illustrates that protection reduces the hazard (the probability of occurrence of an event), but its impact on risk is more complex, because it also increases exposure (here, the capital at risk R). As a result, protection transfers part of the risk from one kind of risk (frequent and low-cost events) to another kind (exceptionnal and high-impact events), a process already stressed in Etkin (1999).

3 Special cases for protection costs

We already made assumptions on the shape of $C(p, R)$, but it is useful to explore two extreme cases for the dependence of $C(p, R)$ to R.

In a first case, we can consider a coast or a river, where additional capital investments are done at a fixed density and are thus using additional land, which in turn requires additional protection. In such a case, the protected areas increases proportionally with the invested capital in the risky zone, and $C(p, R) = R\tilde{C}(p)$. This is the case in Florida, or in the south of France, where population density is low and flood exposure increases mainly through the construction of individuals houses, at low density. This situation can be labeled *"horizontal"* or *"area-increasing"* accumulation.

In a second case, we consider a given risky areas, which is protected against coastal floods and where investment takes place. In such a case, the risky and protected area does not increase with investments, and the cost of protection is independent of the amount of protected capital: $C(p, R) = \tilde{C}(p)$. This is notably the case where additional investments take place through higher concentration and density, on a given area. Examples of such places are the Netherlands, New Orleans, or Manhattan in New York City. This situation can be labeled *"vertical"* or *"density-increasing"* accumulation.

Let us explore the consequence of these two patterns on flood risks.

3.1 Horizontal accumulation

In that case, we can assume that the protection cost function has the form $C(p, R) = R\tilde{C}(p)$. The marginal cost and benefit of protection are equal (Eq. (7)), which means $R\partial_p\tilde{C}(p) = -XR$. Therefore, p is independent of R and constant over time.

Deriving the previous equation with respect to R gives $X + \partial^2_{pR}C = 0$, and we can rewrite Eq. (5) as:

$$e^{\gamma t}\partial_R F(R, S) = \overbrace{\tilde{C}(p) + pX + r}^{r'} \tag{18}$$

Since $\tilde{C}(p)$ is constant and positive, the risk-adjusted marginal productivity r' is also a constant, larger than r. In this case, R evolves like a capital without risk, but with a larger interest rate r' instead of r. Since marginal productivity needs to be larger, the amount of capital is lower in presence of decreasing returns ($R < R^s$), i.e. risk leads to a reduction in capital R. With classical production functions (CES or Cobb-Douglas) and neutral technological change, the capital at risk R increases at the same rate as economic growth. Since p is constant, $\partial_t\mathbb{E}[L] = pX\partial_t R$, and average annual losses grow at the same rate as capital at risk and as risk-free economic growth.

Proposition 4 *In horizontal-accumulation locations — i.e. where flood exposure increases because the developed area at risk is expanded and where protection costs increase therefore proportionally with protected capital — rational decision-making leads to annual flood losses growing at the same rate as economic growth, with a constant flood probability, regardless of how protection costs vary with the residual probability of occurrence.*

3.2 Vertical accumulation

In that case, we can assume that $C(p, R) = \xi p^{-\nu} + C_0$. Assuming $p < p_0$, we can use Eq. (7) to find:

$$p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}} \tag{19}$$

We have $\partial^2_{pR}C = 0$, and thus $X + \partial^2_{pR}C > 0$ if there is risk, so that condition (11) is always verified. As a consequence, we know that p and R evolve in opposite directions: if capital increases (resp. decreases), the protection is improved (resp. weakened) and disaster probability decreases (resp. decreases). We are in the situation where p decreases and R increases when condition (13) is verified, and it can be rewritten:

$$-e^{\gamma t}(\partial^2_R F - \frac{(\partial^2_{RS}F)^2}{\partial^2_S F}) > \frac{X^2 p^{\nu+2}}{\nu(\nu+1)\xi} \tag{20}$$

Also, we know from the general analysis that $\partial_t \mathbb{E}[L]$ is positive and average disaster losses are increasing over time.

Proposition 5 *In locations where flood exposure rises as a result of increased density in a given protected area (e.g., New Orleans), rational decision-making results in a continuous increase in annual flood losses, when flood probability decreases over time.*

To know whether capital at risk and flood losses grow more or less rapidly than economic growth, assumptions are needed on the shape of the production function. This is what is investigated in the next section.

4 Special cases for the production function

In this section, we keep the "vertical-accumulation" assumption, i.e. $C(p, R) = \xi p^{-\nu} + C_0$. All qualitative results are however valid if $C(p, R) = \xi R^c p^{-\nu} + C_0$, where $0 \leq c < 1$.

We will assume first that capital at risk R and safe capital S are separable inputs in the production function, and then show that results can be generalized to other production functions.

If $F(R, S) = f(R) + g(S)$, then $\partial^2_{RS} F = 0$, and all equations can be simplified.

The evolution of the capital at risk is:

$$\partial_t R = \frac{\gamma(r + pX + \partial_R C)}{-e^{\gamma t}\partial^2_R f(R) + \partial^2_R C - \frac{(X+\partial^2_{pR} C)^2}{\partial^2_p C}} \tag{21}$$

In absence of risk and protection, the evolution would be:

$$\partial_t R^s = \frac{\gamma r}{-e^{\gamma t}\partial^2_R f(R^s)} \tag{22}$$

We can also calculate $\partial_t S$ as:

$$\partial_t S = \frac{\gamma r}{-e^{\gamma t}\partial^2_S g(S)} \tag{23}$$

If we now assume that $f(R) = \lambda R^\mu$ and $g(S) = \alpha\lambda S^\mu$, we can solve Eq. (6):

$$S = (\alpha\lambda\mu)^{\frac{1}{1-\mu}} e^{\frac{\gamma t}{1-\mu}} r^{\frac{-1}{1-\mu}} \tag{24}$$

and we have:

$$\frac{\partial_t S}{S} = \frac{\partial_t R^S}{R^S} = \frac{\gamma}{1-\mu} \tag{25}$$

So, with this shape of production function, a productivity growth at rate γ leads to a risk-free economic growth at rate $\frac{\gamma}{1-\mu}$ (i.e. R^s and S grow at rate $\frac{\gamma}{1-\mu}$).

With $p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$, we can rewrite Eq. (5):

$$e^{\gamma t}\lambda\mu R^{\mu-1} = r + pX = r + X\left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}} \quad (26)$$

This equation cannot be solved analytically, but its behavior in special cases can be analyzed.

4.1 Low development level

If productivity is low, capital is also limited (R is small) and thus the probability of occurrence of a disaster is large (p is large).[8] In such an extreme situation, the capital interest rate r is small in the at-risk area compared with the flood-related capital losses pX, and Eq.(26) can be simplified by removing r, leading to the solution:

$$R(t) = R_0 e^{\frac{\gamma}{\frac{\nu}{1+\nu}-\mu}t} \quad (27)$$

If $\mu > \frac{\nu}{1+\nu}$, R decreases and p increases over time. It is likely that this situation corresponds to locations that are abandoned over time, since they are too expensive to protect against natural disasters.

