Policy Research Working Paper 10076

A Welfarist Theory Unifying Monetary and Non-Monetary Poverty Measurement

Benoit Decerf

WORLD BANK GROUP
Development Economics
Development Research Group
June 2022

Policy Research Working Paper 10076

Abstract

Multidimensional poverty measures are increasingly used in practice even though they face strong criticism and generate longlasting debates. These contentions primarily find their origin in the divergence between standard poverty identification practices and a welfarist definition of the poor. This paper fills this gap by constructing a poverty measurement theory that (i) adopts a welfarist definition of the poor, (ii) acknowledges that the relevant welfare function is only partially known and (iii) encompasses both market and non-market dimensions of well-being. The theory shows that standard identification practices are not flexible enough in order to properly account for the multidimensional nature of well-being. This nature implies that an individual is poor when she experiences an extremely low outcome in some dimension or/and when she cumulates moderately low outcomes in several dimensions. The paper proposes a simple refinement that better reflects this insight. The paper uses the theory in order to provide answers to several longlasting debates. The theory provides a conceptual foundation from which practitioners may derive guidance for the many choices they face.

This paper is a product of the Development Research Group, Development Economics. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://www.worldbank.org/prwp. The author may be contacted at bdecerf@worldbank.org.

A Welfarist Theory Unifying Monetary and Non-Monetary Poverty Measurement.*

Benoit Decerf

JEL: I32.
Keywords: Multidimensional Poverty, Poverty Identification, Poverty Measurement.

*Acknowledgments : I am grateful to Francisco Ferreira for several helpful discussions and suggestions. I thank James Foster, Maria Ana Lugo, Berk Ozler and Natalie Nairi Quinn who provided useful comments. I am grateful to Martin Ravallion for an insightful exchange at an early stage of this project. I thank all the participants at the internal seminar of the research departement of the World Bank. All errors remain mine. The findings, interpretations, and conclusions expressed in this paper are entirely those of the author and should not be attributed in any manner to the World Bank, to its affiliated organizations, or to members of its Board of Executive Directors or the countries they represent.

1 Introduction

> "Future progress in devising useful composite indices of development will require that theory catches up with measurement practice." *Ravallion (2012)*

A broad consensus has now emerged that human well-being is multidimensional (Stiglitz et al., 2009). Vast literatures are dedicated to normative indicators of well-being accounting for this multidimensionality (Fleurbaey, 2009). Among these indicators, multidimensional poverty measures are gaining prominence because they are increasingly used in practice, both at national and international levels.[1]

However, multidimensional poverty measurement practices face strong criticism and generate long-lasting debates. The major contentions relate to the way the multidimensionally poor are identified (Ravallion, 2011; Pattanaik and Xu, 2018). The standard practice is to identify an individual as poor when the weighted sum of her deprivation statuses across dimensions exceeds some threshold. Unfortunately, the theory behind this practice (Alkire and Foster, 2011a) offers no guidance to help the practitioners make the necessary choices.[2] This is a major issue because these choices have massive implications for poverty comparisons and trends (Aaberge and Brandolini, 2015).[3] Ravallion (2011) regrets that these measures aggregate dimensions in a way that is unrelated to well-being. In particular, the weights routinely selected are unrelated to marginal rates of substitution.[4] Also, when data are cardinal, measuring outcomes using a dichotomous deprivation status "wastes" well-being relevant information. About the selection of dimensions, a fierce debate still rages on about whether monetary poverty should be integrated with non-monetary poverty (Atkinson, 2016).

We argue that the main cause for these debates is that standard identification practices are not informed by a welfarist definition of the poor. A welfarist definition holds that an individual is poor when her well-being, represented by a welfare function, is too low. The bulk of the literature on multidimensional poverty measurement is not welfarist (Bourguignon and Chakravarty, 2003; Alkire and Foster, 2011a; Datt, 2019).[5] The few welfarist theories (Maasoumi and Lugo, 2008; Decancq et al., 2019) cannot

[1] More than 70 countries have now adopted an official multidimensional poverty measure. International institutions such as the World Bank, the UNDP or the European Commission also adopted their own. Multidimensional poverty measures are also used in order to measure progress towards the Sustainable Development Goals.

[2] The values of weights and cutoffs are considered exogenously determined in Alkire and Foster (2011a). Alkire et al. (2015) do provide guidance for these choices, but this guidance is based on experience and best practices, rather than on a consistent notion of individual well-being, leaving the practitioner without a clear compass.

[3] This issue started a literature devising methods to check the robustness of poverty evaluations to the selection of alternative parameters values (Atkinson and Bourguignon, 1982; Duclos et al., 2006; Bourguignon and Chakravarty, 2019).

[4] Using equal weights is a popular default choice (Alkire et al., 2015). Selecting equal weights resonates with the counting approach (Atkinson, 2003), which counts the number of dimensions for which an individual is deprived and uses a threshold number for identification (Guio et al., 2017).

[5] They typically justify their non-welfarist approaches advancing pragmatic argu-

inform identification practices. The reason is that they do not account for the main information constraint faced by practitioners, namely that they only have partial information on the relevant welfare function. Currently, there exists no theory that can help guide and improve the identification practices inherent to the increasingly popular, though criticized, multidimensional poverty measures.

In this paper, we develop a welfarist theory of poverty measurement that accounts for practitioners' information constraint and derive some of its implications for multidimensional poverty identification practices. We extend the welfarist theory behind monetary poverty measurement to dimensions of well-being for which no market exists. We demonstrate that this extended theory allows shedding new light on several long-standing debates. Our theory reveals that the multidimensional nature of well-being is not properly accounted for by standard identification practices. We show how to refine them in a way that better reflects this multidimensionality. Our extended theory provides the conceptual foundation for taking identification practices beyond what Ravallion (2016) considers to be an essentially ad-hoc aggregation of different dimensions.

The novelty of our poverty measurement theory is to combine three elements: a welfarist definition of the poor, the practitioners' information constraint and the absence of some markets. We assume that the welfare threshold defining poverty is exogenously given, but that the "true" trade-offs between different dimensions at that welfare threshold are imperfectly known to the practitioner. We impose two natural restrictions on these trade-offs that echo key arguments expressed in the literature. In a nutshell, dimensions are to some extent substitutable (Ravallion, 2011), but there is a limit to their substitutability (Sen, 1992). These two restrictions jointly imply that two types of individuals are poor: (i) individuals with an extremely low achievement in at least one dimension ("extremors") and (ii) individuals who are not extremors but cumulate moderately low achievements in several dimensions ("cumulators"). The information constraint implies that the identification method used by practitioners cannot make exactly the same trade-offs between dimensions as the welfare function. The absence of some markets implies that the identification method cannot be welfare-consistent, i.e., some poor individuals are not identified as poor ("exclusion error") or/and some non-poor individuals are identified as poor ("inclusion error"). We propose robust criteria that allow comparing imperfect identification methods as a function of their distance to welfare-consistency.

First, we apply our theory to the case for which non-market dimensions are measured through cardinal variables. The standard practice is to use *linear* identification methods, which aggregate across dimensions by making a weighted sum of the outcomes in each dimension. With cardinal data, the outcomes entering this weighted sum can be measured through alternative information bases, namely achievement, deprivation or (deprivation)

ments, such as the difficulty to measure welfare in practice (e.g., p 308 of Alkire and Foster (2011b)) or the fact that policy makers push towards the use of dimension-specific cutoffs (e.g., p 27 of Bourguignon and Chakravarty (2003)).

status.[6] Monetary poverty identification practices rely on achievements, while non-monetary poverty identification practices, such as the union or the intersection approaches, rely on statuses.[7]

We show that very limited information on the welfare function is sufficient to rank linear identification methods using different information bases. We show that Ravallion (2011) makes a valid point against linear methods using deprivation status, but that his critique does not necessarily justify replacing them by linear methods using achievement. One of our results shows that the limits of the latter methods may outweigh the benefits of using weights corresponding to marginal rates of substitution.[8] In contrast, we show that linear methods using deprivation are superior to those using status, as suggested by Pattanaik and Xu (2018). We show they are also superior to those using achievement, which implies that practitioners should use them when data are cardinal.

Intuitively, the inferiority of linear methods using status or achievement is that they do not provide the flexibility to simultaneously identify cumulators and extremors. For instance, linear methods using achievement are bound to make exclusion errors on some extremors, because they allow that a high achievement in one dimension "compensates" for a low achievement in another dimension. In contrast, the union approach identifies well extremors, but it is bound to make exclusion errors on cumulators because this approach requires using small dimension-specific cutoffs. Another key message from our analysis is that using weights that correspond to the marginal rates of substitution at some bundle is not sufficient for a linear method to make a good identification. Indeed, this only ensures a good *local* approximation of the trade-offs. In the absence of markets, a good identification requires a good approximation at *all* bundles yielding the welfare threshold.

Second, we apply our theory in order to improve identification when non-market dimensions are measured through "ordinal" variables (ordered categorical variables). With ordinal data, linear methods using achievement or deprivation are not meaningful, which only leaves linear methods using status. We show that standard linear methods using status cannot properly account for the multidimensional nature of well-being. The reason is that their single dimension-specific cutoff does not offer the flexibility to correctly identify both cumulators and extremors. Extremors should be identified based on small cutoffs using the union approach while cumulators should be identified based on larger cutoffs using the intersection approach. To improve on this, we propose a small refinement of standard practices that is based on two dimension-specific cutoffs, which lead to three statuses:

[6] The status basis is dichotomous. The achievement and deprivation basis are continuous, even if the latter censors outcomes above dimension-specific deprivation cutoffs.

[7] For instance, consumption poverty measures typically identify the poor by aggregating the quantities (=achievements) of different market goods (=dimensions) consumed using prices as weights. Alternatively, the UNDP-OPHI's global MPI identifies the poor by a weighted sum of the deprivation statuses in each indicator entering its definition.

[8] The limit of using linear methods using achievement is that a sufficiently large achievement in one dimension may "compensate" for an extremely low achievement in another dimension (Alkire and Foster, 2011b).

extreme deprivation, moderate deprivation and no deprivation.

Third, we study identification when both monetary and non-monetary dimensions matter to well-being. We consider three different data constraints. In the absence of any data constraint, we show that identification methods that combine monetary and non-monetary dimensions are superior to both purely monetary and to purely non-monetary methods. The reason is that the latter are bound to make exclusion errors on some individuals who have an extremely low achievement in the ignored dimension. When data come from two separate surveys, the association between monetary and non-monetary outcomes is not observed. We show by means of an example that the two separate surveys may provide a very noisy picture of poverty. This demonstrates the potentially large added-value of collecting achievements on all dimensions in the same survey. When data come from one survey that ignores either the monetary dimensions or the non-monetary dimensions, we study how the weights for the observed dimensions should be "distorted" in order to account for the non-observed dimensions. Some current practices seem to apply such distortions, which could suggest that some practitioners may consider a general notion of well-being, rather than a narrower notion limited to the observed dimensions.

We contribute to the poverty measurement literature on several accounts. First, we provide a welfarist theory that unifies monetary and non-monetary poverty measurement. In our theory, there is no conceptual distinction between a "multidimensional structure" and a "unidimensional structure" to poverty measurement (Chakravarty and Lugo, 2019). The unidimensional structure follows from considering a unique welfare aggregator summarizing achievements in all dimensions. The multidimensional structure follows from considering extreme achievements thresholds below which an individual is poor regardless of her achievements in other dimensions. Considering these elements jointly allows accounting for the main arguments raised by both sides. Second, our results shed new light on longstanding debates around multidimensional poverty measurement. We show that the fact that the weights used in practice do not correspond to marginal rates of substitution is not a sufficient reason to discard these practices. However, when cardinal data are available on at least two dimensions, the practitioner should not identify the poor using the dichotomous status basis. We show that measures combining monetary and non-monetary dimensions better identify the poor than the measures that keep them separate.[9] The former may require more costly data collection than the latter, but such collection may potentially greatly improve identification.

The paper is organized as follows. We present our theory in Section 2. We apply our theory to identification methods under cardinal data in Section 3. We apply our theory to identification methods under ordinal data in Section 4. We apply our theory to study monetary versus non-monetary poverty measurement in Section 5. We conclude in Section 6.

[9] There exists currently no consensus on whether it makes sense to aggregate monetary measures with non-monetary measures (Atkinson, 2016), and if it does make sense, how it should be done. Our result thus provides a foundation for the Multidimensional Poverty Measure introduced by the World Bank (2018), defined in Section 7.1.

2 A welfarist theory for poverty identification

Following Sen (1976), a poverty measure is typically constructed in two steps. In the identification step, the set of individuals is partitioned between those who are identified as poor and those who are not. In the second step, a poverty index is selected in order to aggregate the contributions to poverty of all individuals identified as poor. In this paper, we focus on the identification step.[10]

2.1 Basic framework

There is a continuum $N = [0, 1]$ of individuals, indexed by i. Let $x_i = (x_{i1}, x_{i2}) \in \mathbb{R}^2_+$ denote the bundle consumed by individual i, which summarizes her achievements (or attainments) in the two dimensions. For the sake of simplicity, we restrict our attention to the 2-dimensions case. However, our theory readily generalizes to the m-dimensions case. Let $\boldsymbol{x} = (x_i)_{i\in N}$ denote an allocation.

Individual welfare is formalized by the utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$, which represents a preference relation on the set of bundles. As is standard in the monetary poverty measurement literature (Ravallion, 2016), we assume preference homogeneity, i.e., all individuals hold a common preference. We assume that U is continuous, strictly monotonic in both dimensions and differentiable everywhere.

Bundles are related to individual endowments. Each individual is born with an endowment $\omega_i = (\omega_{i1}, \omega_{i2}) \in \mathbb{R}^2_+$. The distribution of endowments is denoted by $\boldsymbol{\omega} = (\omega_i)_{i\in N}$. Let $F(\boldsymbol{\omega})$ denote the joint distribution of endowments and let $f(\boldsymbol{\omega})$ denote the associated density function. We assume that the density function f has full support and can be differentiated everywhere. Hence, for any bundle $x \in \mathbb{R}^2_+$, there is some individual $i \in N$ with $\omega_i = x$. When a market exists between the two dimensions, we assume that any individual i selects her achievements in the budget set defined by her endowment ω_i and equilibrium prices $p = (p_1, p_2) \in \mathbb{R}^2_+$ in a way that maximizes $U(x_i)$. In the absence of markets, we assume that any individual i consumes her endowment, which means that $x_i = \omega_i$.

