INTERNATIONAL BANK FOR RECONSTRUCTION AND DEVELOPMENT

Economics Staff Working Paper No. 114

PROBLEMS OF APPLYING STABLE POPULATION TECHNIQUES IN ESTIMATING DEMOGRAPHIC MEASURES FOR ARAB COUNTRIES.

September 1971

This paper was presented at the annual meeting of the American Statistical Association, as an invited paper in the session: Methods of Population Growth Estimation. It describes an alternative approach to the problem of adjusting birth rates, estimated by the stable population method, for the effects of recent declines in mortality. Although the method was developed for Arab countries, it has a much wider applicability. It can be applied to any developing country where fertility has been fairly constant, is much simpler than other methods now available in the literature, requires fewer parameters for its application and requires no standard table of mortality corrections for its use.

Population and Human Resources Division
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Introduction

One of the common procedures for estimating birth rate and other fertility measures of a population which lacks reliable birth registration statistics is the stable population method. In this method, the observed age distribution is assumed to be stable and is used along with the rate of population growth or information on the level of mortality to estimate the crude birth rate or gross reproduction rate. The conditions necessary for the application of the method are more or less fulfilled in several Arab countries of North Africa and South West Asia. Therefore, in a recent study of the demographic measures of Arab countries the stable population procedure was widely used.[1/] The application is greatly facilitated by the recent publication of a United Nations manual which not only describes the methodology of the stable population method but also gives numerical examples and discusses many practical problems in applying the method to the data of developing countries.[2/] Yet, there are some problems which the manual has not dealt with adequately and some of these were encountered in the study of the Arab countries. This paper describes the approach taken in that study in meeting one of these problems, namely, the effect of a recent decline in mortality on the estimates of birth rates.

1/ Cairo Demographic Centre, Demographic Measures and Population Growth in Arab Countries, (Cairo) 1970.

2/ United Nations: Manual IV, Methods of Estimating Basic Demographic Measures From Incomplete Data, (New York) 1967.

As mentioned above, the conditions necessary for the application of the stable population approach are more or less fulfilled in most of the Arab countries. All of them except Saudi Arabia, Lebanon and Yemen have taken at least one census in recent years, and single-year or five-year age distributions are available. It is also possible to estimate the rate of population growth for some recent period. Fertility has remained fairly constant and external migration has been negligible. However, mortality has fallen at a comparatively rapid rate during the last 20 to 30 years. The assumption of constant mortality level implied in the stable population method of estimating birth rates is, therefore, not valid for these countries at the present time. The birth rates obtained by assuming a stable age distribution require corrections for the effects of the recent decline in mortality.

A procedure for adjusting the estimate of the birth rate for the effects of mortality decline is given in U.N. Manual IV (pp. 25-28), but its application to a particular situation requires that the period of mortality decline t, and the degree of mortality decline, k, be known and the growth rate, or the mortality level used in the calculation, measure the average situation during a five-year or a ten-year period prior to the census. As a result, several problems arise in applying this procedure to some countries. First, the intercensal period may not be 5 or 10 years. It is 12 years for Algeria, 13 years for the UAR, 8 years for Iraq, etc. Second, it is rather difficult to determine the year of onset of the decline of mortality. Undoubtedly, most of the improvements in mortality took place in very recent years, but mortality has been falling, though slowly, even in earlier years, and the period of its decline may be taken as a relatively short period of 15 or 20 years when most of the improvements took place or a longer one including the incipient period. Third, the rate of mortality decline has been changing. In as much as the period of mortality decline is indefinite, the amount of decline per year is also uncertain.

Empirical rule for mortality corrections when growth is used

For countries which have taken several censuses in recent years (e.g. the UAR, Algeria, India, Mexico, etc.), an alternate approach to the problem of adjustment of birth rate for mortality decline is given below. In this approach there is no need to make any assumption about the period of mortality decline or to consult any standard table of mortality corrections.