In the situation in which protection improves over time, $\mu < \frac{\nu}{1+\nu}$, the growth rate of capital at risk is $\frac{\gamma}{\frac{\nu}{1+\nu}-\mu}$, which is always larger than $\frac{\gamma}{1-\mu}$. In this case, therefore, the capital at risk grows more rapidly than the safe capital and risk-free income. The relative vulnerability of the economy can be measured by the amount of disaster losses when a disaster occurs divided by income or by the "fraction at risk", i.e. the share of capital at risk R in total capital $R+S$. This vulnerability is increasing over time, as shown in Fig.1 with the fraction at risk. Interestingly, the growth in capital at risk is more rapid when ν is smaller, i.e. when the convexity of protection costs is lower and protection costs increase slowly with the protection level.

Average losses pXR have a growth rate equal to:

$$\gamma_L = \frac{\gamma}{1-\mu\frac{1+\nu}{\nu}} \quad (28)$$

In the situation in which protection improves over time, $\mu < \frac{\nu}{1+\nu}$, average losses increase over time, and they increase more rapidly than $\frac{\gamma}{1-\mu}$ and thus more rapidly than

[8] We assume here that p_0 is large, i.e. that the area-at-risk has a large flood probability in absence of protection, and that $p < p_0$. If productivity is so low that $p = p_0$, then pX can be replaced by p_0X, which is independent of R. Then, Eq. (26) can be simplified by replacing its right-hand-side by $r + p_0X = r'$. In this situation, the capital at risk is lower than in absence of risk ($R < R^s$), but it grows at the same rate. When development increases productivity, there is a time when protection is such that $p < p_0$, and the following calculation holds if p_0 is large enough.

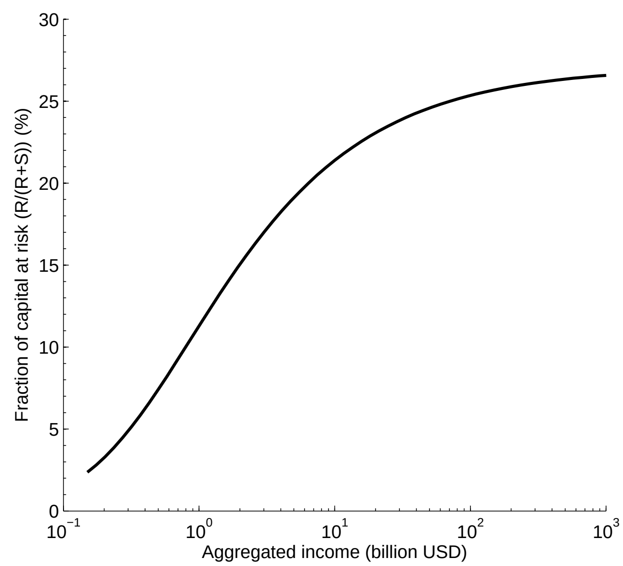

Figure 1: Evolution of the "fraction at risk", i.e. the share of capital at risk R in total capital $R + S$, as a function of time. The fraction at risk increases with development, until it stabilizes at high development level. Calculations using numerical values from New Orleans (see Section 5) and $\alpha = 2$.

risk-free economic growth, i.e. the growth rate of R^s and S (in other terms, the income elasticity of average annual losses is larger than one).

So, in this case, annual disaster losses grow more rapidly than risk-free economic growth at low levels of development, when disaster losses dominate the interest rate in the assessment of the cost of capital, and when the returns on capital at risk are decreasing rapidly, more rapidly than a limit value defined by the shape of the production costs.

In that case, the economic surplus (generated by the at-risk capital) is

$$\mathbb{E}[\pi] = e^{\gamma t} F(R, S) - C(p, R) - (r + pX)R$$

and is growing at the same rate as losses, i.e. $\frac{\gamma}{1-\mu\frac{1+\nu}{\nu}}$. It means that the process of increasing risk-taking leads to a growth in economic surplus that is more rapid than risk-free economic growth. So, increasing risk-taking is also a driver of economic growth (even though the presence of risk leads to a lower output, see proposition 1).

Note that this case is equivalent to the case explored in Hallegatte (2011) where the total amount of capital ($R + S$) is fixed at an exogenous level K, and is independent of the risk level (which is equivalent to $r = 0$ provided that $R \leq K$). It is a situation in which there is no consumption–investment trade-off, and in which the capital at risk can keep increasing more rapidly than growth, until all the capital is located in the risky area. What follows shows that accounting for the consumption–investment trade-off changes significantly the results at high level of development.

4.2 High development level

At a high development level, capital productivity is large, then the amount of capital at risk R is large.[9] As a consequence, the probability of occurrence (p) is small. In such an extreme situation, the capital interest rate r is large compared with flood-related capital losses pX, and Eq.(26) can be simplified by removing pX, leading to the risk-free solution:

$$R(t) = R_0 e^{\frac{\gamma}{1-\mu} t} \tag{29}$$

At high level of development, when disaster probability is very low, the capital at risk grows at the same rate than economic growth. This is why the fraction at risk stabilizes at high income, as shown in Fig.1.

When productivity is high, the average losses pXR has a growth rate equal to:

[9] Using a *reductio ad absurdum* argument, it is easy to show from Eq. (26) that R tends toward infinity when productivity grows. Assume that R is bounded when $t \to +\infty$. In this case, the left-hand-side of Eq. (26) tends toward infinity when t increases, so the right-hand-side has to do the same. In that case, r becomes negligible over time, and the solution of Eq. (26) tends toward Eq. (27), which is not bounded when $t \to +\infty$. This is in contradiction with our initial hypothesis.

$$\gamma_L \underset{t\to+\infty}{\longrightarrow} \frac{\gamma}{1-\mu}\frac{\nu}{(1+\nu)} \tag{30}$$

Average annual losses are thus growing less rapidly than economic growth at high level of development, but they never decrease in absolute terms. At high development level, the growth rate in annual disaster losses is equal to economic growth multiplied by a *"protection factor"* ($\nu/(1+\nu)$), which depends only on the shape of the protection costs and is lower than one.

The protection factor is also the income elasticity of average disaster losses (in a given region, for a given hazard). It should not be confused with the income elasticity of disaster damages (when a disaster occurs), which is larger than one.

In that case, the different terms of the economic surplus from at-risk capital ($\mathbb{E}[\pi] = e^{\gamma t}F(R,S) - C(p,R) - (r+pX)R$) are growing at different rates. The production $e^{\gamma t}F(R,S)$ and capital cost rR are growing at a rate $\frac{\gamma}{1-\mu}$. The protection costs and average losses pXR are growing at a rate $\frac{\gamma}{1-\mu}\frac{\nu}{1+\nu}$, i.e. more slowly than production. It means that when productivity tends to the infinity, the economic surplus is growing at the rate $\frac{\gamma}{1-\mu}$, i.e. at the rate of risk-free economic growth.