An individual is defined as (multidimensionally) poor if her welfare is too low. We assume that there is an exogenously-given level of utility $\underline{U}$ such that the set of poor individuals is $N_p(\boldsymbol{x}) = \{i \in N \mid U(x_i) < \underline{U}\}$. To avoid trivialities, we assume $\underline{U} > U(0, 0)$. Our theory is silent about the selection of the minimal level of utility $\underline{U}$, which is typically a political decision. Graphically, individual i is poor if her bundle is below the indifference curve associated to utility level $\underline{U}$, which we call indifference curve $\underline{U}$ for short.

We impose two minimal restrictions on the shape of indifference curve $\underline{U}$. These restrictions echo positions frequently expressed in the multidi-

[10] Our results have direct implications for poverty indices because these indices require an identification method. However, our framework could also be used to derive implications that are specific to the poverty index. Before investigating these implications, a natural first step is to define a meaningful and defensible identification method.

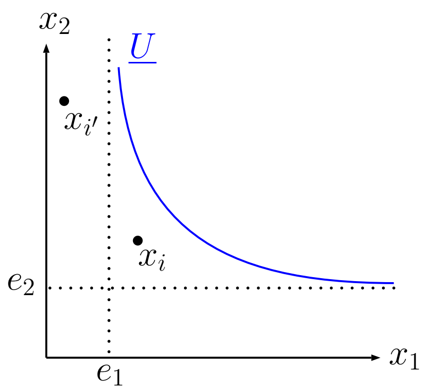

Figure 1: Indifference curve $\underline{U}$ satisfying our two restrictions.
Individual i is a cumulator and i' is an extremor.

mensional poverty measurement literature. First, we assume that there is some substitutability between the two dimensions along indifference curve $\underline{U}$. That is, an individual with welfare level $\underline{U}$ is willing to reduce her achievement in one dimension if her achievement in the other dimension is sufficiently increased. Formally, we require that indifference curve $\underline{U}$ is strictly convex. This restriction reflects the view that individuals on the poverty frontier also make trade-offs between different dimensions (Ravallion, 2011).[11] Second, we assume that there is a limit to the substitutability between the two dimensions along indifference curve $\underline{U}$. That is, for each dimension there is an achievement threshold below which the individual is poor regardless of her achievement in the other dimension. Formally, for any $j \in \{1, 2\}$ there is an extreme achievement threshold $e_j > 0$ such that $U(x) < \underline{U}$ when $x_j \leq e_j$. Graphically, the indifference curve $\underline{U}$ admits asymptotes at these extreme thresholds. This restriction reflects the view that each core dimension of well-being is essential (Sen, 1992; Duclos et al., 2006; Chakravarty and Lugo, 2019).[12] We assume throughout that the shape of indifference curve $\underline{U}$ is fixed but arbitrary as long as it satisfies these restrictions.

Interestingly, our two restrictions imply that we can partition the set of poor individuals into two categories. First, an individual may be poor because her achievement in some dimension is below the extreme threshold, i.e., $x_j \leq e_j$ for some dimension j. We call such poor individual an "**extremor**". Second, an individual with no extreme achievement may be poor because she cumulates moderately low achievements in several dimensions, $x_j > e_j$ for all dimensions j. We call such poor individual a "**cumulator**". In Figure 1, i is a cumulator and i' is an extremor. Distinguishing between these two types of poor individuals not only makes sense from a conceptual

[11] This restriction encapsulates the view that the overlap of low achievements in several dimensions matters for identification (Ferreira and Lugo, 2013).

[12] This restriction is reasonable for core dimensions of well-being, such as consumption, health or freedom. For instance, an individual whose health is very bad has low well-being regardless of her consumption. However, this assumption would be much less compelling for subcategories of consumption, like for instance consumption of rice or consumption of wheat.

perspective, but may also help improve anti-poverty programs. Indeed, unlike cumulators, extremors cannot be lifted out of poverty when some of their moderately low achievements are improved.

2.2 Imperfect knowledge and identification methods

In order to reflect the information constraints faced by practitioners, we assume that the shape of indifference curve $\underline{U}$ is only partially known. This partial knowledge reflects the fact that welfare is challenging to reliably measure in practice (Alkire and Foster, 2011b). At one extreme, it could be that only one bundle on this indifference curve is known. Information constraints imply that the practitioner cannot use indifference curve $\underline{U}$ in order to identify the poor. In other words, she cannot rely on the unidimensional approach to poverty measurement with utility function U as the aggregator and $\underline{U}$ as the target.

In order to partition the set of individuals into two groups, those who are identified as (multidimensionally) poor and those who are not, the practitioner must use an identification method. An identification method evaluates individual bundles by comparing the value returned by some aggregation function $M : \mathbb{R}^2_+ \to \mathbb{R}$ to some identification threshold $k \in \mathbb{R}$. The set of individuals identified as poor is denoted by $\hat{N}_p(\boldsymbol{x}) = \{i \in N \mid M(x_i) < k\}$. For instance, in the case of monetary poverty measurement, for which achievements are quantities, individual i is identified as poor when

$$p_1 x_{i1} + p_2 x_{i2} < k, \tag{1}$$

where $k = p_1 z_1 + p_2 z_2$ is the poverty line, based on a reference bundle z. This identification method makes a linear aggregation of achievements using prices as weights. Graphically, an identification method defines an identification contour below which bundles are identified as poor.[13] The linear identification contour defined in Eq. (1) is illustrated in Figure 2.

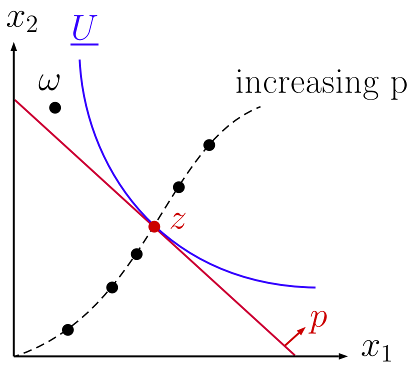

Figure 2: Welfare-consistent identification method with markets. *The identification contour is in red.*

Crucially, the information constraints imply that the shape of the identification contour is in general different from the shape of indifference curve

[13] The identification contour is conceptually different from the "poverty frontier" (Duclos et al., 2006), which in our framework corresponds to the indifference curve $\underline{U}$.

$\underline{U}$. In Figure 2, an individual that would consume endowment ω would not be identified as poor even if endowment ω yields a welfare level smaller than $\underline{U}$. However, this does not necessarily mean that the set of poor individuals is unrelated to the set of individuals identified as poor. When measuring monetary poverty, the objective for the practitioner is to select an identification method that perfectly identifies the poor, i.e., such that $\hat{N}_p(\boldsymbol{x}) = N_p(\boldsymbol{x})$. When it is the case, the identification method is considered *welfare-consistent*.

The monetary poverty measurement literature provides guidance for the practitioner. This literature proposes conditions under which the identification method is welfare-consistent. For instance, the method defined in Eq. (1), which uses prices as weights, is welfare-consistent when bundle z maximizes welfare given prevailing prices and $U(z) = \underline{U}$ (Ravallion, 1998).[14] Prices are directly connected to welfare as they capture marginal rates of substitution (Sugden, 1993). However, the key reason why perfect identification is possible although the identification contour differs from indifference curve $\underline{U}$ is the presence of markets. Indeed, an individual whose endowment is ω will trade part of her achievement in dimension 2 in order to increase her achievement in dimension 1. Optimal behavior implies that all bundles are on an increasing path (see Figure 2). Under the conditions stated above, the identification contour ranks all bundles on the increasing path in the same way as the indifference curve $\underline{U}$.[15]

In the absence of markets, identification methods cannot be welfare-consistent under information constraints. This follows from the fact that (i) the shape of the identification contour differs from the shape of indifference curve $\underline{U}$ and (ii) individuals consume their endowment in the absence of well-functioning markets.[16] In Figure 2, an individual whose endowment is ω would have no choice but to consume ω. This individual would not be identified as poor since her bundle is above the identification contour. This individual would then be mis-identified because her welfare is smaller than $\underline{U}$.[17] More generally, an imperfect identification methods can make two different kinds of mis-identification. An exclusion error arises when a poor individual is not identified as poor. An inclusion error arises when a

[14] Ravallion (1998) describes the following two-step procedure to identify the poor: (1) specify a reference level of utility and (2) derive a money-metric identification cutoff corresponding to this reference level of utility. In practice, the ideal of welfare-consistency remains challenging when prices vary and requires the use of methods for the estimation of the expenditure function (Deaton and Muellbauer, 1980) and the construction of "equivalent-incomes", i.e., money-metric utilities (Samuelson and Swamy, 1974; Dimri and Maniquet, 2019). The most complete review of this literature can be found in Ravallion (2016).

[15] In fact, it is not necessary that weights correspond to prices in order for the method to be welfare-consistent. Indeed, any identification method whose identification contour crosses this increasing path in bundle z (and only in that bundle) is welfare-consistent.

[16] Constraints like ignorance or administrative barriers may hinder the proper functioning of markets and thus force individuals to consume non-optimal endowments.

[17] In other words, in the absence of a market, even if the marginal rate of substitution at bundle z in Figure 2 could be correctly estimated, a linear identification method based on this marginal rate of substitution would not be welfare-consistent.

non-poor individual is identified as poor.[18]

The absence of guidance for multidimensional poverty measurement practitioners follows from the absence of a welfarist theory that acknowledges the existence of these mis-identifications. Non-welfarist theories (Bourguignon and Chakravarty, 2003; Alkire and Foster, 2011a; Datt, 2019) do not consider the existence of an indifference curve $\underline{U}$. Instead, they directly assume that individuals identified as poor are poor.[19] In contrast, welfarist theories (Maasoumi and Lugo, 2008; Decancq et al., 2019) assume that the indifference curve $\underline{U}$ is known and used for identification. As a result, they do not consider an identification contour with a different shape.

2.3 Comparing imperfect identification methods

We propose to rank imperfect identification methods by comparing their "distance" to welfare-consistency. This distance is defined as the total "cost" resulting from all identification errors. This distance thus relies on a cost function $C(x)$, which is continuous, strictly increasing in $U(x) - \underline{U}$ when $U(x) > \underline{U}$ and strictly increasing in $\underline{U} - U(x)$ when $U(x) < \underline{U}$.[20] We assume that $C(x) = 0$ when $U(x) = \underline{U}$ and that C is smooth, i.e. twice differentiable everywhere except maybe when $U(x) = \underline{U}$. Figure 3 provides two examples of such cost functions. The interpretation for the mis-identification costs depends on the purpose of identifying the poor, as discussed in Appendix 7.2.

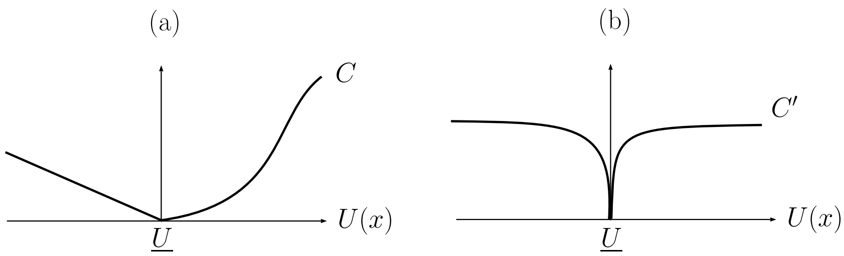

Figure 3: Two different cost functions C and C'.

The two types of error may occur in a given allocation $\boldsymbol{x}$ and the total mis-identification cost sums the costs of exclusion errors and inclusion errors

$$T(\boldsymbol{x}) = \int_{i \,\in\, N_p(\boldsymbol{x})\setminus \hat{N}_p(\boldsymbol{x}) \,\cup\, \hat{N}_p(\boldsymbol{x})\setminus N_p(\boldsymbol{x})} C(x_i)\, di, \tag{2}$$

[18] An inclusion error may arise when a bundle is above indifference curve $\underline{U}$ but below the identification contour (see Section 3).

[19] The counting approach to multidimensional deprivation (Atkinson, 2003) does not always explicitly identify a set of individuals as poor. Some of its refinements, e.g., Aaberge et al. (2019), either do not aim at identifying the poor, or implicitly assume that all individuals with at least one deprivation are identified as poor.

[20] This definition of the cost function $C(x)$ seems to suggest that our theory provides a special importance to the welfare function U, but this is not the case. Our objective here is to present a simple definition for C. Our theory does not rely on the welfare function U, but rather on the preference relation that U represents. Indeed, our comparison criterion requires robustness to all cost functions C.

By definition, a welfare-consistent method is such that $T(\boldsymbol{x}) = 0$ and an imperfect identification method yields $T(\boldsymbol{x}) > 0$. Given some allocation, an identification method is further from welfare-consistency than another method when the former yields a larger total cost than the latter.

A robust comparison of two identification methods should not depend on the particular cost function C selected. Also, it should not depend on the particular allocation considered, and therefore should not depend on the joint distribution of endowments F.[21] We say that an identification method **identifies (the poor) worse** than another method if the total cost of the former is strictly larger than the total cost of the latter for all F and all C. When this is the case, it is clear that the former method should not be used.

Of course, this robust criterion for comparison yields a partial ranking of identification methods. We also define another "weaker" criterion in order discriminate among identification methods. We say that an identification method **may identify (the poor) optimally** in some set of identification methods if there exist some F and some C such that this identification method minimizes the total cost in the set considered. When this is the case, at our level of generality, there is no clear reason to discard this identification method. This second criterion is related to the previous one. Indeed, if a method identifies worse than another method, this method never identifies optimally in any set containing the latter method. However, it can be that a method never identifies optimally in some set even if the method does not identify worse than any particular method in the set.

One limitation of these two criteria is that they depend on the shape of indifference curve $\underline{U}$, which we assume is imperfectly known. However, complete knowledge of this shape is not necessary in order to apply these criteria. Indeed, in the absence of markets, the total cost can alternatively be written as

$$T(\boldsymbol{x}) = \int_{E_{in} \cup E_{ex}} C(x) f(x) dx, \tag{3}$$

where E_{in} and E_{ex} are the sets of bundles on which the identification method makes inclusion and exclusion errors, respectively. For instance, in Figure 2, we have $E_{in} = \emptyset$ and E_{ex} corresponds to the area between the identification contour and indifference curve $\underline{U}$. This second formulation reveals that, regardless of C and F, the total cost is smaller when the sets E_{in} and E_{ex} associated to one method are subsets of the corresponding sets E_{in} and E_{ex} associated to another method. Thus, our main criterion can be applied when the identification contour of the former is an unambiguously better approximation of the indifference curve $\underline{U}$ than the identification contour of the latter. As we show below, our two restrictions on the shape of indifference curve $\underline{U}$ imply that very limited information on this shape is often sufficient when comparing mainstream identification methods.

[21] This is particularly important when the identification method is used in order to construct a multidimensional poverty measure. Indeed, the objective of such measure is to perform pairwise comparisons of different allocations.