From the tables of mortality corrections given in U.N. Manual IV (p. 119) and from other considerations, it is evident that the mortality corrections are negative at younger ages and positive at older ages. In other words, there is an age at which no correction is required. Moreover, this age of no correction varies according to the period for which the growth rate is calculated; the longer the period, the higher the age at which the correction is zero. Thus, in Manual IV when the growth rate is calculated for a 5-year period, for all values of t above 15 years the correction is negative at ages 5 and 10 and positive at higher ages. In this case, the amount of correction is a minimum (almost zero) at age 10 for all values of k (amount of decline) and for all values of t (period of decline) above 10 years. When the growth rate is calculated for a 10-year period, for all values of t above 20 years the correction is negative at ages 5, 10 and 15 and positive at all other ages. In this case, the amount of correction is at a minimum

(almost zero) at age 15, irrespective of the degree and the period of mortality decline. Thus, corresponding to each period (5 years, 10 years, 15 years, etc.) for which the growth rate is calculated, there is an age at which no mortality correction is needed. Alternately, corresponding to each age x (5, 10, 15, ...) there is a period n_x (n_5, n_{10}, n_{15} ....) such that the birth rate estimated from the cumulative age distribution C_x and the growth rate during the period n_x will not require any correction for the effects of mortality decline. The problem of the adjustment of the birth rate for mortality declines is thus reduced to one of finding n_x and estimating the growth rate corresponding to this period.

It is seen that, for values of t above 10 years n_{10} is nearly 5 years and n_{15} is nearly 10 years. Experimental calculations on model populations have shown that for long periods of mortality decline n_5 is not far from zero, n_{20} is nearly 15 years, n_{25} is nearly 20 years, etc. On this evidence, an approximate empirical rule to estimate the birth rate which does not require any significant correction for mortality decline may be stated as follows:

Estimate the birth rate using C_x and the growth rate calculated for the period n_x where

$$n_x = x - 5 \quad ; \qquad x = 5, 10, \ldots. 40 \qquad (1)$$

According to this rule, the birth rates are estimated by using different growth rates at different ages thus eliminating the need for mortality corrections, instead of using the same growth rate at all ages and then correcting the estimates for the effects of the decline. However, it should be noted that the period n_x given against ages 5, 10, 15, etc. in equation (1) are approximate values and more exact values can be obtained empirically.

The growth rates for the 5-year period, 10-year period, 15-year period ... 35-year period prior to the census, are generally not available, and these have to be estimated from intercensal growth rates. The following procedure is recommended:

(i) plot the available intercensal growth rates on a graph (e.g. 1950-1960 at 1955, 1940-50 at 1945, etc.);

(ii) join the points by a smooth curve and extend the curve to the date of the latest census (1960 in the example);

(iii) read off the growth rates at the mid point of each 5-year interval starting from the latest census date (mid point of 1955-60, 1950-55, 1945-50, etc.);

(iv) calculate the average growth rate for longer periods taking the arithmetic mean of the estimated 5-year rates covering the period (e.g. the growth rate for 1940-60 may be taken as equal to the arithmetic mean of the rates of 1940-45, 1945-50, 1950-55 and 1955-60).

In the numerical examples given below the above procedure appears to be satisfactory for all rates except those at the time of the latest census and consequently, the birth rate estimated from C_5 by the empirical rule may not be wholly reliable. The growth rates at higher ages are averages of a number of rates; the errors of interpolation tend to cancel out in the averaging process.

To check whether the procedure proposed in equation (1) gives reasonably accurate estimates of birth rates at ages other than 10 and 15 years, some tests were made and the results are given in Tables 1 and 2.

In Table 1, the difference between the actual birth rate and that estimated by the empirical rule in 27 model quasi-stable (constant fertility rates and declining mortality rates) populations is given. All the models were obtained by the component method of population projection starting with initial stable age distributions with mortality level 5.5 (South Family of Princeton Model[1/]) and GRR 2.5, 3.0 and 3.5 and assuming that the GRR remains constant through the entire period of projection (20 years, 30 years and 40 years) and expectation of life at birth increases at the rate of 0.5, 0.625 and 0.75 year per calendar year. The three initial levels of fertility in combination with three rates of decrease of mortality and three periods of projection give the 27 model quasi-stable age distributions.

The actual birth rates were obtained by dividing the number of births during a period by the average population of that period. For example, the actual birth rate corresponding to age 25 was the ratio of the average annual births during the 25-year period prior to the end of the projection period to the average population of that period. The births and growth rates for periods before the base year of the projection were taken as those of the corresponding stable population. The rates according to the empirical rule were obtained by using the final age distribution for each model and different growth rates as stipulated in the empirical rule (equation 1 above).

It is seen from Table 1 that the birth rates derived according to the empirical rule are very close to the actual rates. The maximum difference between the actual and estimated birth rates is of the order of 0.8 units

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1/ Coale, A.J. and Demeny, P., Regional Model Life Tables and Stable Populations, Princeton University Press, (Princeton) 1966.

