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A New Distribution Sensitive Index for Measuring Welfare, Poverty, and Inequality

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Abstract

Simple welfare indices such as mean income are ubiquitous but not distribution sensitive. In contrast, existing distribution sensitive welfare indices are rarely used, often because they are difficult to explain and/or lack intuitive units. This paper proposes a simple new distribution sensitive welfare index with intuitive units: *the average factor by which individual incomes must be multiplied to attain a given reference level of income.* This new index is subgroup decomposable with population weights and satisfies the three main definitions of distribution sensitivity in the literature. Variants on this index can be used as distribution sensitive poverty measures and as inequality measures, with the same simple intuitive units. The properties of the new index are illustrated using the global distribution of income across individuals between 1990 and 2019, as well as with selected country comparisons. Finally, the index can be used to define the "prosperity gap" as a proposed new measure of "shared prosperity," one of the twin goals of the World Bank.

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A New Distribution Sensitive Index for Measuring Welfare, Poverty, and Inequality

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1 Introduction

Average income per capita and the poverty headcount are among the most common measures used to summarize living standards and to assess progress towards national and international development goals. The popularity of these welfare measures is in no small measure due to their simplicity, which makes them easy to understand and to communicate to wide audiences. However, these measures also have a fundamental limitation: they are silent on the extent of inequality among all individuals (in the case of average income) and inequality among the poor (in the case of the headcount).

This limitation does not sit well with the widely held view that inequality matters: specifically, that welfare measures should be more responsive to changes in income that take place at the bottom of the distribution. This view has at least four justifications. First, with decreasing marginal utility of income, the social preferences of a utilitarian policymaker give more weight to those at the bottom of the distribution.[1] Second, most major philosophical traditions and religions express a special concern for the worse-off (World Bank, 2005). Third, the fact that several of the Sustainable Development Goals have distributional elements reveals a preference among international policymakers for lower inequality.[2] Fourth, direct empirical estimates suggest a high degree of inequality aversion, although the variance of estimates is large (Clark and D'Ambrosio 2015, Kot and Paradowski, 2022).

This limitation is well-understood, and a long literature has generated an impressive variety of distribution sensitive welfare measures to address it.[3] However, these measures are rarely used in practice. To illustrate this, Table 1.1 displays the results of a simple keyword search for leading poverty and welfare measures, in three bodies of documents: academic research papers in EconLit; policy documents of major multilateral development banks as curated at nlp4dev.org; and general writing in the Google N-gram Viewer. Across all three categories of documents, the frequency of references to poverty headcount is two to three orders of magnitude larger than the frequency of references to the most common distribution sensitive poverty measures, i.e., the squared poverty gap and the Watts index. The pattern is similar for welfare indices in the bottom panel: references to average income are two orders of magnitude more common than references to the best-known distribution sensitive welfare measures like the Atkinson and Sen indexes.

[1] This is related to prioritarianism, an ethical theory that gives extra weight to the well-being of the worse-off (Adler and Norheim, 2022).

[2] Such preferences are not just revealed but explicitly stated in international efforts to move beyond per capita GDP as a welfare measure. For example, the Stiglitz-Sen-Fitoussi Commission on the Measurement of Economic Performance and Social Progress criticized measures of per-capita income and wealth because they give no indication of how the available resources are distributed across people (Stiglitz, Sen and Fitoussi 2009).

[3] For example, early critiques of the headcount index for its lack of distribution sensitivity go back to Watts (1969) and Sen (1976). Distribution sensitive poverty and welfare measures have been introduced by numerous scholars, including Watts (1969), Atkinson (1970), Sen (1976), Thon (1979), Takayama (1979), Kakwani (1980), Clark, Hemming, and Ulph (1981), Donaldson and Weymark (1980), Chakravarty (1983), Foster, Greer, and Thorbecke (1984), Hagenaars (1987), Chakravarty (2009).

Table 1.1. Use of Poverty and Welfare Measures in Academic, Policy, and Public Discourse

	Academic Use	Policy Use	Public Discourse
	*EconLit**	*Multilateral Development Banks at nlp4dev.org**	*Google Ngram***
Poverty Measures			
Poverty Rate OR Poverty Headcount ***	40832	26730	40.49
Poverty Gap	4438	4609	6.84
Squared Poverty Gap	473	681	0.32
Watts Index	47	52	0.09
Welfare Measures			
Average income	61493	15492	36.11
Atkinson Index	744	122	0.49
Sen Index	258	70	0.12

Notes: * Number of documents. ** 1 million times fraction of corpus in 2019. *** Search results in last column for "poverty rate" only as Ngram viewer does not support Boolean search. Search results retrieved March 22-24, 2023.

One plausible reason for the limited uptake of distribution sensitive measures is that they are difficult to explain in non-technical terms. For example, it is challenging to define the Atkinson index as anything other than a constant elasticity of substitution aggregate of individual incomes. Compounding the problem, distribution sensitive measures also often have non-intuitive units. For example, the squared poverty gap is measured in squared percent deviations from the poverty line, while the Watts poverty index is measured in log-point differences from the poverty line that cannot easily be interpreted as percent differences unless they are small. While these complications of interpretation and units are of little concern to mathematically minded users, they represent only a small proportion of the audience for welfare measures. For the wider audience of policymakers and the public, "average income in dollars" or "the number of poor people" are far more understandable, and therefore far more effective in influencing policy and public opinion despite their shortcomings. This dilemma is well-summarized by the succinct remark in Watts (1969) that the headcount has "*little but its simplicity to recommend it.*"

To address this problem, we propose a new distribution sensitive welfare index with a simple mathematical formulation and intuitive units. Our basic measure, described more formally in Section 2.1, is the average across all individuals of z/y_i, where y_i is the income of individual i and z is a reference income level. As an average, its formulation is simple to describe. Moreover, the average has very intuitive units. Each term in the average is the factor by which the income of the corresponding individual needs to be multiplied to attain the reference level of income, and the index simply is *the average factor by which individual incomes need to be multiplied to reach the reference level of income*.

In addition to being extremely simple to explain, this measure has several desirable properties. In Section 2.3, we show that it satisfies all three definitions of distribution sensitivity in the literature: Pigou-Dalton sensitivity, transfer sensitivity, and growth sensitivity. In addition, the new index is subgroup decomposable with population weights, which means for example that its value at the world level is simply a population-weighted average of its value in each country in the world. In fact, we show that our measure is unique among all existing named welfare indices we could identify in the literature in that it is both population-weighted subgroup decomposable and satisfies all three concepts of distribution sensitivity.

We develop two variants on our basic measure. The first variant limits attention to individuals with incomes below the reference income level by censoring incomes above it (Section 2.2). This results in a traditional poverty measure that is responsive only to changes in income below the reference income level. The second one transforms our welfare index into an associated relative inequality measure by replacing the absolute level of reference income with average income (Section 2.5). In both cases, the variants retain the simple interpretation of the welfare index: *the average factor by which individual incomes must be multiplied to attain the reference level of income*.

We illustrate the empirical properties of our new welfare index in Section 3. We first document levels and trends in the new measure calculated across the world income distribution between 1990 and 2019. This was a period of strongly pro-poor growth, in the sense that growth was on average much higher in lower percentiles of the world income distribution than in higher percentiles. Consistent with its strong distribution sensitivity, we show our new welfare index improves substantially faster than other welfare measures over this period. In addition, we focus on three pairs of countries, a pair of low-income countries (Burkina Faso and Mali), a pair of middle-income countries (Colombia and Peru) and a pair of high-income countries (France and the United States), showing that our new measure can lead to reordering of welfare comparisons relative to existing measures.

In Section 4, we conclude with a specific policy application of the new welfare index. Since 2013, the World Bank has articulated its twin goals as "*ending extreme poverty and boosting shared prosperity*" (World Bank, 2015). Extreme poverty is measured as the share of the world's population living below the international poverty line, currently set at $2.15 per day in 2017 $PPP, while shared prosperity is measured at the country level as the growth rate of average incomes in the bottom 40 percent. We propose using our new index a new measure of shared prosperity, and refer to it as the global "prosperity gap". In this application, we set the reference income level to $25 per day, a "prosperity standard" roughly corresponding to the median poverty line of high-income countries. The global prosperity gap in 2019 is 5.0, indicating that on average incomes need to be multiplied by a factor of 5.0 to reach the prosperity standard. We show how the prosperity gap varies across countries and over time, and demonstrate that it has better theoretical, empirical, and communications properties than growth in the bottom 40 percent. Readers who are primarily interested in the prosperity gap measure can skip Sections 2 and 3 of the paper and jump directly to Section 4.

2 A New Distribution Sensitive Index

2.1 Basic notation and intuition

The new index is given by:

$$W(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i}, \tag{1}$$

where $i = 1, \ldots, N$ indexes individuals; y_i denotes individual i's economic resources, which we refer to as "income" throughout the paper; $y \equiv (y_1, \ldots y_N)$ denotes the distribution of incomes ordered from smallest to largest; and $z > 0$ is a parameter whose value is interpreted as a reference income level. As discussed further below, z can be a poverty line if it is set at some accepted fixed minimum level; it can be a "prosperity standard" if it is set as some accepted fixed higher aspirational level; or it can be replaced by a linear function of incomes $z(y)$ such as average income to obtain a measure of relative inequality.

The key innovation of our index is to express the contribution of individuals to the welfare index as a ratio of the reference level of income z to their individual income y_i. This ratio represents the factor by which their income must be multiplied to reach z. By contrast, existing poverty indices typically express individual contributions as a difference or gap $z - y_i$. Expressing the contribution of individuals as a ratio has two advantages. First, the ratio formulation $\frac{z}{y_i}$ is a convex function which is distribution sensitive, as we discuss more formally in Section 2.3. By contrast, the simple difference or gap $z - y_i$ is a linear function and is therefore not distribution sensitive. To make it distribution sensitive, a further convex transformation needs to be applied (e.g., exponentiation or minus log), at the cost of complicating interpretation. Second, the ratio formulation leads to two intuitive and appealing interpretations:

(1) The index is literally the average factor by which individual incomes must be multiplied to reach z. It is therefore intimately related to economic growth, capturing the average (gross) growth rate needed to reach z.[4] This property also means that the index responds intuitively to distribution-neutral growth: when all incomes are doubled, the value of the index is divided by two.

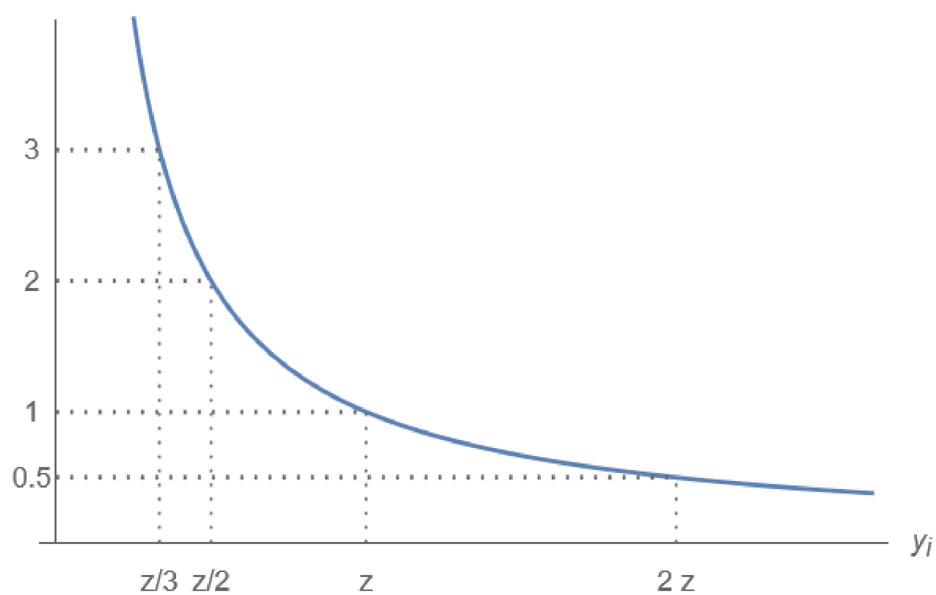
(2) The new index captures the intuition that if person A has half the income of person B, person A is twice as "poor" as person B. The index is a simple average of individual contributions, where the contribution of a person at the threshold z is 1, the contribution of a person with half the threshold z is 2, the contribution of someone with one-third of the threshold z is 3, and so on (Figure 2.1). In this way, individual contributions to the overall index are intuitively and transparently distribution sensitive. In Appendix B we

[4] We note the parallel between this interpretation and Morduch (1998)'s interpretation of the index of Watts (1969) as continuously compounded growth needed to get to z in T years.

demonstrate that our new welfare index is the only symmetric subgroup decomposable welfare index that satisfies this proportionality property.

Finally, the value selected for the reference income z affects the value taken by the index: higher values of reference income naturally imply a greater average multiple needed to reach that reference income level. However, it does not affect the ranking of distributions associated with index W: rather, it only scales the value of the index by a fixed factor equal to z.

Figure 2.1 - Individual contribution (ratio $\frac{z}{y_i}$) as a function of income y_i.



2.2 Censored indices

The new index is inclusive in the sense that it is affected by the incomes of all individuals. This is a typical feature of welfare indices. By contrast, poverty indices conventionally focus exclusively on individuals who earn less than the reference income, i.e. the poverty line. We propose two ways of adjusting the general index in Equation (1) to focus on individuals earning less than the reference income. The first approach is simply to top-code incomes at the reference income z. This yields the following censored version of our measure:

$$C(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i^z}, \qquad (2)$$

where censored income y_i^z is defined as $y_i^z \equiv y_i$ if $y_i < z$ and $y_i^z \equiv z$ if $y_i \geq z$. The interpretation of the index C is the same as that of the index W, except that it applies to the censored distribution of incomes. It retains the simple intuitive interpretation as the *average factor by which incomes* *need to be multiplied to attain the standard of living defined by the threshold* z, although with the minor qualification that no increase, i.e., a factor of one, is needed for people with incomes above the threshold.

The second approach is closer in spirit to the poverty measurement literature because it forces individual contributions to be equal to zero (as opposed to one for C above) for individuals with income greater than or equal to z. To do this, we define the index $P = C - 1$, which yields the expression:

$$P(y,z) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{z}{y_i^z} - 1\right) = \frac{1}{N}\sum_{i=1}^{N_P}\left(\frac{z - y_i}{y_i}\right), \tag{3}$$

where N_P corresponds to the number of individuals with incomes below z.

By definition, both P and C yield the same ranking of income distributions. However, P can be interpreted as a *growth rate* instead of a factor: it is *the average growth rate needed to attain the standard of living defined by the threshold*, again with the qualification that zero growth is needed for people who are above the threshold.