3 Better identification when data are cardinal

We assume in this section that there is no market between the two dimensions and that achievements in the two dimensions are cardinally measured.[22] Consumption is typically cardinally measured.[23] This can also be the case of health when measured in quality-adjusted life years or healthy-years equivalents (Mehrez and Gafni, 1989). This can also be the case of education when measured in learning-adjusted years of schooling (Corral et al., 2021). We use our theory in order to compare linear identification methods that rely on different information bases.

3.1 Three families of linear identification methods

In practice, mainstream identification methods are linear identification methods (LIMs). LIMs have two main characteristics. First, they are based on a reference **deprivation bundle** $z = (z_1, z_2) \in \mathbb{R}^2_{++}$ whose component $z_j > 0$ is dimension j's deprivation cutoff. Second, their aggregation function M makes a weighted sum of outcomes in all dimensions. Under information constraints, a linear aggregation constitutes a natural default choice, especially when it is backed by axiomatic justifications (Alkire and Foster, 2011b).

We distinguish three families of LIMs, which differ by the information bases they use in order to measure outcomes. The mainstream family uses the deprivation-status basis in order to identify the poor (Alkire et al., 2015). For any individual i the deprivation bundle z and her bundle x_i define a vector of **deprivation statuses** $s_i \in \{0, 1\}^2$ such that

$$\begin{aligned} s_{ij} &= 1 \qquad \text{if } x_{ij} < z_j, \\ &= 0 \qquad \text{if } x_{ij} \geq z_j. \end{aligned}$$

This information basis has the key advantage that it can be used both on ordinal and cardinal data. Let $w^S = (w^S_1, w^S_2) \in [0, 1]^2$ be a vector of weights such that $\sum_j w^S_j = 1$. Let $k^S \in [0, 1]$ denote the identification threshold. A **status-based linear identification method** (status-LIM) is defined as a triplet (z, w^S, k^S) that identifies i as poor when

$$w^S_1 s_{i1} + w^S_2 s_{i2} > k^S. \tag{4}$$

For instance, both the UNDP-OPHI's global MPI and the World Bank's MPM use status-LIMs, though they consider more than two dimensions. We illustrate this for the World Bank's MPM in Section 7.1.

A comment is in order. We do *not* consider that the deprivation bundle z is exogenously-given. We rather consider z as a parameter of an LIM.

[22] A dimension is cardinally measured when achievements in this dimension are ordered and the distances between achievements can be meaningfully compared.

[23] Monetary poverty is typically measured using cardinal data. However, when exactly one dimension is cardinally measured and the other dimensions are ordinally measured, we are virtually in a case for which all dimensions are ordinally measured. Indeed, when exactly one dimension is cardinally measured, the identification method cannot take advantage of the cardinal nature of this dimension.

Hence, we do *not* require a priori that the deprivation bundle z corresponds to the extreme achievements bundle e, nor that bundle z provides utility level $\underline{U}$. For this reason, we do *not* call z a "poverty" bundle.[24] There is a priori no special interpretation to being deprived in dimension j, i.e., having achievement in dimension j below z_j, other than having a low achievement in that dimension.

Under two dimensions, any non-trivial status-LIM narrows down to either the **union approach** (union-LIM) or the **intersection approach** (intersection-LIM).[25] [26] Individual i is identified as poor under the union approach if $x_{i1} < z_1$ or $x_{i2} < z_2$, while individual i is identified as poor under the intersection approach if both $x_{i1} < z_1$ and $x_{i2} < z_2$.[27] These two approaches are discussed in the early literature on multidimensional poverty measurement (Tsui, 2002; Bourguignon and Chakravarty, 2003). The identification contours of a union-LIM and an intersection-LIM are respectively illustrated in Figure 4.b and 4.c.

When data are cardinal, the use of the dichotomous deprivation-status basis has been criticized. Ravallion (2011) suggests using instead the achievement basis, which is used in monetary poverty measurement. The achievement basis measures outcomes on a continuous scale and thus avoids wasting potentially relevant information. Let $w^A = (w_1^A, w_2^A) \in [0,1]^2$ be a vector of weights such that $\sum_j w_j^A = 1$. An **achievement-based linear identification method** (achievement-LIM) is defined as a pair (z, w^A) that identifies i as poor when

$$w_1^A x_{i1} + w_2^A x_{i2} < k^A, \tag{5}$$

where the identification threshold $k^A = w_1^A z_1 + w_2^A z_2$ is entirely defined by z and w^A, and is such that $k^A > 0$. In the monetary poverty literature, weights reflect prices (see Eq. (1)) and the identification cutoff k^A corresponds to the monetary line (under normalized prices). The identification contour of an achievement-LIM, defined as the frontier of Eq. (5), is illustrated in Figure 4.a. The achievement basis thus allows defining an identification contour whose trade-off between the dimensions corresponds to the marginal rate of substitution at z, at least if it is known.

A third alternative information basis is the deprivation basis (Maasoumi and Lugo, 2008). Pattanaik and Xu (2018) suggest using linear

[24] Recall that, by definition, an individual i is poor when $U(x_i) < \underline{U}$, regardless of the deprivation bundle z considered.

[25] Under three or more dimensions, status-LIMs are much richer than these two approaches as shown by Alkire and Foster (2011a) (see Section 4.1). Our main results are robust to considering more than two dimensions (see Appendix 7.7).

[26] We consider as trivial a status-LIM for which the deprivation-status of some dimension j never affects identification. This is for instance the case of dimension 2 if $w_2^S < 0.5 < k^S < w_1^S$, because then an individual i is identified as poor if and only if $s_{i1} = 1$. For our purpose, we can safely ignore trivial status-LIMs because they correspond to some LIMs from another family defined below, namely achievement-LIMs for which $w_j^S = 1$ for some j.

[27] A status-LIM narrows down to the union approach when $w_1^S > k^S$ and $w_2^S > k^S$. The intersection approach is a case of linear identification when $w_1^S \leq k^S$, $w_2^S \leq k^S$ and $w_1^S + w_2^S > k^S$.

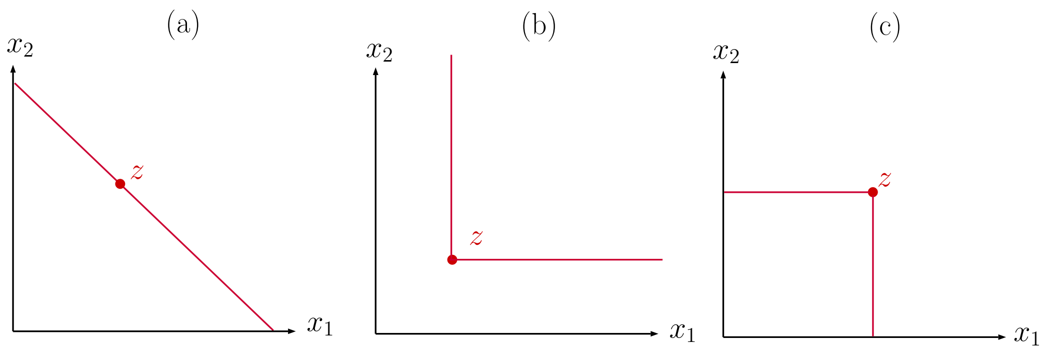

Figure 4: Identification contours for an achievement-LIM (a), a union-LIM (b) and an intersection-LIM (c).
Individuals consuming a bundle in the area between the (red) identification contour and the axes are identified as poor.

identification in the deprivation basis, in order to account for the depth of deprivation. For any individual i the deprivation bundle z and her bundle x_i define a vector of **deprivation** $d_i \in [0,1]^2$ such that

$$\begin{aligned} d_{ij} &= \frac{z_j - x_{ij}}{z_j} && \text{if } x_{ij} < z_j, \\ &= 0 && \text{if } x_{ij} \geq z_j. \end{aligned}$$

Let $w^D = (w_1^D, w_2^D) \in [0,1]^2$ be a vector of weights such that $\sum_j w_j^D = 1$. Let $k^D \in [0,1]$ denote the identification threshold. A **deprivation-based linear identification method** (deprivation-LIM) is defined as a triplet (z, w^D, k^D) that identifies i as poor when

$$w_1^D d_{i1} + w_2^D d_{i2} > k^D. \tag{6}$$

The identification contours of two different deprivation-LIMs are illustrated in Figure 5.a and 5.b.

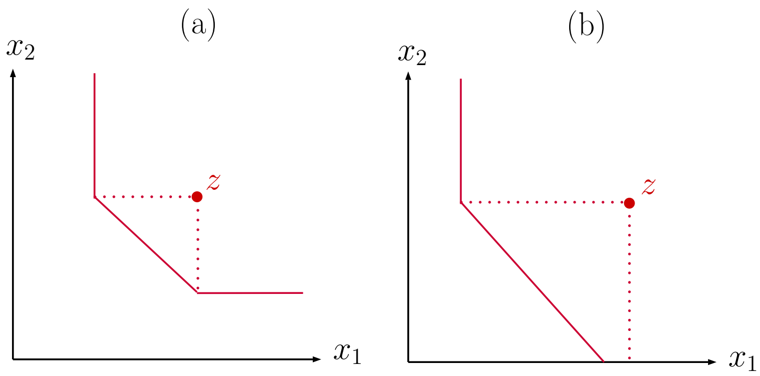

Figure 5: Identification contours for two different deprivation-LIMs.
(a) (z, w^D, k^D) *with* $w_1^D > k^D$ *and* $w_2^D > k^D$. *(b)* $(z, w^{D'}, k^{D'})$ *with* $w_1^{D'} > k^{D'}$ *and* $w_2^{D'} < k^{D'}$.

Two remarks are in order. First, the inequality signs in Eq. (4) and Eq. (6) are reversed with respect to that in Eq. (5). The reason is that welfare is increasing in achievements, but decreasing in deprivations and deprivation-statuses. Second, as we show in Appendix 7.3, deprivation-LIMs subsume both achievement-LIMs and union-LIMs. That is, for any achievement-LIM (z, w^A), there exists a deprivation-LIM (z', w^D, k^D) that defines the same identification contour and therefore always identifies the same individuals as poor. In our terminology, we say in that case that (z, w^A) is *mimicked* by (z', w^D, k^D). In contrast, no deprivation-LIM has the same identification contour as an intersection-LIM. These relationships are illustrated by the Venn diagram in Figure 6.

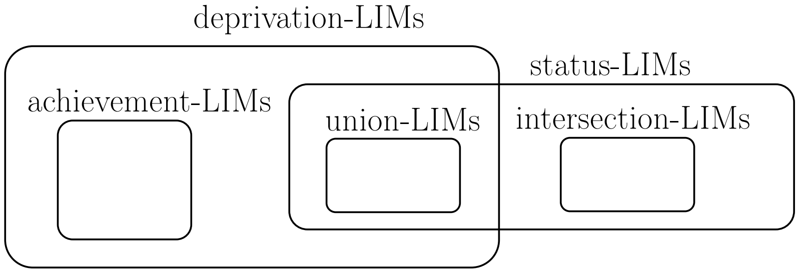

Figure 6: Venn diagram for different types of LIMs.

In the next subsections, we use our welfarist theory in order to compare different LIMs.

3.2 Improving LIMs requires little information

In this subsection, we show by means of an example that very limited information on the shape of indifference curve $\underline{U}$ may be sufficient in order to improve on some LIMs. This may require no more knowledge than a bundle on the indifference curve $\underline{U}$ and the marginal rate of substitution at that bundle, or some extreme achievement thresholds e_j.

Consider an intersection-LIM whose deprivation bundle z is on the indifference curve $\underline{U}$, as illustrated in Figure 7.a. If the practitioner knows the marginal rate of substitution at z, then she can reduce exclusion errors using an achievement-LIM whose identification contour passes through z and whose slope is equal to the marginal rate of substitution at z. These two LIMs do not make inclusion errors, but the achievement-LIM makes fewer exclusion errors. As shown in Figure 7.a, the intersection-LIM makes exclusion errors on the two triangles Int_{ex} and Int'_{ex}, which the achievement-LIM does not make. Hence, the achievement-LIM identifies the poor better than the intersection-LIM.

If the practitioner also has some information on extreme achievement thresholds e_j, she can further reduce exclusion errors by constructing a deprivation-LIM that takes advantage of this information. As illustrated in Figure 7.b, the identification contour of this deprivation-LIM mimics that of the achievement-LIM between these asymptotes.[28] Again, the

[28] Here is how to construct a deprivation-LIM (z', w^D, k^D) from the two bundles x and x' with $x_1 > x'_1$ and $x_2 < x'_2$ that define its identification contour, i.e., bundles x and x' are at the intersection of its three segments. Take the deprivation bundle to be

deprivation-LIM makes no inclusion error and does not make the exclusion errors that the achievement-LIM makes on the areas A_{ex} and A'_{ex}. Hence, the deprivation-LIM identifies the poor better than the achievement-LIM.

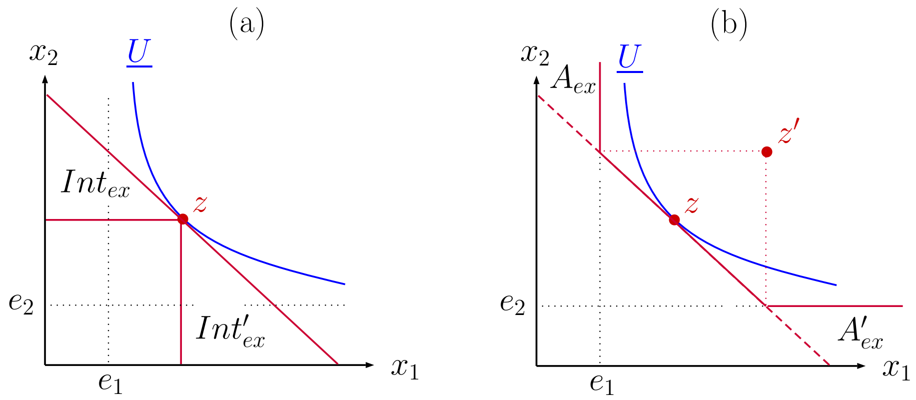

Figure 7: The intersection-LIM identifies worse than the achievement-LIM (a), which in turn identifies worse than the deprivation-LIM (b). *The intersection-LIM and the achievement-LIM are based on z while the deprivation-LIM is based on z'. The identification contour of the achievement-LIM is dashed in (b).*

As graphically illustrated in Figure 7, we have successively improved on the intersection-LIM by considering alternative LIMs whose identification contours are better approximations of the indifference curve $\underline{U}$. The concave identification contour associated to an intersection-LIM does not approximate well a convex indifference curve. The linear identification contour associated to an achievement-LIM provides a better approximation, but not as good as that of the more flexible identification contour associated to a deprivation-LIM.