Table 1: DIFFERENCE BETWEEN ACTUAL BIRTH RATES AND THOSE ESTIMATED BY THE EMPIRICAL RULE IN MODEL POPULATIONS

(Using growth rates)

Age	Rate of increase of expectation of life at birth								
	0.50			0.625			0.75		
	Period of mortality decline			Period of mortality decline			Period of mortality decline		
	20 years	30 years	40 years	20 years	30 years	40 years	20 years	30 years	40 years
	G R R = 2.5								
5	-0.15	-0.04	+0.12	-0.25	-0.17	+0.17	-0.18	-0.08	+0.12
10	-0.05	+0.19	+0.38	-0.07	+0.21	+0.42	-0.05	+0.23	+0.41
15	-0.06	+0.28	+0.56	-0.07	+0.34	+0.53	-0.08	+0.40	+0.58
20	-0.28	+0.35	+0.65	-0.32	+0.36	+0.64	-0.38	+0.43	+0.62
25	-0.35	+0.27	+0.70	-0.45	+0.21	+0.73	-0.48	+0.26	+0.63
30	-0.33	-0.08	+0.56	-0.44	0.00	+0.69	-0.58	-0.12	+0.64
35	-0.39	-0.23	+0.47	-0.52	-0.27	+0.51	-0.64	-0.33	+0.49
40	-0.44	-0.49	+0.37	-0.62	-0.47	+0.08	-0.74	-0.56	+0.09
	G R R = 3.0								
5	-0.21	-0.13	-0.10	-0.29	-0.18	+0.10	-0.36	-0.16	-0.01
10	-0.08	+0.15	+0.29	-0.12	+0.13	+0.34	-0.07	+0.16	+0.29
15	-0.06	+0.27	+0.49	-0.15	+0.36	+0.53	-0.17	+0.36	+0.49
20	-0.25	+0.35	+0.61	-0.33	+0.43	+0.65	-0.44	+0.45	+0.64
25	-0.30	+0.25	+0.75	-0.41	+0.30	+0.65	-0.46	+0.39	+0.75
30	-0.35	+0.08	+0.75	-0.41	-0.04	+0.78	-0.46	+0.01	+0.73
35	-0.40	-0.08	+0.59	-0.53	-0.22	+0.68	-0.55	-0.34	+0.53
40	-0.44	-0.31	+0.29	-0.69	-0.28	+0.37	-0.72	-0.52	+0.23
	G R R = 3.5								
5	-0.40	-0.23	-0.05	-0.34	-0.26	-0.02	-0.36	-0.25	0.00
10	-0.19	+0.05	+0.20	-0.12	+0.06	+0.29	-0.09	+0.08	+0.30
15	-0.15	+0.26	+0.38	-0.10	+0.32	+0.49	-0.09	+0.30	+0.47
20	-0.31	+0.35	+0.54	-0.40	+0.44	+0.59	-0.50	+0.45	+0.62
25	-0.28	+0.30	+0.66	-0.40	+0.39	+0.74	-0.50	+0.41	+0.80
30	-0.30	+0.07	+0.79	-0.35	+0.08	+0.74	-0.39	+0.17	+0.83
35	-0.32	-0.03	+0.31	-0.42	-0.17	+0.74	-0.41	-0.19	+0.72
40	-0.45	-0.15	+0.47	-0.61	-0.21	+0.48	-0.55	-0.46	+0.31

TABLE 2: COMPARISON OF BIRTH RATES ESTIMATED BY THE EMPIRICAL RULE WITH THOSE OBTAINED BY THE MANUAL IV PROCEDURE; FEMALE POPULATION OF INDIA, 1961 AND MALE POPULATION OF MEXICO, 1960.
(Using growth rates)

Age	India, 1961			Mexico, 1960		
	Empirical rule	Manual IV	Difference	Empirical rule	Manual IV	Difference
5	38.9	38.3	0.6	37.5	37.0	0.5
10	46.2	46.5	-0.3	42.0	41.1	0.9
15	44.6	44.6	0.0	43.7	43.4	0.3
20	40.5	40.0	0.5	42.9	43.3	-0.4
25	41.8	41.0	0.8	42.7	43.1	-0.4
30	44.0	43.7	0.0	43.0	43.3	-0.3
35	45.3	45.4	-0.1	42.7	43.1	-0.4
40	45.3	45.7	-0.4	44.1	44.4	-0.3

Note: The estimates by the Manual IV procedure are taken from Manual IV p.69 and 71, and those by the empirical rule are based on West Family of stable populations, C_x values and the growth rates as stipulated by equation (1). For India the intercensal growth rates of 1921-31, 1931-41, 1941-51, and 1951-61 were first calculated, and those of 1951-56, 1946-51 .... 1926-31 were obtained from them by interpolation and of 1956-61 and 1961 by graphical extrapolation. The growth rates for other periods (eg. 1926-1961) were obtained as simple average of the 5-year rates covering that period. For Mexico a similar procedure was followed using the growth rates of 1950-60, 1940-50, 1930-40 and 1921-30.