2.3 Properties

The index is not only easy to interpret and communicate, but it also satisfies two key properties that make it especially useful for policy purposes: it is distribution sensitive according to all three main notions of distribution sensitivity in the literature, and it is subgroup decomposable with population weights. Furthermore, it also satisfies several additional standard properties that are expected of welfare indices.

The literature has advanced three main definitions of distribution sensitivity.[5] First, a measure is *Pigou-Dalton sensitive* if it improves when a progressive transfer is carried out between two individuals.[6] Formally, Pigou-Dalton sensitive measures should satisfy the monotonicity and transfer axioms. Second, a Pigou-Dalton sensitive measure is *transfer sensitive* if it satisfies the transfer sensitivity axiom, which goes one step further, requiring measures to give more weight to equal-sized transfers taking place lower in the distribution. Third, a Pigou-Dalton sensitive measure is *growth sensitive* if the elasticity of the measure with respect to income of the *p*-th percentile is monotonically decreasing in *p*, i.e., a one percentage point growth in income of a poorer person improves the welfare measure by a greater percentage amount than a one percentage point growth for a richer person (Ray ⓡ and Genicot, 2022; Dollar, Kleineberg and Kraay, 2015). This requirement is the "growth" analogue of the transfer axiom and is very much related to the notion of pro-poor growth proposed by Ravallion and Chen (2003). Transfer- and growth-sensitive

[5] For all the mathematical definitions and proofs required for this sub-section, please see Appendix A.

[6] An index satisfying the monotonicity and transfer axioms is often referred to as "distribution sensitive" in the literature. In contrast, we refer to such an index being Pigou-Dalton-sensitive to reserve the term "distribution sensitive" for the larger set of properties encompassing all three notions that we use.

measures satisfy Pigou-Dalton sensitivity by definition, while the opposite is not necessarily true. In contrast, growth sensitivity does not imply and is not implied by transfer sensitivity.

The new index satisfies these three definitions of distribution sensitivity. First, it is Pigou-Dalton sensitive as it satisfies the monotonicity and transfer axioms. Second, it is transfer sensitive because it also satisfies the transfer sensitivity axiom. Finally, it is growth sensitive, i.e., it satisfies the *growth progressivity* axiom proposed by Ray ⓡ and Genicot (2022).[7] The uncensored version of the index (W) satisfies these axioms globally, while the censored version (C, as well as P) satisfies them for incomes below the threshold.

Table 2.1 presents several poverty measures and welfare indices found in the literature, providing their definitions, elasticities with respect to income, as well as showing whether they satisfy the three definitions of distribution sensitivity discussed above. Among poverty measures, the class of Foster-Greer-Thorbecke (FGT) indices is the most popular. FGT indices are Pigou-Dalton sensitive when $\alpha > 1$ and transfer-sensitive when $\alpha > 2$. However, no FGT index is growth sensitive. Table 2.1 shows that the same holds for many standard poverty measures. Among the welfare indices listed by Dollar, Kleineberg and Kraay (2015), some of which are included in Table 2.1, only the Atkinson index with parameter $\varepsilon > 1$ is growth sensitive.

The new index is also subgroup decomposable with population weights, a property shared by many poverty indices but very few welfare indices. In the family of Atkinson welfare indices, only mean income, with parameter $\varepsilon = 0$, is decomposable with population weights. To the best of our knowledge, $W(y, z)$ is the first welfare index proposed in the literature that is decomposable with population weights while satisfying all three definitions of distribution sensitivity.[8]

Our index also enjoys other classical properties. The uncensored version of the index (W) is continuous in all incomes, which formalizes the idea that poverty is not a discrete condition that

[7] Ray ⓡ and Genicot (2022) propose the Growth-Progressivity axiom in the context of upward mobility measurement. They do not study social welfare functions.

[8] To be sure, our index is not the *only* population-weighted subgroup decomposable index that satisfies the three definitions of distribution sensitivity. For instance, this class of indices contains any index obtained by raising an Atkinson index with $\varepsilon > 1$ to the power $1 - \varepsilon$. More generally, any additive welfare index $\frac{1}{N}\sum_{i=1}^{N} f(y_i)$ is population-weighted subgroup decomposable, and will satisfy the three definitions of distribution sensitivity if the derivatives of $f(y_i)$ satisfy the conditions in Appendix A. Among all the members of this class, we single out $W(y, z)$ for two related reasons. First, it has a very simple interpretation as the average factor by which incomes must increase to reach the reference level of income. Second, $W(y, z)$ is homogenous of degree minus one in income: if all incomes double, it is halved. Since $W(y, z)$ is decreasing in income, this is the natural analog of the homogeneity of degree one property enjoyed by most welfare measures that are increasing in income. In Appendix B, we show that $W(y, z)$ is the only welfare measure that is homogenous of degree minus one and population-weighted subgroup decomposable. Admittedly, both homogeneity of degree (minus) one and subgroup decomposability are cardinal properties, which are less desirable than ordinal ones from a normative perspective because they discriminate among indices that are ordinally equivalent. However, cardinal properties may still be desirable from a practical perspective – for instance, when they simplify analysis for a policymaker.

is acquired or lost when one's income crosses a particular threshold (Watts, 1969).[9] Our censored index (C, as well as P) is also continuous because incomes are top-coded to the reference income z.[10] We note that the main axiomatic difference between the uncensored and censored versions of the new index is that the latter satisfy the *focus axiom*, which states that the income distribution of people above the poverty line should not influence the measurement of poverty (Sen, 1976). Policymakers wishing to only focus on incomes below the threshold z can therefore use the censored index as a legitimate measure of poverty.

[9] It is straightforward that the new index also satisfies the symmetry and replication invariance axioms, which imply that any two income distributions having the same cumulative distribution function must have the same poverty value, as well as the scale invariance axiom, which requires the measurement of poverty to be unaffected by the unit and currency against which income is measured.

[10] If instead of top-coding, we drop individuals whose income is larger than z from the summation, then a *discontinuity* arises when the income of an individual crosses the reference income z. Indeed, dropping individuals implies that their ratio drops from 1 to 0 when they cross z.

Table 2.1. Properties of Welfare and Poverty Measures

	Definition	Elasticity	Distribution sensitivity			Subgroup decomposability
		$\varepsilon_{y_i}^{M} = \frac{\partial M/\partial y_i}{M/y_i}$	Pigou-Dalton	Transfer	Growth	
Poverty						
Foster, Greer, and Thorbecke (1984) $\alpha \geq 0$	$FGT_\alpha = \frac{1}{N}\sum_{i=1}^{N_P}\left(1-\frac{y_i}{z}\right)^{\alpha}$	$\varepsilon_{y_i}^{FGT\alpha} = -\frac{\alpha * y_i * \left(1-\frac{y_i}{z}\right)^{\alpha-1}}{N * FGT_\alpha * z}$	If α >1	If α>2	×	✓
Headcount ($\alpha = 0$)	$FGT_0 = \frac{N_P}{N}$	$\varepsilon_{y_i}^{FGT_0} = 0$	×	×	×	✓
Poverty Gap ($\alpha = 1$)	$FGT_1 = \frac{1}{N}\sum_{i=1}^{N_P}\left(1-\frac{y_i}{z}\right)$	$\varepsilon_{y_i}^{FGT_1} = -\frac{y_i}{N * FGT_1 * z}$	×	×	×	✓
Squared poverty gap ($\alpha = 2$)	$FGT_2 = \frac{1}{N}\sum_{i=1}^{N_P}\left(1-\frac{y_i}{z}\right)^{2}$	$\varepsilon_{y_i}^{FGT_2} = -\frac{2 * y_i * \left(1-\frac{y_i}{z}\right)}{N * FGT_2 * z}$	✓	×	×	✓
Sen poverty index	$S = \frac{2}{N}\sum_{i=1}^{N_P}\left(1-\frac{y_i}{z}\right)\left(1-\frac{i}{N_P+1}\right)$	$\varepsilon_{y_i}^{S} = -\frac{2 * y_i * \left(1-\frac{i}{N_P+1}\right)}{N * S * z}$	✓	×	×	✓
Class of Chakravarty (1983) $0 < \beta < 1$	$Chak_\beta = \frac{1}{N}\sum_{i=1}^{N_P}\left(1-\left(\frac{y_i}{z}\right)^{\beta}\right)$	$\varepsilon_{y_i}^{Chak_\beta} = -\frac{\beta * y_i * \left(\frac{y_i}{z}\right)^{\beta-1}}{N * Chak_\beta * z}$	✓	✓	×	✓

Watts (1969) ($\beta \rightarrow 0$)	$Watts = \frac{1}{N}\sum_{i=1}^{N_P}\left(ln(z) - ln(y_i)\right)$	$\varepsilon_{y_i}^{Watts} = -\frac{1}{N * Watts}$	✓	✓	✗	✓
<u>Welfare</u>						
Average income per capita	$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$	$\varepsilon_{y_i}^{\bar{y}} = \frac{y_i}{N * \bar{y}}$	✗	✗	✗	✓
Atkinson welfare index $\varepsilon \geq 0$	$A_\varepsilon = \left(\frac{1}{N}\sum_{i=1}^{N} {y_i}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$	$\varepsilon_{y_i}^{A_\varepsilon} = \frac{1}{N}\left(\frac{y_i}{A_\varepsilon}\right)^{1-\varepsilon}$	If ε >0	If ε >0	If ε >1	If ε =0
Atkinson welfare index ($\varepsilon = 1$)	$A_1 = \left(\prod_{i=1}^{N} y_i\right)^{\frac{1}{N}}$	$\varepsilon_{y_i}^{A_1} = \frac{1}{N}$	✓	✓	✗	✗
Atkinson welfare index ($\varepsilon = 2$)	$A_2 = \left(\frac{1}{N}\sum_{i=1}^{N} {y_i}^{-1}\right)^{-1}$	$\varepsilon_{y_i}^{A_2} = \frac{A_2}{N * y_i}$	✓	✓	✓	✗
Sen welfare index	$Sen = \frac{2}{N^2}\sum_{i=1}^{N}(N + 1 - i) * y_i$	$\varepsilon_{y_i}^{Sen} = \frac{2}{N^2}\frac{(N + 1 - i) * y_i}{Sen}$	✓	✗	✗	✗
<u>Shared Prosperity</u>						
Share of income of Bottom 40%	$\bar{y}^{B40} = \frac{1}{N_{B40}}\sum_{i=1}^{N_{B40}} y_i$	$\varepsilon_{y_i}^{\bar{y}^{B40}} = \frac{y_i}{N * \bar{y}^{B40}}$ for $i \leq N_{B40}$, $\varepsilon_{y_i}^{\bar{y}^{B40}} = 0$ for $i > N_{B40}$	✗	✗	✗	✗
<u>New indices</u>						
$W(y, z)$	$W = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i}$	$\varepsilon_{y_i}^{W} = -\frac{z}{N * W * y_i}$	✓	✓	✓	✓

$C(y,z)$	$C = \frac{1}{N}\sum_{i=1}^{N} \frac{z}{{y_i}^z}$	$\varepsilon_{y_i}^{C} = -\frac{z}{N * C * y_i}$	✓	✓	✓	✓
$P(y,z)$	$P = \frac{1}{N}\sum_{i=1}^{N_p} \left(\frac{z}{y_i} - 1\right)$	$\varepsilon_{y_i}^{P} = -\frac{z}{N * P * y_i}$	✓	✓	✓	✓

Notes: In this table, a property is considered as violated when it is failed within the population deemed relevant by the index (Welfare indices: all individuals; Poverty indices: all individuals below the reference income z, and Share of income of Bottom 40%: all individuals in the bottom 40). The notation N_{B40} corresponds to the number of individuals in the bottom 40% of the distribution. For poverty measures, the elasticity is equal to zero when individual i is non-poor. For the sake of brevity, we do not discuss the welfare measures discussed by Bonferroni (1930) and Donaldson and Weymark (1980), which are not decomposable. For conciseness, we also skip the measures proposed by Thon (1979), Kakwani (1980), and Takayama (1979) as they are directly related to the index of Sen (1976), and we skip the class of poverty measures proposed by Clark, Hemming and Ulph (1981) and Hagenaars (1987) because of their direct relationship with the class of Chakravarty (1983), which has better properties.

2.4 Underlying social welfare function

The new index is ordinally equivalent to a utilitarian social welfare function (SWF) based on an iso-elastic utility function:

$$SWF(y) = \sum_{i=1}^{N} u(y_i) \quad with \;\; u(y_i) = -\gamma y_i^{-1}, \tag{4}$$

where $\gamma > 0$ is an irrelevant constant which for simplicity of interpretation we set to $\gamma = z$.

To interpret this SWF, recall that under the veil of ignorance of Harsanyi (1953), the social planner evaluates an income distribution by the expected utility they get when drawing at random any income in the distribution. Therefore, the degree of inequality aversion in the SWF corresponds to the coefficient of relative risk aversion associated with their utility function.[11] In our case, the degree of inequality aversion of the SWF corresponds to a coefficient of relative risk aversion (CRRA) equal to 2. This is an appealing value as it is widely used in calibrations of macroeconomic models. It is also consistent with direct empirical estimates of the degree of inequality aversion in Kot and Paradowski (2022), who find a global mean estimate of 1.92 (their Table 5). Together, this provides some additional justification for focusing on the specific functional form we propose for $W(y, z)$.[12]

The ordinal equivalence between our index and the SWF defined in Eq. (4) has two implications. First, our index is ordinally equivalent to the Atkinson welfare function for parameter value $\varepsilon = 2$ (A_2), as $W(y, z) = \frac{z}{A_2(y)}$. Hence, $W(y, z)$ shares the same degree of inequality aversion as A_2. Also, given that A_2 corresponds to the harmonic mean of the distribution, our index is ordinally equivalent to the harmonic mean.[13]

Second, our index inherits a nice interpretation associated with Atkinson's SWF. The Atkinson welfare functions are such that $A_\varepsilon(y)$ can be interpreted as the income level that, if earned by all individuals, yields the same welfare as distribution y. This income level is called the equally distributed equivalent income (EDEI(y)), and is thus implicitly defined by:

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[11] This argument assumes that risk preferences are homogenous across individuals. See Eden (2020) for a generalization to heterogenous risk preferences.