3.3 Status-LIMs versus Achievements-LIMs

Ravallion (2011) criticizes the identification inherent to status-LIMs. He argues that status-LIMs used in practice aggregate dimensions in a way that is ad-hoc and unrelated to well-being. In particular, he observes that their weights w^S are typically unrelated to the marginal rates of substitution at z. He suggests using instead achievement-LIMs with weights w^A that correspond to these marginal rates of substitution. In this section, we use our theory in order to compare status-LIMs and achievement-LIMs. Our results show that Ravallion makes a valid point, but that his critique is not necessarily devastating for status-LIMs.

Under two dimensions, status-LIMs are either intersection-LIMs or union-LIMs. Proposition 1 shows that achievement-LIMs are indeed superior to intersection-LIMs. As the proof reveals, the only information that is necessary to construct an achievement-LIM that identifies the poor better than

$(z'_1, z'_2) = (x_1, x'_2)$. The remaining three parameters w^D_1, w^D_2 and k^D solve the following system of three equations: $k^D = w^D_1 * (x_1 - x'_1)/x_1$, $k^D = w^D_2 * (x'_2 - x_2)/x'_2$ and $w^D_1 + w^D_2 = 1$.

an intersection-LIM is the bundles at which the identification contour of the latter intersects the indifference curve $\underline{U}$.

Proposition 1. *Any intersection-LIM identifies the poor worse than some achievement-LIM.*

Proof. See Appendix 7.5. ■

However, at our level of generality, we cannot conclude that achievement-LIMs are superior to union-LIMs. Our theory reveals that union-LIMs have advantages that achievement-LIMs do not have, and the other way around.

Union-LIMs offer the possibility to identify an individual as poor when her achievement is extreme in one dimension, *regardless of her achievements in other dimensions*. This ability allows union-LIMs not to make any exclusion errors on extremors. However, taking advantage of this ability while limiting inclusion errors requires selecting rather small dimension-specific cutoffs z_j. As we show in Appendix 7.4, optimal union-LIMs are such that $U(z) < \underline{U}$. As a result, optimal union-LIMs are bound to make exclusion errors on cumulators.

In contrast, achievement-LIMs are bound to make exclusion errors on some extremors because of the substitutability inherent to their identification contour. Limiting an achievement-LIM's exclusion errors requires selecting rather large dimension-specific cutoffs z_j. As we show in Appendix 7.4, optimal achievement-LIMs are typically such that $U(z) > \underline{U}$.[29] This implies that optimal achievement-LIMs make limited exclusion errors on cumulators, but they still make exclusion errors on extremors.

We illustrate this reasoning in Figure 8 using a particular example for which neither the achievement-LIM nor the union-LIM make inclusion errors. The optimal union-LIMs that does not make inclusion errors has its deprivation bundle equal to the extreme bundle e. The optimal achievement-LIMs that does not make inclusion errors has its deprivation bundle on the indifference curve $\underline{U}$. As shown in Figure 8, the union-LIM makes exclusion errors on cumulators in area U_{ex}, which the achievement-LIM does not make. In contrast, the achievement-LIM makes exclusion errors on extremors in area A_{ex} and A'_{ex}, which the union-LIM does not make. Which of these two LIMs has the largest total cost depends on the respective sizes of these areas, which depends on the degree of complementarity associated to indifference curve $\underline{U}$. This also depends on the relative number of individuals whose bundles lies in these areas, which depends on the joint distribution F.

Proposition 2 formalizes the fact that achievement-LIMs need not identify the poor better than union-LIMs.

Proposition 2. *A union-LIM may identify the poor optimally in the set* {*status-LIMs, achievements-LIMs*}.

[29] More precisely, any optimal achievement-LIM is mimicked by an achievement-LIM whose deprivation bundle z is such that $U(z) > \underline{U}$. Indeed, the same identification contour is associated with different achievement-LIMs defined by the same weights w^A but different deprivation bundles.

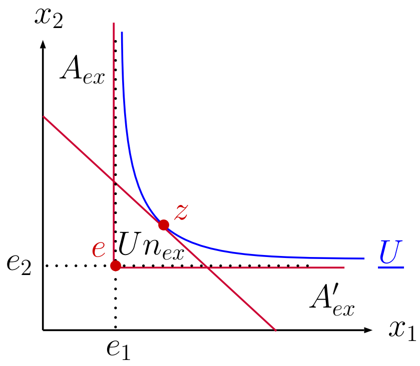

Figure 8: Achievement-LIMs need not identify better than union-LIMs. *The achievement-LIM is based on z and the union-LIM is based on e.*

Proof. See Appendix 7.6. ■

More fundamentally, our theory reveals that it is not sufficient that the weights of an LIM correspond to marginal rates of substitution for this LIM to yield a good identification. Indeed, in the absence of markets, a good identification requires that the *whole* shape of the identification contour be a good approximation of the shape of the indifference curve $\underline{U}$. Having weights equal to marginal rates of substitution allows for a good approximation locally around a particular point of the indifference curve $\underline{U}$. However, this is no guarantee that the approximation remains good further away from this point. Achievements-LIM are not good at identifying extremors because they allow a high achievement in one dimension to compensate for an extremely low achievement in another dimension.[30] In contrast, Union-LIMs are good at identifying extremors because their identification contour approximates well the asymptotes of indifference curve $\underline{U}$. Hence, in the absence of markets, the fact that the weights w^S of status-LIMs differ from marginal rates of substitution is not a sufficient reason to replace status-LIMs by achievement-LIMs.

3.4 Deprivation-LIMs are superior

Pattanaik and Xu (2018) criticize the identification inherent to status-LIMs, on the grounds that it does not account for the depth of deprivation. When data are cardinal, they suggest using instead deprivation-LIMs. Using our theory, we show that not only deprivation-LIMs are superior to status-LIMs, but also to achievement-LIMs.

A deprivation-LIM may appear "strange" at first sight because it does not identify as poor some individuals who are deprived in all dimensions,[31] as can be observed in Figure 5. However, as we show in Appendix 7.4, a general characteristic of optimal LIMs is that their deprivation bundle does not lie on the indifference curve $\underline{U}$. Fundamentally, the reason being that

[30] Achievements-LIM are good at identifying cumulators because their identification contour approximates well the middle of indifference curve $\underline{U}$.

[31] This always happens when $k^D > 0$, the case for which deprivation-LIMs do not correspond to union-LIMs.

all LIMs make misidentification errors and optimal LIMs make a trade-off between inclusion errors, which are limited by selecting smaller dimension-specific cutoffs, and exclusion errors, which are limited by selecting larger dimension-specific cutoffs. Therefore, the deprivation bundle of an optimal LIM is always disconnected from indifference curve $\underline{U}$ and thus at least slightly disconnected from the definition of the poor. This is the reason why we call z a "deprivation" bundle rather than a "poverty" bundle.

Proposition 3 shows that an LIM identifies the poor optimally in the set of LIMs only if it is a deprivation-LIM. Importantly, this result is not the direct consequence of the ability of deprivation-LIMs to mimic achievement-LIMs or union-LIMs (Lemma 1). Indeed, we can always construct deprivation-LIMs that perform strictly better than these LIMs. Deprivation-LIMs perform strictly better than achievement-LIMs because the former allow reducing the exclusion errors that the latter make on extremors. This only requires information on extreme achievement thresholds e_j. The construction is illustrated in Figure 9.a, where the achievement-LIM makes exclusion errors on the areas A_{ex} and A'_{ex} but not the deprivation-LIM. In turn, deprivation-LIMs perform strictly better than union-LIMs because the former allow reducing the exclusion errors that the latter make on cumulators. This only requires information on some bundle on indifference curve $\underline{U}$ that is not identified by the union-LIM and the marginal rates of substitution at that bundle.[32] The construction is illustrated in Figure 9.b, where the optimal union-LIM makes exclusion errors on the areas U_{ex} but not the deprivation-LIM.

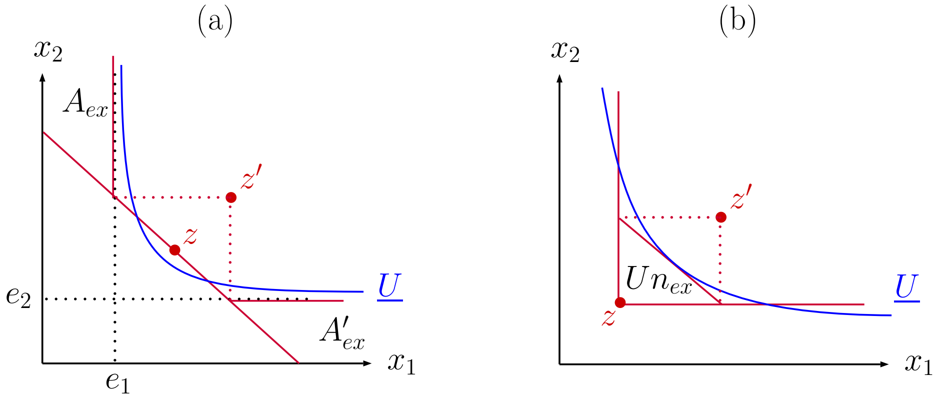

Figure 9: The achievement-LIM identifies the poor worse than the deprivation-LIM (a). The union-LIM identifies the poor worse than the deprivation-LIM (b).
The achievement-LIM is based on z and the deprivation-LIM on z' (a). The union-LIM is based on z and the deprivation-LIM on z' (b).

Finally, Proposition 1 implies that deprivation-LIMs perform better than intersection-LIMs because deprivation-LIMs perform better than achievement-LIMs. Proposition 3 formalizes the above reasoning.

[32] Strictly speaking, the information on the marginal rates of substitution at that bundle is not even necessary in order to design a deprivation-LIM that strictly improves on the union-LIM.

Proposition 3. *Any achievement-LIM identifies the poor worse than some deprivation-LIM. Any status-LIM identifies the poor worse than some deprivation-LIM.*

A priori, the fact that deprivation-LIMs identify the poor better than achievement-LIMs may appear surprising. Indeed, the achievement basis has more information than the deprivation basis, which censors outcomes above the deprivation cutoffs. However, under the constraint of linear aggregation, this censoring provides more flexibility to the identification contour of deprivation-LIMs. As a result, deprivation-LIMs combine the advantage of union-LIMs, which is to identify all extremors, and the advantage of achievement-LIMs, which is to identify a large part of cumulators. Proposition 3 implies that, when the dimensions are measured through cardinal data, the practitioner should not use status-LIMs or achievement-LIMs, but rather deprivation-LIMs. This provides a formal justification for the suggestion of Pattanaik and Xu (2018).

We have only considered the 2-dimensions case, but the main points that we have made in this section should have broader validity, as we discuss in Appendix 7.7.

4 Better identification when data is ordinal

We assume in this section that there is no market between the two dimensions and that achievements in the two dimensions are "ordinally" measured.[33] When data is ordinal, achievement-LIMs and deprivation-LIMs cannot be meaningfully used. We show that the insights gained in Section 3 help improve LIMs based on status.

4.1 The case for two dimension-specific cutoffs

The main limitation with current status-LIMs is that they do not allow to simultaneously identify extremors and cumulators. As we show below, the practitioner is forced to make a choice between making a decent identification of extremors or making a decent identification of cumulators.

Recall that status-LIMs are either union-LIMs or intersection-LIMs in the 2-dimensions case. As explained in Section 3.3, union-LIMs are ideal for the identification of extremors. However, they cannot properly identify cumulators. Indeed, optimal union-LIMs must have a small deprivation bundle, in the sense that $U(z) < \underline{U}$ (see Appendix 7.4). This implies that optimal union-LIMs systematically make exclusion errors on cumulators. This is illustrated in Figure 10.a where the union-LIM identifies the extremor i', but not the cumulator i because in each dimension $j \in \{1, 2\}$ her achievement x_{ij} is just above the deprivation cutoff z_j. In other words, union-LIMs violate the widespread idea that overlapping deprivations mat-

[33] By "ordinally" measured, we mean that they are ordered categorical. That is, achievements are ordered but differences between achievement levels cannot be compared.

ter when identifying the poor (Ferreira and Lugo, 2013; Aaberge and Brandolini, 2015).

In contrast, intersection-LIMs are well-suited for the identification of cumulators. However, they will systematically make exclusion errors on some extremors. Indeed, intersection-LIMs do not identify as poor an individual with an extreme deprivation in one dimension if she has no deprivation in the other dimension. This is illustrated in Figure 10.b where the intersection-LIM identifies the cumulator i, but not the extremor i' because her achievement $x_{i'2}$ is above the deprivation cutoff z_2. In other words, intersection-LIMs violate the widespread idea that, when identifying the poor, there is no presumption of a possible trade-off between dimensions of deprivation (Alkire et al., 2015; Atkinson, 2016).[34] Being deprived in one dimension may be sufficient to be identified as poor, regardless of achievements in the other dimensions.

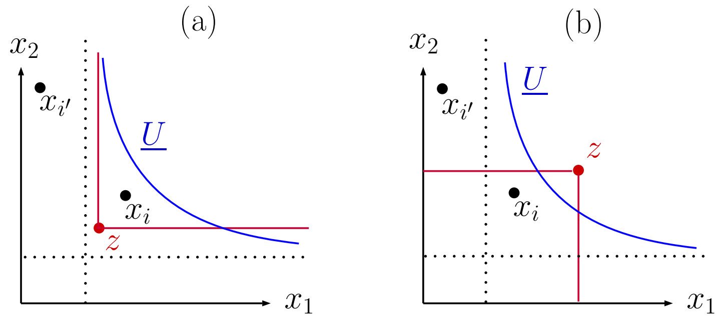

Figure 10: Optimal union-LIMs identify extremors well but not cumulators (a). Optimal intersection-LIMs identify cumulators well but not extremors (b).
Individual i is a cumulator and individual i' is an extremor.

The current status-LIMs are not flexible enough in order to genuinely account for the multidimensional nature of well-being. Indeed, this multidimensional nature implies the existence of two types of poor individuals, namely extremors and cumulators. Identifying extremors implies using the union approach together with a rather small dimension-specific cutoff z_j. In contrast, identifying cumulators implies using the intersection approach together with a rather large dimension-specific cutoff z_j.

The natural solution is for status-LIMs to rely on two dimension-specific cutoffs, one extreme cutoff and one moderate cutoff. There would thus be three deprivation categories in any dimension: extreme deprivation, moderate deprivation and non-deprivation. Extremors would be identified from their extreme deprivation status using the union approach. Cumulators would be identified from their moderate deprivation statuses using the intersection approach.[35]

[34] This relates to the issue whether the depth of an individual's deprivation in one dimension can be sufficient to identify her as poor regardless of her achievements in other dimensions (Duclos et al., 2006; Ferreira and Lugo, 2013).