Source: Demographic Measures and Population Growth in Arab Countries, op cit appendix I, Table 4. p.334.

(less than 2 percent) but most of the differences are of much smaller magnitude. The average absolute deviation varies between 0.2 units and 0.5 units. On the whole, the error in the estimate of the birth rate obtained by the empirical rule is minimum when the period of mortality decline is about 30 years, when the rate of increase of expectation of life at birth has been about 0.5 year per year (Table 4). When the period of mortality decline is less than 30 years the empirical rule tends to underestimate the birth rate and when the period of mortality decline is more than 30 years, the method tends to overestimate the birth rate. The level of fertility does not appear to be a relevant factor. Thus, at least for the range of variation of GRR and the rates and periods of mortality decline implied in the projections, the empirical rule seems to give fairly satisfactory estimates of the birth rate.

In Table 2, comparisons are made in two actual populations, namely, the female population of India, 1961 and the male population of Mexico, 1960. The rates estimated by the Manual IV procedure are taken from that publication. They are close to the values obtained by the empirical rule, the maximum difference being 0.9 units or about 2 percent. The range of variation of the rates is, in fact, slightly smaller for the set of values obtained by the empirical rule. For India, the estimates obtained by this new approach are as good as, if not better than, those obtained by the Manual IV procedure. For Mexico, the Manual IV procedure seems to give more consistent estimates. At the same time, the new approach has an advantage in that there is no need to consult any standard table of mortality corrections.

Empirical rule when mortality level is used

Independent data to estimate the level of mortality were not available for most of the Arab countries. Therefore, the stable population method using mortality level instead of intercensal growth rate was not attempted in that study, although its use in place of the growth rate would have improved the accuracy of the estimates. The use of mortality level, however, would not have eliminated the need for corrections for mortality decline; the estimates obtained by the use of a single mortality level at all ages require correction for mortality decline. The nature of the corrections is somewhat similar to the situation when growth rates are used. They are negative at younger ages and positive at older ages. Corresponding to each mortality level used, there is an age where no correction is required. The longer the period to which the mortality level refers (5 years before the census, 10 years before the census, etc.) the higher is the age at which the transition from negative correction to positive correction takes place.

A working rule similar to that proposed above when growth rate is used may be given when the mortality level is used. It consists in the use of different mortality levels at different ages; the mortality level at the time of the census to be used along with the cumulative age distribution at age 5 (C_5), the average mortality level during the 5-year period prior to the census to be used along with C_{10}, that during the 10-year period prior to the census along with C_{15}, etc.

The validity of this rule is checked with model quasi-stable populations (the same 27 models mentioned earlier) and the results are given in Table 3. The differences between the actual birth rates and those estimated by the empirical rule are on the whole small, none exceeding 0.9.

Table 3: DIFFERENCE BETWEEN ACTUAL BIRTH RATES AND THOSE ESTIMATED BY THE EMPIRICAL RULE IN MODEL POPULATIONS

(Using mortality levels)