4.2.1 Development and disaster trends

Appendix A shows that these results are unchanged if R and S are two production factors in a Cobb-Douglas function. This analysis leads to three conclusions, concerning the trends in capital at risk, average annual losses, and economic surplus.

Proposition 6 *If (i) capital at risk and safe capital can be separated in the production function or are two factors in a Cobb-Douglas function, (ii) protection costs grow less than proportionnally with the amount of capital to protect, and (iii) capital returns are decreasing more rapidly than a threshold that depends on the convexity of protection cost (i.e. if $\mu < \nu/(1+\nu)$), then economic losses in case of disaster (R) grow more rapidly than risk-free economic growth, but their rate of growth converges toward the rate of risk-free economic growth as development proceeds. The relative vulnerability of the economy (the "fraction at risk") is thus increasing over time.*

Proposition 7 *The average annual disaster losses follow a bell-shaped curve in relative terms with income: disaster losses are growing more rapidly than risk-free economic growth at low stages of development and then keep growing in absolute terms but more slowly than risk-free economic growth at higher productivity levels. At high productivity, the growth rate of annual losses is the risk-free economic growth rate reduced by a "protection factor" that depends only on the convexity of protection costs.*

Proposition 8 *The presence of risk reduces the economic surplus. But because of increasing risk-taking, the growth rate of economic surplus is larger than risk-free economic growth at low development level, and it tends to risk-free economic growth at high development level.*

As already stated, this result can be generalized to cases where $C(p, R) = \xi R^c p^{-\nu} + C_0$ and $0 \leq c < 1$. If $c = 1$, then protection is constant and average losses and capital at risk grow at the same rate than economic growth.

Importantly, we do not need capital to be "more productive" in at-risk areas to have capital at risk increasing more rapidly than risk-free economic growth and the capital located in safe areas. We only need a separable production function with two categories of capital (R and S), which are imperfectly substituable and have decreasing returns, and R related to activities located in risky areas. In that case, there is an incentive to invest in at-risk areas to benefit from high marginal returns at low capital levels. Results are then independent of the relative productivity α of the two capitals. The productivity ratio (α) determines the ratio between the two capitals (S/R) at high development level.

5 Numerical application to New Orleans

We apply these formulations to the case of New Orleans, using the following illustrative assumptions:

- The capitals are separable, and we consider only the capital R in the flood-prone area of New Orleans. the capital S is the capital located in safe areas in the rest of the region or the country. The interest rate is $r = 5$ %.

- The area is fixed, and the protection costs depend only on the probability of occurrence p, not on the amount of capital to protect: $C(p, R) = \xi p^{-\nu} + C_0$. In that case, condition (11) is always verified, and p and R evolve in opposite directions.

- In absence of protection, the city would be flooded every year ($p_0 = 1$).

- The cost of protecting New Orleans against category-3 storms is about $3 billion in investment, and we assume a 10% per year maintenance cost; the probability of such a storm is one out of 50 years. The annualized protection cost is $C(1/50) = \$450$ million per year, taking into account the cost of capital and maintenance costs.

- Protecting New Orleans against category-5 hurricane floods would cost about $30 billion, with also a 10% per year maintenance cost, and the probability of such a storm is one out of 200 years, so that $C(1/200) = \$4.5$ billion per year.[10]

[10] State officials estimated the cost of Category 5 protection between $2.5 and $32 billion (Carter, 2005;

Even though each of these assumptions can be discussed, they provide an order of magnitude for the cost of protecting the city. Using these assumptions, we have:

$$C(p) = \xi \left(p^{-\nu} - p_0^{-\nu} \right) \ , \tag{31}$$

with $p_0 = 1$, $\nu = 1.66$, and $\xi = 6.8 \cdot 10^{-4}$.

For New Orleans, we assume that 50% of capital at-risk is lost in case of floods. This large fraction translates the facts that part of the city is under normal sea level and stayed flooded for weeks after Katrina hit the city, amplifying damages to houses and buildings, and that floods occur through dike failure, leading to high water velocity and large damages (RMS, 2005).

The production function is $Y = e^{\gamma t} F(K) = \lambda e^{\gamma t} R^{\mu}$, with $\mu = 0.3$ and $\gamma = 0.015$ (total factor productivity grows by 1.5% per year). The risk-free growth rate, i.e. the growth in the capital S located in safe areas in the rest of the country, is $\frac{\gamma}{1-\mu} \approx 2.1\%$ per year.

The variable Y is the local GDP in the flood-prone areas of New Orleans. Using a New Orleans exposed population of 500,000 people, and the GDP per capita of New Orleans ($24,000 per year in 2009), we have a city GDP of $12 billion. To estimate λ, we solve numerically Eq. (26) to find R as a function of λ, and we chose λ so that the income in absence of disaster is $Y = \lambda R^{\mu} = \$12$ billion (in economic data, protection expenditures are included in income). The value is $\lambda = 3.53$. We find that the optimal capital at risk in New Orleans is $R =$ $59 billion, i.e. about 5 times the local income. Losses in case of flood would be about $30 billion, which is consistent with data for the flood due to Katrina (removing losses due to wind) (RMS, 2005). The optimal probability of flood is found at 2.2% per year, i.e. a return period of 45 years (which is also close to the return period of a category-3 hurricane, i.e. the current protection level in the city).

Then, we can solve numerically Eq. (26) for a series of t, to investigate the dependence of risk to income. We can calculate the trend in $R(t)$ and in average annual loss. We find that R is growing at a rate 2.4% against 2.1% for the growth without risk. Average annual losses due to floods are growing at a rate 1.5%, slower than economic growth. Thanks to increased risk taking, the economic surplus π is growing at a rate 2.2%, i.e. more rapidly than risk-free economic growth.

In the case of New Orleans, therefore, a rational decision-maker would make average disaster losses increase less rapidly than economic growth (1.5 vs. 2.1%), but would increase capital at risk more rapidly than risk-free growth (2.4 vs 2.1%). The consequence is that the average cost of disasters is decreasing relative to regional or national wealth, but the consequences when a flood occurs increase, even relatively to wealth. It means that the New Orleans region evolves toward fewer disasters with consequences that are

Revkin and Drew, 2005; Schwartz, 2005). More recent and detailed estimates by Louisiana Coastal Protection and Restoration (LACPR, led by the U.S. Army Corps of Engineers) reach even larger values.