[35] Our distinction between extreme and moderate deprivations has a similar spirit but is yet conceptually different from the distinction "basic" vs "non-basic" attributes

For instance, in the case of monetary poverty, the extreme cutoff could correspond to the absolute \$1.9 a day international line of the World Bank (Ferreira et al., 2016). In turn, the moderate cutoff could correspond to a relative poverty line such as the Societal line of the World Bank (World Bank, 2018; Jolliffe and Prydz, 2021). Any individual below the international line would be identified as (multidimensionally) poor, regardless of her achievements in other dimensions. In contrast, an individual whose income is above the international line but below the Societal line would only be identified as poor if she is simultaneously deprived in other dimensions. Such identification method would be in line with the idea that being absolutely poor is worse than being only-relatively poor (Decerf and Ferrando, 2021; Decerf et al., 2021).

Our suggestion of using two dimension-specific cutoffs when measuring multidimensional poverty has a deeper motivation than the practice of considering two poverty lines when measuring monetary poverty. In the latter case, the motivation is to perform robustness checks based on alternative poverty lines in order to mitigate the (partial) arbitrariness of any poverty line. In contrast, when measuring multidimensional poverty, two cutoffs are necessary in order to consistently assess low well-being using a unique welfare threshold $\underline{U}$. Indeed, an individual with a moderate deprivation in one dimension may have the *same* welfare as another individual with an extreme deprivation in the same dimension, for instance if the former cumulates moderate deprivations in other dimensions.[36]

Importantly, there still is a benefit of using two cutoffs when measuring multidimensional poverty using more than two dimensions. Indeed, the same tension arises between identifying extremors or cumulators when selecting the cutoff in a given dimension. This is clear when restricting the comparison to the union and the intersection approaches. In practice, the fraction of individuals identified as poor by the union approach (resp. the intersection approach) tends to one (resp. zero) as the number of dimensions considered increases (Rippin, 2010; Dotter and Klasen, 2017). This practical issue is mitigated by selecting small cutoffs when using the union approach and by selecting large cutoffs when using the intersection approach. However, our theory implies that increasing (or reducing) dimensional cutoffs in order to deal with this practical issue leads to increasing mis-identification errors.

As shown by Alkire and Foster (2011a), when at least three dimensions are accounted for, status-LIMs can be much richer than only the union and the intersection approaches. The AF status-LIMs are unions of intersections (of dimensions with deprivation status).[37] Their insight is highly

introduced by Dhongde et al. (2016). For instance, being deprived in a "basic" attribute need not be sufficient to be identified as poor.

[36] When measuring monetary poverty, an individual with income between the two poverty lines must be considered better-off than another individual with income below the lower poverty line.

[37] As Eq. (4) reveals, an individual i is identified as poor if the set of dimensions on which she is deprived, which we denote by D_i, have a total weight $\sum_{j \in D_i} w_j^S$ larger than k^S. Hence, individual i is deprived on the *intersection* of the dimensions in D_i. Then, a status-LIM a la AF identifies as poor the *union* of individuals for whom the

valuable because it allows overcoming the practical issue faced by union-LIMs and intersection-LIMs. This cleared the way for the implementation of multidimensional poverty measures, such as the global Multidimensional Poverty Index (global MPI) (Alkire et al., 2015). However, the extremor versus cumulator tension remains only partially addressed. Indeed, the global MPI measures each dimension using several dichotomous indicators, which are thus one-cutoff indicators. The global MPI's weights are such that a sufficient condition for an individual to be identified as poor is that she is deprived in all the indicators in a given dimension. This implicitly generates an extreme deprivation category in the dimension. Yet, being based on dichotomous indicators, this kind of measure cannot account for the depth of deprivation in a given indicator. Accounting for the depth of deprivation is precisely the objective of using two-cutoffs indicators, as illustrated above for the monetary dimension.

4.2 Refined AF-LIMs

Defining status-LIMs based on two dimension-specific cutoffs only require a small refinement of AF status-LIMs. First, two deprivation bundles are necessary, an extreme deprivation bundle $z^e = (z_1^e, z_2^e)$ and one moderate deprivation bundle $z^m = (z_1^m, z_2^m)$. Second, both extreme and moderate deprivation statuses in any dimension j should be attached a weight. Therefore, we would have $w^e = (w_1^e, w_2^e)$ and $w^m = (w_1^m, w_2^m)$ where $w_j^m < k < w_j^e$ for all $j \in \{1, 2\}$ and k is the identification threshold. A **refined AF-LIM** is then defined as a tuple (z^e, z^m, w^e, w^m, k) that identifies i as poor when

$$w_1^e s_{i1}^e + w_1^m s_{i1}^m + w_2^e s_{i2}^e + w_2^m s_{i2}^m > k, \tag{7}$$

where $s_{ij}^e = 1$ when i is extremely deprived in dimension j and 0 otherwise, and $s_{ij}^m = 1$ when i is moderately deprived in dimension j and 0 otherwise.

Observe that, for the sake of identification, there is in fact no need to select values for w^e. Indeed, any extremely deprived individual is identified as poor, regardless of the exact weights.

The identification contour associated to a refined AF-LIM is illustrated in Figure 11. This refined contour is a mixture of the respective identification contours of an intersection-LIM and a union-LIM. Around the middle of indifference curve $\underline{U}$, the refined contour corresponds to that of the intersection-LIM. Around the two sides of indifference curve $\underline{U}$, the refined contour corresponds to that of the union-LIM.

We have argued that the additional flexibility allowed by refined AF-LIMs allows to genuinely account for the multidimensional nature of well-being. Unsurprisingly, this additional flexibility allows better identifying the poor than when using classical status-LIMs.

Proposition 4. *Any status-LIM identifies the poor worse than some refined AF-LIM.*

Proof. See Appendix 7.8. ■

total weight on their D_i exceeds k^S.

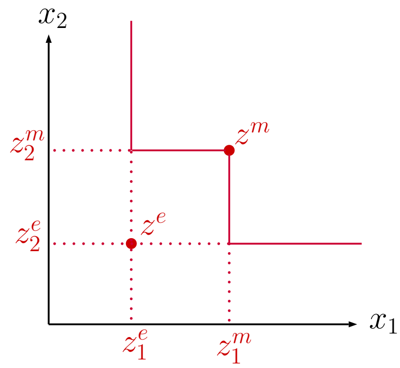

Figure 11: Identification contour for a status-LIM with two dimension-specific cutoffs per dimension.

The Multidimensional Poverty Measure (MPM) introduced in World Bank (2018) implicitly follows the idea that any individual who is extremely deprived in the monetary dimension should be identified as (multidimensionally) poor. The monetary dimension is captured by a single dichotomous variable, which captures whether the individual is below the \$1.9 a day international line. All dimensions are given equal weight. In the 3-dimensions version of the MPM, the monetary dimension has a weight equal to 1/3, which corresponds to the identification threshold $k = 1/3$ selected. In the 5-dimensions version of the MPM, the monetary dimension has a weight equal to 1/5, which corresponds to the identification threshold $k = 1/5$ selected. Hence, the design of the MPM is consistent with the idea of extreme (monetary) deprivation. We have shown above how to improve its design by incorporating the idea of moderate (monetary) deprivation, using a refined AF-LIM.

5 Monetary versus non-monetary poverty

In this section, we use our theory in order to study the identification of the multidimensionally poor from data on monetary and non-monetary outcomes. In particular, we consider identification under different data constraints that the practitioner may face. We show that, in the absence of data constraint, identification methods that neglect either monetary or non-monetary outcomes are inferior to methods combining both. When only marginal distributions are available, we show that estimates of the head-count ratio can potentially be very imprecise. Finally, we show that some current practices can be understood as attempts at improving identification under data constraints.

5.1 No data constraints

One major open question relates to which dimensions should be taken into account when identifying the multidimensionally poor. Our welfarist theory suggests that the dimensions to include are those responsible for low well-being. Any such dimension should thus combine at least two characteristics.

First, the dimension should be important for human well-being. Second, a significant fraction of the population should have low outcomes in the dimension.[38] Whether a particular dimension satisfies these requirements is an empirical question. Yet, there is little doubt that it is typically the case for the monetary dimension and for the health dimension.

Assume that the practitioner faces no data constraints. Hence, there is a survey that collects individual outcomes on both the monetary and the health dimensions. We show that an identification method combining information on both dimensions identifies the poor better than an identification method that ignores one of these two dimensions. This result is a straightforward implication of our theory. This result may in fact appear trivial, but there are substantial debates on whether monetary poverty should be combined with non-monetary poverty. For instance, Atkinson (2016) does *not* recommend the World Bank to track a multidimensional poverty measure that combine both.[39] The dominant practice seems to be to keep monetary poverty measures separated from non-monetary poverty measures. Notable exceptions are the MPM of the World Bank (see definition in Section 7.1) and the official multidimensional poverty measure adopted in Mexico (CONEVAL, 2010).[40]

Formally, let x_1 denote health achievements, as measured by an ordinal variable. Let x_2 denote monetary achievements as measured by the total monetary value of consumption on market dimensions (food, non-food, etc). Assume at this stage that individuals cannot increase their health outcomes by consuming specialized market goods (e.g., drugs do not exist). As we explain below, this assumption is not necessary for our next result. We say that an LIM ignores dimension $j \in \{1, 2\}$ if this LIM attributes a weight $w_j = 0$.

Any LIM that ignores monetary outcomes identifies the poor worse than some LIM that accounts for both dimensions. Indeed, the LIM that ignores monetary outcomes identifies the (multidimensionally) poor simply by selecting some health deprivation cutoff z_1. Provided that the two dimensions are not perfectly correlated, this LIM makes exclusion errors on some extremors. In particular, it makes exclusion errors on individuals whose health achievement is above z_1 but whose monetary achievement is below the extreme monetary threshold e_2. Consider the union-LIM with the same health deprivation cutoff z_1 and whose monetary deprivation cutoff is equal

[38] A dimension for which virtually no-one has low achievement cannot be a major source of well-being losses. For instance, many peasants in medieval Europe where subjected to serfdom, a condition of servitude to the local Lord. There is little doubt that being subjected to serfdom drastically reduces a person's well-being. However, including a freedom-from-serfdom dimension would make little sense when measuring multidimensional poverty in a society where serfdom has been abolished or never existed. In contrast, such dimension would be highly relevant in a society where serfdom is widespread.

[39] See p 170 in Atkinson (2016): "It is not proposed that the [multidimensional] indicator should include a monetary poverty dimension. [. . .] The aim of Recommendations 18 and 19 is to provide indicators that *complement* the monetary indicator, and not to seek to combine the two different approaches."

[40] Other countries that include a monetary dimension in their national multidimensional poverty measure can be found in Tables A.2 and A.3 in Unicef et al. (2021).

to the extreme threshold e_2. As illustrated in Figure 12, this union-LIM does not make exclusion errors on area H_{ex}, unlike the LIM that ignores the health dimension. Moreover, this union-LIM does not make more inclusion errors. Hence, the former identifies the poor better than the latter. The same reasoning also shows that any LIM that ignores the health dimension identifies the poor worse than some LIM that accounts for both dimensions.

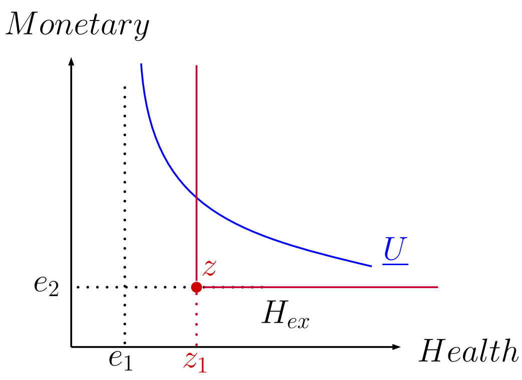

Figure 12: Methods ignoring one dimension identify the poor worse than some methods combining the dimensions.
Ignoring the monetary dimension amounts to identify as poor individuals whose health achievements are smaller than some z_1. *A union-LIM based on* $z = (z_1, e_2)$ *reduces exclusion errors on area* H_{ex}.

Proposition 5 acknowledges the fact that this reasoning is not limited to linear identification methods. This result provides a welfarist foundation for measures combining monetary and non-monetary dimensions. It justifies the aggregation of dimensions of different nature, at least when one is interested in identifying individuals with low well-being.

Proposition 5. *Any identification method that ignores either the monetary dimension or non-monetary dimensions identifies the poor worse than some identification method that accounts for both the monetary and the non-monetary dimensions.*

We have assumed that individuals cannot increase their health outcomes by consuming specialized market goods. In practice, there do exist (costly) treatments for some health conditions. However, Proposition 5 is robust to more realistic assumptions on health-related expenses. Indeed, the argument rests on the existence of individuals who have a high achievement in one dimension and an extremely low achievement in the other dimension. It is obvious that some individuals in perfect health have extremely low monetary achievements, because one cannot "sell" their own good health. It should also be clear that some rich individuals are in extremely bad health, because some health conditions have no (effective) treatment. We provide in Appendix 7.9 a more detailed discussion on the robustness of Proposition 5 to the existence of health-related expenses.

5.2 Only marginal distributions

We consider the case for which two separate surveys exist, one on monetary achievements and one on health achievements. These separate surveys provide the marginal distribution in each dimension, but no information on the correlation that may exist between monetary and health achievements.

It is literally not possible to identify a subset of households as poor when relying on two marginal distributions coming from separate surveys. Indeed, identifying the poor requires partitioning one set of households into two categories. Such partition cannot be done because the two separate surveys contain not one but two sets of households, which may overlap or not. Of course, one could identify the poor by only considering one of these two sets of households. This can for instance be done by entirely ignoring one survey, or by only using this survey in order to (somehow) impute the missing achievements in the other survey. We consider the former case in the next subsection and the latter case is equivalent to having no data constraint, at least if one assumes that the imputation is reliable.

In this section, we study the implications of using two separate surveys for poverty *measurement*. We focus on the poverty index most directly related to identification, namely the (multidimensional) head-count ratio. An important question is whether the two separate surveys contain most of the relevant information for precisely estimating the true fraction of poor individuals. In other words, we ask to which extent the ignored correlation between achievements may affect the head-count ratio.

A standard approach consists in computing upper- and lower-bounds on the head-count ratio by looking at all distributions that match the marginal distributions observed. We consider a stylized example. For simplicity, assume in line with Section 4 that achievements in each dimension are either extreme deprivation (E), moderate deprivation (M) or non-deprivation (N). An individual is identified as poor if she has one extreme deprivation or two moderate deprivations. In each survey, three individuals are surveyed, one is extremely deprived, one is moderately deprived and one is non-deprived, which yields the following two marginal distributions

$$(E_1, M_1, N_1),$$
$$(E_2, M_2, N_2),$$

where subscripts refer to dimensions. The following two allocations are consistent with these two marginal distributions

$$\boldsymbol{x} = (\underbrace{E_1, N_2}_{indiv.\ 1}; \underbrace{M_1, M_2}_{indiv.\ 2}; \underbrace{N_1, E_2}_{indiv.\ 3}),$$
$$\boldsymbol{x'} = (\underbrace{E_1, E_2}_{indiv.\ 1}; \underbrace{M_1, N_2}_{indiv.\ 2}; \underbrace{N_1, M_2}_{indiv.\ 3}).$$

with respective head-count ratios $H(\boldsymbol{x}) = 1$ and $H(\boldsymbol{x'}) = 1/3$.