[12] For further justification, note that if people have iso-elastic preferences, which is consistent with empirical evidence (Chiappori and Paiella, 2011), a CRRA of 2 corresponds to the inverse of the elasticity of intertemporal substitution (EIS), i.e., 0.5, which corresponds almost exactly to the average EIS obtained in the meta-analysis of Havranek et al. (2015). That study collected and analyzed 2,735 estimates of the EIS in consumption from 169 published studies that cover 104 countries. They estimate an average EIS of 0.492, with a standard deviation of 1.298. This provides some additional empirical support for the degree of inequality aversion implied by the new index and its underlying SWF. Finally, please note that $W(y, z)$ can easily be generalized to vary the degree of inequality aversion, α, using the ratio formulation $(\frac{z}{y_i})^\alpha$. However, this comes at the expense of simplicity and ease of interpretation.

[13] The relationship between our index and Atkinson's welfare function connects our index to a measure of pro-poor growth proposed by Foster and Székely (2008). These authors suggest that growth be considered "pro-poor" when the percentage change in mean income is smaller than the percentage change in some generalized mean, like the geometric or the harmonic mean. Hence, when the percentage reduction in our index is larger than the percentage increase in mean income, growth can be considered "pro-poor."

$$A_\varepsilon(y) = A_\varepsilon\big(EDEI(y), \dots, EDEI(y)\big). \tag{5}$$

Hence, we can straightforwardly compute the EDEI that our index associates with any distribution as $EDEI(y) = \frac{z}{W(y,z)}$. For instance, the two-person income distribution $y' = \left(\frac{z}{3}, z\right)$ is such that the ratio of individual 1 is $\frac{z}{y_1} = 3$ and the ratio of individual 2 is $\frac{z}{y_2} = 1$, which means that $W(y'; z) = 2$. The EDEI of distribution y' can thus be computed as $EDEI(y') = \frac{z}{W(y\prime,z)} = \frac{z}{2}$. By definition, the fact that its EDEI is equal to $\frac{z}{2}$ means that the equal distribution $y'' = (z/2, z/2)$ yields the same welfare as distribution y', i.e., $W(y'';\ z)$=$W(y';\ z) = 2$.[14]

2.5 Associated inequality measures

Atkinson (1970) pointed out that any social welfare function automatically defines an associated inequality index that quantifies how much welfare is lost due to the inequality observed in the distribution. Following this insight, we can construct an inequality index associated with our welfare index. When income is equally distributed, all individuals earn an income equal to the mean. Thus, our welfare index becomes an inequality index when z is set equal to mean income, i.e.,

$$I(y, \bar{y}) = \frac{1}{N} \sum_{i=1}^{N} \frac{\bar{y}}{y_i}, \tag{6}$$

where $\bar{y} \geq 0$ denotes mean income in the distribution y. $I(y, \bar{y})$ is a measure of inequality because it is homogeneous of degree zero (i.e, it is unchanged when all incomes in a distribution are multiplied by a common factor) and it satisfies the transfer axiom. One noteworthy feature of $I(y, \bar{y})$ is that it is equal to one when all incomes are equal, as opposed to zero for most inequality indices. This difference reflects the fact that our inequality index contrasts individual incomes to mean income in terms of ratios rather than in terms of differences. When individual incomes are equal to mean income, the ratios are equal to one, whereas the differences are equal to zero.[15]

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[14] With a small value for z, our index admits another interesting interpretation, related to the headcount ratio. Consider a "two-groups" society that is divided between a "poor" group, where everyone earns z, and a "non-poor" group, where everyone earns a very large income. Now, consider any distribution y for which $W(y,z) < 1$, which is always possible when selecting a sufficiently small value for z. Then, the value $W(y,z)$ corresponds to the fraction of individuals in the "poor" group that the two-group society should have in order to have the same welfare as distribution y, which is $W(y,z) = W\big((z, \dots, z, \infty, \dots, \infty), z\big)$.

[15] We note that $I(y, \bar{y})$ is related to the Generalized Entropy class of inequality measures $GE(\alpha) = \frac{1}{\alpha(\alpha-1)} \frac{1}{N} \sum_{i=1}^{N} \left(\left(\frac{y_i}{\bar{y}}\right)^\alpha - 1\right)$ of Shorrocks (1980). Specifically, we have $I(y, \bar{y}) = 2GE(-1) + 1$. We are grateful to Gaurav Datt for pointing this out to us.

Unlike many existing inequality measures, the inequality index $I(y, \bar{y})$ has a simple interpretation with intuitive units: it is the average factor by which incomes must be multiplied to attain mean income $\bar{y}$, recognizing that this factor is less than one for individuals above the mean. We note that $I(y, \bar{y})$ is ordinally equivalent to the inequality index AI_2 proposed by Atkinson (1970), which has rarely been used in practice, perhaps because it lacks a straightforward interpretation.

It follows from Equation (6) that our welfare and inequality indices are closely related. Specifically, we can write our welfare index as a product of inequality and (the inverse of) mean income, i.e. $W(y,z) = I(y, \bar{y})\left(\frac{z}{\bar{y}}\right)$. This in turn implies that we can decompose the growth in $W(y,z)$ into the difference between the growth rates of inequality, $I(y, \bar{y})$, and mean income, $\bar{y}$.

This inequality index admits a *multiplicative* decomposition into within-group and between-group inequality. Let G denote the number of groups, indexed by g. Let I_g denote inequality within group g as measured by our index. Let I_{btw} denote between-group inequality, which corresponds to the inequality measured by our index in a counterfactual distribution y^c obtained from the initial distribution y by replacing the income of each individual by the mean income in their group. Formally, we have $I_g = I(y_g, \bar{y}_g)$ where y_g denotes the distribution of income and $\bar{y}_g$ denotes mean income in group g and $I_{btw} = I(y^c, \bar{y}^c)$ where y^c is defined for all groups g and individual i in g as $y_{ig}^c \equiv \bar{y}_g$. Our index decomposes into:

$$I(y, \bar{y}) = I(y^c, \bar{y}^c)\left(\sum_{g=1}^{G} w_g \, I\big(y_g, \bar{y}_g\big)\right), \tag{7}$$

with weights:

$$w_g = \frac{N_g/\bar{y}_g}{\sum_{g'=1}^{G} N_{g'}/\bar{y}_{g'}}, \tag{8}$$

where N_g denotes the number of individuals in group g and the weights satisfy $\sum_{g=1}^{G} w_g = 1$.

This decomposition has a simple interpretation.[16] $I(y_g, \bar{y}_g)$ represents the average factor by which income in each group g must be multiplied to reach average income within the group. The within-group contribution to overall inequality is simply a weighted average of these factors, $\sum_{g=1}^{G} w_g \, I(y_g, \bar{y}_g)$, which assigns greater weight to groups that are larger and/or have lower mean income. If incomes are equally distributed within all groups, this term is equal to one. The first term $I(y^c, \bar{y}^c)$ captures between-group inequality in an obvious way – it simply is our inequality measure applied to the counterfactual distribution where inequality within groups has been

[16] Amartya Sen, referring to the Theil index, which is a member of the General Entropy Class of inequality with a parameter value of one, says: "But the fact remains that [the Theil index] is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income is not a measure that is exactly overflowing with intuitive sense." (Sen & Foster, 1973, page 36)

eliminated by setting everyone's income to group average income. If there are no differences in mean income across groups, this term also is equal to one. Overall inequality simply is the product of these two terms.

Finally, the index $I(y, \bar{y})$ generalizes to a class of inequality indices, whose members only differ by the definition given to z. Each member in the class defines z to be equal to the mean income among the X% richest individuals:

$$I(y, \bar{y}^X) = \frac{1}{N}\sum_{i=1}^{N}\frac{\bar{y}^X}{y_i}, \tag{9}$$

where $\bar{y}^X \geq 0$ denotes mean income among the X% richest individuals. The index $I(y, \bar{y}^X)$ is a measure of inequality because it is homogeneous of degree zero and it satisfies the transfer axiom. The index $I(y, \bar{y})$ corresponds to the extreme case where X=100%, i.e., $I(y, \bar{y}) = I(y, \bar{y}^{100\%})$. The other extreme member in the class defines z to be the largest income in the distribution. This latter measure can be interpreted as the factor by which incomes need to be multiplied to attain the income of the richest person in the distribution. Any member of this class admits an interpretation that is related to a Kuznets ratio (Kuznets, 1955). Indeed, a Kuznets ratio measures the *ratio of average income* among the top X% to average income among the bottom 1-X% of the distribution.[17] In contrast, index $I(y, \bar{y}^X)$ measures the *average of income ratios*, namely those of the average income among the top X% to each individual income. This illustrates that index $I(y, \bar{y}^X)$ is more distribution sensitive than the corresponding Kuznets ratio.

3. Empirical Applications

3.1 Data

We use data from the World Bank's Poverty and Inequality Platform (PIP) (World Bank, 2022a) to illustrate the empirical properties of the new welfare measures proposed in this paper and contrast them with existing ones. The PIP is a large compendium of over 2000 household surveys covering 168 countries and over 97 percent of the world's population.[18] For most of the analysis in this section, we rely on the "lined-up" version of the PIP data which interpolates/extrapolates between/beyond survey years assuming distribution-neutral growth to create a continuous annual dataset covering 1990-2019.[19] This is the same dataset used by the

[17] A closely related measure is the Palma ratio, defined as the ratio of the top 10 percent income share to the bottom 40 percent share (Palma, 2011). Setting $X = 10\%$ in $I(y, \bar{y}^X)$ so that it measures the average factor by which all incomes must increase to reach the mean income of the top 10% of the distribution is similar in spirit to the Palma ratio and moreover gives greater weight to the shortfall of the poorest.
[18] For documentation on PIP, see pip.worldbank.org and for technical details on the data and estimates used in PIP, see the Global Poverty Monitoring Technical Notes at https://pip.worldbank.org/publication.
[19] For details on the methodology used for the distribution-neutral shifts to the survey distribution to line up surveys to a common reporting year, see the PIP methodological handbook (https://datanalytics.worldbank.org/PIP-

World Bank for its headline global poverty monitoring. For ease of reproducibility, the calculations reported here are performed on the "binned" version of the dataset created by Mahler ⓡ et. al. (2022), where the income distribution in each country-year is represented by data grouped into 1000 bins of equal population size per country and year, rather than on the underlying microdata itself.[20]

Due to differences in the underlying survey instruments, the PIP combines surveys in which consumption is the main household-level measure of well-being (covering about three-quarters of the world's population) with surveys in which income is the measure of well-being (covering the remaining quarter of the world's population). This presents challenges, since conceptually consumption and income are very different measures of well-being, and these differences are likely to be more pronounced in the tails of the distribution. For example, it is biologically impossible to survive with zero consumption, but it is possible to survive with zero income if households can draw down stocks of wealth. This conceptual difference is apparent in the data. For example, zero values for reported consumption are virtually nonexistent, while it is common for income surveys to have some mass of observations with zero or negative incomes.[21] Beyond these extremes in the lower tail of the distribution, household saving introduces conceptual and empirical differences between income and consumption throughout the income distribution. We follow the practice of the World Bank's global poverty monitoring by abstracting from the difference between income- and consumption-based surveys when combining distributions across countries. For terminological convenience we will refer to the measure of well-being from the survey as "income" irrespective of whether it is drawn from an income or a consumption survey and note that it is measured in constant 2017 $PPP, per person per day.

An additional challenge is how to handle the effect of very low or zero measured income when constructing distribution-sensitive welfare measures. Specifically, our welfare measures are based on the ratio z/y_i which is undefined when reported income is zero and can be very large when reported income is close to zero, to the point of dominating the index calculated as the average of these ratios across all individuals.[22] To address this problem, we bottom-code all binned observations at $0.50 per day. We arrive at this value by applying the consumption floor methodology outlined in Ravallion (2016) to estimate minimum consumption levels. Applying this method to consumption surveys in PIP, we find that essentially no country has a daily per

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Methodology). For countries without any household survey data, it is assumed that their income distribution is equal to the population-weighted average income distribution in their region.

[20] The dataset is available at https://datacatalog.worldbank.org/search/dataset/0064304/1000-Binned-Global-Distribution.

[21] When zeros in consumption are reported in surveys, they typically are treated as unit non-response. Regarding incomes, Hlasny, Ceriani and Verme (2022) note that more than 75 percent of the 354 surveys from the Luxembourg Income Study that they examine, contain income values taking the value of zero.

[22] The concern that distribution sensitive measures are highly sensitive to low incomes is not new. Cowell and Victoria-Feser (1996) discuss this issue in the context of data contamination (e.g., measurement error), using influence functions to demonstrate the extent to which a poverty or inequality measure is changed by small errors in the data.

person consumption floor below this level. This procedure results in bottom-coding 0.3% of the population covered by both income and consumption surveys in 2019.

3.2 The evolution of global welfare, inequality, and poverty over time

We begin by illustrating the features of our new measures in the setting of the world interpersonal income distribution. Table 3.1 reports our new welfare index W, comparing it with mean income in Panel A with the help of our new inequality measure I (Panel B). We also compare our new censored poverty measures C and P with the headcount ratio (Panel C). While we compare our new measures to other distribution sensitive welfare indices and poverty measures later in this section, we start with the mean and the headcount index as comparators, precisely because they are simple, commonly used, and not distribution sensitive by any of the three definitions in the literature (see Table 2.1). W is calculated using $z = \$6.85$ as the reference income level, corresponding to the World Bank's poverty line for upper-middle income countries, while the censored measures in Panel C are presented for $z = \$6.85$ as well as $z = \$2.15$, the World Bank's international poverty line capturing extreme poverty.

Table 3.1: Global Welfare, Poverty, and Inequality Measures 1990-2019

	1990	2019	Avg Annual Change, %		1990	2019	Avg Annual Change, %
Panel A: Welfare Measures (z=$6.85 per day)				**Panel B: Inequality Measures**			
W	2.95	1.36	-2.67	I	5.02	3.58	-1.16
Mean	11.6	18.0	1.51				
Sen	3.53	6.88	2.30	Gini	0.70	0.62	-0.41
A_1	4.29	8.76	2.46	AI_1	0.63	0.51	-0.71
A_2	2.32	5.03	2.67	AI_2	0.80	0.72	-0.36
Panel C: Poverty Measures							
(i) z = $6.85 per day				**(ii) z = $2.15 per day**			
C	3.15	1.66	-2.20	C	1.31	1.05	-0.74
P	2.15	0.66	-4.06	P	0.31	0.05	-5.95
FGT_0	0.69	0.47	-1.34	FGT_0	0.38	0.08	-5.20
FGT_1	0.45	0.21	-2.56	FGT_1	0.14	0.03	-5.84
FGT_2	0.32	0.12	-3.36	FGT_2	0.07	0.01	-6.06
Watts	0.85	0.34	-3.18	Watts	0.20	0.04	-5.91

Notes: This table reports the global estimates of $W(y,z)$, $I(y,\bar{y})$, $C(y,z)$, and $P(y,z)$ and selected comparators. The welfare measure, W, is reported using the \$6.85 per person per day threshold in 2017 \$PPP (Panel A). C and P are reported for the \$6.85 and \$2.15 thresholds (per person per day in 2017 \$PPP). The *Avg Annual Change* column reports the average annual log differences. Abbreviations: A_ε – Atkinson Social Welfare Index with $\varepsilon = 1\ or\ 2$; FGT – Foster-Greer-Thorbecke poverty measure with $\alpha = 0, 1, or\ 2$; AI_ε – Atkinson Inequality Index with $\varepsilon = 1\ or\ 2$.