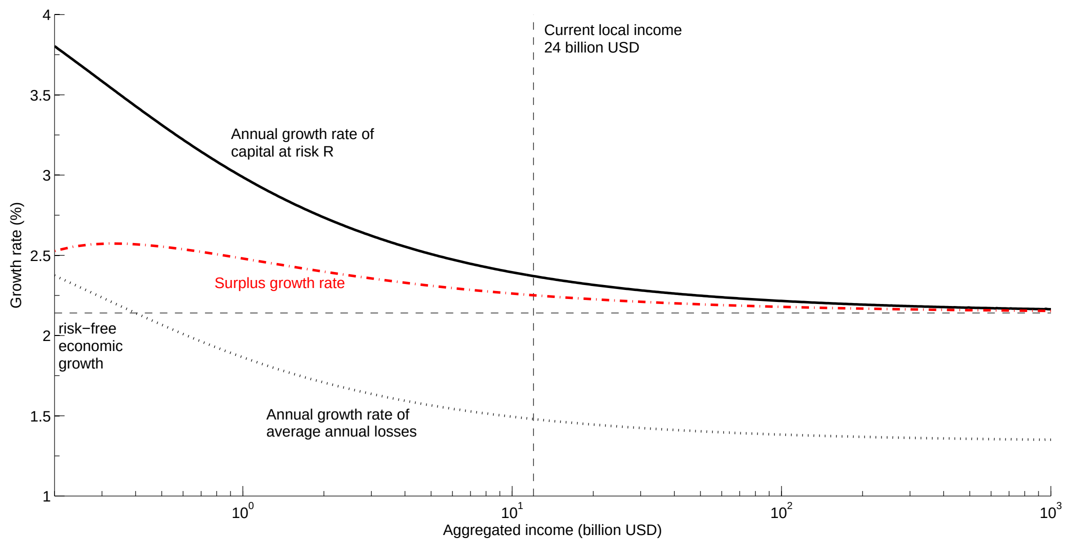

Figure 2: Annual growth rate of capital at risk R, average annual disaster losses, and economic surplus growth rate, as a function of local income. The horizontal line is the rate of risk-free economic growth. Growths in capital at risk and in economic surplus are more rapid than economic growth at early development stage, and these growth rates converge toward economic growth rate over time. Growth in average annual losses is faster than risk-free economic growth at low income and slower at high income. The vertical dashed line shows the current income in New Orleans.

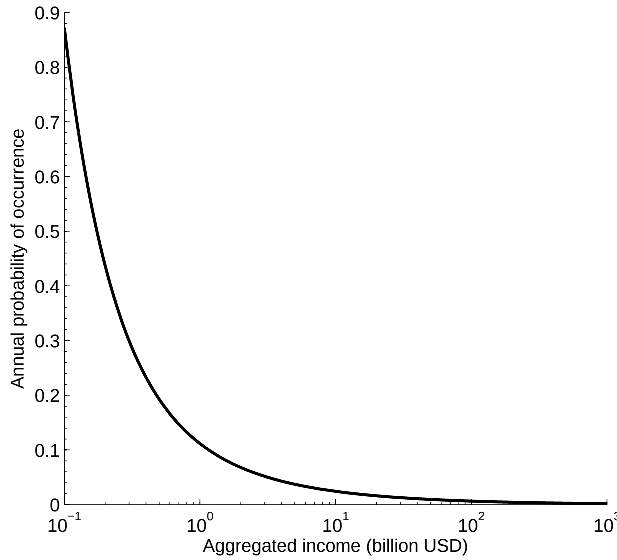

Figure 3: Optimal disaster probability of occurrence in New Orleans, as a function of local income.

growing relative to income, leading to an increased need for recovery and reconstruction support.

One can investigate how this result depends on the local income in New Orleans (and thus on total factor productivity), assuming that everything else remains unchanged (including protection costs $C(p)$ and the fraction of capital at risk lost in case of flood X). Results are reproduced in Fig. 2 and 3. Figure 2 shows the annual growth rate of capital at risk R, of average annual losses (pXR), and of the economic surplus, as a function of local income Y. The vertical line shows the current income in New Orleans, and the horizontal line the rate of risk-free economic growth. At very low productivity, the growth in capital at risk R would be 4.6% per year, i.e. more than twice the rate of risk-free economic growth. In this situation, the annual probability of flood would exceed 80% (see Fig. 3). This growth then converges toward the rate of risk-free economic growth (the horizontal dashed line) as development proceeds. The economic surplus growth rate is also larger than the risk-free economic growth rate at low development level, and it converges toward it as development proceeds.

Figure 1 shows how the fraction at risk — i.e. the fraction of capital located in at risk area — increases with income, assuming $\alpha = 2$ (i.e., that in the absence of risk, this fraction would be 38 percent, according to Eq. (24)).

For average annual disaster losses, the growth rate is 2.8% per year at low development level, i.e. 40% faster than economic growth. At an income of 40 million USD per year in the city, the growth rate of annual losses is equal to the rate of economic growth (2.1%), and this growth rate keeps decreasing until 1.3% per year, which is economic growth (2.1%) corrected by the "protection factor", equal to 0.62 in the case of New Orleans.

In this case, therefore, the income elasticity of average disaster losses would be equal to 0.62 (i.e. a 1% growth in the US would lead to a 0.62% growth in average annual losses) and the income elasticity of disaster losses (when a disaster occurs) would be equal to 1.1 (i.e. a 1% growth in the US would lead a 1.1%-increase in the size of disasters when they occur).

6 Taking into account risk aversion

The present analysis does not include risk aversion, following Arrow and Lind (1970). It thus assumes that the social planner who determines the appropriate level of risk ensures that (i) aggregate losses remain limited compared with national income; (ii) there is risk sharing across individuals in the country to avoid large individual losses[11]; and (iii) there is temporal smoothing of disaster losses, through savings and borrowing (self-insurance) or reinsurance. In absence of these elements, risk aversion needs to be taken into account. This is the case, for instance, in small countries where the entire economy can be affected (as in Grenada after hurricane Ivan in 2004 where losses reached 200% of GDP) and where the risk-free level of capital R (R^s) would be large compared with the rest of the economy.