This reveals that the ignored correlation may have big implications on the head-count ratio, which in our example lies somewhere in the wide bracket $[1/3, 1]$. This example also reveals that the head-count ratio is not necessarily monotonic in the correlation between achievements. Allocation $\boldsymbol{x}$ has its outcomes perfectly *negatively* correlated, but has the highest head-count ratio. The impact that the association between outcomes have on the head-count ratio depends on whether this association pushes individuals above or below the identification threshold. In our example, a positive correlation in extreme deprivation *reduces* the head-count ratio because an individual is identified as poor as soon as she has one extreme deprivation. In contrast, a positive correlation in moderate deprivations *increases* the head-count ratio because an individual is identified as poor when she cumulates two moderate deprivations.

Together, these considerations suggest that two separate surveys only provide a very noisy picture of the head-count ratio. There is potentially a large added value in collecting in the same survey the achievements in all relevant dimensions.

5.3 Only one marginal distribution

We consider the case for which only one survey exists. This survey is assumed "specialized" in the sense that it collects information either only on monetary poverty, or only on non-monetary poverty. We ask how to best identify individuals with low well-being from such a specialized survey. More precisely, we ask how to set the weights in order to decrease the (expected) mis-identification errors.

We need to consider a slightly more general framework with m-dimensions. The first ℓ dimensions are market goods and they form the market domain. The remaining $m - \ell$ dimensions are non-market goods and they form the non-market domain. We assume that $\ell \in \{2, \ldots, m-1\}$, which implies that both domains are non-trivial. Any bundle x can be decomposed into (x^m, x^{nm}) where $x^m = (x_1, \ldots, x_\ell)$ is the market bundle and $x^{nm} = (x_{\ell+1}, \ldots, x_m)$ is the non-market bundle. The non-market bundle corresponds to non-market endowment while the market bundle is optimally selected in the individual's budget set. Individual i's monetary aggregate corresponds to the monetary value of her market bundle, i.e., to $\sum_{j=1}^{\ell} p_j x_{ij}$.

Our main point is that the weights given to the observed dimensions should reflect how their achievements are related to achievements on the non-observed dimensions.

Identifying low well-being from monetary data

We consider the case of a specialized survey covering the monetary domain. We show by means of an example that the weights of an achievement-LIM that minimizes mis-identification errors may deviate from market prices, at least when market behavior is non-separable from non-market endowment. This non-separability arise when the welfare function U cannot be written as $U(x) = U(u(x^m), x^{nm})$, where function $u : \mathbb{R}_+^\ell \to \mathbb{R}$ represents behavior on the market domain.[41]

When the welfare function U is non-separable, the market bundles are not on a single increasing path. This is for instance illustrated on Figure 13 for a case with two market dimensions and one non-market dimension on which achievements are assumed dichotomous (either low or high achievement in the non-market dimension). The bundles are on two different increasing paths, one increasing path for each category in the non-market dimension. In this simple case, behavior on the market domain completely reveals an individual's non-market achievement.

In our example, we show that using an achievement-LIM whose weights correspond to prices may identify the poor worse than some achievement-LIM with other weights. Our example is illustrated in Figure 13. Assume for simplicity that, in each of the two categories for dimension 3, there are individuals whose welfare is above $\underline{U}$ and individuals whose welfare is below $\underline{U}$.[42] The monetary aggregate necessary to have welfare level $\underline{U}$ is smaller for those having high achievement in dimension 3 than for those having low achievement. In Figure 13, an individual with low (resp. high) achievement in dimension 3 is not poor when her bundle is above bundle x_L (resp. x_H). Clearly, x_L corresponds to a larger monetary aggregate than x_H. It is thus not possible to perfectly identify the poor in our example when weights correspond to prices. Consider the identification contour drawn in red in Figure 13, which is associated to an achievement-LIM whose weights differ from prices. This achievement-LIM perfectly identifies the poor. Thus, this achievement-LIM identifies the poor better than an achievement-LIM whose weights correspond to prices.

Here is the intuition for the suboptimality of equating weights to prices. Assume that the non-market dimension 3 is health and that a low achievement means that the person cannot move without a wheelchair. Assume that market dimension 1 captures books consumption and market dimension 2 captures sport material consumption. The fact that an individual spends a larger fraction of her budget on books may be a signal of her low health achievements. When achievement-LIMs are limited to the market domain, it is possible to take advantage of this signal in order to decrease expected mis-identification errors. This can be done in our example by attributing lower relative weights to books than their relative prices. When its weights differ from prices, an achievement-LIM implicitly attribute different monetary lines to individuals with different non-market achievements.

This suggests that, when trying to identify the multidimensionally poor from a survey covering the monetary domain, health expenditures could

[41] Formally, we say that U is **separable** if for all x^m, x^{nm}, $x^{m'}$ and $x^{nm'}$ we have

$$U(x^m, x^{nm}) \geq U(x^{m'}, x^{nm}) \Leftrightarrow U(x^m, x^{nm'}) \geq U(x^{m'}, x^{nm'}).$$

Graphically, the indifference map on the market domain does not depend on achievements on the non-market domain, even though the utility level attached to each indifference surface typically depends on them.

[42] This assumption violates our requirements on the indifference curve $\underline{U}$, namely the existence of an extreme threshold. One can construct a more complicated example that satisfies these requirements, although it would be a bit more involved.

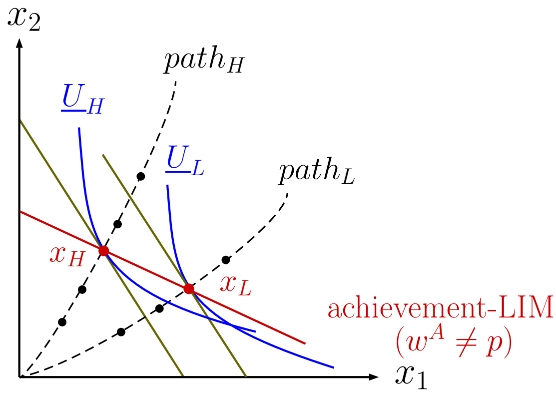

Figure 13: Equating weights to market prices needs not yield an optimal identification of the poor.
Budget lines for bundles x_H and x_L are in green, the two market dimensions (shown) are not separable from the third non-market dimension, which admits values L or H.

be excluded from total consumption. Such exclusion would correspond to setting the weights associated to health expenditures to zero. Again, the rationale being that health expenditures signal low unobserved health achievements.

Identifying low well-being from non-monetary data

We consider the case of a specialized survey covering the non-monetary domain. We explain that, in oder to minimize (expected) mis-identification errors, the weight of a non-market dimension should depend on the correlation that is expected between achievements in this dimension and monetary aggregates.

Non-market achievements are not chosen, but they may still signal something useful about market achievements. This is at least the case when the strength of the correlation between the monetary aggregate and non-market dimensions varies across non-market dimensions. Assume for instance that the correlation between education and income is perfect while health and income are uncorrelated. Under this assumption, someone who has low education also has low income, but an individual with bad health may have low income or high income.

The signal that non-market achievements provide on the ignored monetary aggregate may help improve identification in expected terms. For simplicity, assume that there are four dimensions, where the market domain is $\{1, 2\}$. Dimension 3 is perfectly correlated with the monetary aggregate while dimension 4 is not correlated with the monetary aggregate. Assume that all dimensions are cardinally measured and all are equally important to welfare, e.g., $U(x) = x_1 + x_2 + x_3 + x_4$.[43] Without data constraints, it would be possible to perfectly identify the poor with an achievement-LIM

[43] This assumption violates both of our requirements on the indifference curve $\underline{U}$. One can construct a more complicated example that satisfies these requirements, although it will be a bit more involved.

that attributes equal weights to each of the four dimensions. Now, it is still possible to perfectly identify the poor with an achievement-LIM that ignores the market domain, at least if the weight that it attributes to dimension 3 is three times the weight attributed to dimension 4. Hence, the weight for dimension 3 reflects not only the welfare importance of dimension 3 but also the welfare importance of the non-observed market domain.

This shows that, when constructing an LIM limited to the non-market domain, the relative weights between non-market dimensions should be distorted in the direction of giving a higher relative weight to the dimensions that are most positively correlated with the monetary aggregate. This may explain why non-monetary measures such as the global MPI give a relatively high importance to the education dimension, even if some doubt that this dimension has a high intrinsic value to well-being (Ravallion, 2011).

6 Concluding remarks

We argue that the lack of conceptual guidance for practitioners and the long-lasting debates surrounding the standard practices aimed at identifying the multidimensional poor both find their origin in the absence of a welfarist theory able to inform these practices. In order to be able to inform these practices, a welfarist theory must acknowledge that welfare is at best partially known to the practicioner. This paper builds such a theory as an extension of the welfarist theory sustaining the identification of the monetary poor. Several results show that this theory can fruitfully be used to inform these debates as well as help improve the standard identification practices.

When constructing a multidimensional poverty measure, the practitioner is confronted with many non-trivial choices. Arguably the most salient of these choices are the selection of dimensions, the selection of weights and the selection of the identification threshold. These choices cannot be informed by the theory proposed by Alkire and Foster (2011a) because their theory treats them as exogenous. However, a welfarist definition of the poor provides conceptual guidance on the characteristics of the relevant dimensions, as observed in Section 5. Exactly which dimensions meet these characteristics in practice is an empirical question, whose answer likely depends on the context considered. Our theory can also provide conceptual guidance on the selection of weights and identification threshold. These parameters must be selected in such a way that the shape of the identification contour is as close as possible to the shape of the indifference curve $\underline{U}$. For simplicity, assume that the practitioner knows a bundle x on this indifference curve, i.e., $U(x) = \underline{U}$. Assume that some information is available on the welfare function U, namely that we know the ranking of bundle x with several other bundles. This information provides indications on the shape of indifference curve $\underline{U}$. This information is thus useful to select values for the weights and identification threshold because these parameters define the shape of the identification contour. In a nutshell, our welfarist theory implies that these choices can be informed by information on how individuals rank bundles around the threshold welfare level $\underline{U}$.

We illustrate by means of an example that our theory can provide insights about the relationships existing between the parameters defining optimal LIMs. Consider the simplified case for which inclusion errors entail zero cost and assume the practitioner only considers achievement-LIMs. In this case, the practitioner can do no better than selecting the deprivation bundle z on the indifference curve $\underline{U}$ and have weights w^A correspond to the marginal rate of substitution at z. As illustrated in Figure 14, different bundles on the indifference curve $\underline{U}$ lead to different achievement-LIMs. Exactly which of these achievement-LIMs minimizes the total cost depends on the joint distribution F. However, the convexity of indifference curve $\underline{U}$ implies the existence of relationships between the parameters of these achievement-LIMs. First, the larger is the deprivation cutoff z_1, the smaller must be the deprivation cutoff z_2.[44] Second, the larger is the deprivation cutoff z_1, the smaller must be the weight w_1^A attributed to dimension 1.[45] Indeed, all else equal, being deprived in some dimension affects welfare less when its deprivation cutoff is larger. This discussion suggests that selecting equal weights between dimensions *regardless of the choice of deprivation cutoffs*, as is popular, cannot be in line with a welfarist definition of the poor. It also suggests that our theory may have implications for the design of tests aimed at checking the robustness of poverty comparisons to alternative parameter values. This is an area for future research.

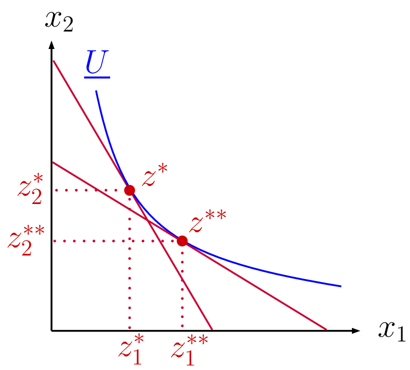

Figure 14: Relationships between cutoffs and weights for Achievement-LIMs.
Achievement-LIMs (z^*, w^{A*}) *and* (z^{**}, w^{A**}) *are such that* $z_1^* < z_1^{**}$, $z_2^* > z_2^{**}$ *and* $-\frac{w_1^{A*}}{w_2^{A*}} < -\frac{w_1^{A**}}{w_2^{A**}}$.

[44] In Figure 14, we have indeed that $z_2^{**} < z_2^*$.

[45] In Figure 14, we have indeed that the slope of the identification contour associated to the achievement-LIM with bundle z^{**} is less steep, meaning its weight w_1^A is smaller (and w_2^A is larger given that $w_2^A = 1 - w_1^A$).

7 Appendix

7.1 The World Bank's MPM

For illustrative purposes, we present and comment on the definition of the World Bank's Multidimensional Poverty Measure (MPM). The MPM considers three dimensions, namely monetary poverty, education and access to basic infrastructure. Each dimension is captured by one or more indicators, whose respective weights are summarized in Figure 15. The MPM identifies as (multidimensionally) poor any individual who lives in a household whose total deprivation is at least as large as the identification threshold $k^S = 1/3$. This is for instance the case if her household is monetary poor. This is also the case if her household lacks access to two basic infrastructures and is deprived in one education indicator, because its total deprivation is then equal to $1/9+1/9+1/6$, which is larger than k^S.

Dimension	Parameter	Weight
Monetary poverty	Daily consumption or income is less than US$1.90 per person.	1/3
Education	At least one school-age child up to the age of grade 8 is not enrolled in school.	1/6
	No adult in the household (age of grade 9 or above) has completed primary education.	1/6
Access to basic infrastructure	The household lacks access to limited-standard drinking water.	1/9
	The household lacks access to limited-standard sanitation.	1/9
	The household has no access to electricity.	1/9

Figure 15: World Bank's Multidimensional Poverty Measure: indicators and weights. Source: WorldBank (2020).

We comment on several characteristics of this definition. First, the MPM uses a status-based linear identification method. We discuss this in the case of the monetary poverty indicator. Consider a household whose achievement in this dimension is equal to \$1 per person. Given that the dimensional cut-off is \$1.9 per person, the household's deprivation is equal to $\frac{1.9-1}{1.9}$ and its deprivation-status is equal to 1. Only deprivation-status is used by the MPM's identification method, which ignores the household's exact achievement or deprivation. Indeed, changes in its achievements or deprivation that do not affect its deprivation-status cannot affect whether or not the household is identified as poor.