Age	Rate of increase of expectation of life at birth								
	0.50			0.625			0.75		
	Period of mortality decline			Period of mortality decline			Period of mortality decline		
	20 years	30 years	40 years	20 years	30 years	40 years	20 years	30 years	40 years
	G R R = 2.5								
5	-0.35	-0.39	-0.21	-0.09	-0.42	-0.20	-0.42	-0.32	-0.21
10	-0.35	-0.20	-0.13	-0.12	-0.23	-0.12	-0.39	-0.24	-0.13
15	-0.30	-0.16	-0.05	-0.26	-0.19	-0.08	-0.44	-0.21	-0.09
20	-0.40	-0.19	-0.02	-0.39	-0.20	-0.06	-0.54	-0.21	-0.08
25	-0.41	-0.21	-0.02	-0.52	-0.26	-0.05	-0.64	-0.30	-0.08
30	-0.30	-0.33	-0.06	-0.60	-0.41	-0.08	-0.71	-0.47	-0.11
35	-0.05	-0.45	-0.11	-0.66	-0.55	-0.16	-0.78	-0.65	-0.22
40	+0.23	-0.49	-0.26	-0.72	-0.73	-0.36	-0.84	-0.86	-0.46
	G R R = 3.0								
5	-0.38	-0.33	-0.24	-0.47	-0.35	-0.25	-0.52	-0.37	-0.23
10	-0.36	-0.23	-0.14	-0.42	-0.26	-0.14	-0.45	-0.27	-0.15
15	-0.35	-0.18	-0.05	-0.41	-0.20	-0.08	-0.48	-0.21	-0.10
20	-0.41	-0.18	-0.03	-0.47	-0.20	-0.02	-0.57	-0.21	-0.07
25	-0.43	-0.20	0.00	-0.52	-0.23	-0.03	-0.59	-0.26	-0.04
30	-0.26	-0.28	-0.02	-0.58	-0.36	-0.02	-0.70	-0.42	-0.05
35	+0.10	-0.36	-0.05	-0.64	-0.46	-0.08	-0.75	-0.56	-0.14
40	+0.43	-0.38	-0.17	-0.70	-0.62	-0.24	-0.80	-0.75	-0.33
	G R R = 3.5								
5	-0.49	-0.34	-0.26	-0.52	-0.38	-0.23	-0.58	-0.41	-0.24
10	-0.43	-0.24	-0.15	-0.46	-0.28	-0.15	-0.48	-0.30	-0.16
15	-0.38	-0.18	-0.07	-0.44	-0.22	-0.11	-0.50	-0.25	-0.10
20	-0.44	-0.18	-0.03	-0.52	-0.19	-0.06	-0.60	-0.23	-0.07
25	-0.38	-0.18	+0.02	-0.52	-0.21	-0.04	-0.61	-0.26	-0.03
30	-0.24	-0.26	+0.02	-0.58	-0.35	-0.02	-0.69	-0.40	-0.03
35	+0.21	-0.28	+0.02	-0.62	-0.40	-0.03	-0.72	-0.52	-0.10
40	+0.63	-0.29	-0.10	-0.69	-0.52	-0.18	-0.77	-0.69	-0.27

TABLE 4: AVERAGE ABSOLUTE DEVIATIONS OF ACTUAL AND ESTIMATED BIRTH RATE BY LEVEL OF FERTILITY, RATE OF INCREASE OF EXPECTATION OF LIFE AT BIRTH, AND PERIOD OF MORTALITY DECLINE

GRR	Δe_o	Period of mortality decline 20	30	40
		Using growth rates		
2.5	0.500	0.26	0.24	0.47
2.5	0.625	0.34	0.25	0.47
2.5	0.750	0.39	0.31	0.45
3.0	0.500	0.26	0.20	0.48
3.0	0.625	0.37	0.24	0.51
3.0	0.750	0.40	0.30	0.46
3.5	0.500	0.30	0.18	0.42
3.5	0.625	0.34	0.24	0.51
3.5	0.750	0.36	0.29	0.51
		Using mortality levels		
2.5	0.500	0.30	0.30	0.11
2.5	0.625	0.42	0.37	0.14
2.5	0.750	0.60	0.41	0.17
3.0	0.500	0.34	0.27	0.09
3.0	0.625	0.53	0.34	0.11
3.0	0.750	0.61	0.38	0.14
3.5	0.500	0.40	0.24	0.08
3.5	0.625	0.54	0.32	0.10
3.5	0.750	0.62	0.38	0.12

Source: Calculated from Tables 1 and 3.

Most of the differences are negative suggesting that the empirical rule, in general, underestimates the birth rate. The difference varies from one age to another and from one model population to another. The empirical rule appears to be most accurate when the period of mortality decline is about 40 years, the rate of increase of expectation of life at birth is about 0.5 years, and when GRR is 3.5 (Table 4). However, the influence of the rate of mortality change and the level of fertility is, on the whole, negligible compared with the influence of the period of mortality decline.

The tests given in Tables 1, 2 and 3 do not establish the general validity of the empirical rule. In fact, the procedure may not be valid for all situations, but it appears to be satisfactory as a working rule for populations for which the GRR ranges between 2.5 and 3.5 and the expectation of life at birth has increased at a rate of one-half to three-fourths of a year per year for about 20 to 40 years, a situation which is closely approximated in most of the developing countries.