To take into account risk aversion, we need to introduce an utility function, which we assume to depend on the economic surplus $u(\pi)$ and to have decreasing returns $u'(\pi) > 0$ and $u''(\pi) < 0$. The utility cost of disasters can be approximated by the insurance premium δ that the region would be ready to pay to avoid all losses, which is defined by:

$$u(\pi_0 - \delta) = pu(\pi_0 - L) + (1-p)u(\pi_0) \tag{32}$$

where π_0 is the surplus in absence of disaster and is equal to $e^{\gamma t}F(R,S) - C(p,R) - r(R+S)$. This equation defines a function $\delta(p,R,S,t)$, which replaces pXR in Eq. (3) when risk aversion is accounted for.

$$\mathbb{E}[u] = \overbrace{e^{\gamma t}F(R,S) - C(p,R) - r(R+S) - \delta(p,R,S,t)}^{\tilde{\pi}} \tag{33}$$

And the maximization program becomes:

[11] Since fatalities and casualties cannot be shared, it means that forecasts and early warning systems reduce human losses, as is observed in most developed countries where economic losses have increased while human losses have decreased.

$$\max_{p,R,S}\ \widetilde{\pi} \quad s.t. 0 \leq p \leq p_0 \tag{34}$$

We use a constant relative risk aversion (CRRA) utility function under the form $u = \frac{\pi^{1-\rho}}{1-\rho}$, then we have:

$$\delta = \pi_0 - \left[p(\pi_0 - RX)^{1-\rho} + (1-p)\pi_0^{1-\rho}\right]^{\left(\frac{1}{1-\rho}\right)} \tag{35}$$

Since $\delta > pRX$ in presence of positive risk aversion, the taking into account of risk aversion makes perceived risk larger and creates a non-linearity between R and risk. To go further, the optimization program can be solved numerically with the parameters and functional forms from Section 5 on New Orleans, and using the same methodology to calibrate λ. Since risk aversion introduces total income in the equations of R and p, it creates a link between R and S even when the two capitals are separable in the production function. It means that the value of S (i.e. α) also needs to be calibrated. In practice, the value of S depend on how disaster risks in New Orleans are shared with risk-free capital (or capital that is subject to a risk independent of hurricane risk). As an illustration, equations are solved assuming that $\alpha = 2$.

Results for the fraction at risk are presented in Fig. 4, for a risk aversion $\rho = 2$. It shows that risk aversion leads to locating less capital in at risk areas, at all development levels.

Figure 5 shows that risk aversion has an ambiguous impact on the probability of occurrence: at low development level, the capital at risk is so much smaller with risk aversion that it is optimal to increase the probability of occurrence; at higher development level, risk aversion leads to better protection and to a decrease in the probability of occurrence. Finally, Fig. 6 shows that capital at risk still grow more rapidly than income at all development levels, and the growth rate converges toward the risk-free growth rate. At high development level and in this simulation, the capital at risk is lower but increases more rapidly with risk aversion than without risk aversion. At high development level, average annual losses grow at a lower rate than risk-free economic growth, like in the case without risk aversion.

Numerical simulations suggest therefore that the qualitative results in the case without risk aversion remain valid with risk aversion. An exploration of various values of risk aversion (ρ) and of various risk sharing level (modeled through α here) confirms that results are robust to the presence of risk aversion.

7 Taking into account biases in risk perception

The assumption of rational and perfectly informed decision-making is clearly an oversimplification of reality. The analysis of disaster protection and risk in such a framework

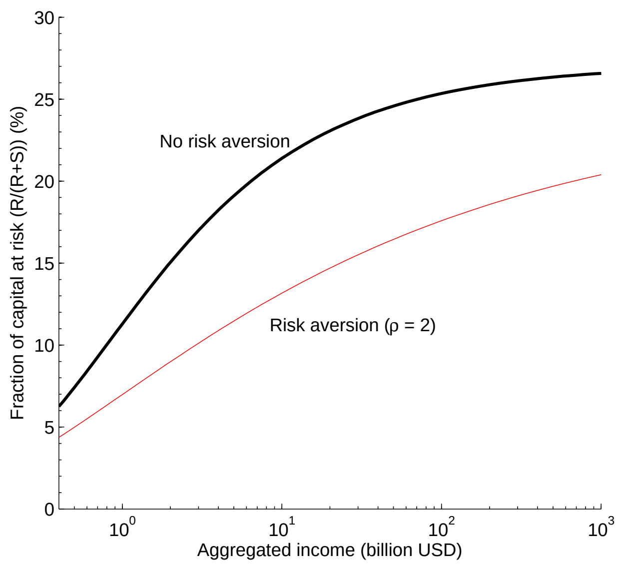

Figure 4: Evolution of the "fraction at risk", i.e. the share of capital at risk R in total capital $R+S$, as a function of time, with and without risk aversion. Risk aversion reduces the fraction at risk at all development levels. Calculations using numerical values from New Orleans (see Section 5) and $\alpha = 2$.

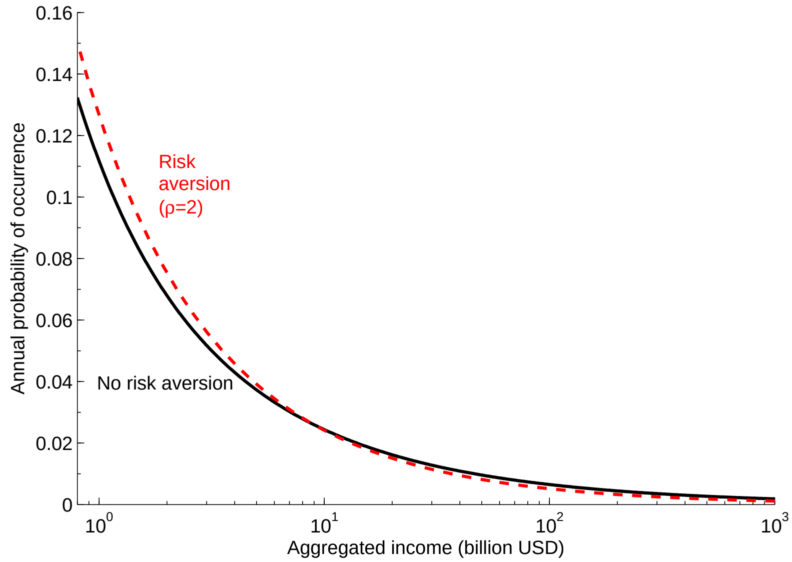

Figure 5: Evolution of the annual probability of occurrence, without risk aversion and with risk aversion ($\rho = 2$).

suggests that — in a normative context — it is optimal from an economic perspective to increase risk taking with development (provided that fatalities can be avoided) and disasters should become rarer and more costly as income grows.

But interpreting real-world disaster loss data series would require a descriptive approach, where realistic characteristics of decision-making are considered.[12] To do so, the analysis needs to include behavior imperfections.

First, there are information and transaction costs. Since the information on natural hazards and risk is not always easily available, households and businesses may decide not to spend the time, money and effort to collect them, and disregard this information in their decision-making process (Camerer and Kunreuther, 1989; and Hogarth and Kunreuther, 1995).

Second, risk perceptions are sometimes biased by cognitive failures. Indeed, individuals do not always react rationally when confronted to small probability risks, and they defer choosing between ambiguous choices (Tversky and Shafir 1992; Trope and Lieberman, 2003). Moreover, private and public investment decisions do not always adequately take long and very long-term consequences into account (Kunreuther et al. 1978; Thaler, 1999). Kahneman and Tversky (1979) proposed the prospect theory to better explain

[12]Other features may be needed to fully explain the current trends in disaster losses, such as the role of externality and moral hazard (e.g., Laffont 1995).