Welfare and inequality indices

In 1990, our new welfare measure W was equal to 2.95, indicating that on average, incomes of people around the world needed to be multiplied by a factor of about three to attain the reference income level of \$6.85 (Table 3.1, Panel A). Our proposed inequality measure, I (Table 3.1, Panel B) was equal to 5.02, meaning that on average, incomes needed to be multiplied by a factor of about five to reach mean income, which was equal to \$11.6 per person per day for the world in 1990 (Table 3.1, Panel A). By 2019, W had fallen by over half from its value in 1990, to 1.4, i.e., on average, individual incomes needed to be multiplied only by a factor of 1.4 to reach \$6.85 in 2019. I had declined to 3.6, while mean income had increased to \$18.0 over those three decades.

Column 3 in Table 3.1 indicates an average annual decline (i.e., improvement) of -2.7 percent per year for W. In contrast, average per capita income $\bar{y}$ increased from \$11.6 to \$18.0, an average improvement of 1.5 percent per year. To understand the difference in the time trends of these two measures between 1990 and 2019, recall from Section 2.5 that the growth rate of W is the growth rate of our inequality measure, I, minus the growth rate of mean income, $\bar{y}$. Panel B of Table 3.1 indicates that our inequality measure declined at a rate of 1.2 percent per year, which accounts for the faster rate of improvement in W as compared with mean income.

The concept of equally distributed equivalent income (EDEI) discussed earlier in Section 2.4 is helpful to interpret how the numerical value of W reflects its distribution sensitivity. Recall that EDEI is the level of income that, if earned equally by all individuals, would generate the same level of welfare as the actual distribution of income. For our measure, $EDEI = z/W$, which means that our welfare measure would value the actual world income distribution in 1990 the same as a counterfactual one in which every individual had an income equal to $\frac{\$6.85}{2.95} = \$2.32$. This is roughly one-fifth of actual world average income in 1990, indicating a substantial degree of inequality aversion. By 2019, the $EDEI$ had increased to $\frac{\$6.85}{1.36} = \$5.04$. This represents 28 percent of world average income in 2019, again reflecting the same considerable degree of inequality aversion, but now applied to the lower level of inequality prevailing in 2019.[23]

To better understand the sources of this reduction in inequality that contributes to the improvement in W relative to the mean, a simple decomposition is helpful. Define $g_i \equiv \Delta ln y_i$ as the growth rate of income of individual i, and $g \equiv \Delta ln\bar{y}$ as the growth rate of average income. The growth rate of W can be decomposed as follows:

$$-\Delta lnW \approx g + \sum_{i=1}^{N} \varepsilon_i^W (g_i - g) \qquad (10)$$

[23] In Appendix C, we further quantify the distribution sensitivity of our indices by considering the fraction of their value that is contributed by individuals below each percentile and by contrasting this fraction with that obtained for other subgroup decomposable indices.

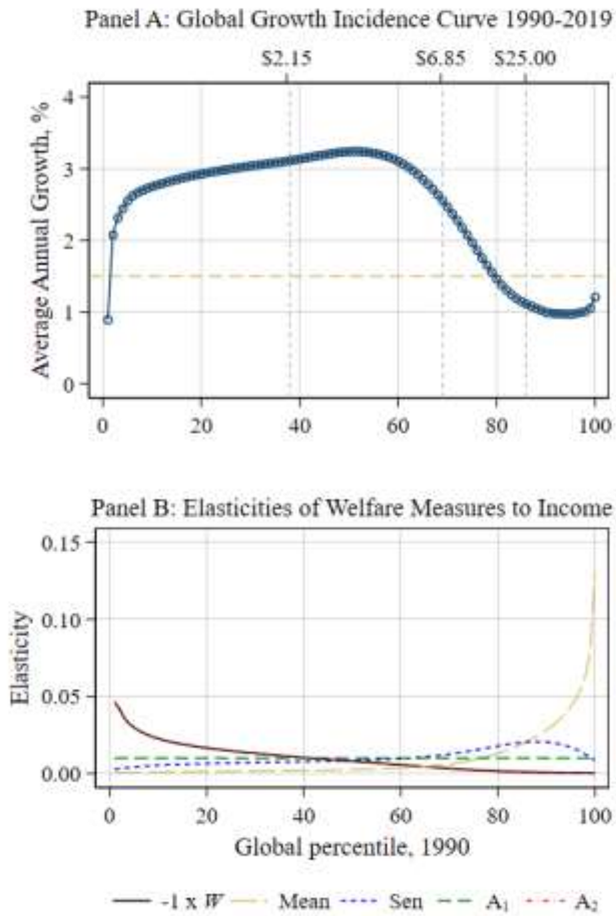
where ε_i^W is the elasticity of the welfare measure with respect to the income of individual i given in Table 2.1.[24] The first term in Equation (10) is the growth rate of average income, while the second term is a first-order approximation to the growth rate of inequality, decomposing it into a weighted average of relative growth rates across the income distribution, $g_i - g$, with weights reflecting the sensitivity of the welfare measure to growth at each point in the income distribution, ε_i^W. For W, these weights are proportional to y_i^{-1}, assigning a higher weight to growth that occurs lower in the income distribution, which is what makes W a growth sensitive measure.

Panel A of Figure 3.1 reports the distribution of growth rates across the world income distribution, plotting the average annual growth rate over the period 1990-2019 at each percentile of the initial income distribution in 1990, i.e., the global growth incidence curve.[25] World average growth is shown as a horizontal line at 1.5 percent, and the relative growth rates $g_i - g$ are the deviations of the growth incidence curve from this global average growth rate. Virtually every percentile in the bottom 80 percent of the world income distribution experienced higher-than-average growth over this period. Since our measure assigns uniformly higher weights to growth in lower percentiles of the income distribution (Panel B of Figure 3.1), it is not surprising that W improves faster than average income over this period. Conversely, since average income assigns higher weights to lower growth rates observed near the top of the income distribution, it follows that it improves more slowly than W over this period.

[24] Please see Appendix D for the derivation of this decomposition.

[25] Note that this is an "anonymous" growth incidence curve in that it tracks the growth rate of percentiles, not the growth rate of individuals (which would require true panel data for all individuals in the world and does not exist). As is well-known, the anonymous growth incidence curve understates (overstates) growth at the bottom (top) of the income distribution relative to non-anonymous growth rate that tracks the growth rate of individuals starting at different points in the income distribution (Jenkins and Van Kerm (2006, 2016), Grimm (2007), Van Kerm (2009), Bourguignon (2010), and Kraay and Van der Weide (2022)). For terminological convenience, in the main text we refer to "individuals" and "percentiles" interchangeably even though this is not strictly correct.

Figure 3.1: Growth Incidence and the Sensitivity of Welfare Measures to Growth



Notes: Panel A reports the average annual growth for each global income percentile for the period 1990 to 2019. The horizontal line at 1.5% shows the average annual growth in the global mean over the same period. Panel B shows the elasticity of various welfare measures with respect to income at each percentile of the global income distribution in 1990. See Table 2.1 for the expressions for these elasticities.

Equation (10) can be used to decompose growth in the other welfare functions in Table 3.1, which are also distribution sensitive. All we need to do is replace ε_i^W with the corresponding elasticity, ε_i^X, for $X \in \{Sen, A_1, A_2\}$, as reported in Table 2.1. This clarifies that the only source of difference in the growth rates of these welfare indices compared with that of W is the fact that the relative weights ε_i^X they assign to different points along the growth incidence curve are different from ε_i^W. Table 3.1 also shows that the average annual changes in the Sen, A_1, and A_2 indices are 2.30, 2.46, and 2.67, respectively (Panel A, Rows 3-5, Column 3). This indicates that these other measures on average place less weight on the faster growth observed in lower percentiles of the world income distribution over this period. This can be seen clearly in Panel B of Figure 3.1, which plots the elasticities of all five measures shown in Panel A of Table 3.1. As noted above, the elasticity for mean income is increasing with income, while leading distribution sensitive measures such as the Sen index and the Atkinson index with $\varepsilon = 1$ also do not place uniformly higher weight on growth rates in poorer percentiles. Only the Atkinson index with $\varepsilon = 2$ (A_2) assigns the same downward-sloping weights to growth across the income distribution. This is because, as discussed in Section 2.4, $W = z/A_2$. For this reason, also we see in Panel A of Table 3.1 that its growth rate is the same (in absolute value) as the growth rate of W.

Finally, recall from Section 2.5 that our inequality measure I can be expressed as a product of between-group and within-group inequality. Applying this decomposition to the world income distribution in 1990, we find that between-country inequality is 3.4 (indicating that the average factor by which country average incomes must be multiplied to reach world average income is 3.4), while within-country inequality is 1.5 (indicating that the average factor by which individual incomes in each country in the world need to increase to reach their corresponding country average is 1.5). The product of these two, 3.4 x 1.5, gives overall inequality of $I = 5$ as shown in Table 3.1. By 2019, the between-country component of global inequality had declined by a third to 2.3, while the within-country component had hardly changed, at 1.6. Consistent with the evidence for the decomposition of global inequality using the generalized entropy index with $\alpha = 0$ in Mahler ⓡ et. al. (2022), the decomposition above indicates that most of the decline in global inequality between 1990 and 2019 is due to a narrowing of gaps between countries.

Poverty indices

Panel C of Table 3.1 reports the censored measures, C and P, comparing them to the Foster-Greer-Thorbecke poverty measures (headcount ratio, poverty gap, and squared poverty gap), as well as the Watts index. Recall that C is the average factor by which incomes need to be multiplied to attain the reference income level z (equal to \$6.85 or \$2.15 in Panel C), setting this factor equal to one for individuals at or above the poverty line. At the threshold of $z = \$6.85$, C was 3.15 in 1990 and improved to 1.66 in 2019.[26] We also report $P = C - 1$, the average *percent increase* in individual incomes needed to bring everyone below the poverty line to the poverty line, which was 2.15 (or 215%) in 1990. We use P for comparisons of changes over time with other poverty measures because, like those measures, it is also bounded by zero from below, while the minimum value taken by C is one when poverty is eliminated.[27]

Between 1990 and 2019, the average annual decrease in the headcount index relative to a \$6.85 per day poverty line was 1.3 percent per year (dropping from 0.69 to 0.47), while the same figure was 4.0 percent per year for P (dropping from 2.15 to 0.66). The rates of decline of the other poverty measures are between these two extremes. As with the welfare measures discussed above, these differences reflect the interaction between the elasticities of these measures with the distribution of growth rates across percentiles.

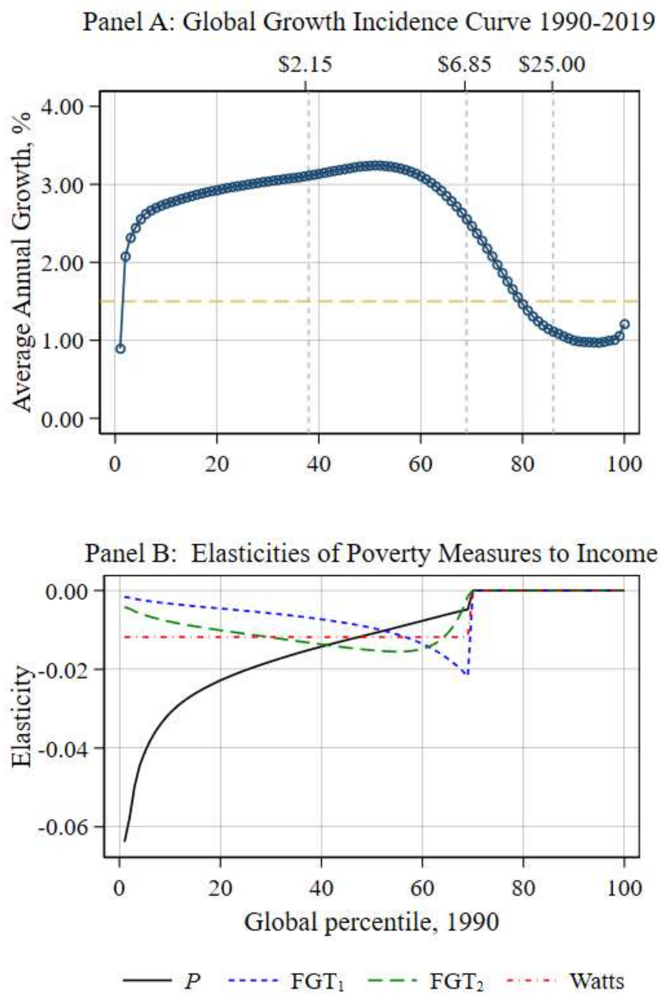
Panel B of Figure 3.2 graphs the elasticities of the various poverty measures with respect to income, shown here for the \$6.85 poverty line, while Panel A repeats the global growth

[26] $C = 3.15$ is only slightly higher than $W = 2.95$ in 1990, because less than a third of incomes are censored and W attaches a relatively low weight to these incomes.

[27] The growth rate of C mechanically is lower than the growth rate of P by a factor of $\frac{C-1}{C} < 1$, as $C = P + 1$, which makes its growth rate less comparable with that of the other poverty measures.

incidence curve for reference purposes. The elasticities are all negative, since all the poverty measures decline when incomes increase. However, only our proposed *P* measure gives a consistently higher weight (in absolute terms) to income growth in poorer percentiles. The elasticity of the Watts index does not vary with income (thus failing the growth progressivity axiom defined in Section 2). The elasticity for the poverty gap is increasing throughout (in absolute terms) as the poverty gap measures mean income below the poverty line, while for the squared poverty gap it starts to decline only as incomes approach the poverty line.

Figure 3.2: Growth Incidence and the Sensitivity of Poverty Measures to Growth



Notes: Panel A reports the average annual growth for each global income percentile for the period 1990 to 2019. The horizontal line at 1.5% shows the average annual growth in the global mean over the same period. Panel B shows the elasticity of various poverty measure with respect to income at each percentile of the global income distribution in 1990 using the $6.85 per person per day threshold in 2017 $PPP. See Table 2.1 for the expressions for these elasticities.