Second, the three dimensions have no formal existence from the perspective of the MPM's identification method. That is, the MPM's identification method works as-if six dimensions are considered, each corresponding to one of the six indicators. For any given set of weights, this method simply sums the weights across indicators in which the household is deprived, regardless of the dimensions captured by these indicators. The only impact that these three dimensions might potentially have is to influence the weights attributed to each indicator. Indeed, a popular convention is to attribute the same (total) weight to each dimension and the same weight to each indicator in a given dimension.

7.2 Interpretation for the mis-identification costs

The interpretation for the cost of mis-identifying depends on the purpose of identifying the poor.

First, assume that social transfers are targeted to individuals who are identified as poor. In the case of inclusion errors, citizens may call into question these transfers if non-poor individuals benefit too much from these transfers. Alternatively, for a fixed budget, the social transfers received by poor individuals decrease with the size of the inclusion errors. In the case of exclusion errors, some poor individuals are (mistakenly) not eligible and do not receive the help the government would like to extend to them.

Second, assume that the aim is to evaluate the *trend* of multidimensional poverty. Then, the larger the mis-identification costs, the larger the potential for capturing spurious trends. Assume for simplicity that the poverty index is the head-count ratio, which corresponds to the fraction of those identified as poor. The poverty level might be correct at some point in time when the number of inclusion errors exactly compensates for the number of exclusion errors. However, the larger are these errors, the larger the potential mistake in the poverty trend if the inclusion errors evolve over time at a different speed than exclusion errors.

7.3 Relationships between families of LIMs

Lemma 1. *Any achievement-LIM is mimicked by some deprivation-LIM. Any union-LIM is mimicked by some deprivation-LIM.*

Proof. The deprivation-LIM (z', w^D, k^D) has the same identification contour as the union-LIM (z, w^S, k^S) when $z' = z$ and $k^D = 0$. In turn, the deprivation-LIM (z'', w^D, k^D) has the same identification contour as the achievement-LIM (z, w^A) when for some $a > 1$ we have $z'' = (a\frac{k^A}{w_1^A}, a\frac{k^A}{w_2^A})$, $w_1^D = w_2^D = 0.5$ and $k^D = 1 - \frac{1}{2a}$.[46] ■

7.4 Deprivation bundle z of optimal LIMs

Lemma 2 shows that, for any particular type of LIM that we consider, selecting a deprivation bundle that lies on the indifference curve $\underline{U}$ is not optimal.

Lemma 2. *(i) The union-LIM (z, w^S, k^S) identifies the poor optimally in the set of union-LIMs only if $U(z) < \underline{U}$. (ii) The intersection-LIM (z, w^S, k^S) identifies the poor optimally in the set of intersection-LIMs only if $U(z) > \underline{U}$. (iii) The achievement-LIM identifies the poor optimally in the set of achievement-LIMs only if it is mimicked by an achievement-LIM (z, w^A) such that $U(z) > \underline{U}$. (iv) The deprivation-LIM (z, w^D, k^D) identifies the poor optimally in the set of deprivation-LIMs only if $U(z) > \underline{U}$.*

[46] If we have $w_j^A = 0$ for some $j \in \{1, 2\}$, then the construction is not well-defined (division by zero). In that case, the construction is $z'' = z$, $w^D = w^A$ and $k^D = 0$.

Proof. Consider statement (i).

First, consider any union-LIM (z, w^S, k^S) with $U(z) > \underline{U}$. We show that this union-LIM identifies the poor worse than a union-LIM (z', w^S, k^S) such that $U(z') = \underline{U}$ and such that bundle z' is dominated by bundle z, which is $z'_j < z_j$ for all $j \in \{1, 2\}$. Neither of these LIMs makes an exclusion error, but the set E'_{in} on which (z', w^S, k^S) makes inclusion errors is a subset of the set E_{in} on which (z, w^S, k^S) makes inclusion errors. Therefore, the total costs of (z', w^S, k^S) is always strictly smaller than the total costs of (z, w^S, k^S), as desired.

There remains to consider any union-LIM (z, w^S, k^S) with $U(z) = \underline{U}$. We show that there exists no cost function C and joint distribution F such that (z, w^S, k^S) is optimal in the set of union-LIMs. Consider the contradiction assumption that such C^* and F^* do exist. We derive a contradiction by showing that there exists another union-LIM (z', w^S, k^S) that yields a smaller total cost under C^* and F^*. The union-LIM (z', w^S, k^S) will be such that $U(z') < \underline{U}$ and such that bundle z' is dominated by bundle z. The proof of the existence of such (z', w^S, k^S) is illustrated in Figure 16.a.

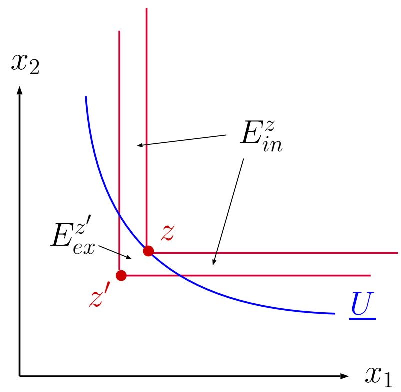

Figure 16: The union-LIM (z, w^S, k^S) with $U(z) = \underline{U}$ does not identify optimally in the set of union-LIMs.

The union-LIM (z, w^S, k^S) does not make exclusion errors but makes more inclusion errors (on areas E^z_{in}) than (z', w^S, k^S).[47] Hence, (z, w^S, k^S) has a larger total cost if its additional inclusion errors are more costly than the exclusion errors made by (z', w^S, k^S) (on area $E^{z'}_{ex}$). We show that this must be the case for some z' sufficiently close to z. For some z', let $T^{z'}$ denote the exclusion costs associated to (z', w^S, k^S) and let T^z denote the additional inclusion costs associated to (z, w^S, k^S) on E^z_{in}, which by Eq. (3) correspond to

$$T^{z'}(\boldsymbol{x}^*) = \int_{E^{z'}_{ex}} C^*(x) f^*(x) dx \qquad \text{and} \qquad T^z(\boldsymbol{x}^*) = \int_{E^z_{in}} C^*(x) f^*(x) dx$$

where $\boldsymbol{x}^*$ and f^* are respectively the allocation and density function corresponding to F^*. Recall that the density function f^* is smooth and has

[47] The union-LIM (z, w^S, k^S) must make inclusion errors because this indifference curve $\underline{U}$ is strictly convex.

full support. Recall that the cost function C^* is smooth and tends to zero when bundles tend to z. The area of $E^{z'}_{ex}$ is smaller than the area of E^{z}_{in}. The density f^* on these areas cannot tend to zero or infinity as z' tends to z because f^* has full support. In contrast, the cost C^* on $E^{z'}_{ex}$ tends to zero when z' tends to z, whereas it is not the case on E^{z}_{in}.[48] This implies that $T^{z'} < T^{z}$ for some z' sufficiently close to z, and thus the total cost of (z', w^S, k^S) is smaller than the total cost of (z, w^S, k^S) under C^* and F^*, the desired contradiction.

The proofs for statements (ii), (iii) and (iv) follow a similar reasoning and are thus omitted.

■

7.5 Proof of Proposition 1

Take any intersection-LIM (z, w^S, k^S) that identifies optimally in the set of intersection-LIMs. By Lemma 2, we have that $U(z) > \underline{U}$. We construct an achievement-LIM (z', w^A) that identifies the poor better than (z, w^S, k^S). The construction is illustrated in Figure 17.

As deprivation bundle z of the intersection-LIM is such that $U(z) > \underline{U}$, its identification contour intersects twice this indifference curve $\underline{U}$.[49] Consider any achievement-LIM (z', w^A) whose identification contour passes through these two intersections. As illustrated in Figure 17, this achievement-LIM makes fewer inclusion errors and fewer exclusion errors than the intersection-LIM. In contrast to the intersection-LIM, the achievement-LIM does not make inclusion errors on the triangle Int_{in}. In contrast to the intersection-LIM, the achievement-LIM does not make inclusion errors on the triangles Int_{ex} and Int'_{ex}. This shows that the total cost of the achievement-LIM is smaller than the total cost of the intersection-LIM for all C and F, the desired result.

7.6 Proof of Proposition 2

We show for a particular joint distribution F^* and cost function C^* that a union-LIM has negligible total cost while any achievement-LIM has non-negligible total cost.

The proof is illustrated in Figure 18. As indifference curve $\underline{U}$ is strictly convex, there must exist three bundles x_i, x'_i and x''_i such that any achievement-LIM that correctly identify the extremors i' and i'' as poor makes an inclusion error on the non-poor individual i. In contrast, there exists a union-LIM that identifies the extremors i' and i'' as poor but does not identify i as poor. For instance, it is the case of the union-LIM whose deprivation bundle is e.

[48] The cost C^* on $E^{z'}_{ex}$ tends to zero when z' tends to z because all bundles in $E^{z'}_{ex}$ tend to z, which implies that they all tend to provide utility $\underline{U}$. This is not the case on E^{z}_{in}, which always contains bundles that are not in the immediate neighborhood of z and thus do *not* tend to provide utility $\underline{U}$ as z' tends to z.

[49] Indeed, the second restriction on the shape of indifference curve $\underline{U}$ implies that this curve has two asymptotes.

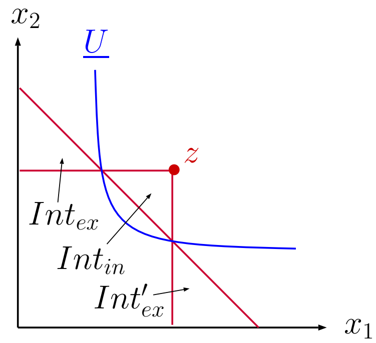

Figure 17: The intersection-LIM (z, w^S, k^S) with $U(z) > \underline{U}$ identifies the poor worse than the achievement-LIM.

From these three bundles, we can construct F^* and C^* such that a union-LIM has negligible total cost while any achievement-LIM has non-negligible total cost. Assume that one third of individuals has bundle x_i, one third has bundle x'_i and the remaining third has bundle x''_i. Such joint distribution would not have an associated density function with full support, violating our hypothesis. However, there exists a joint distribution F^* that comes arbitrarily close to it and that has full support. Take the cost function C^* to be arbitrarily close to attributing the same cost to any mis-identification.

Given that the union-LIM does not make mis-identification errors on bundles x_i, x'_i and x''_i, its total cost is negligible with respect to that any achievement-LIM, which is bound to make a mis-identification error on any of those three bundles.

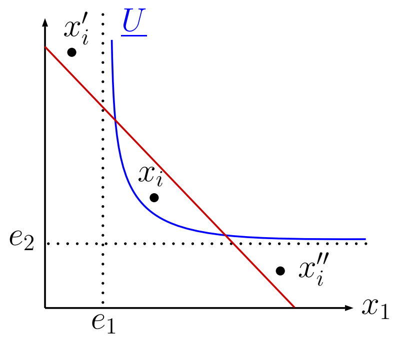

Figure 18: Any achievement-LIM makes a mis-identification error on at least one of the tree bundles x_i, x'_i and x''_i.

7.7 Deprivation-LIMs are superior with m-dimensions

We discuss the reasons why some insights presented in Section 3 should generalize to m dimensions.

When there are two dimensions, status-LIMs a la Alkire-Foster (AF) are either intersection-LIMs or union-LIMs. When there are at least three dimensions, this is no longer the case because then status-LIMs a la Alkire-

Foster (AF) are unions of intersections (of dimensions with deprivation status).[50] Three illustrations of status-LIMs in the 3 dimensions case are drawn in Figures 19.a, 19.c and 19.e. For instance, the status-LIM whose poverty surface is illustrated in Figure 19.c identifies as poor the union of individuals who are simultaneously deprived in any two dimensions.[51]

We comment on how three key insights generalize to m dimensions when there are no markets. First, achievement-LIMs do not identify the poor better than status-LIMs, and the other way around. Each of these two types of LIMs still keeps its respective advantages and limitations. Achievement-LIMs always allow a high achievement in some dimension to compensate for extreme achievements in other dimensions. As a result, achievement-LIMs are bound to make exclusion errors on extremors whose achievements are high on other dimensions. In turn, status-LIMs a la AF do not allow *the achievement level in dimension j below which an individual is identified as poor* to depend on her achievements x_{-j} in the other dimensions. That is, for fixed x_{-j}, there is a unique achievement level (equal to z_j) at which the identification of an individual may change as her achievement x_j decreases. As a result, these status-LIMs cannot simultaneously (i) identify an individual as poor *because* she experiences an extreme achievements in dimension j and (ii) identify an individual as poor *because* she cumulates moderate deprivations, including one in dimension j.[52]

This shows that, from the perspective of identifying the poor, no decisive argument can be made that achievement-LIMs are preferable to status-LIMs at our level of generality. Ravallion (2011) and Ferreira and Lugo (2013) rightfully recall that relative prices represent marginal rates of substitution, which are intimately related to welfare. However, when there are no markets, using an achievement-LIM whose weights capture marginal rates of substitution at the deprivation bundle needs not improve on status-LIMs, even when the weights of these status-LIMs do not reflect marginal rates of substitution. Capturing correctly the marginal rates of substitution does not prevent from making exclusion errors on individuals who have extreme achievements.

Second, little information is sufficient in order to construct a deprivation-LIM that identifies the poor better than an achievement-LIM. Indeed, the social planner only needs to know extreme achievement thresholds e_j. Then, it is straightforward to construct a deprivation-LIM that mimics the achievement-LIM everywhere above extreme achievement thresholds,

[50] As Eq. (4) reveals, an individual i is identified as poor if the set of dimensions on which she is deprived, which we denote by D_i, have a total weight $\sum_{j\in D_i} w_j^S$ larger than k^S. Hence, individual i is deprived on the *intersection* of the dimensions in D_i. Then, a status-LIM a la AF identifies as poor the *union* of individuals for whom the total weight on their D_i exceeds k^S.

[51] The status-LIM whose poverty surface is illustrated in Figure 19.a identifies as poor the union of individuals who are deprived in dimension 3 and individuals who are simultaneously deprived in dimensions 1 and 2. The status-LIM whose poverty surface is illustrated in Figure 19.e identifies as poor the union of individuals who are deprived in any dimension.

[52] Indeed, when $w_j^S < k^S$, the status-LIM cannot implement (i); and when $w_j^S > k^S$, the status-LIM cannot implement (ii).