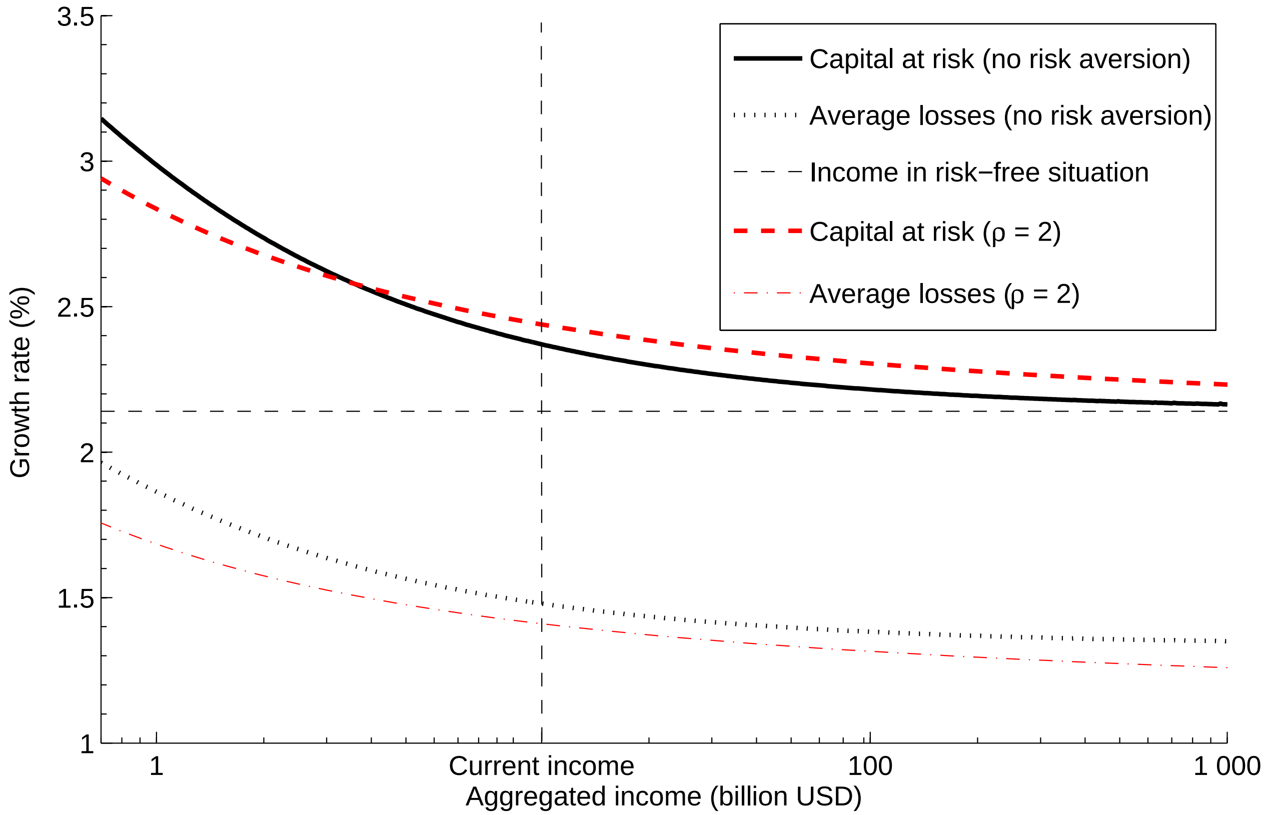

Figure 6: Evolution of capital at risk R and average annual disaster losses as a function of time, without risk aversion and with risk aversion ($\rho = 2$).

individual behaviors, taking into account observed behaviors and experimental results.

In the following, we modify the model to account for some of these effects. We assume that capital investment decisions are made with imperfect knowledge, or with biases in risk perception or behavior. This assumption is consistent with the observation that most capital investment decisions are not made using all available disaster risk information.

On the other hand, we model protection decisions as made with perfect knowledge of natural risks and assuming (wrongly) that capital investment decisions will then also be made optimally and with perfect knowledge. There is thus an inconsistency in the model between protection decisions and capital investment decisions. This hypothesis is justified by the fact that (public and private) decisions concerning disaster protections (e.g. the design of a dike system) are most of time designed through sophisticated risk analyses, taking into account all available information and assuming optimal behaviors.

We borrow from the prospect theory (Kahneman and Tversky, 1979) the idea that people weight different outcomes of a decision not using the probability of the outcome (here, the probability of occurrence p), but a "decision weith" π. In a classical decision-making framework — based on expected utility maximization — π can be interpreted as the perceived probability of disaster occurrence, and the difference between p and π is the bias in risk perception. In a prospect theory framework, the decision weights "should not be interpreted as measure of belief" but they can also be influenced by other factors such as ambiguity (Kahneman and Tversky, 1979). In this case, there are thus two distinct issues: the misestimation of probability of occurrence (due for instance to biased risk perception) and over- or under-weighting of some possible outcomes (which is a preference, not a mistake).

In the model, π is used by investors to decide of the amount of capital to install in at-risk areas. In this section, we assume however that potential losses remain small enough for the utility function (in an expected utility maximization framework) or the value function (in a prospect theory context) to remain linear.[13] Equation (5) therefore becomes:

$$e^{\gamma t}\partial_R F(R,S) - \partial_R C - (\pi X + r) = 0 \tag{36}$$

We investigate two ways of modeling the decision weight π. The first model assumes a static relationship between the actual probability of occurrence p and the decision weight π. The second model takes a dynamic view on risk perception and introduces myopic adaptive expectations.

[13] And the reference point used by decision-makers to assess a situation is a situation with zero economic surplus.

7.1 Systematic perception bias or decision weighting

A first model can be proposed where the relationship between p, the real probability of occurrence, and π, the decision weight, can be represented as:

$$\pi = B(p) \tag{37}$$

Kahneman and Tversky (1979) propose that (i) the decision weight is higher than probability for low probabilities; (ii) "subcertainty" means that π is less sensitive to change in probabilities than perceived probability; (iii) π changes abruptly near the end-points, with $\pi(0) = 0$ and $\pi(1) = 1$. One function that satisfies these conditions is:

$$\pi(p) = \begin{array}{ll} 0 & \text{if } p < p_{min} \\ 1 & \text{if } p > p_{max} \\ p_b + p^\beta & \text{otherwise} \end{array} \tag{38}$$

Events with a probability below p_{min} are considered impossible; low-probability events with probability higher than p_{min} are overweighted; and non-certain higher-probability events are underweighted.