The interaction between global growth incidence curve (Figure 3.2, Panel A) and these elasticities (Figure 3.2, Panel B) accounts for the different rates of reduction across the different poverty measures. Taking advantage of the fact that poverty measures are additively sub-group decomposable, Figure 3.3 plots the contribution of growth at each percentile of the income distribution to the percent change in the poverty measure for the $6.85 poverty line. The contribution of each percentile is expressed in percent changes, and the average of all points along each line represents the overall percent change in each poverty measure.[28]

[28] See Appendix E for the precise formulation of the measures plotted in this graph.

Changes in poverty measures reflect both individuals crossing the poverty line and growth in incomes below the poverty line (except for the headcount, which reflects only the former). The downwards step pattern in the line for the headcount ratio in Figure 3.3 corresponds to the change in the poverty rate over this period. The growth of percentiles between the 47th and 69th (the poverty rate in 1990 and 2019, respectively, at the poverty line of $z = \$6.85$) contributes to the reduction in the headcount ratio, and each percentile contributes equally, since the headcount measures only whether people cross the poverty line. The growth of percentiles below the 47th percentile that did not cross the poverty line contributes nothing to the change in the headcount.

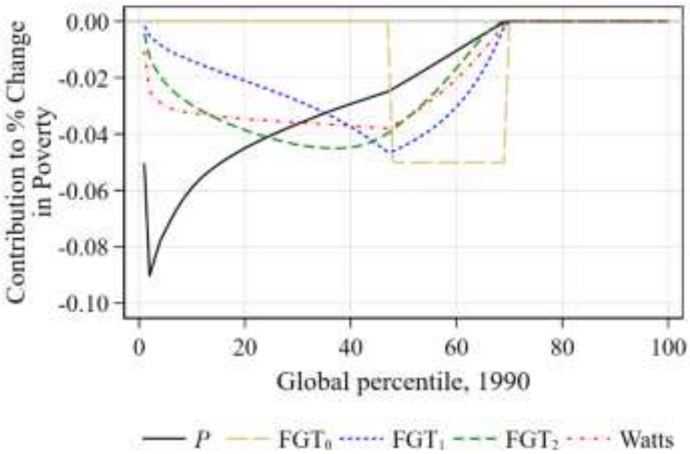
Compared to the headcount ratio, all other poverty measures are less responsive to people who cross the poverty line. This is because they are continuous while the head-count ratio features a discontinuous jump at the poverty line. Instead of merely recording poverty incidence, the other poverty measures all capture the distance from the poverty line in some way. For our P measure, the contribution of each percentile is $\frac{z}{y_i} - 1$, which approaches 0 as y_i approaches the poverty line. The main difference between the growth rate of P and the growth rate of the headcount can be seen in the region below the 47th percentile, i.e., all people who do not cross the poverty line and therefore do not contribute to a reduction in the headcount, but who still contribute to the reduction in the distribution sensitive poverty measure P. Overall, this explains why the percent change in these measures is bigger (in absolute terms) than the percent change in the headcount ratio. The contribution of growth to the reduction in poverty measures differs most starkly in the bottom tail of the distribution, where growth has been relatively low. Because of the high weight that P assigns to the poorest, they make a large contribution to the overall reduction in the measure, despite their relatively low growth.

The Watts index is an intermediate case. Below the 47th percentile, the Watts mimics the growth incidence curve (with a negative sign) since it assigns equal weight to everyone (Panel B of Figure 3.2). Comparing P with the Watts, growth in the bottom 30 contributes more to reducing P than the Watts, while the opposite is true for percentiles 30 to 70. As a result, P and the Watts both fall proportionately by similar amounts (decline of 4.0 percent and 3.1 percent, respectively).

Finally, it is interesting to note that the change in the headcount ratio is much more like the change in the distribution sensitive measures when using the \$2.15 line (Table 3.1, Panel C). The poverty rate at \$2.15 declined from 38 percent to 8 percent. As before, the headcount is the most responsive measure between the 8th and 38th percentile, which is counteracted by the distribution-sensitive measures considering the income growth of the population that is always poor.[29] However, the latter corresponds to only the bottom 8 percent of the distribution that remains below the \$2.15 line, rather than 47 percent with the \$6.85 line. Since growth in this region of the distribution contributes most to the difference between the growth rates of the poverty measures, these differences largely disappear when this region is smaller. As a result, the growth rates of all the poverty measures are quite similar in Panel C of Table 3.1.

[29] Figure E.1 replicates Panel B of Figures 3.1 and Figure E.2 replicates Figure 3.3 for the \$2.15 line.

Figure 3.3: Contributions of Growth at Different Percentiles to Changes in Poverty



Notes: This figure reports the contributions of each global percentile to the percent change in overall poverty from 1990 to 2019 for various poverty measures reported in Table 3.1. Poverty measures are reported using the \$6.85 per person per day threshold in 2017 \$PPP. See Appendix E for the expressions of the various contributions.

3.3 Cross-country comparisons of welfare, inequality, and poverty

We conclude this section with cross-country comparisons of three pairs of countries around 2019 – one low-income pair (Burkina Faso and Mali); one middle-income pair (Colombia and Peru); and one high-income pair (France and the U.S.) – using the measures that were presented in Table 3.1. W, C, P, and FGT_0 are estimated at the \$2.15, \$6.85, and \$25 per person per day thresholds, respectively. The U.S. and Colombia, even though they have higher mean incomes than their pairwise comparators, France and Peru, respectively, also have higher values of W, because they have higher inequality, as measured by I. In the case of United States, incomes on average need to be multiplied by more than three, indicating the high level of inequality there, in contrast with countries like France (1.4) with much lower inequality. Within each of the three country pairs, the difference in P is substantially larger than that indicated by the headcount ratio, FGT_0. The table serves to demonstrate that welfare judgments across countries (or regions or groups) at a given point in time can also vary meaningfully if our proposed measure is used instead of some of the other commonly used measures of welfare**.**

Table 3.2: Cross-country Comparison of the Simple Welfare, Poverty, and Inequality Measures

	Low-income, consumption		Middle-income, income		High-income, income	
	Burkina Faso	Mali	Colombia	Peru	France	United States
	2018	2018	2019	2019	2019	2019
Panel A	z = \$2.15 per day		z = \$6.85 per day		z = \$25 per day	
W	0.82	0.66	1.14	0.92	0.58	0.93
Mean	4.85	4.77	15.6	13.9	58.9	83.6
I	1.85	1.47	2.59	1.86	1.36	3.09
Panel B						
C	1.15	1.05	1.47	1.26	1.04	1.46
P	0.15	0.05	0.47	0.26	0.04	0.46
FGT_0	0.31	0.17	0.35	0.29	0.10	0.11

Notes: This table reports the estimates of $W(y,z)$, mean, and $I(y,\bar{y})$ in Panel A and $C(y,z)$, $P(y\ ,z)$, and the poverty headcount ratio (FGT_0) in Panel B. W, C, P, and FGT_0 are estimated at the \$2.15, \$6.85, and \$25 per person per day thresholds in 2017 \$PPP for the low-, middle-, and high-income groups respectively. Mean is expressed in 2017 \$PPP per person per day. Consumption is used to measure welfare in Burkina Faso and Mali while income is used in other countries.

4 The Prosperity Gap: a New Measure of Shared Prosperity

In this section of the paper, we present a policy application of our new welfare measures. We show how our new welfare measure W can be used as a natural measure of "shared prosperity", the promotion of which is one of the two main institutional goals of the World Bank. We first provide background on the concept of shared prosperity as it is understood at the World Bank. We then interpret W as a global "prosperity gap" in incomes relative to a prosperity standard set at \$25 per day, roughly equal to the median poverty line among high-income countries. We show how the prosperity gap varies across countries and over time, highlighting the benefits of it being both distribution sensitive and subgroup decomposable. For the benefit of readers who are primarily interested in the prosperity gap and are less interested in the technical properties of our new welfare measures, this section is designed to be self-contained and may be read separately from the rest of the paper.

4.1 Background on Shared Prosperity

Since 2013, the World Bank has articulated its twin goals as "*ending extreme poverty and boosting shared prosperity*" (World Bank, 2015). Extreme poverty is measured as the share of the world's population living below the international poverty line. This line is currently set at \$2.15 per day in 2017 \$PPP, corresponding to the median national poverty line across low-income countries. The concept of "shared prosperity" lacks an accepted precise definition but is clearly intended to capture improvements in a distribution-sensitive welfare measure. This can be seen in the following explanation of the concept:

> *"One way to think about the World Bank's new shared prosperity goal is as an alternative to average income as the benchmark of development progress. Instead of assessing and measuring economic development in terms of the overall average growth in a country, the shared prosperity goal places emphasis on the bottom 40 percent of the population. In other words, good progress is judged to occur not merely when an economy is growing, but, more specifically, when that growth is reaching the least well-off in society. Thus, the shared prosperity goal seeks to increase sensitivity to distributional issues, shifting the common understanding of development progress away from average per capita income and emphasizing that good growth should benefit the least well-off in society."* (World Bank (2015), p.10)

As indicated in the quote above, shared prosperity is measured as the growth rate of average incomes in the bottom 40 percent of the population in each country. This in turn implies that the underlying distribution-sensitive welfare measure currently used by the World Bank to measure "shared prosperity" is mean income in the bottom 40 percent of the income distribution in each country.

Average income in the bottom 40 percent is a welfare measure that is simple to explain. However, it has two key shortcomings. First, it is not subgroup decomposable with population weights. This implies, for example, that average income in the bottom 40 percent of the world cannot be expressed as a population-weighted average of average income in the bottom 40 percent of each country in the world. This is simply because not every person in the bottom 40 percent of the world income distribution is in the bottom 40 percent of the income distribution of the country where they live, and vice versa.[30] This makes the welfare measure less interpretable, because it is not possible to have a global measure that corresponds to a simple aggregation of the same measure at the country level, and similarly, it is not possible to simply decompose the measure across groups of interest within a country.

Second and more importantly, average income in the bottom 40 percent only partially satisfies one of the three definitions of distribution-sensitivity in the literature and fails to satisfy the other two. It is Pigou-Dalton sensitive only with respect to transfers from a person in the top 60 percent to a person in the bottom 40 percent, but not within the bottom 40 percent since the measure is simply mean income within this group. It does not satisfy transfer sensitivity or growth sensitivity. In fact, within the bottom 40 percent, average incomes of the bottom 40 percent inherit the upward-sloping pattern of growth elasticities of mean income, making it a "pro-rich growth" rather than a "pro-poor growth" measure within the bottom 40 percent.

4.2 A New Measure of Shared Prosperity

To address these shortcomings, we propose a new "prosperity gap" to measure shared prosperity. The prosperity gap is defined as the global average shortfall in income from a standard

[30] In fact, 39 percent of people in the bottom 40 percent of the world income distribution in 2019 are in the top 60 percent of the income distribution of their country, and conversely, 26 percent of the people in the top 60 percent of the world income distribution are in the bottom 40 percent of the income distribution of their country.

of prosperity set at \$25 per day (adjusted for differences in purchasing power across countries). Formally, it is an application of the censored version of the index, $W(y,z)$ defined in Section 2.2:

$$Prosperity\ Gap \equiv W(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i} \tag{11}$$

where the "prosperity standard" z set at \$25 per day for reasons discussed further below.

The prosperity gap has the same simple interpretation discussed earlier in the paper: it is *the average factor by which incomes must be multiplied to reach the prosperity standard*. For a person below the prosperity standard, multiplying their income y_i by a factor $\frac{z}{y_i}$ will raise income to the prosperity standard z. For a person above (or at) the prosperity standard, the factor is less than (or equal to) one, indicating that they have already attained the prosperity standard. The prosperity gap is the average across all people of these factors. A value greater than one indicates a prosperity gap or shortfall in prosperity, since it means that incomes need to increase by an average multiple equal to the prosperity gap to achieve the prosperity standard.

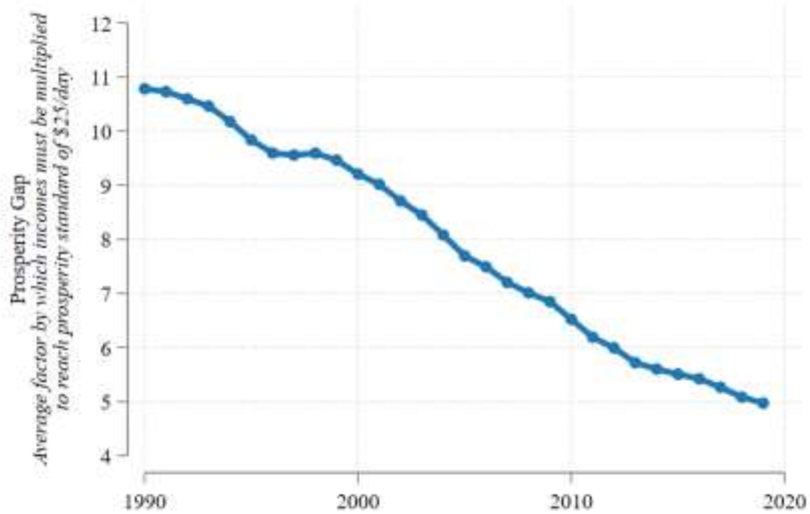
The prosperity gap is subgroup decomposable with population weights. This means, for example, that the global prosperity gap can be written as a population-weighted average of the prosperity gaps at the country level. This makes it a useful tool for measuring shared prosperity at the country level while having a global measure of the prosperity gap measured in the same terms. Similarly, it means that the prosperity gap can readily be decomposed across groups of interest within a given country.

The prosperity gap also is readily decomposable into average income, $\bar{y}$, and the inequality measure discussed in Section 2.5 of the paper, i.e., $W(y,z) = I(y,\bar{y})\left(\frac{z}{\bar{y}}\right)$. The inequality measure $I(y,\bar{y})$ in turn has a simple interpretation as the average factor by which incomes must increase to attain mean income (recognizing that this factor is below one for incomes above the mean). This clarifies how reductions in inequality and/or increase in average income will lead to improvements in the prosperity gap. Specifically, the growth rate of the prosperity gap is simply the difference between the growth rate of inequality $I(y,\bar{y})$ and the growth rate of mean income $\bar{y}$.

4.3 Empirical Implementation

We empirically implement the prosperity gap using the same data discussed in Section 3.1. In 1990, the prosperity gap for the world was 10.8, meaning that incomes would have to increase nearly 11-fold on average to reach the prosperity standard of \$25 per day. By 2019 the prosperity gap had fallen to 5.0, meaning that incomes would on average have to increase 5-fold to reach the prosperity standard (Figure 4.1). As discussed in Section 3.2, the prosperity gap improves (i.e., falls) faster than average income during this period, reflecting the fact that growth was concentrated in lower percentiles of the income distribution. Specifically, the prosperity gap improved at an average annual rate of 2.7 percent, while average incomes increased by 1.5 percent per year. The difference between these represents the contribution of a 1.2 percent per year reduction in global interpersonal income inequality (as measured by $I(y, \bar{y})$) to the improvement in the prosperity gap.