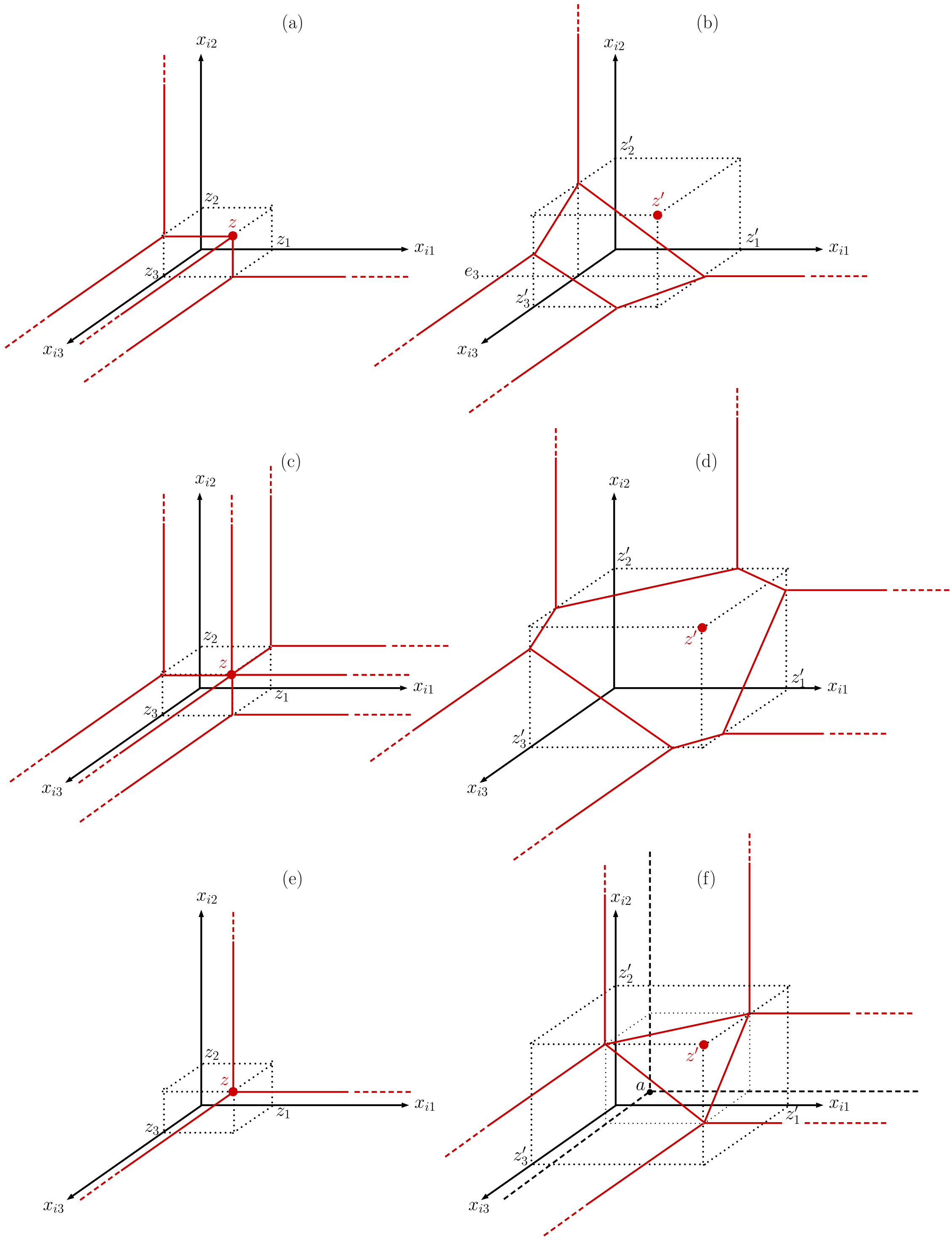

Figure 19: Identification contours corresponding to status-LIMs (a), (c), (e). Identification contours corresponding to deprivation-LIMs (b), (d), (f). The identification contour of the deprivation-LIM is drawn as a better approximation of some (undrawn) indifference surface $\underline{U}$ than the corresponding identification contour of the status-LIM, where (a) corresponds to (b), (c) corresponds to (d) and (e) corresponds to (f).

but that also identify as poor any individual with an extreme achievement. (Such construction has been illustrated in the case of two dimensions in Figure 7.) Observe that exact knowledge of the extreme achievement thresholds is not necessary. Indeed, it is sufficient to know a lower bound for these extreme achievement thresholds in order to construct a deprivation-LIM that identifies the poor better than the achievement-LIM.

Third, little information is sufficient in order to construct a deprivation-LIM whose identification contour is a better approximation of the indifference surface $\underline{U}$ than the identification contour of a status-LIM. Indeed, the social planner only needs to have information on marginal rates of substitution. The identification contours of status-LIMs will always be very rough approximations of the (convex) indifference curve $\underline{U}$ because they are not convex. As status-LIMs identify the poor using a union of intersections, they suffer from the fact that identification contours of intersections are not convex. In the case of two dimensions, we showed how to construct achievement-LIMs (and thus deprivation-LIMs) that identify the poor better than intersection-LIMs, by using information on the marginal rate of substitution in a way that reduces exclusion errors. (Such construction has been illustrated in the case of two dimensions in Figure 7.) Essentially, this construction takes advantage of the convex identification contours of achievement-LIMs (deprivation-LIMs). In the case of k dimensions, deprivation-LIMs are also convex and can thus better approximate a given indifference curve $\underline{U}$, which implies making fewer mis-identification errors. We illustrate graphically in Figure 19 that identification contours of deprivation-LIMs can be better approximations of convex indifference curve $\underline{U}$. Figure 19.b relates to Figure 19.a, Figure 19.d relates to Figure 19.c and Figure 19.f relates to Figure 19.e.[53]

7.8 Proof of Proposition 4

Status-LIM are either union-LIMs or intersection-LIMs. Any optimal union-LIM has a deprivation bundle z such that $U(z) < \underline{U}$. Therefore, such union-LIM makes exclusion errors on some cumulators. A refined AF-LIM whose extreme bundle $z^e = z$ and whose moderate bundle z^m is slightly larger than z will reduce these exclusion errors without making additional inclusion errors.

In turn, any intersection-LIM, regardless of its deprivation bundle z, makes exclusion errors on the extremors that are not deprived in one dimension. A refined AF-LIM whose moderate bundle $z^m = z$ and whose moderate bundle z^e corresponds to e will reduce these exclusion errors without making additional inclusion errors.

[53] Consider Figure 19.f. Observe that, above the deprivation cutoff in any dimension, the poverty surface defines, in the plane of the other two dimensions, a constant identification contour with the same shape as in the two dimensions case. Information on marginal rates of substitution in those planes allow constructing deprivation-LIMs that are better approximations of the iso-welfaresurface $\underline{U}$.

7.9 Proposition 5 and health expenditures

The consumption of some market goods like drugs do improve one's health outcome.

This does not mean that we should consider that there exists a market between the consumption dimension and the health dimension. As writen in the text, no treatment exists for some health conditions. This implies that some people can do no better than "consume" their low health endowments ω_1, where dimension 1 is health.

Here is how our framework should be understood in the presence of health-related market goods. First, an individual's health achievement x_1 need not be equal to her endowment ω_1, but is rather obtained as a combination of her endowment and the efficacy of her health expenditures. Second, there is currently no consensus on how to define an individual's monetary achievement x_2 in the presence of such goods. Two focal alternatives are to either include health expenditures in x_2 or to exclude them from x_2.[54] When measuring multidimensional poverty, it would seem natural to define the monetary achievement x_2 by excluding health expenditures in order to avoid a form of double-counting. However, whichever of these two definitions is used, our assumptions on the shape of indifference curve $\underline{U}$ are still relevant. There would thus still be extremors who have a high achievement in some dimension. And the argument sustaining Proposition 5 applies.

References

Aaberge, R. and Brandolini, A. (2015). Multidimensional poverty and inequality. In *Handbook of income distribution*, volume 2, pages 141–216. Elsevier.

Aaberge, R., Peluso, E., and Sigstad, H. (2019). The dual approach for measuring multidimensional deprivation: Theory and empirical evidence. *Journal of Public Economics*, 177:104036.

Alkire, S. and Foster, J. (2011a). Counting and multidimensional poverty measurement. *Journal of Public Economics*, 95(7-8):476–487.

Alkire, S. and Foster, J. (2011b). Understandings and misunderstandings of multidimensional poverty measurement. *The Journal of Economic Inequality*, 9(2):289–314.

Alkire, S., Foster, J. E., Maria, F., Paola, E., and Seth, S. (2015). *Multidimensional Poverty Measurement and Analysis*. Oxford University Press.

[54] These alternative definitions will not only change the observed individual achievements x_2. Indeed, any of these two alternatives definitions may lead to a slightly different shape for the indifference curve $\underline{U}$. Among other things, this implies that the optimal cutoff z_2 of an LIM that ignores the health dimension will likely be different under these alternative definitions.

Atkinson, A. (2016). *Monitoring Global Poverty: Report of the commission on global poverty*. The World Bank, Washigton, DC.

Atkinson, A. B. (2003). Multidimensional deprivation: contrasting social welfare and counting approaches. *The Journal of Economic Inequality*, 1(1):51–65.

Atkinson, A. B. and Bourguignon, F. (1982). The comparison of multidimensioned distributions of economic status. *The Review of Economic Studies*, 49(2):183–201.

Bourguignon, F. and Chakravarty, S. R. (2003). The measurement of multidimensional poverty. *Journal of Economic Inequality*, 1(1):25.

Bourguignon, F. and Chakravarty, S. R. (2019). Multidimensional poverty orderings: theory and applications. In *Poverty, Social Exclusion and Stochastic Dominance*, pages 143–166. Springer.

Chakravarty, S. R. and Lugo, M. A. (2019). Multidimensional indicators of inequality and poverty. *Poverty, Social Exclusion and Stochastic Dominance*, pages 223–259.

CONEVAL (2010). Methodology for multidimensional poverty measurement in mexico. *Consejo Nacional de Evaluacion de la Politica de Desarrollo Social*.

Corral, P., Dehnen, N., D'Souza, R., Gatti, R., and Kraay, A. (2021). The world bank human capital index. In *Measuring Human Capital*, pages 55–81. Elsevier.

Datt, G. (2019). Distribution-sensitive multidimensional poverty measures. *The World Bank Economic Review*, 33(3):551–572.

Deaton, A. and Muellbauer, J. (1980). An almost ideal demand system. *The American economic review*, 70(3):312–326.

Decancq, K., Fleurbaey, M., and Maniquet, F. (2019). Multidimensional poverty measurement with individual preferences. *Journal of Economic Inequality*, 17(1):29–49.

Decerf, B. and Ferrando, M. (2021). Unambiguous trends combining absolute and relative income poverty: new results and global application. *Forthcoming in the World Bank Economic Review*.

Decerf, B., Ferrando, M., and Natalie, Q. (2021). Fair and Welfare-Consistent Global Income Poverty Measurement: Theory and Application. *World Bank Discussion Paper Series*, (9844).

Dhongde, S., Li, Y., Pattanaik, P. K., and Xu, Y. (2016). Binary data, hierarchy of attributes, and multidimensional deprivation. *The Journal of Economic Inequality*, 14(4):363–378.

Dimri, A. and Maniquet, F. (2019). Income poverty measurement in india: defining group-specific poverty lines or taking preferences into account? *The Journal of Economic Inequality*, pages 1–20.

Dotter, C. and Klasen, S. (2017). The multidimensional poverty index: Achievements, conceptual and empirical issues. Technical report, Discussion Papers.

Duclos, J.-Y., Sahn, D. E., and Younger, S. D. (2006). Robust multidimensional poverty comparisons. *The economic journal*, 116(514):943–968.

Ferreira, F. H., Chen, S., Dabalen, A., Dikhanov, Y., Hamadeh, N., Jolliffe, D., Narayan, A., Prydz, E. B., Revenga, A., Sangraula, P., Serajuddin, U., and Yoshida, N. (2016). A global count of the extreme poor in 2012: data issues, methodology and initial results. *Journal of Economic Inequality*, 14(2):141–172.

Ferreira, F. H. and Lugo, M. A. (2013). Multidimensional poverty analysis: Looking for a middle ground. *The World Bank Research Observer*, 28(2):220–235.

Fleurbaey, M. (2009). Beyond gdp: The quest for a measure of social welfare. *Journal of Economic literature*, 47(4):1029–75.

Guio, A.-C., Marlier, E., and Pomati, M. (2017). Evolution of material deprivation over time: the impact of the great recession in eu countries. *Monitoring social inclusion in Europe*, page 367.

Jolliffe, D. and Prydz, E. B. (2021). Societal poverty: A relative and relevant measure. *The World Bank Economic Review*, 35(1):180–206.

Maasoumi, E. and Lugo, M. A. (2008). The information basis of multivariate poverty assessments. In *Quantitative approaches to multidimensional poverty measurement*, pages 1–29. Springer.

Mehrez, A. and Gafni, A. (1989). Quality-adjusted life years, utility theory, and healthy-years equivalents. *Medical decision making*, 9(2):142–149.

Pattanaik, P. K. and Xu, Y. (2018). On measuring multidimensional deprivation. *Journal of Economic Literature*, 56(2):657–72.

Ravallion, M. (1998). *Poverty lines in theory and practice*, volume 133. World Bank Publications.

Ravallion, M. (2011). On multidimensional indices of poverty. *The Journal of Economic Inequality*, 9(2):235–248.

Ravallion, M. (2012). Mashup indices of development. *The World Bank Research Observer*, 27(1):1–32.

Ravallion, M. (2016). *The Economics of Poverty: History, Measurement, and Policy*. Oxford University Press.

Rippin, N. (2010). Poverty severity in a multidimensional framework: the issue of inequality between dimensions. *Courant Research Centre: Poverty, Equity and Growth-Discussion Papers.*

Samuelson, P. A. and Swamy, S. (1974). Invariant economic index numbers and canonical duality: survey and synthesis. *The American Economic Review*, 64(4):566–593.

Sen, A. (1976). Poverty: an Ordinal Approach to Measurement. *Econometrica*, 44(2):219–231.

Sen, A. (1992). *Inequality Reexamined.* Harvard University Press, Cambridge.

Stiglitz, J. E., Sen, A., Fitoussi, J.-P., et al. (2009). Report by the commission on the measurement of economic performance and social progress.

Sugden, R. (1993). Welfare, resources, and capabilities: a review of inequality reexamined by amartya sen. *Journal of Economic literature*, 31(4):1947–1962.

Tsui, K.-y. (2002). Multidimensional poverty indices. *Social Choice and Welfare*, pages 69–93.

Unicef et al. (2021). A roadmap for countries measuring multidimensional poverty.

World Bank (2018). *Poverty and Shared Prosperity 2018: Piecing Together the Poverty Puzzle.* World Bank, Washington, DC.

WorldBank (2020). Poverty and shared prosperity 2020: Reversals of fortune.