Calculation from Section 4 can be redone with decision weight, leading to replace Eq. (26) by:

$$e^{\gamma t} \lambda \mu R^{\mu - 1} = r + \pi X \tag{39}$$

At very high development level, protection is so high that $p < p_{min}$ and disasters are considered impossible. This may be the situation in the Netherlands for most decision-makers. In that case, as in previous cases, the capital at risk R grows as fast as risk-free economic growth. At higher development level, $p > p_{min}$ and we have:

$$e^{\gamma t} \lambda \mu R^{\mu - 1} = r + X p_b + X \left(\frac{X}{\nu \xi} \right)^{-\frac{\beta}{1+\nu}} R^{-\frac{\beta}{1+\nu}} \tag{40}$$

In that case, a high productivity still leads to a situation where the fixed term $r + X p_b$ dominates the right hand side of Eq. (40), and the capital at risk R still grows as fast as risk-free economic growth.

So at high development level, risk perception bias or a prospect theory decision framework still leads to a growth rate in capital at risk that is as fast as risk-free economic growth, and thus to a growth rate in average annual losses that is lower than risk-free economic growth (namely, the risk-free rate reduced by the protection factor, like in previous cases).

Bias in risk perception or different decision-making however changes results at lower development level. If the fixed term $r + X p_b$ is dominated in the right hand side of Eq. (40), then the growth rate in capital at risk R becomes:

$$\gamma_R = \frac{\gamma}{1 - \mu - \frac{\beta}{1+\nu}} \tag{41}$$

in locations where capital at risk R and the protection level would be increasing with economic growth (i.e., where the probability of occurrence p decreases), the presence of a large underestimation (or underweighting) of risk can lead to the opposite outcome, that is a decrease of capital at risk and protection level over time.

If the growth rate of R is positive ($\mu < 1 - \frac{\beta}{1+\mu}$), then this growth rate increases with β. It means that capital at risk increases more rapidly if hazards are more under-weighted. Regardless of risk perception, however, the growth in capital at risk is larger than capital in a risk-free situation, and the economy evolves toward more risk taking.

Average losses pXR have a growth rate equal to:

$$\gamma_L = \frac{\gamma}{1 - \mu\frac{1+\nu}{\nu} + \frac{1-\beta}{\nu}} \tag{42}$$

In the situation in which protection improves over time, $\mu < \frac{\nu}{1+\nu}$, and in presence of risk under-weighting, average losses increase over time, and they increase more rapidly than with perfect information ($\beta = 1$), and thus more rapidly than risk-free economic growth, i.e. the growth rate of R^s and S. In short, the introduction of a systematic bias in risk perception (or of decision weights instead of probabilities) does not change the main conclusion of this paper, namely that development leads to more risk taking (capital at risk increases more rapidly than risk-free economic growth). Risk under-weighting or under-estimation can however amplify this effect.

7.2 Myopic expectations

This modeling is a simplification, especially because it disregards the dynamics of risk perception. In practice, it is likely that perceived risk is higher than actual risk during the years following an event, and lower after some times and when the memory of disaster losses has lost its acuteness. A second model can be proposed where decisions on the amount of capital to install in the risky area are based on a disaster probability that is estimated empirically, based on previous disasters (Hallegatte 2011). The empirically estimated disaster probability is π and is given by:

$$\pi(t) = \frac{1}{\tau}\int_{u=-\infty}^{u=t} e^{-\frac{t-u}{\tau}} F(u)du \tag{43}$$

Where $F(u)$ is equal to one if a disaster occurred at time u, and zero otherwise.

This modeling corresponds to backward-looking adaptive expectation, in which past events have an exponentially decreasing weight (with time scale τ). In other terms, agents assess future disaster risks from past events, with a memory characteristic time τ. The consequence is that the estimated disaster probability is higher than the real one just after a disaster, and lower than the real one when no disaster has occurred for a while. This behavior appears consistent with many observations (e.g., Kunreuther and Slovic, 1978; Tol et al., 1998).

The efficiency of this empirical process depends on the disaster probability. If there are many disasters over a period τ (i.e. if $1/p << \tau$), the estimated probability remains close to the real one. If the memory is too short, i.e. if τ is too low, then the estimated probability will often be different from the real one.

Here, we are interested in the dynamics between two disasters.[14] Assuming that the last disaster occurs at time t_0, we have $F(t) = 0$ for $t > t_0$, and:

$$\pi(t) = \frac{1}{\tau}\int_{u=-\infty}^{u=t_0} e^{-\frac{t-u}{\tau}} F(u)du + \frac{1}{\tau}\int_{u=t_0}^{u=t} e^{-\frac{t-u}{\tau}} F(u)du = \pi(t_0)e^{-\frac{t-t_0}{\tau}} \tag{44}$$

Replacing p by π in Eq. (26) gives:

$$e^{\gamma t}\lambda\mu R^{\mu-1} = r + \pi X = r + X\pi(t_0)e^{-\frac{t-t_0}{\tau}} \tag{45}$$

At high development level and if economic actors know that disaster probability is low, then πX is very small compared with r, and the introduction of myopic expectations does not change anything: capital at risk increases as fast as economic growth, like in all other cases. This is also a case in which periods between disasters are the longest, and therefore the risk is most underestimated. Fortunately, it corresponds to situation where risk is a minor component in the cost of capital, and the consequences of this underestimation are minor.

But at low development levels, and where risk perception is high (π is large), then Eq. (45) can be simplified, and the growth rate in capital at risk R is equal to:

$$\gamma_R = \frac{\gamma + \frac{1}{\tau}}{1-\mu} = \gamma_S + \frac{1}{\tau(1-\mu)} \tag{46}$$

In that case, therefore, capital at risk increases faster than the risk-free economic growth (γ_S), like in other cases. But in this case, the growth rate is independent of the shape of the protection cost function (ν), and only depends on the expectation timescale τ.

Since protection is done with perfect knowledge about the amount of installed capital, the actual probability of occurrence remains equal to $p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$, and we have:

$$\gamma_L = \frac{\gamma}{1-\mu}\frac{\nu}{(1+\nu)} + \frac{1}{\tau(1-\mu)}\frac{\nu}{(1+\nu)} \tag{47}$$

[14] A more complete dynamical analysis is made using a numerical model in Hallegatte (2011).

The last term of the equation is due to myopic expectations, and it leads to an increase in the growth rate of average annual losses. Since the risk-free growth is equal to $\frac{\gamma}{1-\mu}$, the growth rate of annual losses can be either slower or faster than risk-free economic growth, depending on the values of ν and τ.

At low development level and with myopic expectations, annual average losses are growing more rapidly than risk-free growth if:

$$\nu > \gamma\tau(1-\mu) \tag{48}$$

This is the case if protection costs increase rapidly with the desired safety standard (ν is large), if expectation are short-sighted (τ is small), but also if economic growth is slow (γ is small) or if the production function of capital at risk is close to constant return (μ is close to one). Using parameter values from our New Orleans case study, this condition is met if τ is lower than 158 years, which is a very long timescale. It seems therefore possible that at low development level, when natural risks dominate in the cost of capital, a dynamic bias in risk perception leads to mean annual losses that increase more rapidly than risk-free economic growth.