Figure 4.1: The Global Prosperity Gap



Notes: This figure reports the global estimate of $W(y, z)$ using the prosperity threshold of $25 per person per day in 2017 $PPP for the period 1990 to 2019.

Globally, 81 percent of the world's population, or about 6.2 billion people, falls short of the $25 per day prosperity standard in 2019. In low-income countries, 99 percent of people fall short of this standard, with a *prosperity gap* of 12.4 – meaning that incomes would have to increase more than twelve-fold on average to attain the $25 per day standard. In lower-middle-income (upper-middle-income) countries, 98 (86) percent of people also fall short of this standard, with a *prosperity gap* of 6.8 (3.1). Even in high-income countries, 19 percent of people fall below the prosperity standard, although the *prosperity gap* for them is much smaller at 0.9 (Table 4.1). This reflects the fact that most people in high-income countries have incomes that exceed the prosperity standard, so that their contribution to the prosperity gap is less than one.

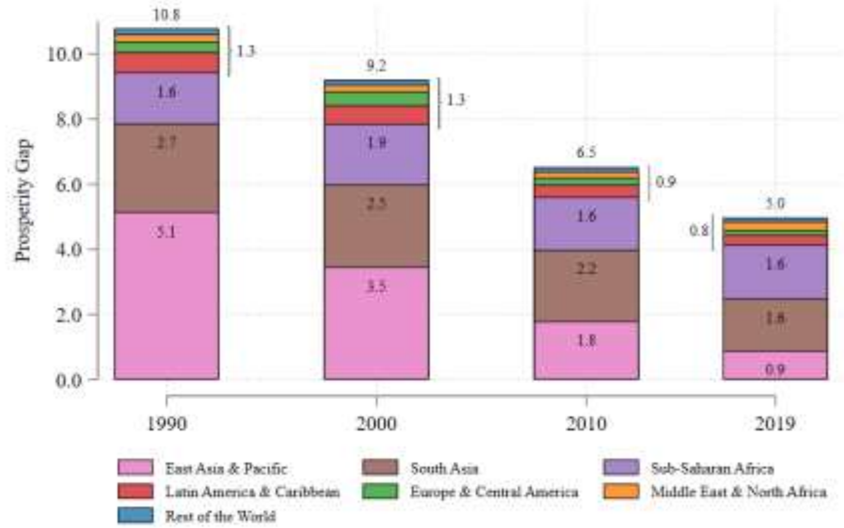
Figure 4.2 illustrates the subgroup decomposability property of the prosperity group, decomposing the global prosperity gap into regional contributions. The largest contributors to the prosperity gap are the 5 billion people in East Asia, South Asia, and Sub-Saharan Africa contributing 88% of the global multiple in 1990 and 84% in 2019. It is discouraging to note that, in contrast with East Asia & Pacific and South Asia, the absolute contribution of Sub-Saharan Africa to the global prosperity gap has not declined since 1990, with its share increasing from less than 15% to more than 32%.

Table 4.1: The Prosperity Gap in 2019 – By Regions, by Income Groups, and for Selected Countries

		Prosperity Gap, Multiple by Which Income Must Increase	Share of Population below Prosperity Threshold, %	Millions of People below Prosperity Threshold
Income group	Low income	12.4	99.2	663
	Lower middle income	6.8	98.3	2,864
	Upper middle income	3.1	85.9	2,463
	High income	0.9	18.9	234
Region	East Asia & Pacific	3.2	90.0	1,893
	Europe & Central Asia	2.5	75.5	374
	Latin America & Caribbean	3.5	81.1	521
	Middle East & North Africa	5.0	93.8	370
	South Asia	6.7	99.1	1,820
	Sub-Saharan Africa	11.4	98.8	1,094
	Rest of the world	0.8	13.9	153
Select countries	Burkina Faso	9.3	98.1	20
	Bangladesh	6.5	99.4	162
	Bolivia	2.5	76.3	9
	Brazil	3.6	77.7	164
	China	2.8	88.5	1,246
	Colombia	4.2	85.8	43
	Germany	0.5	7.7	6
	Ethiopia	8.2	99.6	112
	France	0.6	9.8	7
	Indonesia	5.2	98.1	265
	India	6.9	99.1	1,354
	Japan	0.8	16.5	21
	Mali	7.3	99.5	20
	Nigeria	10.1	99.9	201
	Peru	3.4	88.7	29
	United States	0.9	11.0	36
	South Africa	7.8	88.1	52
	Global	5.0	81.0	6,224

Notes: This table reports the global estimate of $W(y,z)$ using the prosperity threshold of $25 per person per day in 2017 $PPP, the share of the population living below the prosperity threshold, and the number of people living below the prosperity threshold in 2019 for various income groups, regions, and select countries. Income groups are classified according to World Bank FY21 definitions, which uses data from 2019 (see https://datahelpdesk.worldbank.org/knowledgebase/articles/906519-world-bank-country-and-lending-groups). Regions are classified according to definitions used in the Poverty and Inequality Platform (see https://datanalytics.worldbank.org/PIP-Methodology/lineupestimates.html#regionsandcountries).

Figure 4.2: Regional Contributions to the Global Prosperity Gap

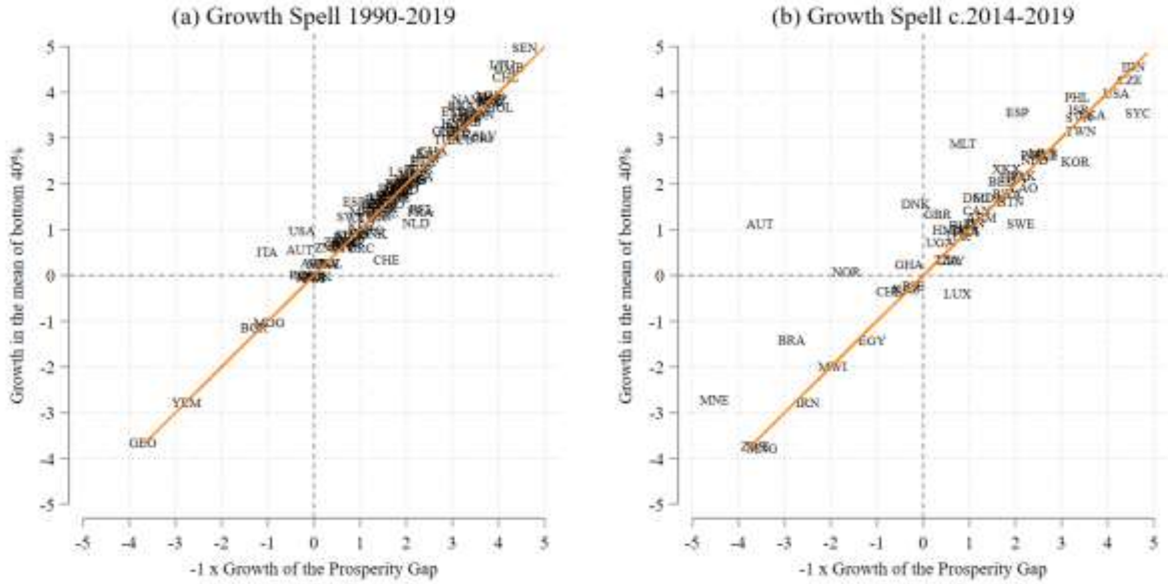


Notes: This figure reports the decomposition of the global estimate of $W(y,z)$ into contributions by regions using the prosperity threshold of $25 per person per day in 2017 $PPP. See also Table 4.1.

Although they measure different concepts, empirically changes over time in the prosperity gap and growth in the bottom 40 percent are similar at the country level. This can be seen in Figure 4.3, which compares average annual growth in the bottom 40 percent (on the vertical axis) against growth in the prosperity gap (on the horizontal axis and multiplied by -1 so that positive growth corresponds to an improvement in the prosperity gap). The left panel of Figure 4.3 considers the full period 1990-2019 using the lined-up data discussed in Section 3.1, while the right panel focuses on a smaller set of 53 “spells” of various lengths calculated between actual survey years around 2014 and around 2019.[31] Each panel includes the 45-degree line, with points above (below) the line corresponding to episodes where average incomes in the bottom 40 percent improved faster (slower) than the prosperity gap. Overall, the correlation between the two measures is high, with most countries close to the 45-degree line.

[31] The comparable spells can be found in the Global Dataset for Shared Prosperity here: https://www.worldbank.org/en/topic/poverty/brief/global-database -of-shared-prosperity. Fifty-three countries have growth spells between -5% and 5%.

Figure 4.3: Growth in the Prosperity Gap and Mean Income of the Bottom 40%



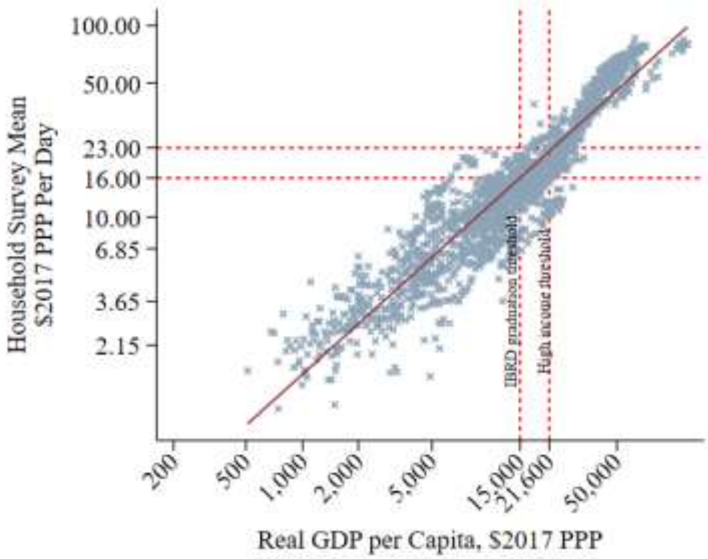
Notes: The left panel of this figure reports annualized growth between 1990 and 2019 for all countries with data in PIP. The right panel reports the same information using the latest comparable spells for each country from the 10th edition of the Global Database of Shared Prosperity (World Bank, 2022b) covering circa 2014-2019. For visual clarity only cases of annual growth between -5% and 5% are reported.

4.4 Rationale for the Prosperity Threshold

We conclude with a brief discussion of the rationale for the prosperity threshold, which we set at $25 per day at 2017 $PPP. There are two complementary rationales for this figure. First, it is approximately equal to the median poverty line among high-income countries, which is close to $24.4 per day (Jolliffe ⓡ et. al. 2022, Table B7). Second, it is close to the average level of household per capita income in countries that cross the per capita GDP threshold that the World Bank uses to distinguish middle- and high-income countries. This can be seen in Figure 4.4, which plots average household daily per capita income (on the vertical axis) against annual GDP per capita (on the horizontal axis), with both measured in constant 2017 $PPP and each point corresponding to a survey-year observation for all surveys in the Poverty and Inequality Platform. The solid upward-sloping line shows the line of best fit, with the vertical line at $21,600 ($15,000) in 2017 $PPP indicating the high-income (IBRD graduation) threshold.[32] The intersection of the high-income threshold with the line of best fit occurs at $23 per day, with the intersection of the IBRD graduation threshold occurring at $16 per day.

[32] The high-income threshold and the IBRD graduation threshold in the Bank's income classification for operational purposes is defined in terms of US dollars at market exchange rates and is currently $13,205 and $7,445 respectively. Hamadeh et. al. (forthcoming) propose a conversion of the income classification system to PPP terms. Preserving the same number of countries in each group of the operational classification results in a high-income threshold of approximately $23,745 and an IBRD graduation threshold of $16,500 in PPP-adjusted terms using 2021 dollars, which is equivalent to $21,599 and $14,977 in 2017 dollars.

Figure 4.4: High Income Threshold as Justification for $25 per day Prosperity Standard

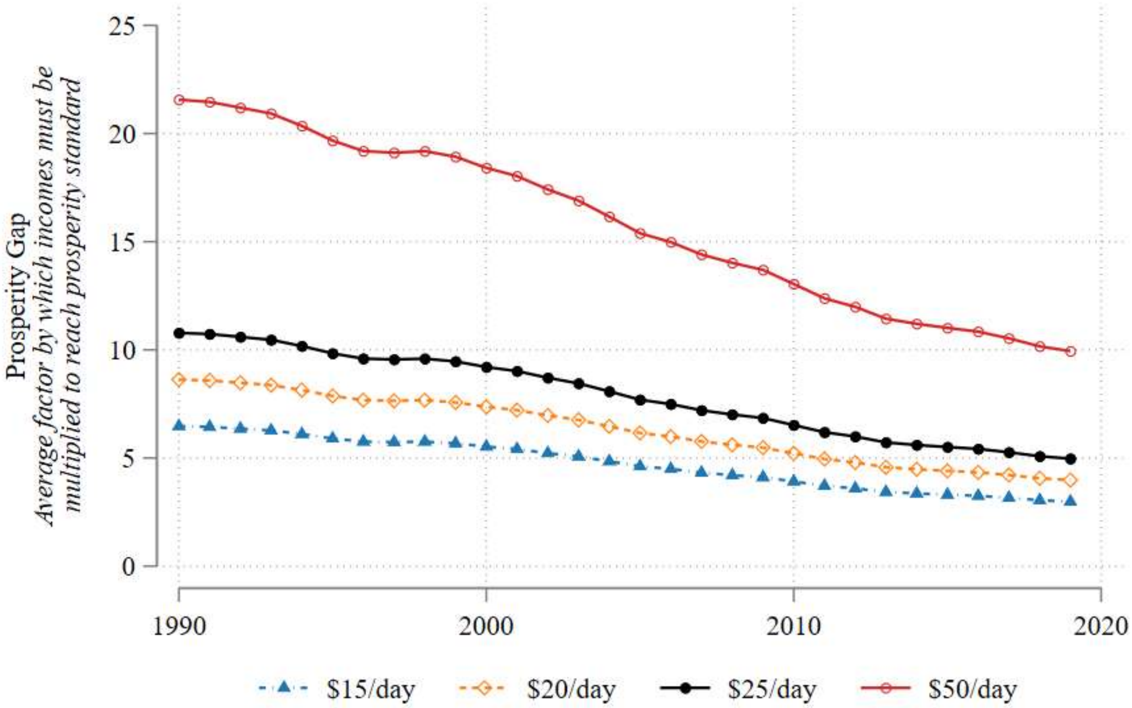


Notes: The solid upward-sloping line shows the line of best fit between average household daily per capita income (on the vertical axis) and annual GDP per capita (on the horizontal axis). Each observation represents a survey available in the Poverty and Inequality Platform. The vertical line at $21,600 indicates the high income threshold and the vertical line at $15,000 indicates the IBRD graduation threshold in PPP-adjusted terms (updated using Hamadeh et al. forthcoming).