8 Conclusion and discussion

This paper proposes an economic analytical framework to analyze the trade-off between disaster losses and investment returns in areas at risk from natural hazards. This trade-off is analyzed under various assumptions on decision-making, including the presence of risk aversion, biases in risk perception, and alternative decision theories such as the prospect theory.

Under conditions that ensure that protection improves over time, the presence of risk and the possibility to protect against disasters lead to a lower amount of capital in risky area (compared with the risk-free situation), but it also increases the growth rate of capital at risk where protection costs increase less rapidly than the amount of protected capital (i.e. where investments are at least partly done by increasing capital density and concentration).

By improving protection, economic development drives the economy toward more risky behaviors (i.e. a growing share of capital is installed in at-risk areas). Protection reduces the probability of occurrence of an event, but its impact on risk is more complex. In particular, it transfers part of the risk from one kind of risk (frequent and low-cost events) to another kind (exceptionnal and high-impact events).

Reciprocally, the increasing in risk-taking is found to accelerate economic growth. Along an optimal growth pathway, increasing risk-taking is thus both a driver and a consequence of economic development. This interlinkage between development and risk taking suggests that risk should not be reduced at all cost.

By focusing on average losses that mix probability and loss (e.g., Toya and Skidmore, 2007; Rashky, 2008), econometric studies have left out of their analysis the potential increase in damages *when a disaster occur*. Current trends in disaster losses appear however consistent with the prediction of fewer but larger disasters (e.g., Etkin, 1999; Nordhaus, 2010; Bouwer et al., 2007; Pielke et al., 2008; Bouwer, 2011; Schumacher and Strobl, 2011). These results are also in line with UN-ISDR (2009), which observes that poor countries suffer from frequent and low-cost events, while rich countries suffer from rare but high-cost events.

The paper suggests that natural disasters will become less frequent but more costly with development and economic growth, and this result has some policy-relevant consequences. In particular, it means that development requires more resilience, i.e. an improved ability to deal with and recover from rare events, which exceed the protection capacity. The Tohoku Pacific earthquake could thus be an illustration of the type of events the world will have to deal with in the future. Such a trend toward larger disasters translates into a strong and increasing need for crisis management and post-disaster support, through (1) forecasts and early warning to mitigate human losses (e.g., Subbiah et al., 2008; Hallegatte 2012); (2) rainy-day funds and insurance and reinsurance schemes to support reconstruction (e.g., Ghesquiere and Mahul, 2010; Jaffee et al., 2010; Michel-Kerjan, 2010); and (3) new international instruments for post-disaster support and solidarity (e.g., Linnerooth-Bayer et al. 2009). Finally, the growing role of exceptional disasters, on which knowledge and data is the scarcest, call for decision-making processes that are able to cope with deep uncertainty and the so-called “black swans” (Lempert and Collins, 2007; Paté-Cornell, 2012; Hallegatte et al., 2012). Such low-probability high-impact events represent a challenge for the design of development policies.

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A Capital at risk and safe capital as substituable inputs in a Cobb-Douglas function

If R and S are imperfectly substituable, we can assume that $F(R,S) = \lambda R^{\mu_1} S^{\mu_2}$. This section demonstrates that this situation is similar to the situation where R and S are separable in the production function.

In this case, the marginal productivity of S gives us:

$$S = \left(\frac{e^{\gamma t}\lambda \mu_2}{r}\right)^{\frac{1}{1-\mu_2}} R^{\frac{\mu_1}{1-\mu_2}} \tag{49}$$

With $p = \left(\frac{RX}{\nu\xi}\right)^{-\frac{1}{1+\nu}}$, we have:

$$pX = X\left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}} \tag{50}$$

and the marginal productivity of R gives us:

$$\lambda\mu_1\left(\frac{\lambda\mu_2}{r}\right)^{\frac{\mu_2}{1-\mu_2}} e^{\frac{\gamma t}{1-\mu_2}} R^{\frac{\mu_1+\mu_2-1}{1-\mu_2}} = r + X\left(\frac{X}{\nu\xi}\right)^{-\frac{1}{1+\nu}} R^{-\frac{1}{1+\nu}} \tag{51}$$

Here, we can use the same approach as before.

A.1 Low development level

At low level of development, and using the same assumptions on p_0, pX is larger than r, and the equation can be approximated by assuming that $r << pX$, which gives:

$$R(t) = R_0 e^{\frac{\gamma}{1-(\mu_1+\mu_2)-\frac{1-\mu_2}{1+\nu}} t} \tag{52}$$

So R is increasing if $1-(\mu_1+\mu_2) > \frac{1-\mu_2}{1+\nu}$, i.e. if $\nu > \frac{\mu_1+2\mu_2}{1-\mu_1-\mu_2}$. Using classical values for decreasing return (i.e. $\mu_1+\mu_2 \approx 0.3$), and assuming that the capital at risk and the safe capital have the same exponent, it leads to $\nu > 0.64$, which is the case if protection costs are convex.

Since economic growth in absence of risk would be $\frac{\gamma}{1-(\mu_1+\mu_2)}$, the capital at risk increases more rapidly than risk-free economic growth. Average losses $\mathbb{E}[L] = pXR$ are growing at a rate:

$$\gamma_L = \frac{\gamma}{1-(\mu_1+\mu_2)-\frac{1-\mu_2}{1+\nu}} \frac{\nu}{\nu+1} \tag{53}$$

Average losses increase more rapidly than risk-free economic growth if:

$$\frac{\gamma}{1-(\mu_1+\mu_2)-\frac{1-\mu_2}{1+\nu}}\frac{\nu}{\nu+1} > \frac{\gamma}{1-(\mu_1+\mu_2)} \tag{54}$$

If R is increasing, then the denominator is positive, and this inequality is always verified. So, in this setting, at low level of development and under mild conditions insuring that the probability of occurrence decreases with time, average disaster losses increase more rapidly than risk-free economic growth.

A.2 High development level

At high level of development, pX is very small compared with r, and the equation can be solved by assuming at $pX = 0$:

$$R(t) = R_0 e^{\frac{\gamma}{1-(\mu_1+\mu_2)}t} \tag{55}$$

Which is also the rate of risk-free economic growth. Average losses $\mathbb{E}[L] = pXR$ are then growing at a rate:

$$\gamma_L = \frac{\gamma}{1-(\mu_1+\mu_2)}\frac{\nu}{\nu+1} \tag{56}$$

In this case, the growth rate of disaster losses is lower than the rate of risk-free economic growth. Indeed, the growth rate in annual disaster losses is equal to economic growth multiplied by the same *"protection factor"* $\nu/(1+\nu)$ as in the case of a separable production function.

Proposition 9 *The case where R and S are substituable inputs in a Cobb-Douglas function is equivalent to the case where the production function is separable.*