Naturally, the prosperity threshold could be set at other levels as appropriate to the application, in the same way that different poverty lines can be deployed when measuring poverty in different contexts. For example, a higher threshold such as $50 per day could be justified based on being approximately equal to median income in high-income countries – an unambiguously high global standard of prosperity. On the other hand, a lower threshold around $15 per day could be justified because it is close to the IBRD graduation threshold. Figure 4.5 illustrates how the prosperity gap would change with different values of the prosperity threshold.

Finally, we note that a key feature of our measure is that changing the prosperity threshold z only changes the units of the prosperity gap, and otherwise has no effect on cross-country or over-time comparisons of the measure. In fact, converting the prosperity gap from a measure based on a prosperity threshold of $z = \$25$ to any other threshold z^* requires nothing more than multiplying the prosperity gap by $z^*/25$. This greatly adds to the simplicity and interpretability of the proposed new prosperity gap measure of shared prosperity.

Figure 4.5: The Global Prosperity Gap with Different Prosperity Thresholds



Notes: This figure reports the global estimate of the prosperity gap using $15, $20, $25, and $50 per person per day prosperity thresholds for the period 1990 to 2019. The thresholds are expressed in 2017 $PPP.

5 Conclusions

The most used measures of welfare, mean income and the poverty headcount, are not distribution sensitive. Despite widespread agreement that inequality is important, the large menu of existing distribution sensitive welfare measures is rarely used in academic, policy, or public discourse. We argue that a main reason for this is their complexity and lack of intuitive units. To remedy this shortcoming, we have introduced a new family of distribution sensitive welfare, poverty and inequality measures that has a simple mathematical formulation and simple units. The key ingredient of these measures is the ratio of reference income z to individual individual incomes, z/y_i, which captures the factor by which individual incomes must be multiplied to attain the reference level of income. This ratio formulation ensures that our measures satisfy all three definitions of distribution sensitivity in the literature. Our measures are based on averages of this ratio, giving a simple "average factor by which incomes must increase" interpretation to our measures. This simple average property also ensures that our measures are sub-group decomposable with population weights, further enhancing its ease of communication.

We demonstrate the empirical properties of our measures in the context of the world interpersonal income distribution over the period 1990-2019, as well as in selected country comparisons. Consistent with their strong distribution sensitivity, we find that our welfare and poverty measures decline more strongly than their non-distribution sensitive counterparts in a setting that has featured a substantial decline in global inequality. Finally, we exploit the simple structure and units of our new measure to propose a new "prosperity gap" measure that the World Bank could use to track progress towards its goal of promoting shared prosperity.

One area that requires further work to facilitate the implementation of these new measures is a more careful consideration of the treatment of low incomes in household surveys. As discussed in Section 3, by virtue of being based on z/y_i, which makes them distribution sensitive, our measures also are sensitive to mismeasurement of very low or, at the extreme, zero incomes. In this paper, we bottom-coded income at $0.50 per day, at which we arrived by applying the consumption floor methodology outlined in Ravallion (2016) to estimate minimum consumption levels. Further work is needed on different approaches to address the issue of mismeasurement of income (or consumption) at the bottom of the distribution.

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Contributors: *Sterck had the original idea for the welfare index, which aggregates individual contributions to the index in the form of z/y_i. Decerf, Kraay, Özler, and Sterck drafted Sections 1 and 2, which were revised by all authors. Jolliffe, Kraay, Lakner, and Yonzan drafted Sections 3 and 4, which were revised by all authors. All authors interpreted the data, contributed to writing the paper, and approved the final version.*

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Appendix

A Definitions and Axioms

In this section, we provide formal definitions for three notions of distribution sensitivity that have been proposed in the literature and present the mathematical conditions that they impose on welfare indices.

Basic notation

The notation is as follows. Let $y = (y_1, \dots, y_N)$ denote an ordered income distribution whose incomes are sorted in increasing order, i.e., $y_{i+1} \geq y_i$ for all $i \in \{1, \dots, N-1\}$. Let $\aleph$ denote the set of natural numbers and let R^+ denote the set of strictly positive reals. Let $Y = \cup_{N \in \aleph} (R^+)^N$ denote the set of all income distributions whose individual incomes are all strictly positive, where the dependence of N on distribution y is omitted in the notation (i.e., we simply write N instead of $N(y)$). Let a measure M be any function $M: Y \rightarrow R_0^+$ be such that $M(y) > 0$ for some $y \in Y$.[33]

Definitions of three notions of distribution sensitivity

We present the formal definitions of three central notions of distribution sensitivity. These notions are defined based on axioms. Before presenting these axiomatic definitions, we make two remarks.

First, welfare indices can either increase in incomes, like social welfare functions, or decrease in incomes, like poverty measures. As an improvement is marked by an increase in the index value in the former case and by a reduction in the latter case, the exact definition for their axioms is slightly different. As our index belongs to the latter category, we consider here the case of a "decreasing" welfare index, whose value is decreasing in incomes. This is without loss of generality.

Second, some decreasing welfare indices, like poverty measures, are only affected by incomes that are smaller than some reference income threshold, i.e., they satisfy the Focus axiom proposed by Sen (1976).[34] For the sake of presentation, we present axioms so that they apply to all individuals in the distribution. It is straightforward to adapt the version of the axioms we present in such a way that they only apply to poor individuals. When discussing distribution sensitivity of indices that satisfy the Focus axiom, we consider only their application to poor individuals.

DEFINITION 1: A welfare index is *Pigou-Dalton sensitive* if it satisfies both the Monotonicity and the Transfer axioms.[35]

[33] To avoid trivialities, we rule out any measure that takes value 0 for all income distributions.

[34] The Focus axiom could be defined as follows: For all $y \in Y$, $\delta > 0$ and i for whom $y_i \geq z$ we have $M(y) = M(y_1, \dots, y_{i-1}, y_i + \delta, y_{i+1}, \dots, y_N)$.

[35] This definition is standard in the literature, see for instance Zengh (1997).

Monotonicity requires that the measure is reduced by an increment in the income of an individual.

(A1) Monotonicity: For all $y \in Y, \delta > 0$ and $i \in \{1, \ldots, N\}$, we have: $M(y) > M(y_1, \ldots, y_{i-1}, y_i + \delta, y_{i+1}, \ldots, y_N)$.

Transfer requires that the measure is reduced by a progressive equal-sized transfer.

(A2) Transfer: For all $y \in Y$, $\delta > 0$ and $i, j \in \{1, \ldots, N\}$ with $y_i - \delta \geq y_j + \delta$, we have: $M(y) > M\big(y_1, \ldots, y_{j-1}, y_j + \delta, y_{j+1}, \ldots, y_{i-1}, y_i - \delta, y_{i+1}, \ldots, y_N\big)$.

DEFINITION 2: A welfare index is *transfer sensitive* if it is a *Pigou-Dalton-sensitive* index that satisfies the Transfer Sensitivity axiom.

Transfer sensitivity requires that a progressive equal-sized transfer reduces the measure more when it takes place further down the distribution.

(A3) Transfer Sensitivity: For all $y \in Y, \delta > 0$ and $i, j, k, l \in \{1, \ldots, N\}$ with $j > k$, $y_i - \delta \geq y_j + \delta$ and $y_k - \delta \geq y_l + \delta$, we have: $M\big(y_1, \ldots, y_{j-1}, y_j + \delta, y_{j+1}, \ldots, y_{i-1}, y_i - \delta, y_{i+1}, \ldots, y_N\big) > M(y_1, \ldots, y_{l-1}, y_l + \delta, y_{l+1}, \ldots, y_{k-1}, y_k - \delta, y_{k+1}, \ldots, y_N)$.

DEFINITION 3: A welfare index is *growth sensitive* if it is a *Pigou-Dalton-sensitive* index that satisfies the Growth Progressivity axiom.

The Growth Progressivity axiom requires that a transfer of *growth rate* from a richer to a poorer individual reduces the measure (Ray r and Genicot, 2022).

(A4) Growth progressivity: For all $y \in Y, \delta > 1$, and $i, j \in \{1, \ldots, N\}$ with $i > j$, $y_{i+1} > \delta y_i$ and $y_{j+1} > \delta y_j$ we have: $M(y_1, \ldots, y_{i-1}, \delta y_i, y_{i+1}, \ldots, y_N) > M\big(y_1, \ldots, y_{j-1}, \delta y_j, y_{j+1}, \ldots; y_N\big)$.

Mathematical relationships between the three notions of distribution sensitivity

We now discuss the conditions under which a welfare index that is both symmetric and decomposable in population weights satisfies these three notions of distribution sensitivity. Both symmetry and decomposability are highly desirable for policy work. Moreover, concentrating on symmetric and decomposable indices yields simpler conditions.

A welfare index is symmetric if its value is unaffected when we permute the names given to individuals. This is the standard anonymity requirement.

(A5) Symmetry: For all $y, y' \in Y$ such that y' is obtained from y through a permutation of incomes across individuals, we have $M(y') = M(y)$.

A welfare index is decomposable in population weights if its value corresponds to the weighted sum of the value that it takes on sub-groups, where weights correspond to the population shares of these subgroups.

(A6) Decomposability (in population weights): For all $y = (y^A, y^B) \in Y$ we have:

$$M(y) = \frac{N^A}{N} M(y^A) + \frac{N^B}{N} M(y^B).$$

A decreasing welfare index that is symmetric and decomposable in population weights must have its expression given by:[36]

$$M(y) = \frac{1}{N} \sum_{i=1}^{N} f(y_i)$$

where function $f: R^+ \rightarrow R_0^+$ is decreasing in y_i.

The mathematical conditions for the first two notions are well-known. As observed by Zengh (1997), a symmetric and decomposable welfare index M is:

- **Pigou-Dalton sensitive** if and only if:

  $\frac{\delta f}{\delta y_i}$ <0 and $\frac{\delta^2 f}{\delta {y_i}^2}$ >0 for all $y_i > 0$, (condition 1)

- **Transfer sensitive** if and only if:

  $\frac{\delta f}{\delta y_i}$ <0, $\frac{\delta^2 f}{\delta {y_i}^2}$ >0 and $\frac{\delta^3 f}{\delta {y_i}^3}$ <0 for all $y_i > 0$. (condition 2)

We next present the mathematical conditions for the third notion. By definition, a growth-sensitive measure must satisfy the requirements of a Pigou-Dalton-sensitive index (condition 1). Moreover, the growth progressivity axiom requires that for all 0< $y_j < y_i$, we have that a given growth rate in income y_j reduces function f more than the same growth rate in income y_i, which is $y_j \frac{\delta f}{\delta y_j} < y_i \frac{\delta f}{\delta y_i}$. This inequality is equivalent to requiring that the first derivative of $y_i \frac{\delta f}{\delta y_i}$ with respect to y_i be strictly positive, which yields $\frac{\delta^2 f}{\delta {y_i}^2} y_i + \frac{\delta f}{\delta y_i} > 0$ for all $y_i > 0$. For a Pigou-Dalton-sensitive decomposable measure, it is only if function f is sufficiently convex ($\frac{\delta^2 f}{\delta {y_i}^2}$>>0) that this condition can hold (because $\frac{\delta f}{\delta y_i} < 0$).

Hence, we have that a symmetric and decomposable welfare index M is:

- **Growth sensitive** if and only if:

[36] See the proof of Proposition 1 for details.

$\frac{\delta f}{\delta y_i}$ <0, $\frac{\delta^2 f}{\delta {y_i}^2}$ >0 and $\frac{\delta^2 f}{\delta {y_i}^2}\ y_i\ +\ \frac{\delta f}{\delta y_i}\ > 0$ for all $y_i > 0$. (condition 3)

As can easily be seen from their respective conditions, growth-sensitive measures and transfer-sensitive measures are necessarily Pigou-Dalton sensitive. The contrary is not necessarily true. We also note that some growth-sensitive measures are not transfer sensitive, while some transfer-sensitive measures are not growth sensitive. The two notions are not logically related.

Finally, we observe here that the growth progressivity axiom is equivalent to another property proposed by Dollar, Kleineberg, and Kraay (2015), namely that the measure's elasticity to growth is strictly decreasing in income. By definition, the elasticity to growth is $E_g = \frac{\delta M}{\delta y_i}\frac{y_i}{M}$, where $E_g < 0$ for any measure satisfying monotonicity. The property proposed by these authors thus requires that E_g becomes less negative with income, i.e., $\frac{\delta\, E_g}{\delta y_i} > 0$, which yields $\frac{\delta^2 M}{\delta {y_i}^2}\frac{y_i}{M} + \frac{\delta M}{\delta y_i}\frac{1}{M} > 0$ for all $y_i > 0$. This last inequality yields for a symmetric and decomposable measure that $\frac{\delta^2 f}{\delta {y_i}^2}\ y_i\ +\ \frac{\delta f}{\delta y_i}\ > 0$ for all $y_i > 0$, as desired. As a result, we can deduct from the sign of the derivate of their elasticity to growth (given in Table 2.1) that the poverty indices of FGT, Sen and Chakravarty, the welfare indices of Sen and Atkinson ($\varepsilon \leq 1$) and the shared prosperity indicator mean of bottom 40 are not growth sensitive because for some y_i we have $\frac{\delta\, E_g}{\delta y_i} \leq 0$.

In contrast, we show that our index W satisfies the three notions of distribution sensitivity. By the above discussion, we need to show that it satisfies conditions 1, 2 and 3. By definition, our decomposable welfare index is such that $f(y_i) = z/y_i$, which yields:

$$\frac{\delta f}{\delta y_i} = -z\left(\frac{1}{y_i}\right)^2, \qquad \frac{\delta^2 f}{\delta {y_i}^2} = 2z\left(\frac{1}{y_i}\right)^3, \qquad \text{and} \qquad \frac{\delta^3 f}{\delta {y_i}^3} = -6z\left(\frac{1}{y_i}\right)^4.$$

The respective signs of these three mathematical expressions directly yield that our index satisfies Pigou-Dalton sensitivity (condition 1) and transfer sensitivity (condition 2). Our index is also growth sensitive as:

$$\frac{\delta^2 f}{\delta {y_i}^2}\ y_i\ +\ \frac{\delta f}{\delta y_i} = \frac{z}{{y_i}^2}$$

which has the appropriate sign for condition 3, as desired.[37]

B Axiomatic derivation

In this section, we show that our uncensored index W is fully jointly characterized by a

[37] From these reasonings, it is straightforward to see that our two indices C and P also satisfy conditions 1, 2 and 3 for the population that they deem relevant (i.e., for individuals whose income is smaller than z).

proportionality axiom together with symmetry and decomposability. The proportionality property of a welfare index $M(y)$ can be formally defined by the following axiom:

(A7) Proportionality: For all $y \in Y$ and all $\delta > 0$, we have $M(\delta y) = \frac{1}{\delta} M(y)$.

Equivalently, A7 is the requirement that the welfare index is homogenous of degree -1.

Axioms A5, A6 and A7 directly lead to the following result.

Proposition 1: A welfare index M satisfies Decomposability, Symmetry and Proportionality if and only if $M = W$.

Proof

“if part”. The proof that W satisfies all three axioms is straightforward and thus omitted.

“only if part”. We must show that only measure W satisfies these three axioms.

Take any measure M that satisfies these three axioms and any $y \in Y$. By the iterative application of *Decomposability*, we get:

$$M(y) = \frac{1}{N} \sum_{i=1}^{N} f_i(y_i),$$

where function $f_i: R^+ \to R$ could be specific to individual i. By *Symmetry*, function f_i must be the same for all individuals. As the image set of M is R_0^+, we must have $f: R^+ \to R_0^+$. By *Proportionality*, function f must be homogeneous of degree -1, which implies that for all $y_i > 0$ we have:

$$f(y_i) = \frac{z}{y_i}$$

for some factor $z > 0$. We cannot have $z = 0$, otherwise we would have M(y)=0 for all distributions, a contradiction to the definition of a measure M. This shows that $M = W$, which concludes the proof of the Proposition ■

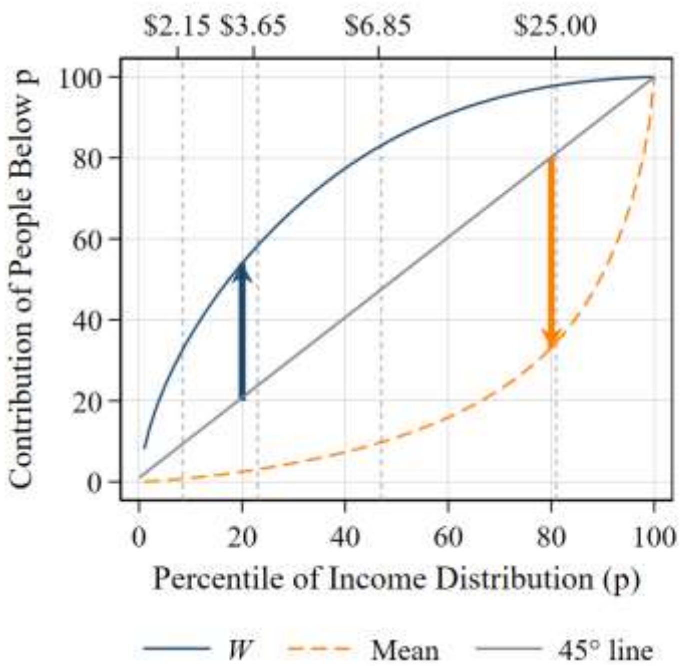
C Quantifying the distribution sensitivity of our indices

In Section 3.2 we used the concept of equally distributed equivalent income to interpret the distribution sensitivity of our index W. However, some readers may still fear that our index W is so distribution sensitive that it basically only accounts for low incomes in the first few percentiles. If that is the case, then W would not be highly robust to measurement errors of incomes at the bottom of the distribution, and even in the absence of measurement error, might de facto be focused on only the very poorest to the exclusion of everyone else. In this section, we take advantage of the subgroup decomposability of W to quantify the reliance of W on incomes in the bottom of the distribution.

Figure C.1 shows the fraction of the value of W for the world in 2019 that is contributed by individuals below percentile p. Clearly, this fraction is one when considering percentile $p = 100$ and zero when considering percentile $p = 0$. The graph shows that individuals below percentile $p = 20$ contribute slightly more than 50% of the value of W. This implies that almost half of the value of W is contributed by individuals above percentile $p = 20$. Yet, individuals below percentile $p = 20$ contribute more than twice their population share, which provides some quantified sense of W's distribution-sensitivity. To fix ideas, we compare W's sensitivity to the bottom of the distribution to mean income's sensitivity to the top of the distribution. This is a meaningful comparison given that W is *decreasing* with individual incomes while mean income is *increasing* with individual incomes. As shown in the figure, individuals above percentile $p = 80$ contribute more than 60% of the value of mean income. This suggests that W is not necessarily more sensitive to the bottom of the distribution than mean income is sensitive to the top of the distribution.

Clearly, the global distribution is particularly unequal. When repeating the same exercise for all countries in the world in 2019, we find that for the average country, individuals below percentile $p = 20$ contribute 43% of the value of W. In contrast, in the average country, individuals above percentile $p = 80$ contribute 44% of the value of mean income.

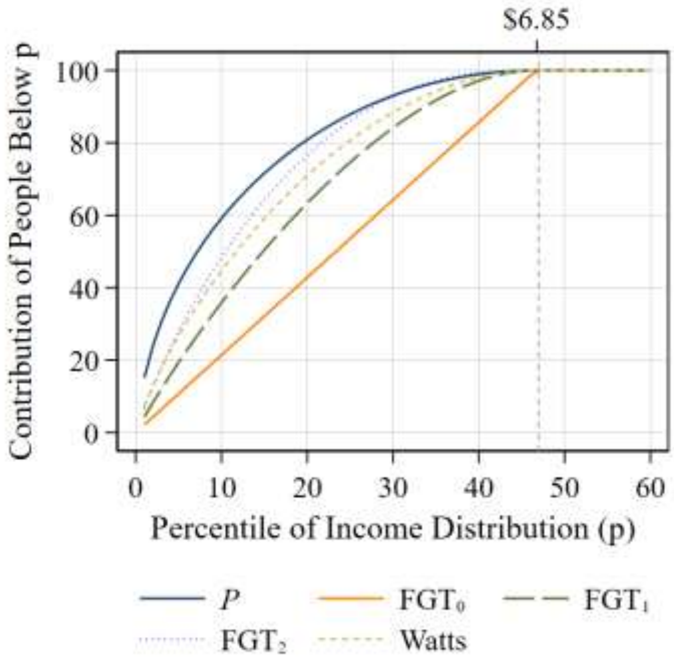
Figure C.1: Contributions to $W(y, z)$, and the Mean by Individuals Below each Percentile



Notes: This figure reports the cumulative contribution of each global percentile p to the overall $W(y, z)$ and to the overall mean in the 2019 global income distribution. Vertical lines show the various poverty thresholds for low, lower-middle, upper-middle, and high-income countries in per person per day using 2017 $PPP.

Figure C.2 provides the same information for our censored measure P. The difference is that the contribution to P reaches one when considering the percentile $p = 47$ at which incomes become equal to the reference income $6.85, which serves as the poverty threshold. The graph also shows the same information for the headcount ratio (FGT_0), poverty gap (FGT_1), the poverty gap squared (FGT_2) and the $Watts$. These poverty measures are interesting comparison points because they are not distribution-sensitive enough to satisfy one definition of distribution-sensitivity: FGT_1 is at the frontier of being Pigou-Dalton sensitive; FGT_2 is at the frontier of being transfer sensitive; and $Watts$ is at the frontier of being growth sensitive. Thus, by contrasting the graph for P (i.e., censored W) to the graphs for these poverty measures, we get a sense of how much each of the three definitions of distribution-sensitivity matter. The fraction contributed by individuals below percentile $p = 20$ is equal to 62%, 71%, 77% and 81% respectively for FGT_1, $Watts$, FGT_2 and P.

Figure C.2: Contribution to $P(y,z)$, and Various Poverty Measures by Individuals Below each Percentile



Notes: This figure reports the cumulative contribution of each global percentile p to the overall $P(y,z)$, headcount ratio, poverty gap, poverty gap squared and Watts index in the 2019 global income distribution. Vertical line shows the \$6.85 per person per day poverty threshold in 2017 \$PPP used to estimate the various poverty measures.

D Decomposing growth in welfare

We provide in this appendix the derivation for Eq. (10) in Section 3.2, which decomposes the growth in uncensored welfare.

Let $M(y)$ denote a generic uncensored existing welfare function from among those listed in Table 2.1, as well as our new index W. The first-order Taylor series approximation to the change over time in logarithm of M is:

$$\Delta lnM = \frac{\Delta M}{M} \approx \frac{1}{M}\sum_{i=1}^{N}\frac{\partial M}{\partial y_i}\frac{\partial y_i}{\partial t}\Delta t = \sum_{i=1}^{N}\frac{\partial M}{\partial y_i}\frac{y_i}{M}\frac{\partial y_i}{\partial t}\frac{1}{y_i}\Delta t = \sum_{i=1}^{N}\varepsilon_i^M\frac{\partial lny_i}{\partial t}\Delta t$$

$$= \sum_{i=1}^{N}\varepsilon_i^M(g+g_i-g) = g + \sum_{i=1}^{N}\varepsilon_i^M(g_i-g)$$

where the third-last equality uses the definition of the elasticity $\varepsilon_i^M \equiv \frac{\partial M}{\partial y_i}\frac{y_i}{M}$ (reported in Table 2.1 for the various welfare functions); the second-last equality uses the definitions of individual and average income growth $g_i \equiv \frac{\partial ln y_i}{\partial t}\Delta t$ and $g = \frac{\partial ln \bar{y}}{\partial t}\Delta t$; and the last equality uses the fact that $\sum_{i=1}^{N} \varepsilon_i^M = 1$ for the uncensored welfare functions since they are homogenous of degree one in incomes.

E Unpacking growth in the censored indices

In this Appendix, we explain Figure 3.4 in more detail and replicate Figures 3.3 and 3.4 for the $2.15 poverty line.

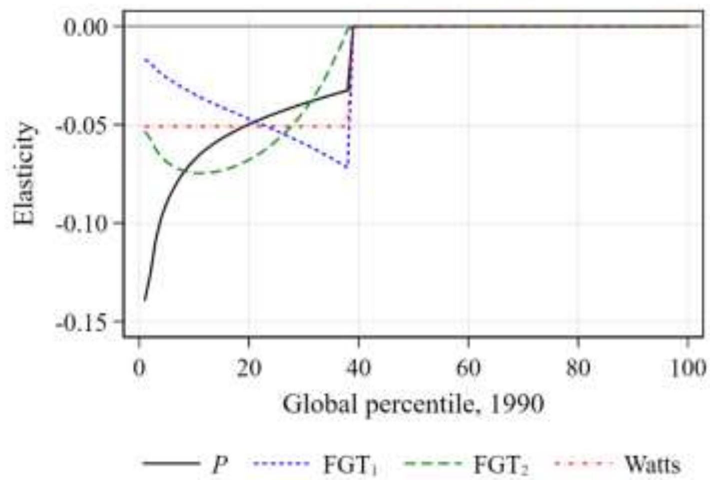
Figure 3.4 has been derived as follows. Each of the poverty measures can be written as $X(y_t) = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}_{y_{it}<z} f^X(y_{it})$ where $\mathbb{I}_{y_{it}<z}$ is the indicator function and $f^X(y_{it})$ is the appropriate function for each poverty measure. The functions for the four poverty measures we consider are simply $f^P(y_{it}) = \frac{z}{y_{it}} - 1$, $f^{FGT(0)}(y_{it}) = 1$, $f^{FGT(1)}(y_{it}) = \frac{z-y_{it}}{z}$, $f^{FGT(2)}(y_{it}) = \left(\frac{z-y_{it}}{z}\right)^2$, and $f^{Watts}(y_{it}) = \ln(\mathrm{z}) - \ln(\mathrm{y_{it}})$. The percent change in poverty is given by:

$$\frac{X(y_t) - X(y_{t-1})}{X(y_{t-1})} = \frac{1}{N}\sum_{i=1}^{N} \frac{\left(\mathbb{I}_{y_{it}<z} f^X(y_{it}) - \mathbb{I}_{y_{it-1}<z} f^X(y_{it-1})\right)}{X(y_{t-1})}$$

Figure 3.4 graphs each term in this average across all percentiles of the income distribution. The average of the points along each line gives the total percent change in the poverty measure.

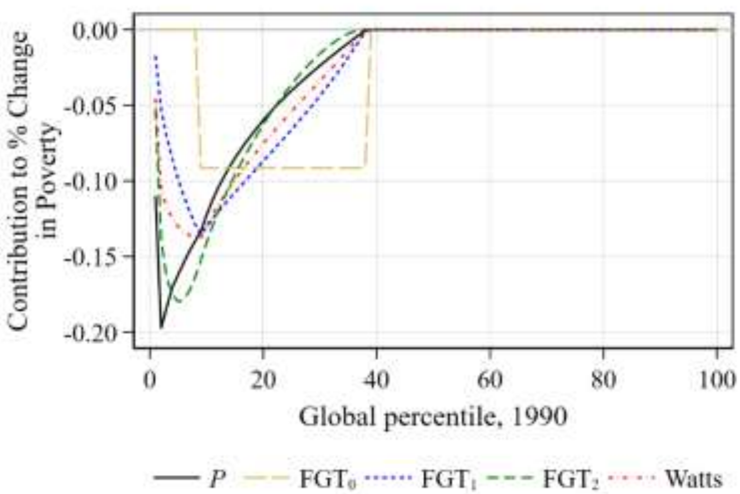
Figure E.1 shows the elasticities of the different poverty measures using $2.15 as the poverty line. This graph is very similar to Figure 3.2, Panel B in the main text which is drawn at $6.85, except that now the non-zero elasticities are compressed into a smaller region corresponding to the lower poverty rate. Figure E.2 replicates Figure 3.3 using $2.15 as the poverty line. Again, qualitatively the graph is very similar to Figure 3.3 in the main text, but is compressed into a smaller region below the lower poverty line.

Figure E.1: Elasticities of Poverty Measures using the $2.15 per Person per Day Threshold



Notes: This figure shows the elasticity of various poverty measures for each percentile of the global income distribution in 1990 using the $2.15 per person per day threshold in 2017 $PPP. See also Table 2.1.

Figure E.2: Contribution to Percent Change in Poverty for Various Poverty Measures



Notes: This figure reports the contributions of each global percentile to percent change in poverty overall poverty from 1990 to 2019 for various poverty measures reported in Table 3.1. Poverty measures are reported using the $2.15 per person per day threshold in 2017 $PPP